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Optimizing C-RAN backhaul topologies: A resilience-oriented approach using graph invariants

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Abstract: Trends in wireless networks are proceeding toward increasingly dense deployments, supporting resilient interconnection for applications that carry ever higher capacity and tighter latency requirements. These developments put increasing pressure on network backhaul and drive the need for a re-examination of traditional backhaul topologies. Challenges of impending networks cannot be tackled by star and ring approaches due to their lack of intrinsic survivability and resilience properties, respectively. In support of this re-examination, we formulate backhaul topology optimization as a graph optimization problem by capturing both the objective and constraints of optimization in graph invariants. Our graph theoretic approach leverages well studied mathematical techniques to provide a more systematic alternative to traditional approaches to backhaul design. Specifically, herein we optimize over some known graph invariants, such as maximum node degree, topology diameter, average distance, and edge betweenness, and also over a new invariant called node Wiener impact, in order to achieve baseline backhaul topologies that match the needs for resilient future networks.

Keywords: Cloud-radio access networks, survivability, resilience, graph theory, graph invariants, topology optimization, node Wiener impact

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1 Introduction

The capabilities envisioned for the fifth generation of telecommunication networks (5G) demand a broad-reaching service platform that supports traffic volume, edge data rate, latency, and reliability in both heterogeneous and dense scenarios [1]. As such, 5G represents an evolution of more classical network paradigms with new demands on backhaul networks [2, 3] that must be able to carry several heterogeneous radio access technologies to power data centers. Notably, the concept of Cloud-Radio Access Networks (C-RANs) employs optical backhaul for transfer of radio signals for direct processing in data centers, with minimal processing at base stations. As such, C-RANs represent some of the most demanding use causes for backhauling in 5G and therefore provide our focus herein.

Cheaper radio equipment composed solely by array of antennas, digital to analog converters, and usually an optical wired interface, a.k.a. a Remote Radio Head (RRH), will gradually replace current radio base stations in C-RANs. The RRHs have the capability to upload all their sampled signal workload to be processed at cloud data-centers, , a.k.a. Baseband Unit (BBU) pools. Far away from the edge, BBU pools centralize the processing of workload, which provides several benefits, including better energy consumption, simpler synchronization, and less complicated realizes for new network functions, such as coordinated multipoint (CoMP) and Joint Transmission (JT).

A C-RAN approach to network design is hampered by four main challenges: (*i*) high link capacity demands, (*ii*) scalability limitations, (*iii*) strict latency requirements, and (*iv*) survivability. In the first, the workload transmitted from RRH to BBU requires a massive quantity of data to be represented, i.e., samples at rates from 0.6 to 24.3 Gbps for each RRH antenna using Common Public Radio Interface protocol [4]. In the second, C-RAN systems are envisioned to accommodate the workload of high density RANs in BBU pools ranging from 100 to 1000 RRHs, which demands intricate approaches to designing and managing a large number of data streams and processing points. In the third, the workload of RRHs must be processed in time for any responses to be transmitted from the RRH, in accordance with the radio protocol definition [5]. For example, given the processing of the Hybrid Automatic Repeat reQuest (HARQ) protocol, the RRHs workload must be processed within 3ms to meet protocol requirements [6]. Furthermore, C-RAN systems must provide guarantees about the degree to which operation can remain unaltered during element failure or unexpected events; that is, C-RAN must offer some level of survivability [7].

Here, we note that a C-RAN must both continue basic operation in the face of failures or unexpected events and deliver some degree of quality in doing so. That is, the nature of C-RAN backhauling demands that a BBU is both able to communicate with its associated RRH and do so within accordance to the radio protocols employed at the RRH. A basic definition of survivability would require simply that the backhaul of a C-RAN include a set of paths which supports communication between every BBU and RRH pair after f failures. Instead we prefer the notion of resilience which requires that after f failures, there is a "good" set of paths which enables communication between every pair. We conjecture that the notion of resilience better captures the survivability requirements of C-RAN backhaul than the more basic formulation.

Given this notion of survivability for C-RAN backhauls, we also distinguish between the properties of basic survivability and resilience in backhaul networks. Here we describe these properties in the case that $f = 1$, but the concepts extend to any number of failures. Basic network survivability is guaranteed by 2-connectivity between nodes. Every 2-connected graph survives any single node or link failure, in the sense that it remains connected, so there is still a path to communicate after the failure. Noting that latency provides the primary requirement for the goodness of C-RAN backhaul, we consider a network to be resilient if it has two disjoint paths of limited length connecting any pair of nodes. So, the survivability is guaranteed by the 2-connectivity and the operation in accordance with radio protocols after failure is guaranteed because the diameter and the average distance (and, consequently, the latency) are bounded after a failure.

Survivability in C-RAN has been lightly investigated in prior literature. Existing models tend to include this notion into the consideration of other problems, such as (*i*) a BBU placement problem or (*ii*) a control function split problem. In the first, proposals highlight that better positioning BBUs in the network enable distribution of the processing capabilities of a C-RAN to tackle latency and scalability issues, as well as achieve better survivability in case of unexpected delay occurrence. Whereas, in the second, the radio protocol functionalities are split between BBUs and RRHs based on the trade-off between the rate required by the underlying infrastructure and the degree to which processing may be inserted at the edge of the C-RAN. As a secondary effect of optimizing this split of functionalities, both the link capacity and latency requirements are eased, in turn improving the margin for absorption of failures. Subsequently, the survivability of C-RAN networks that are optimized in this manner improves. Extending beyond such existing work, we assume a broader perspective on survivability by examining the properties intrinsically associated to the topology planning and refining of backhauling to achieve increased survivability in C-RANs.

In this paper, we propose a method for systematic refinement of existing optical networks toward service of C-RAN backhaul grounded in graph theoretic optimization. Our approach leverages graph invariants to quantify the requirements of wireless networks and constraints of backhauling infrastructure. These invariants provide the means to capture several important features of a backhaul topology including the number of nodes or edges as well as the maximum degree of nodes, the topology diameter, and the average distance (or average hop count). Note that these invariants do not depend on the specific technology adopted by the backhauling network and instead focus on the structure of the topology. Importantly, this aspect of invariants enables their use in determining the fundamental ability of a backhaul topology to meet latency and resilience requirements. Moreover, a grounding in graph theory enables the use of mature tools. Therefore, a basis in graph theory enables our approach to provide fundamental insight into the ability of a backhaul design to meet the requirements of wireless network and systematically refine this ability. Through refining existing optical network topologies in service of C-RAN backhaul, we jointly consider the needs of wireless networks and the associated optical operation.

This paper is organized as follows. In Section 2, we describe our method and present a new distance-based graph invariant which is related to resilience aspects of the backhaul topology design. Some case studies of the proposed method are presented in Section 3. Section 4 draw our conclusions and points out future research directions.

2 Mapping network requirements to graph invariants

The problem of designing network topologies using graph theory may be stated as determining means of linking a given number n of nodes, where both wireless objectives and backhauling constraints are expressed in terms of graph invariants. The definitions of all the graph invariants used in this work can be found in the Appendix A.

In the design of any backhaul network, the most basic constraint is that each element of the network be connected to the network; in graph theoretic terms, at least one path must connect each pair of nodes, resulting in the network being a connected graph. Therefore, this requirement outlines a search space for our optimization. The size of this space grows rapidly with the number of nodes considered within a network: there are 853 possible connected graphs with 7 nodes and if we simply double the number of nodes, this number grows to 29003487462848061 [8]. Such a rapid growth of search space prevents the purely exhaustive search for telecommunication topologies. Given the plug'n play nature of C-RAN, this is even more difficult to be performed, mainly because the graph is changing dynamically, where RRHs can be dynamically associated to a different BBU pool when the backhaul become overloaded, preventing pre-processing techniques to be applied.

Adding constraints related to interconnection of backhauling elements further bounds our search space. For example, the practical constraint of the number of edges that may be deployed maps into a limitation of the size of a topology. Alternatively, the number of edges supported by any single node may be constrained through limiting the degree of nodes. Applied in this manner, invariants provide the means to restrict the search on the basis of the interconnection constraints of a backhaul network. Even during dynamic changes occurring in C-RAN, the graph invariants remains as a rule to be followed preventing the problem aforementioned.

Once the constraints of potential backhaul interconnection define a space, wireless objectives may be mapped into graph theoretic terms to guide a search. For example, the betweenness of a edge e in a graph G measures the proportion of the number of shortest paths among all node pairs in G that pass through e; as such, this invariant can be used as measurement of potential network congestion. As another example, since latency is fundamentally limited by propagation delay, the diameter and the average distance invariants provide the means to describe the fundamental latency performance of a network. Minimization of these invariants finds topologies with the highest potential for supporting wireless networks with strict latency requirements. On the usage of the betweenness invariant, the latency requirements for C-RAN, such as posed by the HARQ protocol, can be better managed defining the exactly topology which would give the better survivability in case of backhaul overloads, preventing C-RAN to lose connectivity and performance.

Alternatively, the resilience requirements of a wireless network may be described in terms of the effect of alterations to a topology. For example, traditional tree topologies lack fault tolerance due to collapse upon loss of any edge or non-leaf node. A need for survivability may be translated into the search for k-connected graphs, where the value of k is proportional to the level of backhaul fault tolerance required by the wireless network. Given the dynamic capabilities of C-RAN to readjust the RRHs workload distribution to BBU pools on the fly, changing the communication topology to achieve better fault tolerance is not just feasible but required. Therefore, to enhance survivability, the C-RAN topology can be reconfigured online through different graph invariants to achieve better fault tolerance and resilience.

By describing backhaul interconnection constraints and wireless objectives in this manner, our approach allows the refinement of existing topologies to address novel challenges. For example, the traditional ring topology is commonly used for its fault tolerance, but exhibits undesirable latency properties. Specifically, the ring topology has the maximum diameter of all 2-connected graphs with n nodes [9] and the diameter of a ring increases from $\lfloor n/2 \rfloor$ to $n-1$ after the removal of any edge.

Our approach enables the refinement of the ring topology toward one more suitable for supporting wireless networks with tight latency requirements. In this case, we may constrain our search space to contain only topologies with two disjoint paths for each pair of nodes, while minimizing the diameter of the resultant topology. To this end, we propose a new invariant called node Wiener impact, which provides a measure for a 2-connected graph G of the impact on the distances between node pairs in the remaining graph when a node v is removed. This new node invariant is based on well-known invariants: the graph Wiener index [10] and the node transmission [9].

The Wiener index of a given graph is the half sum of its distance matrix, which represents the total number of hops that are necessary to interconnect all pair of nodes using the shortest path between them. If the load is homogeneously distributed among all the node pairs and the routing algorithm considers the shortest paths, this invariant can offer an intuition on the cost to route data in a network: the lower the Wiener index, the more likely a network is to route data using a smaller number of hops. The transmission of a node, on the other hand, is the sum of the distances from this node to all other nodes.

In an ideal solution, all the distances remain the same after a vertex removal and, consequently, there are at least two disjoint geodesics interconnecting any pair of nodes. This can be achieved if the Wiener index of the resultant graph is equal to the Wiener index of the original graph subtracted by the transmission of the removed vertex, for any vertex of the graph. Formalizing this idea, the node Wiener impact of a node v in a 2-connected graph G, denoted as τ_v, is defined as:

$$\tau_v = W(G-v) - W(G) + T(v), \tag{1}$$

where $W(G)$ is the Wiener index of the original graph G, $W(G-v)$ is the Wiener index of the graph obtained by removing v from G, and $T(v)$ is the transmission of the node v.

Therefore, the Wiener impact can be used as a more robust way of measuring network resilience in 2-connected networks: the lower the Wiener impact of a node, the lower is the growth of the distances between node pairs in the resultant graph when that node is removed. In our approach, we consider that a network is resilient if it has two disjoint paths of limited length connecting any pair of nodes. The authors of [11] provide a further study of betweenness, Wiener index, and transmission.

Moreover, our approach offers an extensible method for backhaul design. That is, additional constraints or objectives may be represented through the incorporation of new invariants. Development of new graph invariants increase the power of our approach to backhaul design to capture desirable topological properties.

Table 1 summarizes this discussion by showing the mapping of some common network features into graph invariants.

Table 1: Network requirements and invariants.

Feature	**Invariant**
Cost	Order, size, and maximum/average degree
Maximum latency	Diameter
Average latency	Average distance
Network congestion	Edge betweenness, and degree variance
Survivability	2-connectivity
Resilience	Node Wiener impact

3 Case studies

In this section, we explore some case studies of backhaul topology design using graph optimization. To support invariants computation and graph optimization, we selected the AutoGraphiX III[1] (AGX) software in [12, 13]. Initially we examine tree topologies, which are still very popular in access networks due to their low cost. Although they do not offer survivability, we use this simple architecture to illustrate how graph optimization can lead to better topological solutions for meeting C-RAN future network requirements. We then move on to investigate ring-based networks, which offer survivability but not resilience, showing how graph optimization can improve them. Finally, a real-world network topology is studied to show the practical implications of graph optimization in the decision making process regarding the C-RAN underlying topology design.

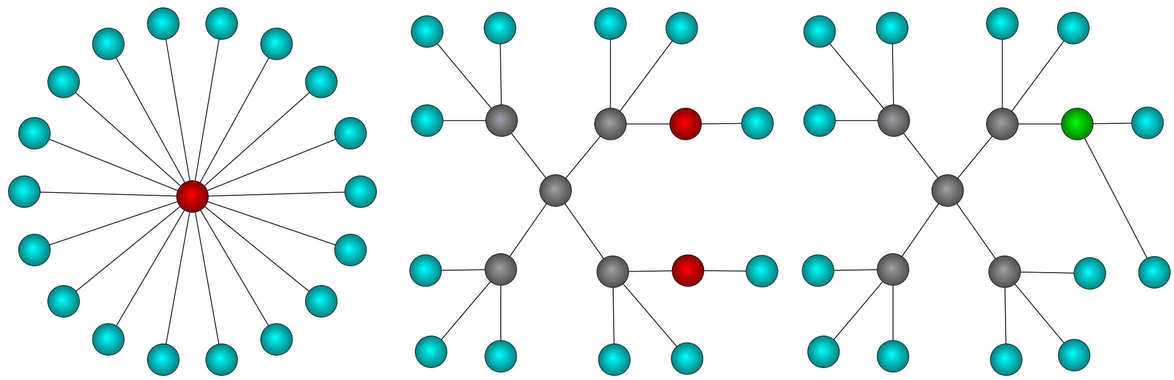
3.1 Non survivable topologies: trees

This first case study focuses on node interconnection and latency requirements. Table 2 collects the invariants values of the proposed topologies.

In the search for topologies for a network with 19 nodes, basic interconnection constraints and a minimal latency objective could be addressed with a classic star topology (tree graph family), shown in Figure 1(a), which contains only 18 edges (low edge cost), has diameter 2 (low maximum latency), has average distance 1.89 (low average latency), and has Wiener index 324. Unfortunately, analysis shows that this topology is highly dependent on a central node (maximum degree is 18, and degree variance is 14.41), whose removal stops the communication between all other nodes. Moreover, the high maximum degree brings implementation barriers.

Although a high centralized topology would bring some impacts to the network interconnectivity, for C-RAN such centralization would be perfect for reusing the resources at datacenters. The more centralized a BBU pool is positioned, bigger is the potential to receive offloaded workload from other RANs to be processed in the same BBU pools. It means that the operator would not have to spent to much revenue with new BBU pools by reusing the already deployed ones.

[1] Available at *https://www.gerad.ca/gilles.caporossi/agx.*



(a) (b) (c)

Figure 1: Tree based topologies: (a) Star, (b) Expert [14], (c) Improved solution (AGX).

Table 2: Tree based topologies (19 nodes).

Invariants	**(a) Star**	**(b) Expert [14]**	**(c) Improved solution (AGX)**
Max. degree	18	4	4
Average degree	1.89	1.89	1.89
Degree variance	14.41	1.67	1.78
Diameter	2	6	5
Average distance	1.89	3.18	3.16
Wiener index	324	544	540
Max. / Min. edge betweenness	18 / 18	70 / 18	78 / 18

Despite the benefits that C-RAN would afford from such topology, to minimize the centralization impacts, an expert could propose the solution shown in Figure 1(b) [14]. This solution effectively reduces the maximum degree and the degree variance from 18 to 4 and from 14.41 to 1.67, respectively, with the downside of increasing the diameter from 2 to 6, and increasing the Wiener index from 324 to 544.

Our method allows the further systematic refinement of even the expert's solution. From the solution presented in Figure 1(b) and using AGX, our approach obtains the graph shown in Figure 1(c). The optimization applied here minimizes the diameter and the average distance, subject to maximum degree less than or equal to 4. The maximum degree constraint ensures that the gains originally achieved by the expert are maintained. The result of this optimization only changes one edge to reduce the diameter from 6 to 5 while keeping almost all other invariants, with the only disadvantage of slightly increasing the load on some edges (maximum edge betweenness goes from 70 to 78, and degree variance goes from 1.67 to 1.78). Alternatively, we could consider different objective functions and constraints in order to improve other topological parameters. For example, if the constraint on the maximum degree is not so critical to the network design, we could use maximum degree less than or equal to 5 in order to reduce the diameter to 3. That is, our approach to topology design enables the systematic exploration of potential topologies and enumeration of major trade-offs.

3.2 From survivable to resilient topologies: 2-connected graphs

This second case study is focused on survavibility requirements. Information on the invariants values of the proposed topologies can be found in Table 3.

At first sight, the search for resilient topology solutions for a network with 14 nodes could lead to the 2-connected ring topology shown in Figure 2(a), which contains only 14 edges and has average degree 2 (low cost). However, as discussed in Section 2, the ring is a poor solution considering latency and resilience requirements, because the diameter is high (7, in this case) and changes to 13 in case of any node removal. Moreover, given a fixed number of nodes, the ring is the 2-connected graph maximizing the Wiener index [9]. For a ring with 14 nodes, the Wiener index is 343, and the node Wiener impact of each of its nodes is 70.

With the goal of minimizing these impacts, an expert could propose the solution shown in Figure 2(b) [15], which attempts to combine the benefits from ring and star topologies. This solution reduces the latency related invariants (the average distance, from 3.77 to 1.71, and the diameter, from 7 to 2) and the Wiener index by adding 12 edges from each node to a central node. On the other hand, its benefits are still dependent on a single node, which is related to the high values for maximum degree (13) and degree variance (6.63). The maximum and minimum values for Wiener impact (130 and 0, respectively) also demonstrate the significance of removing this central node on the transmission of the other nodes and, thus, show that the solution is not resilient.

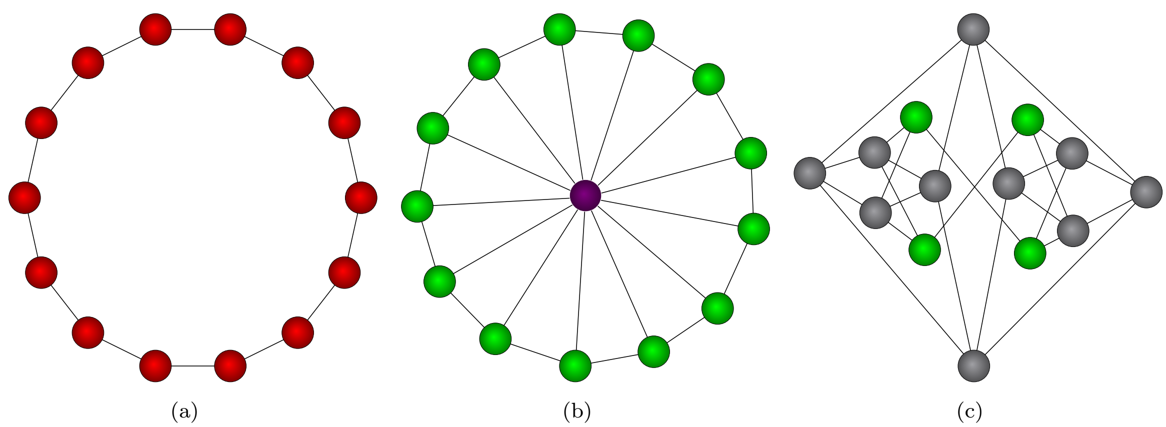


Figure 2: 2-connected topologies: (a) Ring, (b) Wheel [15], (c) Improved solution (AGX).

Table 3: 2-connected topologies (14 nodes).

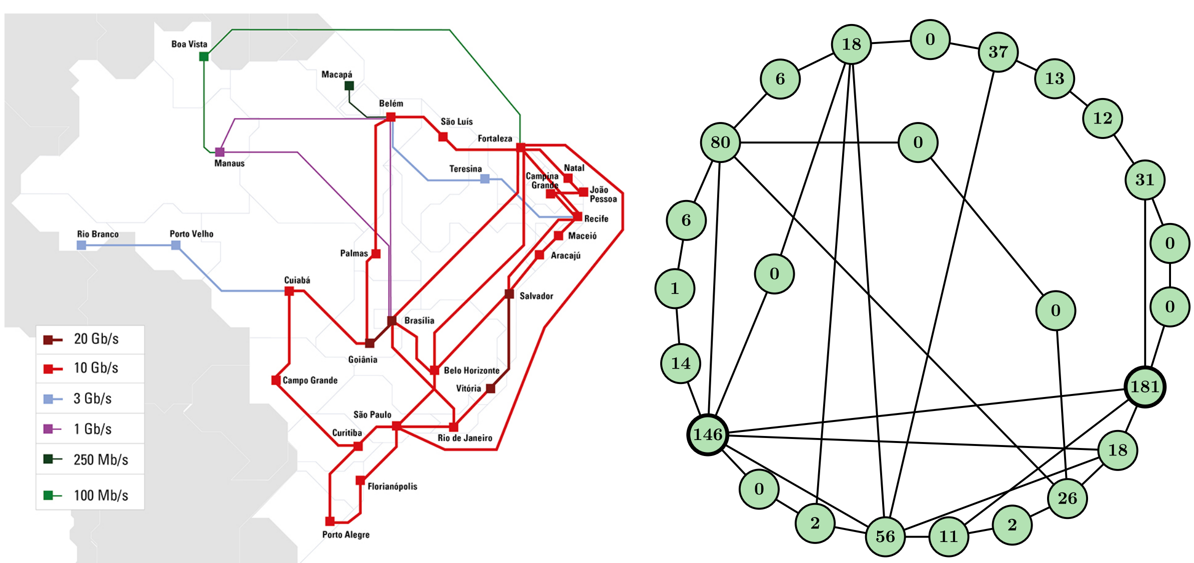
Invariants	**(a) Ring**	**(b)Wheel [15]**	**(c) Improved solution (AGX)**
Nr. of edges	14	26	26
Max. degree	2	13	4
Average degree	2	3.71	3.71
Degree variance	0	6.63	0.20
Diameter	7	2	3
Average distance	3.77	1.71	2.04
Wiener index	343	156	186
Max. / Min. edge betweenness	24.5 / 24.5	10 / 2	12 / 6.5
Max. / Min. node Wiener impact	70 / 70	130 / 0	6 / 0

Once again our solution supports the refinement of this topology to address its faults. Starting with the solution presented in Figure 2(b) and using AGX, we obtained the graph shown in Figure 2(c). Here the optimization minimizes the Wiener impact, and the average distance, subject to a 2-connected graph of both diameter and maximum degree less than or equal to 4. The number of edges is also fixed to 26, which allows a fair comparison between both solutions with respect to interconnection parameters. The maximum degree constraint is an example requirement from [14]. Our optimized solution has few drawbacks: the average distance goes from 1.71 to 2.04, the diameter goes from 2 to 3, and the Wiener index goes from 156 to 186. On the other hand, it has great positive impact on the maximum degree (from 13 to 4) and the degree variance (from 6.63 to 0.20). Moreover, our method heavily improves the topology resilience as shown by the reduction on the maximum Wiener impact (from 130 to 6).

3.3 From survivable to resilient topologies: a real-world network

In C-RAN, its underlying infrastructure presents two problems: (i) massive initial investment; (ii) ossified infrastructure. In the former, BBU pools need to be connected to RRHs using redundant optical links (e.g., ring links) that require site installation and long length links (e.g., $\geq$ 10 Km) per antenna, hindering the centralization of BBU pools due to high cost. In the later, the underlying infrastructure after installed cannot be easily repositioned needing new investments and man-working, compromising the scalability of C-RAN. Based on these problems, reusing already deployed underlying infrastructure (backhaul) enables the cut of initial investments also giving new possibilities of deployment, merging C-RAN's underlying infrastructure with backhaul.

Although the reuse of the backhaul is crucial to the realization of C-RAN, the survivability of such hybrid underlying infrastructure may be compromised, mainly because of the different already deployed infrastructure purposes, such as interconnecting cities or provide connectivity to Internet Service Providers. Therefore, the goal of this case study is to extend our resilience analysis to a real-world backbone network in the realization of C-RAN. To this end, we optimize the Brazilian National Research and Educational Network (RNP) topology,[2] which is composed by 28 points of presence, as shown in Figure 3(a). Since the resilience analysis only makes sense in a 2-connected graph, Figure 3(b) shows the representation of the resultant 25-node topology after the removal of the three nodes that are not part of any cycle. Information on the invariants values of the proposed topologies can be found in Table 4.

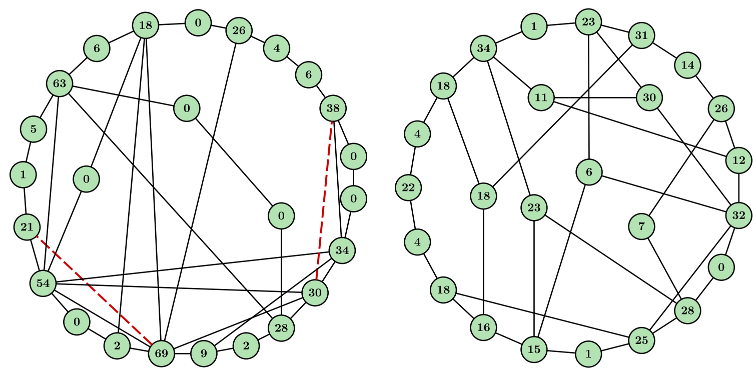


(a) (b)

Figure 3: RNP backbone: (a) Geographical distribution with 28 nodes, (b) Resultant 2-connected graph with 25 nodes.

An analysis of Figure 3(b) shows that the highlighted nodes have high Wiener Impact values (181 and 146), which tell us that the impact on the overall network resilience of removing these nodes is high. This diagnosis can drive an expert to connect extra edges in the network in order to reduce the impact of their disconnection. For example, in the graph of Figure 4(a), the addition of only two extra edges (highlighted in red dashed lines) to the original graph of Figure 3(b) reduces the maximum Wiener Impact from 181 to 69. This example

[2]Available at *https://www.rnp.br/servicos/conectividade/rede-ipe*.



(a) (b)

Figure 4: Optimization of topology shown in Figure 3(b): (a) Addition of two edges, (b) Minimization of the maximum Wiener Impact.

Table 4: RNP backbone (25 nodes).

Invariants	**Original**	**Addition of two edges**	**Minimization of max. WI**
Nr. of edges	38	40	38
Max. degree	7	7	5
Average degree	3.04	3.20	3.04
Degree variance	2.12	2.48	0.68
Diameter	5	5	5
Average distance	2.84	2.74	2.76
Wiener index	852	822	828
Max. / Min. edge betweenness	55.17 / 6.50	45.85 / 6.23	32.74 / 12.33
Max. / Min. node Wiener Impact	181 / 0	69 / 0	34 / 0

shows how the analysis of node invariants can be used as an incremental and informed method for increasing network resilience in an existing network that does not support disruptive changes.

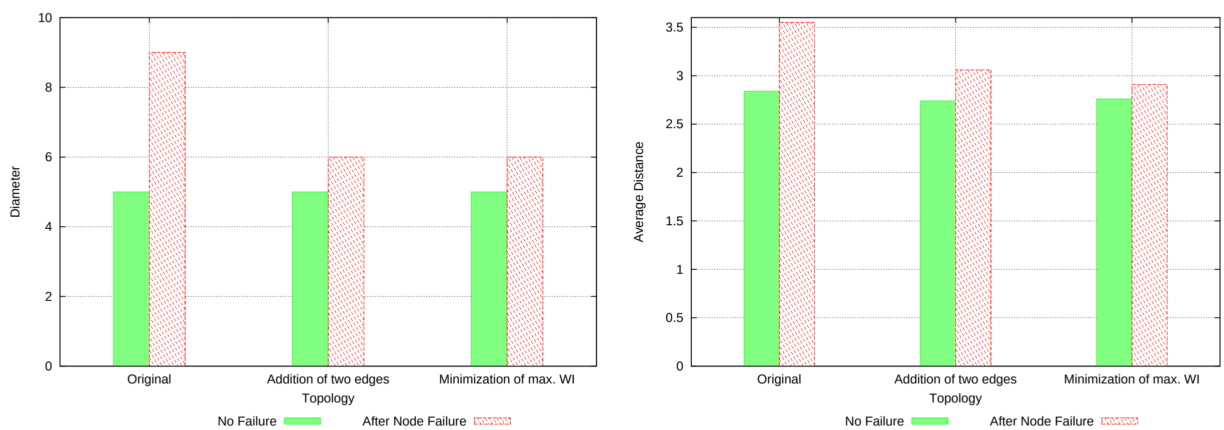
On the other hand, if the goal is to design a new topology from scratch that focuses on resilience, we could fix the original number of nodes (25) and edges (38) and try to minimize the maximum Wiener Impact. With this objective function and these constraints in AGX, we were able to find the topology shown in Figure 4(b) that presents a maximum Wiener Impact value of 34 (against the original 181 value of the original topology). We can also check in Table 4 that all the other analyzed invariants were improved when compared to the original topology.

In order to better understand the effects of minimizing the node Wiener impact on the topological parameters, a worst case analysis was performed for the same topologies analyzed in Table 4. For that end, for each topology, the node maximizing the Wiener impact was removed from the topology, and the invariants were computed again. The results are presented in Table 5. Notice that, whereas the diameter of the original topology has increased from 5 to 9 after the worst case node failure, the diameter of both the improved topologies has only increased from 5 to 6. Moreover, the average distance of these improved topologies increases much less (11.68% for the topology shown in Figure 4(a) and 5.43% for the one shown in Figure 4(b)) than the average distance of the original topology (25%). These results are graphically presented in Figure 5. In addition, this figure shows that the diameter and the average distance invariants from the optimized topology of Figure 4(b) are similar to the topology of Figure 4(a), but without the need for adding two additional edges.

Table 5: RNP backbone after the worst case node failure (max. WI).

Invariants	Original	Addition of two edges	Minimization of max. WI
Nr. of edges	33	33	34
Max. degree	6	6	5
Average degree	2.75	2.75	2.83
Degree variance	1.85	1.52	0.89
Diameter	9	6	6
Average distance	3.55	3.06	2.91
Wiener index	979	844	802
Max. / Min. edge betweenness	97.5 / 6.58	53.27 / 7.00	39.38 / 10.83

When we optimize the original graph, the output is a improved node interconnection solution that suits better the defined constraints and objective functions. A final step to implement the solution in real networks is to map the nodes from the optimized solution into the original node geographical distribution. This is another problem that can also be tackled with the guidance of graph invariants. For example, a heuristics may sort the original geographical nodes by traffic load or population aspects and map them to the nodes in the optimized solution that have the greater values for some specific graph invariant (e.g., degree or Wiener impact). In addition, this heuristics has to ensure that neighbors in the optimized solution are close in terms of geographic distance to minimize the cabling costs.



(a) (b)

Figure 5: Invariants after the worst case node failure (max. WI): (a) Diameter, (b) Average distance.

4 Conclusion and future research directions

In this paper, we proposed a method to the design of resilient backhaul topologies, based on graph invariants. We also proposed a new invariant, the node Wiener impact, which measures the impact of a node failure to the distances among the remaining node pairs. We are currently working on a version of the node Wiener impact invariant for analyzing the impact of link failures.

As demonstrated herein, our graph theoretic approach to backhaul topology design enables the systematic refinement of backhaul topologies on the basis of wireless network objectives. Furthermore, our method enables the direct comparison of topologies and examination of trade-offs in network design. Moreover, this manner of designing network topologies allows us to bring the large knowledge base available within the graph theory community to overcome telecommunications challenges.

In summary, our solution could provide guidance in the decision making about the impacts on the network when new entities (edges and nodes) are added or removed in the topology. For the C-RAN, this is fundamental to raise the survivability and make feasible the usage of the backhaul as part of the C-RAN underlying infrastructure.

This work can be extended in many ways. For instance, in order to implement the interconnection solutions obtained by our method, an important practical aspect to be further studied is the embedding of the nodes of a improved solution into the nodes of the original solution. Also, one could use the interconnection solution as the basis to design an ILP (Integer Linear Program) model that takes into account traffic demands.

Appendix A: Definitions

This appendix defines the main graph theory concepts used in this paper. Basic concepts not defined here follow the definitions of [16].

Definition 1 (Graph, $G = G(V, E)$) *A graph $G = G(V, E)$ consists of a set $V = V(G)$ of vertices or nodes, and a set $E = E(G)$ of edges or links, where each edge uv, connects a pair of vertices $u, v \in G$. The order of G is $n = n(G) = |V(G)|$, and the size of G is $m = m(G) = |E(G)|$.*

Definition 2 (Connected graph) *A graph G is connected if there is at least one path between each pair of vertices $u, v \in V(G)$.*

Definition 3 (k-connected graph) *A graph is k-connected if and only if there are at least k vertex-disjoint paths between each pair of vertices $u, v \in V(G)$.*

Definition 4 (Vertex degree) *The number of edges incident to a vertex $v \in V(G)$ defines the degree of v.*

Definition 5 (Geodesic or shortest path) *The shortest path (in number of hops) connecting two vertices $u, v \in V(G)$ is called a uv geodesic.*

Definition 6 (Distance between two vertices) *The distance between two vertices u and v in a connected graph G, denoted as $dist(u, v)$, is the length of a geodesic between u and v in G.*

Definition 7 (Graph distance matrix) *Let G be a connected graph of order n. The distance matrix of G is an order n matrix such that:*

$$Dist[i,j] \;=\; \begin{cases} dist(v_i, v_j), & v_i, v_j \in V(G),\ i \neq j; \\ 0, & otherwise. \end{cases} \tag{2}$$

Definition 8 (Graph diameter) *The diameter of a connected graph G is the length of the greatest geodesic in G. More formally, it is given by:*

$$diam(G) \;=\; \max\{dist(u,v);\; u,v \;\in V(G)\}. \tag{3}$$

Definition 9 (Vertex transmission [9]) *Let G be a connected graph. The transmission of a vertex $v \;\in V(G)$ is defined as:*

$$T(v) \;= \sum_{u\in V} dist(u,v). \tag{4}$$

Definition 10 (Graph Wiener index [10]) *Let G be a connected graph. The Wiener index of G is given by:*

$$W(G) \;= \sum_{u\in V(G)} \;\sum_{v\in V(G),\; v<u} dist(u,v). \tag{5}$$

Definition 11 (Graph average distance) *The average distance of a connected graph G is the Wiener index of G over the number of pairs of vertices in G.*

Definition 12 (Node Wiener impact) *Let G be a 2-connected graph. The Wiener impact of a vertex $v \;\in V(G)$ is defined as:*

$$\tau_v \;=\; W(G-v) - W(G) + T(v), \tag{6}$$

where $G - v$ refers to the graph obtained by removing v from G.

Definition 13 (Edge betweenness) *The betweenness of an edge $uv \in E(G)$ is given by:*

$$b_{uv} \;= \sum_{k\in V(G)} \;\sum_{l\in V,\; k<l} \frac{s^{kl}_{uv}}{s^{kl}};\; \forall\; uv \in E(G), \tag{7}$$

where s^{kl}_{uv} is the number of geodesics between vertices k and l $(k < l)$ passing through edge uv, and s^{kl} is the total number of geodesics between k and l.

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