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Robust solutions for the DARP with variations in transportation time

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Abstract: The Dial-a-Ride Problem (DARP) consists of designing a set of routes to transport clients from pickup node to delivery node, taking into account vehicle capacity, time windows and riding time constraints. The Stochastic DARP (SDARP) is a practical variant of the DARP in which the travel time between nodes is a random variable related to traffic jam, road maintenance or weather conditions. The solution robustness is the probability that a solution remains feasible considering different distribution laws and the driving policy which defines the driver procedure to follow when vehicle arrives earlier or later than the planned schedule. Three policies are introduced to model the wide spread transportation industry behaviours. For stochastic routing problems, classical approaches require either simulation based evaluation method to provide a useful estimation of the robustness or analytical methods. Both are too time consuming to be efficiently integrated into an iterative improvement scheme. A new method is introduced to estimate the robustness. It relies on an indirect approach using a specific criterion which is shown to be correlated to the robustness and which can be easily computed. This indirect approach is embedded into both an Evolutionary Local Search (ELS) metaheuristic and a multi-criteria population-based method. The solutions obtained at the end of the process are highly robust with respect to this criterion. Their robustness can be evaluated and confirmed by simulation. Numerical results are presented to analyse the robustness of the best published solutions for the DARP in the context of stochastic travel times. Many of them appear to be very sensitive to random variations. Computational results for the ELS and the population-based algorithm for the SDARP are then reported. They show the robustness can be significantly improved, without significant impact on the cost of the solution.

Keywords: Dial-A-Ride, transportation, stochastic travel time, robustness, gamma distribution

Résumé : Le Problème de Transport à la Demande (PTAD) consiste à définir un ensemble de tournées pour acheminer des clients de leur point d'origine à leur point de destination tout en respectant des contraintes de capacité, de fenêtre de temps et de durée. Le PTAD Stochastique (PTADS) est une variante du PTAD dans laquelle le temps de trajet entre deux points est soumis à incertitude. Il est modélisé par une variable aléatoire pouvant représenter la variabilité des conditions de trafic (congestion due à un afflux, à des travaux de maintenance, à des conditions climatiques, etc.). La robustesse d'une solution peut être vue comme la probabilité de rester réalisable en tenant compte à la fois de la loi de distribution et de la politique du conducteur en réponse aux variations par rapport au planing initial. Trois politiques représentant des pratiques dans l'industrie sont considérées. Les principales approches pour les problèmes de transport stochastiques reposent sur l'estimation de la robustesse par simulation ou sur des méthodes analytiques. Toutes deux sont trop coûteuses pour être intégrées dans un schéma itératif de résolution. Nous proposons une approche indirecte pour estimer la robustesse. Elle repose sur la proposition d'un critère spécifique, corrélé avec la robustesse et qui peut être aisément calculé. Cette approche est intégrée dans une Evolutionary Local Search (ELS) et dans une métaheuristique multi-critère à base de population. Les solutions obtenues sont très robustes par rapport à ce critère et leur robustesse est confirmée par simulation. La robustesse des meilleures solutions du PTAD publiées dans la littérature est évaluée dans le contexte stochastique. La plupart est très sensible aux variations aléatoires. Les solutions fournies par l'ELS et l'algorithme à base de population sur les mêmes instances sont significativement plus robustes avec un impact modéré sur le coût de transport.

Mots clés : Transport à la Demande, transport, temps de trajet stochastique, robustesse, loi Gamma

1 Introduction

The Dial-a-Ride Problem (DARP) consists of designing a set of routes to transport clients from their pickup node to their delivery node, taking into account vehicle capacity, time windows and riding time constraints. This problem is central in the transportation of persons by a flexible fleet of vehicles (taxi, on-demand services for medical transportation, etc.) and two main versions of the DARP are considered in the literature: in the static case, transportation requests are known in advance, while they are iteratively revealed in the dynamic case. The DARP is NP-hard as an extension of the Vehicle Routing Problem (VRP) and was first investigated in the late 70s by [22] who focuses on the scheduling part, *i.e.* assigning clients to vehicles. Partitioning techniques with simple rules are proposed for the static case. They are then extended for the dynamic context. For the single vehicle version, [20] proposes a heuristic, while [16] presents a dynamic programming approach. Then [21, 14, 25] develop heuristics in the multiple vehicle case. However few publications focus on real-life instances: [1] address the transportation of disabled people in Berlin, [17] consider dial-a-ride problems in Bologna. Another recent publication on stochastic routing addresses the Distance Constrained VRP with stochastic travel and service time on trip [10]. Authors propose an approach dedicated to PH Distribution to model both service and transport time. The stochastic time model can easily take into account a given service level.

Our contribution is to propose a new approach for the DARP with stochastic travel time which uses an indirect estimation of the robustness and which addresses the driver policies. The robustness can be defined as the probability to satisfy the time windows constraints for some specific law and driver policy. Note that in recent publications, authors have included semi-triangular distribution [11] or shifted gamma distribution [23, 24] in addition to distribution probabilities studies. Gamma distribution is of interest since the parameter of the distribution allows a wide range of behavior, from exponential distribution for low values to approximately normal distribution for large values. Our proposal can handle any probability law, but in practice it is limited to continuous laws with a strictly increasing distribution function, and with a finite variance.

Because simulation-based approach to obtain evaluation of the robustness can be time-consuming, a new criterion is defined, and a new trip evaluation function is defined extending the Firat and Woeginger's algorithm [9] to maximize the robustness. For the mono-objective approach, this criterion is aggregated with the solution cost and it is optimized by an ELS metaheuristic. The relative importance between the criterion and the cost is adapted during the metaheuristic iterations. A population-based method is also introduced to provide a Pareto front. Then, the robustness of existing known solutions for the DARP for two well-known benchmarks is evaluated in the stochastic context. The numerical experiments prove that solution robustness can be significantly improved without significant impact on the solution cost.

The remainder of this paper is organized as follows: a model for the DARP, its extension to the SDARP and driving policies are presented in Section 2. The robustness of a solution is defined in Section 3. It involves the reformulation of time constraints, and both an evaluation by simulation and the probabilistic evaluation of a trip are proposed. The ELS metaheuristic and the population-based method for computing robust solutions for the SDARP are detailed in Section 4. Numerical results in Section 5 focus on both the robustness of best known solutions from the DARP and the behavior of the two proposed methods on two well-known benchmarks from the literature. Concluding remarks are set in Section 6.

2 Model for the SDARP

2.1 DARP definition

The static DARP [4] is defined on a complete graph $G = (V, A)$ corresponding to a weighted directed transportation network. The fleet of K vehicles is assumed to be homogeneous, *i.e.* all the vehicles have the same capacity Q. The set of n transportation requests is known in advance. The following notations are used for data: $V = \{0, 1, \ldots, 2n, 2n+1\}$ is the set of nodes with $1, \ldots, n$ the pickup nodes and $n+1, \ldots, 2n$ the delivery nodes. Two copies 0 and $2n+1$ of the depot model respectively the beginning and the end of the trips. For convenience, given a transportation request $i = 1, \ldots, n$, let $i^+ = i$ be its pickup node and $i^- = i + n$ be its delivery node. Given an arc $(i, j) \in A$, t_{ij} and c_{ij} are respectively the transportation time and the transportation cost to go directly from i to j. A time window $[e_i;\ l_i]$ is associated to each node $i \in V$; e_i is the earliest starting time of service while l_i is the latest starting time of service. The service duration is sd_i and q_i is the number of customers to handle at node i. Thus, given a transportation request i, $q_i > 0$ at its pickup node and $q_{i+n} = -q_i < 0$ at its delivery node.

Each customer must be picked up at his pickup node and dropped off at his delivery node while no transfer is allowed. Each trip starts and ends at the depot. The capacity constraint of any vehicle must hold at any node of the trip and the beginning of service must fit the time windows at any visited node, including nodes 0 and $2n + 1$. A waiting delay is allowed before the beginning of service at any node. This can be used to satisfy the time window constraints, the riding time constraints or the driving time constraints. No waiting delay is allowed at the node after the service time and the vehicle leaves the node just after finishing its service.

The problem consists of routing at most K vehicles by assigning a node list λ to each vehicle k. Let $\lambda(i)$ be the i^{th} node of the list and n_λ be the number of nodes corresponding to a transportation request. Thus $\lambda(0) = 0$, $\lambda(n_\lambda + 1) = 2n + 1$ and $\lambda(n_\lambda)$ is the last node on which to perform an operation. Obviously $\lambda(1)$ is a pickup node and $\lambda(n_\lambda)$ is a delivery node. Let $\beta(i)$ represent node i position in the list λ. Thus, for a transportation request i, $\beta(i)$ and $\beta(i+n)$ are respectively the position of the pickup node and of the delivery node in λ. The following variables are used for any node $\lambda(i)$ (see Figure 1):

- A_i, arrival time;
- B_i, beginning of service;
- D_i, departure time, defined as $D_i = B_i + sd_i$;
- W_i, waiting delay, defined as $W_i = B_i - A_i$;
- R_i, riding time for transportation request i, *i.e.* the time between the end of service at $\beta(i)$ and the beginning of service at $\beta(i + n)$. It is defined as $R_i = B_{\beta(i+n)} - D_{\beta(i)}$.

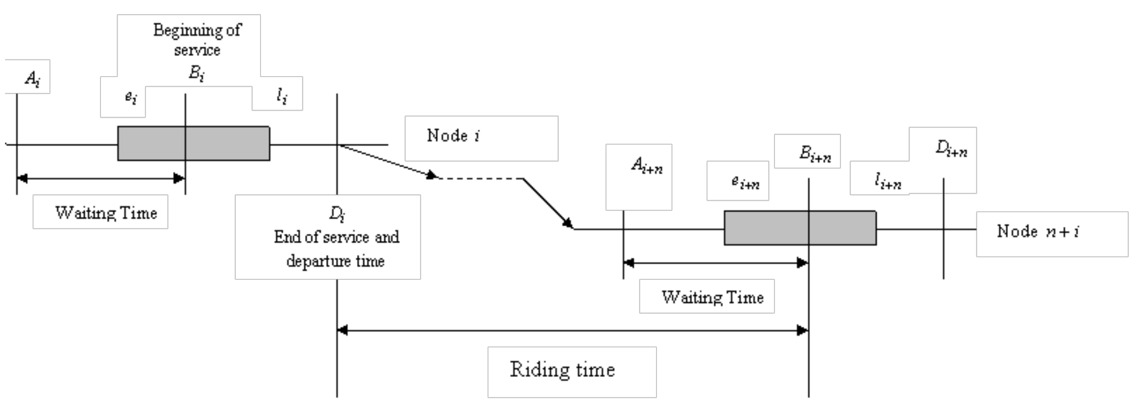


Figure 1: Description and notations [3].

Given a node list λ, B^λ is the departure time from the depot and E^λ is the return time to the depot. The objective function is to minimize the total cost of a solution s:

$$c(s) = \sum_{k=1}^{m(s)} \sum_{i=0}^{n_{\lambda_k}} c_{\lambda_k(i)\lambda_k(i+1)}$$

where $m(s)$ is the number of trips of the solution, *i.e.* the number of node lists in λ. Then, $m(s) = K$ if all the vehicles are used. A solution must satisfy the following constraints:

(C1) The number of customers in the vehicle at each node cannot exceed the capacity Q;

(C2) The beginning of service at any node i belongs to its time window, $e_i \leq B_i \leq l_i$;

(C3) The riding time for a transportation request i cannot exceed the time limit, $R_i \leq L$;

(C4) The trip duration for each vehicle is at most H.

Constraints (C3) and (C4) model the quality of service respectively for the users and for the vehicles. As stressed by [4], the time constraints may be often tight in practice and several authors have allowed a partial relaxation for the local search in order to investigate "unfeasible" solutions.

2.2 Stochastic DARP

In practice, as stressed by [27, 19], many parameters are subjected to uncertainty, including customer requests, transportation times, vehicle availability, etc. The Stochastic DARP (SDARP) variant addressed in this paper is similar to the static DARP, except that the transportation time t_{ij} is not constant and is modeled as a positive random variable T_{ij}. The costs c_{ij} related to the arcs remain deterministic. A common – yet approximate – way to solve the stochastic routing problem subject to transportation time variations consists of considering the expected value $E(T_{ij})$ for the transportation times. Based on the definition of [12], robust solutions of stochastic problems are solutions expected to perform well with variations on uncertain data. While the objective of the DARP is to find a minimal cost solution, the objective of the SDARP is to find a sufficiently robust minimal cost solution.

Given an arc $(i,j) \in A$, the following notations are used:

- t_{ij} is the transportation time in the original DARP;
- T_{ij} is the random transportation time in the SDARP;
- $T_{ij}(\omega)$ is a realization of T_{ij} (event ω).

Then, given a node list λ related to a trip and given a node at position i in λ:

- the deterministic transportation time from $\lambda(0)$ to $\lambda(i)$ is $\gamma(i) = \sum_{k=0}^{i-1} t_{\lambda(k)\lambda(k+1)}$;
- the random transportation time from $\lambda(0)$ to $\lambda(i)$ is $\Gamma(i) = \sum_{k=0}^{i-1} T_{\lambda(k)\lambda(k+1)}$;
- the sum of waiting durations from $\lambda(0)$ to $\lambda(i)$, where $A_0 = e_0$, is $X(i) = \sum_{k=0}^{i} W_{\lambda(i)}$.

Due to the stochastic nature of the transportation times $T_{ij}(\omega)$, some DARP constraints hold or not depending on random event ω and some variables should be updated including for example the waiting time on node. If $T_{ij}(\omega) < t_{ij}$, the vehicle should arrive at node k earlier than e_k, *i.e.* $\beta(k) \geq \beta(j)$, increasing the waiting time and ensuring constraints feasibility. Note that, aside from (C2), constraints (C3) and (C4) can be enforced by increasing the waiting delay at node j to keep the preliminary arrival time. On the other hand, if $T_{ij}(\omega) > t_{ij}$, the vehicle may arrive later than l_k at some node k such that $\beta(k) \geq \beta(j)$. In such situation the trip λ becomes unfeasible since some constraints do not hold. This example is illustrated in Figure 2.

Since a variation in transportation times can create a time windows violation, a customer maximal riding time violation or a total trip duration violation, the respective criteria $w(s)$, $t(s)$ and $d(s)$ [4] are usually defined. They are sued to maximize the robustness of a solution minimizing the probability that one constraint does not hold. An unfeasible solution is then characterized by $w(s) > 0$ and/or $t(s) > 0$ and/or $d(s) > 0$.

To model variations around the expected value, the random transportation times T_{ij} on arc (i,j) are assumed to follow a Gaussian law. In practical applications, the transportation time on an arc is the consequence of a large number of elementary vehicle transportation time variations in the traffic. Because each measure results from a large number of small independent sources, the central limit theorem applies and the random transportation time can be efficiently modeled by a Gaussian random variable, with one restriction since the transportation time should remain positive.

Assuming the transportation time between two nodes is modeled by a Gaussian distribution $\mathcal{N}(t_{ij}, \sigma_{ij}^2)$, the distribution is truncated to avoid negative values and, without loss of generality, the standard deviation

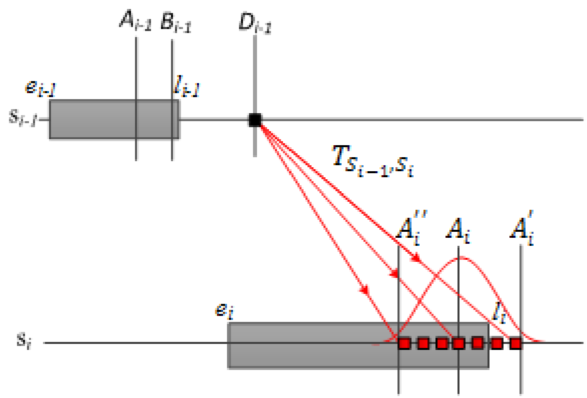


Figure 2: Stochastic transportation times and time windows constraints [3].

parameter is arbitrarily set to $\sigma_{ij} = t_{ij}/\psi$ where ψ defines the relative importance of standard deviation regarding the average value (for example $\psi = 10$ defines a standard deviation σ_{ij} which is about 10 % of t_{ij}).

Because, random variables T_{ij} for transportation time are assumed to be independent, the total stochastic transportation time Γ_i from the depot to the beginning of service at node $\lambda(i)$ of the trip λ can be modeled by a Gaussian distribution $\mathcal{N}(\gamma_{\lambda(i)}, \sigma^2_{\lambda(i)})$. Assuming that the deterministic transportation times are suitable approximations of the random ones, the standard deviation parameter is set to $\sigma_{\lambda(i)} = \sqrt{\sum_{k=0}^{i-1}(t_{\lambda(k)\lambda(k+1)}/10)^2}$. Using the Gaussian laws additivity for independent random variables, Γ_i itself is a Gaussian with parameter $\gamma_{\lambda(i)}$.

2.3 Driving policies

A trip is defined by the sequence of visited nodes, the arrival time A_i and the beginning of service B_i at each node i. When a realization ω of a random variables occurs, the initial schedule might be disturbed. Since the driver is supposed to drive at the speed limit, the speed of the vehicle cannot be modified. On the other hand, those variabilities can be handled by increasing or reducing the waiting delays as well as by updating the starting times. These decisions are supposed to be based on a driver policy defined by the supervisor in order to favor feasible schedule with respect to the time constraints, after the realization of the random variables.

Let us consider the deterministic schedule of Figure 3, where, for sake of simplicity, the service time is supposed to be zero. The vehicle starts its operations at node s_{i-1} at time B_{i-1}, and immediately leaves s_{i-1} for the current node s_i with time window $[e_i;\ l_i]$. The travel duration is $t_{s_{i-1}s_i}$ and the vehicle arrives at node s_i at time A_i. It starts its operations at time B_i after a waiting duration W_i. Such scenario clearly defines a "route schedule" which does not address explicitly the random events.

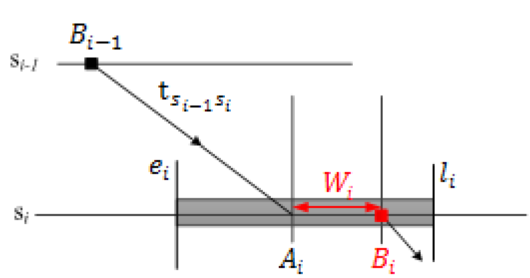
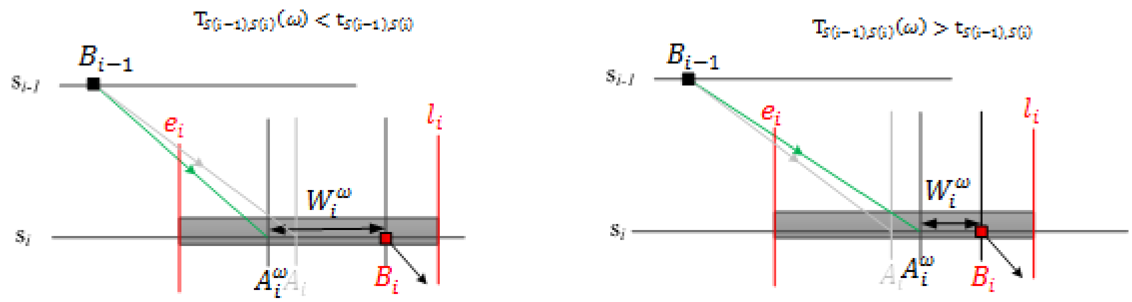


Figure 3: Stochastic transportation times and time windows constraints [3].

After the realization $T_{s_{i-1}s_i}(\omega)$, the arrival time is $A_i^\omega = B_{i-1} + T_{s_{i-1}s_i}(\omega)$. Three policies are investigated:

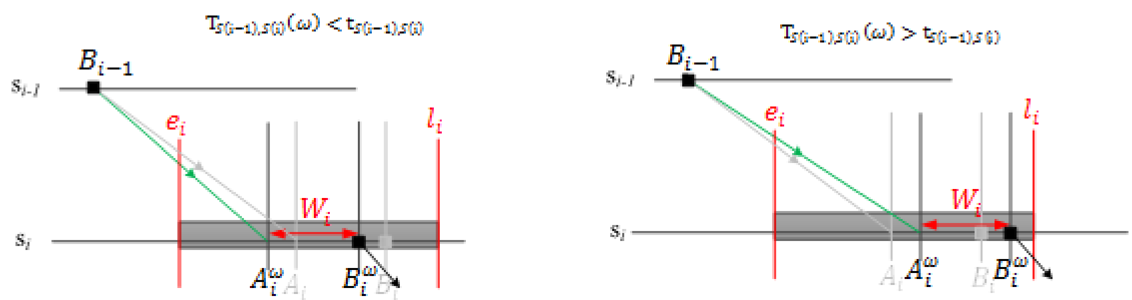
P1 “Keep the starting time B_i”: if $A_i^\omega < B_i$, the vehicle arrives before B_i and the driver has to wait until B_i; otherwise, the driver immediately starts his operations, *i.e.* $B_i^\omega = A_i^\omega$. The first case occurs in early arrival ($A_i^\omega \leq A_i$) and preserves the beginning of service B_i since $A_i < A_i^\omega \leq B_i$, as stressed on Figure 4a. The second case occurs for extra delay in transportation time and leads to situations where the arrival time is greater than the scheduled starting time B_i, see Figure 4b. Whatever the arrival date $A_i^\omega > B_i$, the vehicle is assumed to leave the node at the earliest possible time. Such policy is common in routing problems where the service time depends on both vehicle arrival time and on the availability of extra resources. This includes skilled workers or handling tools at some specific unloading docks which must be booked ahead. This situation is widespread in the supply of supermarkets where the driver cannot take advantage of an earlier arrival date.



(a) early arrival **(b) mild lateness**

Figure 4: Impact of P1 policy.

P2 “Keep the waiting duration W_i”: the driver must set a waiting duration $W_i = B_i - A_i$ as defined in his initial schedule, whatever the realization $T_{s_{i-1}s_i}(\omega)$. This might lead to infeasibility in case of late arrival, *i.e.* when $A_i^\omega + W_i > l_i$. The same situation occurs for an early arrival for which $A_i^\omega + W_i < e_i$. The two situations of early arrival and mild lateness are illustrated in Figure 5. This policy is widespread in periodic transportation systems, including subways or any automated lines. A fixed waiting time on stations is required to handle boarding and leaving passengers.



(a) early arrival **(b) mild lateness**

Figure 5: Impact of P2 policy.

P3 “Keep the waiting duration for early arrival, keep the starting time otherwise”: the decision differs according to the situation. If the vehicle arrives earlier, the waiting duration W_i is kept. Otherwise, the driver waits until B_i after a late arrival (Figure 6). In case of significant delay, *i.e.* $A_i^\omega > B_i$, the service at node i is started immediately. This policy can be seen as hybridization between P1 and P2.

Other policies could be considered, depending on the context. For instance, the lower bound e_i of the time window could be taken into account to shift backwards the service time B_i. Another approach could

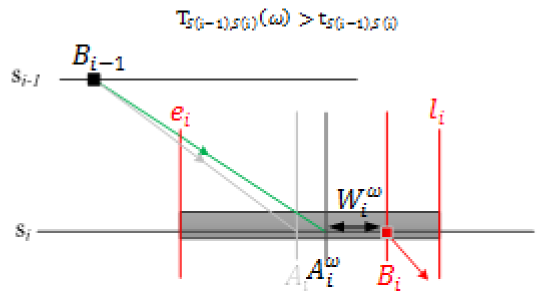


Figure 6: Impact of P3 policy.

be to measure the feasibility of the current schedule after arriving at each node and to update the arrival times and the waiting durations dynamically and iteratively. However, this could no longer be considered as a policy, since the decision process would be too complex to be taken by the driver himself. In all situations, the delay or the advance following $T_{s_{i-1}s_i}(\omega)$ might lead to infeasibility on the time windows, on the riding time constraints or on the total trip duration constraints.

3 A methodological approach for solution robustness optimization

3.1 General framework

Optimizing a problem $\min_{x\in\mathcal{X}}\{h(x)\}$ subject to stochastic variations requires the evaluation of either $h(x)$ –if an analytical tractable formulation exists– or $\overline{h(x,r)}$ for any solution x where r defines the number of replications required to obtain a suitable evaluation $\overline{h(x)}$ of $\overline{h(x)}$. The computation of $\overline{h(x)}$ and/or $\overline{h(x,r)}$ is often time-consuming and it cannot be embedded efficiently into metaheuristic optimization processes for the evaluation of each solution.

Our proposition consists in defining an indirect criterion $\rho(x)$, fast to compute and correlated to $\overline{h(x,r)}$, which can be used as surrogate in a metaheuristic, since a lot of solutions need to be evaluated. This framework is illustrated in Figure 7. A function f' is first defined to assign one ρ to each solution $x \in \Omega$ and to consider $\overline{h(x,r)}$ only at the end of the optimization.

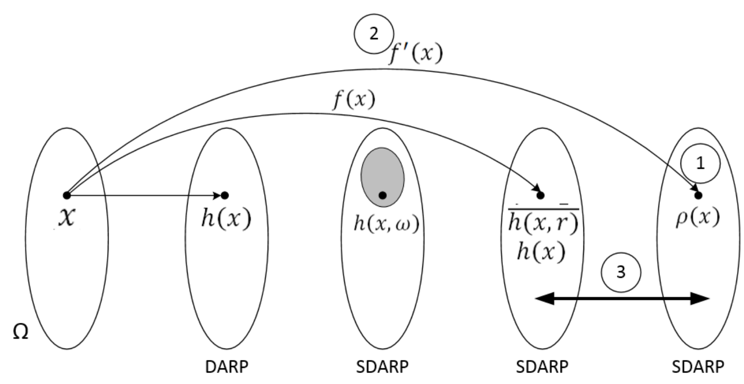


Figure 7: General framework for SDARP.

In the context of a deterministic DARP, any solution $x \in \Omega$ is related to $h(x)$. Typically this corresponds to the total travel time. In SDARP, this also depends on the random event ω leading to the realization $T_{ij}(\omega)$ and one evaluation becomes $h(x,\omega)$. As stressed before, the value $h(x)$ used for a deterministic problem must be replaced by $\overline{h(x,r)}$. The evaluation of $\overline{h(x,r)}$ can be replaced by a function $f'(x)$ which computes an indirect criterion $\rho(x)$. This criterion must be correlated to $\overline{h(x,r)}$ to ensure that high quality solutions regarding $\rho(x)$ are also high quality solutions regarding $\overline{h(x,r)}$. The key-points of this approach are:

- definition of an indirect criterion $\rho(x)$;
- definition of a function $f'(x)$.

The estimation of the correlation between the criteria $\overline{h(x)}$ or $\overline{h(x,r)}$ and the indirect criterion $\rho(x)$ only requires a large-enough number r of replications, since the comparison between $\overline{h(x,r)}$ and $\rho(x)$ is a postmortem analysis of the $\rho(x)$ adequacy.

3.2 Definition of the indirect criterion: Theoretical considerations

In order to be feasible, a node list λ (which corresponds to a trip) in the DARP has to satisfy different time constraints as stressed by [4]. A beginning of service time B_i must be defined for each node $i \in \lambda$ such that constraints (C2), (C3) and (C4) hold. As mentioned in [9], B_i can be defined as the sum of waiting durations X_i and transportation times γ_i, $B_i = X_i + \gamma_i$. Thus $X_i = B_i - \gamma_i$.

Four constraints must hold for B_i (Figure 1):

(D1) Maximal time window: $B_i \geq e_i$, $\forall i \in \lambda$
(D2) Minimal time window: $B_i \leq l_i$, $\forall i \in \lambda$
(D3) Maximal trip duration: $B_{n_k} - B_1 \leq H$
(D4) Maximal request riding time: $B_{\beta(\lambda(i)+n)} - B_{\lambda(i)} \leq sd_{\lambda(i)} + L$, $\forall i \in \lambda$

This set of constraints leads to the subsequent set of constraints introduced by [9] using X_i as variables:

(D1) $(e_{\lambda(i)} - \gamma_i) \leq X_i$
(D2) $(l_{\lambda(i)} - \gamma_i) \geq X_i$
(D3) $(H - \gamma_{n_k}) \geq X_{n_k} - X_1$
(D4) $(sd_{\lambda(i)} + L - (\gamma_j - \gamma_i)) \geq X_j - X_i$

In the SDARP, the deterministic value γ_i becomes a random variable Γ_i and the inequalities must be rewritten, considering a random event ω leading to $\Gamma_i(\omega)$.

(D1’) $(e_{\lambda(i)} - \Gamma_i(\omega)) \leq X_i$
(D2’) $(l_{\lambda(i)} - \Gamma_i(\omega)) \geq X_i$
(D3’) $(H - \Gamma_{n_k}(\omega)) \geq X_{n_k} - X_1$
(D4’) $(sd_{\lambda(i)} + L - (\Gamma_j(\omega) - \Gamma_i(\omega))) \geq X_j - X_i$

Since inequalities depend on the event ω, each inequality has a probability $\mathbb{P}(Dx')$ to hold:

- $\mathbb{P}(D1') = \mathbb{P}\left((e_{\lambda(i)} - \Gamma_i(\omega)) \leq X_i\right)$
- $\mathbb{P}(D2') = \mathbb{P}\left((l_{\lambda(i)} - \Gamma_i(\omega)) \geq X_i\right)$
- $\mathbb{P}(D3') = \mathbb{P}\left((H - \Gamma_{n_k}(\omega)) \geq X_{n_k} - X_1\right)$
- $\mathbb{P}(D4') = \mathbb{P}\left((sd_{\lambda(i)} + L - (\Gamma_j(\omega) - \Gamma_i(\omega))) \geq X_j - X_i\right)$

The node list λ is stated valid if and only if $\mathbb{P}(D1') \geq \rho$, $\mathbb{P}(D2') \geq \rho$, $\mathbb{P}(D3') \geq \rho$ and $\mathbb{P}(D4') \geq \rho$, with ρ a positive constant. This corresponds to a reliability level and it is a parameter of the trip evaluation. Since $\mathbb{P}(D1')$, $\mathbb{P}(D2')$, $\mathrm{P}\mathbb{P}(D3')$ and $\mathbb{P}(D4')$ depend on X_i, the problem is to set a value for each X_i considering:

- $\mathbb{P}(D1') \geq \rho \rightarrow \mathbb{P}\left(\Gamma_i \geq -X_i + e_{\lambda(i)}\right) \geq \rho$
- $\mathbb{P}(D2') \geq \rho \rightarrow \mathbb{P}\left(\Gamma_i \leq -X_i + l_{\lambda(i)}\right) \geq \rho$
- $\mathbb{P}(D3') \geq \rho \rightarrow \mathbb{P}\left(\Gamma_{n_k} \leq X_1 - X_{n_k} + H\right) \geq \rho$
- $\mathbb{P}(D4') \geq \rho \rightarrow \mathbb{P}\left(\Gamma_j - \Gamma_i \leq X_i - X_j + sd_{\lambda(i)} + L\right) \geq \rho$

If the transportation times are Gaussian random variables, $\Gamma_i \sim \mathcal{N}(\gamma_i, \sigma_i^2)$ and the first constraint can be rewritten:

$$\mathbb{P}\left(\Gamma_i \geq -X_i + e_{\lambda(i)}\right) \geq \rho \Leftrightarrow \mathbb{P}\left(\frac{\Gamma_i - \gamma_i}{\sigma_i} \geq \frac{(-X_i + e_{\lambda(i)}) - \gamma_i}{\sigma_i}\right) \geq \rho \tag{1}$$

$$\Leftrightarrow \mathbb{P}\left(\mathcal{N}(0,1) \geq \frac{(-X_i + e_{\lambda(i)}) - \gamma_i}{\sigma_i}\right) \geq \rho \tag{2}$$

Similar remark holds for the other constraints, which leads to the following inequalities:

- $\mathbb{P}\left(D1'\right) \geq \rho \rightarrow \mathbb{P}\left(\mathcal{N}(0,1) \geq \frac{-X_i+e_{\lambda(i)}-\gamma_i}{\sigma_i}\right) \geq \rho$
- $\mathbb{P}\left(D2'\right) \geq \rho \rightarrow \mathbb{P}\left(\mathcal{N}(0,1) \leq \frac{-X_i+l_{\lambda(i)}-\gamma_i}{\sigma_i}\right) \geq \rho$
- $\mathbb{P}\left(D3'\right) \geq \rho \rightarrow \mathbb{P}\left(\mathcal{N}(0,1) \leq \frac{X_1-X_{n_k}+H-\gamma_{n_k}}{\sigma_{n_k}}\right) \geq \rho$
- $\mathbb{P}\left(D4'\right) \geq \rho \rightarrow \mathbb{P}\left(\mathcal{N}(0,1) \leq \frac{X_i-X_j+sd_{\lambda(i)}+L-(\gamma_j-\gamma_i)}{\sqrt{\sigma_j^2+\sigma_i^2}}\right) \geq \rho$

The cumulative probability of $\mathcal{N}(0,1)$ is $\phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt$. Let $c = \phi^{-1}(\rho)$, which is known if $\rho \in [0,1[$. Under these conditions, the probability inequalities $\mathbb{P}\left(\mathcal{N}(0,1) \leq y\right) \geq \rho$ are equivalent to inequalities $y \geq c$ and probability inequalities $\mathbb{P}\left(\mathcal{N}(0,1) \geq y\right) \geq \rho$ are equivalent to inequalities $y \leq -c$. Thus,

$$\mathbb{P}\left(\mathcal{N}(0,1) \geq \frac{-X_i + e_{\lambda(i)} - \gamma_i}{\sigma_i}\right) \geq \rho \Leftrightarrow \frac{-X_i + e_{\lambda(i)} - \gamma_i}{\sigma_i} \leq -c \Leftrightarrow X_i \geq c\sigma_i + e_{\lambda(i)} - \gamma_i$$

A similar remark holds for the other constraints, which leads to the following set of constraints:

(D1") $X_i \geq e_{\lambda(i)} - \gamma_i + c\sigma_i, \quad \forall i = 1, \ldots, n_k$
(D2") $X_i \leq l_{\lambda(i)} - \gamma_i - c\sigma_i, \quad \forall i = 1, \ldots, n_k$
(D3") $X_{n_k} - X_1 \leq H - \gamma_{n_k} - c\sigma_{n_k}$
(D4") $X_j - X_i \leq sd_{\lambda(i)} + L - (\gamma_j - \gamma_i) - c\sqrt{\sigma_j^2 + \sigma_i^2}, \quad \forall (i,j)|\lambda(j) = \lambda(i) + n$

Thus, a node list λ is considered valid if and only if $(D1'')$, $(D2'')$, $(D3'')$ and $(D4'')$ hold and a valid constraint (Dx''), $x = 1, \ldots, 4$, induces $(Dx') \geq \rho$. If $\rho = 0.5$, then $c = 0$ and the SDARP constraints are identical than deterministic DARP constraints.

To conclude, the main hypothesis is that the probability $\mathbb{P}(t)$ of a trip t being feasible depends strongly on probability $\mathbb{P}(Cx) \geq \rho$, $x = 1, \ldots, 4$. Thus, the probability $\mathbb{P}(s)$ of the solution s to be feasible is also connected to $\mathbb{P}(t)$ for all the trips $t \in s$. As a consequence, computing a robust solution can be done by maximizing the value ρ associated with its trips. Note that the ρ criterion is deterministic.

3.3 Trip evaluation

The $f'(x)$ function (trip_evaluation() procedure) uses a binary search (Algorithm 1) to compute the largest reliability level ρ such that a node list λ is feasible. This algorithm uses the linear time evaluation algorithm from [9], initially introduced for the deterministic DARP to check the feasibility of a node list λ and which is based on a conjunctive graph built with constraints $(D1)$, $(D2)$, $(D3)$ and $(D4)$. For the SDARP, a similar graph built with constraints $(D1'')$, $(D2'')$, $(D3'')$ and $(D4'')$ is used. The Firat_feasibility(λ, ρ) evaluation takes a node list λ and a constant ρ as input. It returns true if there exists values for X_i satisfying all the constraints with a probability $\mathbb{P}(Ci') \geq \rho$, otherwise it returns false.

Checking a node list λ feasibility is a part of the evaluation algorithm (lines 2, 4, 10) but the real challenge is to find the largest probability $\rho \in [0.5, \rho_M]$ such that λ is feasible. It is computed by a binary search. It depends on λ and it is used to measure the trip robustness. The algorithm returns the existence of a

Algorithm 1 Trip_evaluation()

Input: λ: node list; ρ_M: maximal reliability; δ: numerical precision
Output: t: trip feasiblity; $\rho \in [0.5, \rho_M]$: largest reliability; cost: trip cost

cost = compute_cost(λ)
test = Firat_feasibility(λ,ρ_M)
if test = false **then**
  test = Firat_feasibility(λ,0.5)
  if test = true **then**
    $\rho_{max} = \rho_M$
    $\rho_{min} = 0.5$
    while $\rho_{max} - \rho_{min} > \delta$ **do**
      $\rho = (\rho_{max} + \rho_{min})/2$
      test = Firat_feasibility(λ,ρ)
      if test = true **then**
        $\rho_{min} = \rho$
      else
        $\rho_{max} = \rho$
      end if
    end while
    t = true; return {t, ρ_{min}, cost}
  else
    t = false; return {t, 0.0, cost}
  end if
else
  t = true; return {t, ρ_M, cost}
end if

reliability level ρ such that λ is feasible (boolean t), the largest ρ such that λ is feasible and the cost of the trip.

The solution evaluation (Algorithm 2) consists of evaluating all trips sequentially. The robustness ρ^* of a solution is then defined as the product of the value ρ for all the trips. A robust solution is assumed to have a higher ρ^* value and methods presented in the next section investigates solutions with high value $\rho^*(s)$. Such an approach complies with the hypothesis of Section 3.1. In practice, the loop in lines 1-7 stops as soon as f_t=false. In this case, the return values for ρ^* and cost are meaningless.

Algorithm 2 Solution_evaluation()

Input: s: solution
Output: f: solution feasiblity; $\rho^* \in [0.5, \rho_M]$: largest reliability; cost: trip cost

1: **for** all trip t $\in$ s **do**
2:   $\{f_t,\rho_t,c_t\}$ = trip_evaluation(t)
3:   f = f $\wedge$ f_t
4:   $\rho^* = \rho_t^*$
5:   cost = cost + c_t
6: **end for**
7: return $\{f_t,\rho_t,c_t\}$

4 ELS and NSGA-II for the SDARP

The Evolutionary Local Search (ELS) has been first proposed by [26] and has been applied to numerous routing problem including the VRP by [15]. It basically extends the Iterated Local Search (ILS) defined by [13]. The ELS we propose takes advantage of an indirect solution representation – an ordered list of pickup nodes – and it relies on the ELS fully described in [2]. The second algorithm is a bi-objective population-based optimization scheme based on the NSGA-II algorithm which computes successive generations of a population divided into non-dominated fronts as designed by [5, 6].

5 Numerical results

The experiments have been done on a PC with an Intel Core i7-3770 CPU 3.40 GHz with about 2529 MFlop/s according to the Linpack (http://www.roylongbottom.org.uk) and [8]. The benchmark uses the 20 Cordeau and Laporte's instances which define the largest instances in the DARP literature. The solutions computed by the approach are available online at http://www.isima.fr/~duhamel/SDARP/index.html.

5.1 Quality of $\overline{h_p(s,r)}$ depending on r

For convenience, let us note $\overline{h_p(s,r)}$ the average probability estimated after r replications on the solution s with driver's policy p and let us note $h_p(s,\omega)$ one stochastic evaluation of s. Each replication depends on the distribution law [7]. Depending on r, *i.e.* on the number of replications, $\overline{h_p(s,r)}$ provides an estimation of $\overline{h_p(s)}$, *i.e.* the probability of a solution s to remain feasible with respect to the distribution law on travel times and the driver's policy p. The random variables on travel times are assumed to be independent and the probability $\mathbb{P}(s)$ that solution s remains feasible is the product of the trip probability of being feasible, *i.e.* $\mathbb{P}(s) = \prod_{t \in s} \mathbb{P}(t)$.

The convergence of $\overline{h_p(t,r)}$ towards the theoretical value $\overline{h_p(t)}$ is illustrated on the second trip of the best known solution for instance pr01 of [4]. This trip corresponds to the following sequence of nodes:

$$\{0,9^+,17^+,9^-,8^+,20^+,1^+,17^-,7^+,7^-,9^+,20^-,2^+,1^-,5^+,13^+,5^-,2^-,16^+,4^+,4^-,13^-,19^+,23^+,13^-,23^-,0\}$$

The Figure 8 displays the evolution of $\overline{h_{P1}(t,r)}$ with the number r of replications under P1 driver's policy. Namely, 10,000 replications seem to be sufficient to obtain a high quality approximation. 5 runs are reported, illustrating that the quality of $\overline{h_p(t,r)}$ does not depend on the random generator seed.

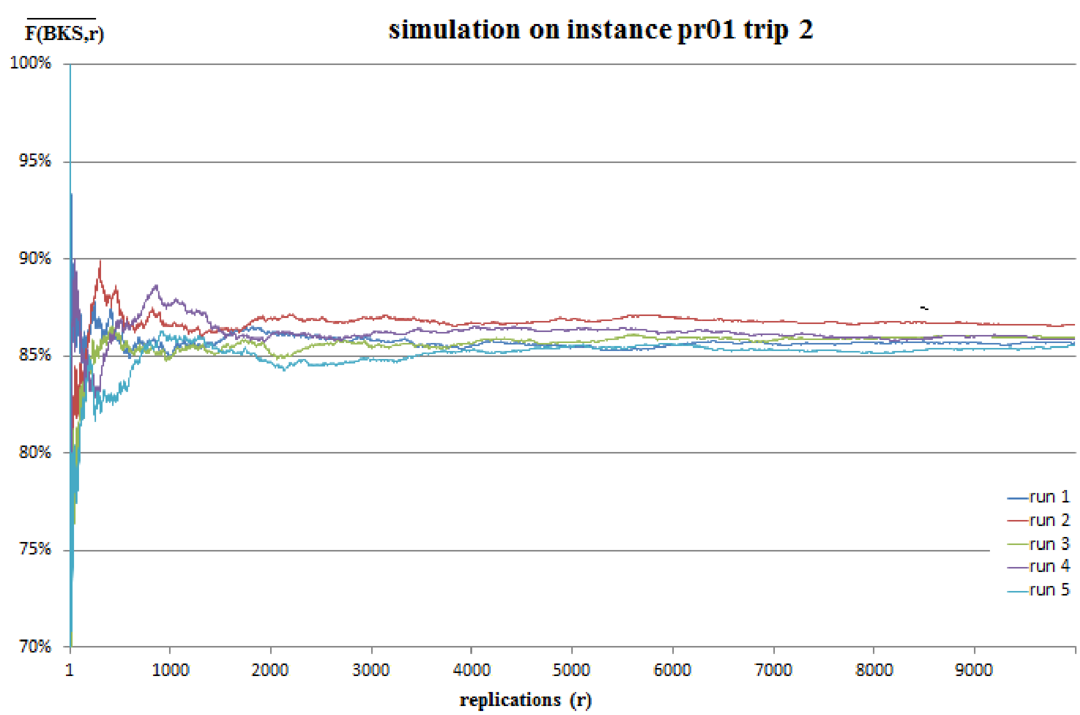


Figure 8: Simulation on pr01, trip 2, with $10,000$ replications.

5.2 Robustness of the best known DARP solutions (Normal distribution)

The robustness evaluation we propose is applied to the best known solutions (BKS) of the 20 instances introduced by [4] and Table 1 provides an estimation of the BKS using the 3 driver policies with the estimation

Table 1: **Robustness for the best known solutions of [4] instances.**

inst.	$c(BKS)$	ρ^*_{P1}	$\overline{h_{P1}(BKS,r)}$	ρ^*_{P2}	$\overline{h_{P2}(BKS,r)}$	ρ^*_{P3}	$\overline{h_{P3}(BKS,r)}$
pr01	190.02	74.8%	86.4%	66.8%	25.9%	66.8%	50.0%
pr02	301.34	76.8%	85.4%	48.6%	28.4%	48.6%	72.6%
pr03	532.00	4.8%	0.1%	3.1%	0.0%	3.1%	0.0%
pr04	570.25	14.3%	0.3%	9.4%	0.0%	9.4%	0.1%
pr05	626.93	13.1%	0.7%	5.0%	0.0%	5.0%	0.7%
pr06	785.26	3.9%	0.0%	3.0%	0.0%	3.0%	0.0%
pr07	291.71	21.9%	14.7%	14.7%	0.3%	14.7%	7.9%
pr08	487.84	8.9%	0.2%	5.9%	0.0%	5.9%	0.2%
pr09	658.31	9.4%	0.4%	4.3%	0.0%	4.3%	0.3%
pr10	851.82	1.3%	0.0%	0.6%	0.0%	0.6%	0.0%
pr11	164.46	63.2%	61.0%	57.6%	36.1%	57.6%	69.0%
pr12	295.66	32.0%	5.8%	25.0%	0.2%	25.0%	3.7%
pr13	484.83	32.5%	16.5%	19.3%	0.8%	19.3%	6.2%
pr14	529.33	59.3%	64.6%	35.6%	9.2%	35.6%	27.8%
pr15	577.29	37.9%	72.0%	17.9%	1.9%	17.9%	25.1%
pr16	730.67	16.2%	11.1%	7.5%	0.2%	7.5%	3.4%
pr17	248.21	26.8%	1.6%	23.8%	0.3%	23.8%	0.9%
pr18	458.73	62.2%	72.9%	43.8%	17.0%	43.8%	54.4%
pr19	593.49	5.8%	0.0%	3.5%	0.0%	3.5%	0.1%
pr20	785.68	3.8%	0.0%	2.8%	0.0%	2.8%	0.1%
avg.	508.19	28.5%	24.7%	19.9%	6.0%	19.9%	16.1%

of $\overline{h_p(BKS,r)}$ achieved with $r = 10,000$ replications. For each instance, the total travel distance $c(s)$ of the BKS is provided with the value ρ^*_p of the robustness criterion and the estimation $\overline{h_p(BKS,r)}$ for each policy $p \in \{P1, P2, P3\}$.

$\overline{h_{P1}(BKS,r)}$ gives an average feasibility probability about 24.7% which is quite low, but significant differences occur over the instances. For example, for the pr01 instance, the best known solution has a feasibility probability about 86.4% while, for the instance pr08, the probability for the best know solution to be feasible is less than 0.2%. Thus the former solution will remain feasible two thirds of the time, while the latter will seldom be feasible. Similar remarks hold for policies P2 and P3 with respective average feasibility probabilities 6.0% and 16.1%. Note that $\rho^*_{P2} = \rho^*_{P3}$ in our context.

The correlation between the indirect criterion and the stochastic criterion can be stated as satisfactory. For example, for the pr01 the BKS is one of the solutions with the highest feasibility probability (about 86%) according to $\overline{h_{P1}(BKS,r)}$ and this solution has one of the highest values of ρ^*_{P1} (about 74.8%). The average gap between $\overline{h_{P1}(BKS,r)}$ and ρ^*_{P1} is about 3.7%. Thus the correlation is satisfactory and the graphical representation in Figure 9 of both ρ^*_p and $\overline{h_p(BKS,r)}$ confirms the global trend. Similar comments hold for all the instances from [18]. The full set of results is available at: `http://www.isima.fr/~duhamel/SDARP/index.html`.

5.3 SDARP with a Normal distribution

Tables 2 and 3 introduce the ELS results on the instances from [4, 18] respectively. The two criteria (total travel distance and robustness criterion) are aggregated, the latter being prioritized, and optimized. For each instance and each policy p, the total travel cost $c(s)$ of the best found solution s, its relative gap to the total travel cost of the BKS (*gap*), the value of the robustness estimator ρ^*_p, the value of the estimation $\overline{h_p} = \overline{h_p(s,r)}$ and the CPU time in seconds are reported with $r = 100,000$. Since the CPU time is identical for al policies, it is only reported once.

From Table 2, the ELS is able to find highly robust solutions for most of the instances from [4]. It provides a nearly 100% feasibility for all instances, and for all policies. Only solutions for the pr10 instance with policies $P2$ and $P3$ seems to lack some robustness (63.3% and 96.2% feasibility probability respectively). The values for the robustness criterion are correlated with the value of the estimation. At the same time, the relative gap of the ELS solution increases on average by 13.19% for policy $P1$ and by 18.30% for policies

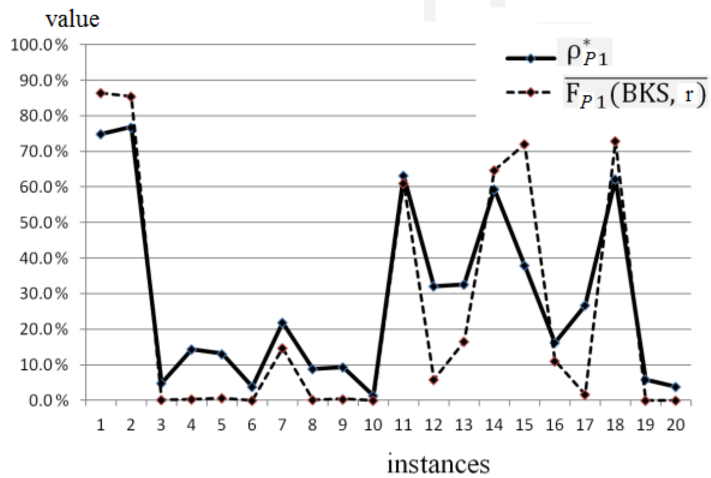


Figure 9: Correlation between ρ^*_p and $\overline{h_p(BKS, r)}$ for the instances of [4].

Table 2: ELS results on [4] instances.

inst.	$c(s)$	gap	ρ^*_{P1}	$\overline{h_{P1}}$	$c(s)$	gap	$\rho^*_{P2/P3}$	$\overline{h_{P2}}$	$\overline{h_{P3}}$	time
	$P1$ policy				$P2$ and $P3$ policies					
pr01	200.48	5.50%	100.0%	100.0%	202.98	6.82%	100.0%	100.0%	100.0%	0.25
pr02	307.11	1.92%	100.0%	100.0%	314.59	4.40%	100.0%	100.0%	100.0%	1.25
pr03	587.96	10.52%	100.0%	100.0%	643.53	20.96%	97.9%	96.1%	99.9%	2.30
pr04	643.75	12.89%	100.0%	100.0%	665.65	16.73%	99.5%	99.2%	100.0%	7.37
pr05	744.92	18.82%	100.0%	100.0%	767.45	22.41%	98.3%	97.3%	99.6%	12.07
pr06	979.78	24.77%	100.0%	100.0%	1109.04	41.23%	94.5%	91.2%	98.8%	21.92
pr07	306.69	5.13%	100.0%	100.0%	328.96	12.77%	100.0%	100.0%	100.0%	0.47
pr08	573.65	17.59%	100.0%	100.0%	592.72	21.50%	95.9%	93.4%	99.7%	2.68
pr09	774.53	17.65%	99.7%	100.0%	801.65	21.77%	95.4%	94.8%	99.4%	11.25
pr10	1049.23	23.18%	99.4%	99.5%	1085.67	27.45%	78.7%	63.3%	96.2%	21.33
pr11	168.81	2.64%	100.0%	100.0%	171.43	4.24%	100.0%	100.0%	100.0%	0.28
pr12	310.66	5.07%	100.0%	100.0%	324.75	9.84%	100.0%	100.0%	100.0%	1.37
pr13	527.10	8.72%	100.0%	100.0%	544.54	12.31%	100.0%	100.0%	100.0%	3.70
pr14	634.16	19.80%	100.0%	100.0%	630.64	19.14%	99.9%	99.9%	100.0%	10.20
pr15	634.08	9.84%	100.0%	100.0%	702.38	21.67%	100.0%	100.0%	100.0%	19.93
pr16	891.86	22.06%	100.0%	100.0%	882.28	20.75%	99.4%	99.4%	100.0%	32.32
pr17	266.83	7.50%	100.0%	100.0%	271.55	9.40%	100.0%	100.0%	100.0%	0.58
pr18	494.54	7.81%	100.0%	100.0%	560.83	22.26%	100.0%	100.0%	100.0%	4.32
pr19	699.70	17.89%	100.0%	100.0%	738.65	24.46%	98.5%	97.4%	99.6%	12.43
pr20	978.61	24.56%	100.0%	100.0%	989.00	25.88%	97.1%	95.5%	99.8%	31.45
avg.	588.72	13.19%	100.0%	100.0%	616.41	18.30%	97.8%	96.4%	99.7%	9.87

$P2$ and $P3$. Thus it is possible to obtain solutions which are far more robust than the BKS with a limited increase on the cost (13.19% for policy $P1$ and 18.30% for policies $P2$ and $P3$).

The improvement on the robustness is also significant for the [18] set A instances, even if lower values are obtained. The ELS provides solutions with an average feasibility probability of about 80% for policy $P1$, 45% for policy $P2$ and 75% for policy $P3$. The variation in the robustness obtained over the instances is also higher. This difference in the results from Table 2 may come from the stronger riding time limit and the narrower pickup and delivery time windows in the [18] set A instances. Thus, the probability of a realization ω to make a solution unfeasible is higher. One can also note the robustness is lower with policy $P2$, which means $P1$ and $P3$ are better alternatives in this context.

5.4 Bi-objective SDARP with a Normal distribution

The two criteria (total travel distance and robustness) are addressed independently in the NSGA-II algorithm. The performance of the algorithm is assessed with respect to each criterion. Thus, the solution of the Pareto

Table 3: ELS results on [18] instances.

		$P1$ policy				$P2$ and $P3$ policies				
inst.	Opt	$c(s)$	gap	ρ^*_{P1}	$\overline{h_{P1}}$	$c(s)$	gap	$\rho^*_{P2/P3}$	$\overline{h_{P2}}$	$\overline{h_{P3}}$
a2-16	294.25	310.85	5.64%	59.1%	71.7%	305.32	3.76%	57.6%	44.4%	67.7%
a2-20	344.83	357.73	3.74%	76.5%	75.9%	361.75	4.91%	66.8%	53.5%	80.8%
a2-24	431.12	467.56	8.45%	72.0%	67.2%	463.88	7.60%	61.5%	23.9%	49.8%
a3-24	344.83	383.45	11.20%	93.0%	99.7%	391.39	13.50%	88.5%	87.8%	99.1%
a3-30	494.85	537.39	8.60%	50.9%	63.1%	525.89	6.27%	40.1%	4.1%	18.7%
a3-36	583.19	618.84	6.11%	69.7%	92.9%	630.97	8.19%	57.9%	67.6%	95.7%
a4-32	485.50	519.73	7.05%	79.1%	93.3%	518.30	6.76%	73.1%	63.8%	87.7%
a4-40	557.69	637.01	14.22%	92.8%	99.7%	651.52	16.82%	69.5%	55.8%	91.2%
a4-48	668.82	728.09	8.86%	67.4%	97.9%	752.59	12.52%	53.9%	33.8%	83.8%
a5-40	498.41	601.63	20.71%	96.6%	99.9%	603.24	21.03%	86.1%	81.4%	99.6%
a5-50	686.62	769.83	12.12%	84.2%	90.1%	741.23	7.95%	65.9%	54.6%	96.6%
a5-60	808.42	906.39	12.12%	41.6%	9.7%	890.61	10.17%	32.4%	0.1%	2.4%
a6-48	604.12	718.95	19.01%	97.1%	100.0%	698.84	15.68%	87.1%	83.5%	99.0%
a6-60	819.25	959.36	17.10%	56.8%	77.5%	917.37	11.98%	41.8%	13.2%	37.6%
a6-72	916.05	1024.69	11.86%	63.9%	55.1%	1051.81	14.82%	44.1%	20.0%	62.3%
a7-56	724.04	892.27	23.23%	83.5%	67.9%	894.21	23.50%	62.0%	64.6%	89.6%
a7-70	889.12	1091.01	22.71%	70.8%	82.6%	1030.98	15.95%	51.9%	38.6%	85.9%
a7-84	1033.37	1190.63	15.22%	67.1%	91.2%	1181.75	14.36%	47.5%	28.6%	83.8%
a8-64	747.46	896.71	19.97%	90.8%	78.5%	867.39	16.04%	78.5%	78.1%	98.0%
a8-80	945.73	1100.76	16.39%	71.9%	94.4%	1096.67	15.96%	57.0%	37.9%	93.7%
a8-96	1229.70	1396.82	13.59%	54.1%	74.1%	1398.94	13.76%	33.9%	14.5%	62.3%
avg.	671.77	767.13	13.23%	73.3%	80.1%	760.70	12.45%	59.86	45.2%	75.5%

front with the lowest total travel distance and the solution of the Pareto front with the highest robustness are considered. They correspond respectively to the leftmost solution and the rightmost solution when the Pareto front is displayed on the two criteria space. Table 4 presents the results for the [4] instances. For each extreme solution (leftmost / rightmost) of the Pareto front, the total travel distance $c(s)$, its relative gap to the total travel distance of the BKS, the value of the ρ^*_{P1} robustness estimator and the evaluation $\overline{h_{P1}(s,r)}$ are reported. The time in seconds spent by NSGA-II is reported in the last column.

Table 4: NSGA-II results on [4] instances with policy $P1$.

		Leftmost solution				Rightmost solution				
inst.	$c(BKS)$	$c(s)$	gap	ρ^*_{P1}	$\overline{h_{P1}}$	$c(s)$	gap	ρ^*_{P1}	$\overline{h_{P1}}$	time
pr01	190.02	190.02	0.00%	74.8%	87.3%	197.42	3.89%	100.0%	100.0%	14.95
pr02	301.34	311.51	3.37%	97.7%	99.9%	317.95	5.51%	100.0%	100.0%	74.75
pr03	532.00	556.60	4.62%	50.7%	71.0%	622.37	16.99%	100.0%	100.0%	137.54
pr04	570.25	620.31	8.78%	69.8%	62.2%	641.66	12.52%	100.0%	100.0%	440.73
pr05	626.93	675.98	7.82%	30.7%	16.1%	691.59	10.31%	100.0%	100.0%	721.79
pr06	785.26	833.78	6.18%	7.0%	0.3%	857.31	9.17%	100.0%	100.0%	1310.82
pr07	291.71	297.36	1.94%	72.2%	75.5%	304.29	4.31%	100.0%	100.0%	28.11
pr08	487.84	505.43	3.61%	33.5%	37.3%	546.28	11.98%	100.0%	100.0%	160.26
pr09	658.31	717.53	9.00%	45.0%	61.9%	758.78	15.26%	100.0%	100.0%	672.75
pr10	851.82	918.80	7.86%	12.7%	4.4%	1005.33	18.02%	100.0%	100.0%	1275.53
pr11	164.46	168.51	2.46%	72.1%	68.9%	171.44	4.24%	100.0%	100.0%	16.74
pr12	295.66	304.18	2.88%	19.0%	0.5%	312.65	5.75%	100.0%	100.0%	81.93
pr13	484.83	520.95	7.45%	93.0%	96.8%	531.94	9.72%	100.0%	100.0%	221.26
pr14	529.33	574.28	8.49%	45.9%	34.7%	584.81	10.48%	100.0%	100.0%	609.96
pr15	577.29	589.36	2.09%	96.7%	48.4%	601.13	4.13%	100.0%	100.0%	1191.81
pr16	730.67	781.06	6.90%	30.1%	16.4%	803.26	9.94%	100.0%	100.0%	1932.74
pr17	248.21	262.41	5.72%	21.8%	1.4%	273.73	10.28%	100.0%	100.0%	34.68
pr18	458.73	482.44	5.17%	95.9%	100.0%	514.56	12.17%	100.0%	100.0%	258.34
pr19	593.49	642.90	8.32%	32.9%	51.9%	677.33	14.13%	100.0%	100.0%	743.31
pr20	785.68	860.79	9.56%	22.9%	2.5%	935.25	19.04%	100.0%	100.0%	1880.71
avg.	508.19	541.06	5.65%	51.5%	46.7%	567.45	10.39%	100.0%	100.0%	590.44

On average, the cost of the leftmost solution is better than the cost of the solution computed by the ELS (5.65% vs. 13.19%). At the same time, its robustness is worse (47.7% vs. 100%). This result comes from the fact that ELS optimizes the robustness first, while the leftmost solution is the one with the best solution considering the total travel cost in the Pareto front. On the other hand, the rightmost solution reaches the same average level of robustness (100%) than ELS solution, but its total travel cost is lower (10.39% on average against 13.19%). Thus NSGA-II seems to be a valuable methods to provide quality solutions. In addition it competes with mono-criteria approaches, even if its computational time is higher.

5.5 SDARP with a Gamma distribution

The uncertainty in travel time can be modelled using other distributions with no consequence on the proposed approach nor the design of the algorithms. Namely, any distribution can be used in our framework, provided first it is defined on $\mathbb{R}^+$ and second any value X such that $\mathbb{P}(T_{ij} \leq X) = Y$ can be computed. For instance, lately [23, 24] introduced shifted gamma distribution with shape parameter n and scale parameter λ. In order to be consistent with the normal distribution used in the main part of the work, λ is set to obtain the same standard deviation, *i.e.* $\sigma_{ij} = t_{ij}/10$. Since the standard deviation and the expected value are connected in the gamma distribution ($\mathbb{E}(\Gamma(n,\lambda)) = \sigma_{ij}\sqrt{n}$), a shift has to be done to obtain t_{ij} as expected value. Thus the shift is set to $t_{ij} - \sigma_{ij}\sqrt{n}$.

The shifted gamma distribution is a parametric law and the shape parameter n defines the number of exponential distributions used in the composition. Thus, if $n \in \mathbb{N}$, the gamma distribution is an Erlang distribution. For $n = 1$, the gamma distribution is no more than an exponential distribution with rate parameter 1λ. For large enough n (usually $n \geq 10$) the gamma distribution converges towards a Gaussian distribution with expected value $n\lambda$ and standard deviation $\lambda\sqrt{n}$.

Table 5 reports the ELS results on the [4] instances, with $n \in \{4, 16\}$. For each instance and each parameter n, the total travel distance $c(s)$, the relative gap to the BKS, the estimation ρ^*_{P1} and the estimation by simulation $\overline{h_{P1}(s,r)}$ are reported. The $P1$ policy is used.

Table 5: ELS results on [4] instances for the policy $P1$ with 2 gamma distributions.

	$(G,4,n)$				$(G,16,n)$			
inst.	$c(s)$	gap	ρ^*_{P1}	$\overline{h_{P1}}$	$c(s)$	gap	ρ^*_{P1}	$\overline{h_{P1}}$
pr01	210.94	11.01%	97.9%	99.9%	210.48	10.77%	99.0%	100.0%
pr02	317.00	5.20%	96.0%	100.0%	324.64	7.73%	98.4%	100.0%
pr03	628.59	18.16%	76.3%	99.7%	645.29	21.29%	81.9%	99.5%
pr04	663.68	16.38%	82.2%	99.3%	679.67	19.19%	86.3%	98.3%
pr05	752.20	19.98%	75.6%	99.8%	745.58	18.93%	79.1%	99.8%
pr06	993.02	26.46%	63.9%	99.1%	918.08	16.91%	66.9%	99.6%
pr07	320.29	9.80%	95.2%	100.0%	322.17	10.44%	97.4%	100.0%
pr08	607.81	24.59%	69.6%	99.3%	602.77	23.56%	71.8%	99.6%
pr09	790.74	20.12%	50.9%	93.8%	798.34	21.27%	52.8%	100.0%
pr10	1052.57	23.57%	33.0%	76.6%	1017.61	19.46%	33.2%	92.6%
pr11	178.97	8.82%	100.0%	100.0%	175.91	6.96%	100.0%	100.0%
pr12	334.41	13.10%	99.9%	100.0%	341.08	15.36%	100.0%	100.0%
pr13	605.94	24.98%	97.9%	100.0%	561.61	15.84%	99.1%	100.0%
pr14	638.52	20.63%	94.2%	100.0%	644.46	21.75%	97.2%	100.0%
pr15	701.94	21.59%	94.2%	99.8%	682.42	18.21%	98.4%	100.0%
pr16	934.23	27.86%	82.7%	99.9%	891.59	22.02%	86.7%	100.0%
pr17	294.83	18.78%	99.5%	100.0%	288.76	16.34%	100.0%	100.0%
pr18	530.68	15.68%	94.0%	100.0%	564.12	22.97%	95.9%	100.0%
pr19	712.33	20.02%	85.8%	99.9%	706.43	19.03%	84.8%	100.0%
pr20	972.21	23.74%	64.9%	99.8%	928.72	18.21%	58.2%	98.5%
avg.	612.05	18.52%	82.7%	98.4%	602.49	17.31%	84.4%	99.4%

With $n = 16$, the shifted gamma distribution is close to the Normal distribution and the reported results are close to the ones previously introduced in Table 2. The robustness is slightly better with the Normal distribution but the difference is marginal. For small n values, for example $n = 4$, the shifted gamma distribution has a larger standard deviation. As a consequence, for $n = 1$, the probability of a solution being feasible is around 92.3% which is less than with $n = 4$ or $n = 16$.

6 Conclusions

The objective of this work is to consider the DARP with uncertainty on the transportation time to address more accurately real-life situations. The main contribution consists in defining an indirect criterion of robustness in order to avoid time-consuming robustness evaluations during optimization. The theoretical contribution allows us to define a generic approach for computing robust solutions of DARP with a wide scope of distributions. The proposed robustness criterion is optimized into both an ELS based framework and a bi-objective method based on NSGA-II algorithm.

The numerical experiments are done on the classical DARP instances, first considering Normal distributions and second Gamma distributions. They lead to the following results: (i) the robustness criterion we defined is correlated to the robustness; (ii) the best known solution of the deterministic DARP have poor probability to be feasible; (iii) much more robust solutions can be obtained with minimal consequences on the cost.

The framework is generic enough to be able to handle a large number of distribution laws, including but not limited to the normal and the shifted gamma distributions. Besides, with no modification, it could handle correlated random variables to address predictable congestion due to commuting. It could also handle uncertainty in service time as well. Our work is now directed into extensions of DARP with heterogeneous fleets.

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