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Stochastic strategic planning of open-pit mines, with ore selectivity recourse

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Abstract: A computational framework has been developed that extends stochastic strategic mine planning algorithms, improving the representation of ore selection decisions. Existing algorithms result in block extraction sequences that may violate the mineral processing capacity; under such geological scenarios, the excess ore is subject to a recourse action which is not as profitable as regular processing, hence decreasing the net present value. This decrease has previously been represented as a linear penalty on excess ore, which may be loosely related to stockpiling or outsourcing costs, but is difficult to quantify, and does not accurately represent actual operations. The new framework considers the most typical recourse action, which is to increase the cutoff grade into the mineral process. This approach is demonstrated by hybridizing a Variable Neighborhood Descent (VND) method with a Continuous Knapsack formulation. Sample computations are performed using data from a copper ore deposit, consisting of 9953 blocks, under 20 geological scenarios.

Key Words: Strategic mine planning, open-pit, stochastic optimization, variable neighborhood descent, continuous knapsack.

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1 Introduction

For well over a decade, stochastic optimization techniques have incorporated geological uncertainty into open-pit strategic mine planning (Dimitrakopoulos 2011), through the simultaneous consideration of several orebody scenarios. Generally, the resulting extraction plans are neither optimal for any single scenario, nor for the "average" scenario, but are expected to perform well for the entire distribution of possible orebody realizations. In comparison to the deterministic techniques, the stochastic approaches have been shown to enhance the net present value (NPV) of production schedules by as much as 25% (Dimitrakopoulos 2011; Godoy and Dimitrakopoulos 2004), which may correspond to hundreds of millions of dollars.

These methods typically consider the NPV of excavating individual blocks of ore at each time period. Such economic values are somewhat artificial, since the individual blocks are not inherently marketable; they require some amount of processing before any tradable product may be extracted from them. The valuation of a single block must be taken in the context of a hypothetical production plan, and is contingent on the downstream mineral processing.

For instance, there may be two competing extraction plans, x and x' (Figure 1). It may be that x' would be expected to yield a higher total NPV than x, except that x' is likely to overwhelm the mill capacity. Thus, x' must be penalized, albeit in a fair and realistic way. Existing approaches incorporate a linear penalty term for the excess ore tonnage in each period (Lamghari et al. 2013; Lamghari et al. 2014; Montiel and Dimitrakopoulos 2013; etc.), which differs from scenario to scenario. This penalty is related to a so-called recourse (corrective) action that will be described below. The current paper presents an alternative representation of the recourse action that allows the mill feed grade to change, and does not rely on a linear penalty term.

Extraction plans may be evaluated with an objective function f that considers NPV, as well as mineral processing capacities and other constraints. In a stochastic framework, x' is superior to x if and only if $f(x')$ is expected to be greater than $f(x)$,

$$[x' \succ x] \Leftrightarrow \Big[\mathbb{E}\left[f\left(x'\right)\right] > E\left[f\left(x\right)\right]\Big] \tag{1}$$

An optimization algorithm constructs candidate solutions x' that are compared to the incumbent solution x. When the algorithm encounters a candidate x' that supersedes x, the former becomes the new incumbent solution, $x \leftarrow x'$. The case when $\mathbb{E}[f(x')] = \mathbb{E}[f(x)]$ is handled differently, depending on the particular implementation.

In a standard approach, the objective may be to maximize

$$\mathbb{E}\left[f\left(x\right)\right] = -\sum_{t=1}^{n_{\mathrm{T}}} c_t(x) + \frac{1}{n_{\mathrm{S}}} \sum_{t=1}^{n_{\mathrm{T}}} \sum_{s=1}^{n_{\mathrm{S}}} \left(v_{ts}(x) - p_t e_{ts}(x)\right) \tag{2}$$

in which $c_t(x) > 0$ is the discounted mining cost in period t; $v_{ts}(x)$ is the discounted net value from processing ore in time t and under scenario s (assuming infinite processing capacity, in this case); $e_{ts}(x) \geq 0$ is the amount of excess ore that is handled in time t and under scenario s, and is subject to the recourse penalty factor $p_t > 0$. The number of time periods and scenarios are given by n_{T} and n_{S}, respectively.

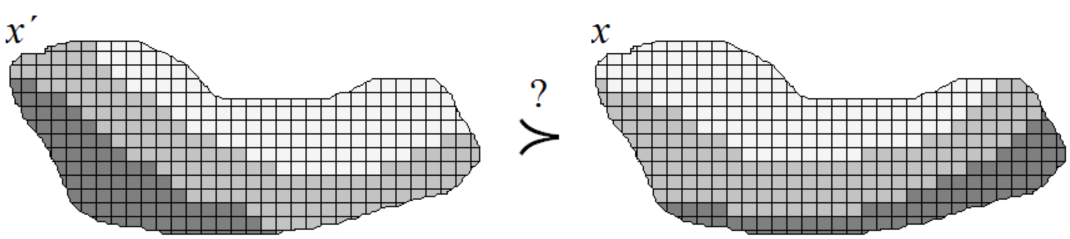

Figure 1: Schematic of two extraction plans, x' and x, that are compared to determine whether or not x' is superior to x, i.e. whether or not $x' \succ x$.

Equation 2 is conducive to Mixed Integer Linear Programming (MILP) formulations that use binary variables x_{bt}, such that $x_{bt} = 1$ if block b is extracted in period t. Indeed, $c_t(x)$ can be expressed directly as a linear combination of x_{bt}; likewise for $v_{ts}(x)$, assuming that there is no long-term (strategic) stockpiling (Lamghari et al. 2013). To represent the excess ore $e_{ts}(x)$, continuous variables are introduced into the MILP, which soften the process capacity constraints (Godoy and Dimitrakopoulos 2004; Lamghari et al. 2013).

The resulting MILP formulations permit the downstream capacity to be violated ($e_{ts} > 0$), but the recourse penalty p_t causes the algorithm to favor plans for which the e_{ts} values are relatively small. As the p_t becomes increasingly large, the algorithm becomes unstable, as it is increasingly sensitive to each geological scenario. In the extreme case, as $p_t \to \infty$, a good plan x' can be rejected because it violates the mill capacity in a single period of a single scenario; the violation may be so small that it would be easily handled in reality. Indeed, large sums of money may be misallocated if an unlikely scenario is given undue importance. It is therefore important that p_t not be too large. Yet if the penalties p_t are too small, the downstream resources can be overwhelmed, also leading to significant costs.

It is a rather unattractive feature of Equation 2 that it relies heavily on penalty values p_t that are difficult to quantify. On a case-by-case basis, the penalty might be related to stockpiling contingencies; however, long-term stockpiles are not necessarily the most economical recourse (Asad 2005), as they represent an illiquid inventory. There may also be unusual cases where a given mine suffers from unexpectedly large amounts of ore, while there is a distant processing facility that is short of feed; the recourse action is then to ship the excess ore to the distant facility, hence a transportation cost. There is thus a connection between p_t and outsourcing or stockpiling costs, but a linear relationship is not always forthcoming.

Moreover, the linear penalty p_t may actually be erroneous in many cases, as it does not represent the most basic recourse for excess ore. When a mill is overwhelmed with feed, the natural policy is often to increase the selectivity (Asad 2013). This usually implies the rejection of lower grade ores, resulting in a higher mill head grade. This concept has been implemented in deterministic mine planning software such as the Whittle package (Whittle 2004; Whittle 2009), but remains underutilized in the stochastic context (Dimitrakopoulos and Godoy 2014).

This paper presents an algorithm that strictly adheres to the downstream capacities ($e_{ts} = 0$),

$$\mathbb{E}\left[f\left(x\right)\right] = -\sum_{t=1}^{n_\mathrm{T}} c_t(x) + \frac{1}{n_\mathrm{S}} \sum_{t=1}^{n_\mathrm{T}} \sum_{s=1}^{n_\mathrm{S}} v_{ts}(x) \tag{3}$$

The resulting mine plan x^* is not permitted to violate the downstream capacity for any of the scenarios. Instead, the downstream value function $v_{ts}(x)$ is constrained to automatically adjust the feed grade for the individual scenarios. As the mineral processing capacity is reached in a given scenario, the process becomes more selective and potentially valuable ore is turned away. There is hence an implied recourse penalty, which may be more appropriate than earlier approaches (Equation 2).

The new approach can be extended to explicitly include stockpiling and outsourcing options, depending on the particular situation. More importantly, the stability of the new framework does not depend on a linear penalty term that is technically artificial (Kolman and Beck 1995).

2 Adaptation of the VND method of Lamghari et al.

MILP formulations have allowed major advances in stochastic strategic open-pit mine planning, and continue to be an important benchmark (Dimitrakopoulos 2011; Lamghari et al. 2013, 2014). In recent years, however, metaheuristics have been developed, which have allowed faster resolution times. These newer methods do not guarantee a truly optimal solution, but may be computationally more efficient than the MILP approaches.

The current implementation is an adaptation of the Variable Neighborhood Descent (VND) technique that was developed by Lamghari et al. (2013, 2014). This method respects the block precedence constraints and mining capacity, as will be described below. However, the earlier VND method applied a linear penalty term to implement mineral processing capacity (Equation 2).

Numerous metaheuristic approaches have been attempted, based on Simulated Annealing (Dimitrakopoulos 2011; Montiel and Dimitrakopoulos 2013), Tabu Search (Lamghari and Dimitrakopoulos 2012), Ant Colony Optimization (Soleymani and Sattaryand 2015), etc. Nonetheless, the VND technique is preferred over other approaches for two main reasons. Firstly, it has been used on industrial data, with a standard desktop computer, to obtain nearly optimal results within a few hours, and sometimes in a matter of minutes (Lamghari et al. 2013, 2014); this is remarkably faster than any corresponding MILP implementation. Secondly, Lamghari's algorithm does not present any extraneous computation parameters. Other methods require some amount of tuning before they can be successfully applied to a given problem instance. For example, the Simulated Annealing requires "temperature" parameters; it is not yet clear which temperatures are best suited for the various types of orebodies.

Following the work of Lamghari et al. (2013, 2014), the current paper does not consider long-term stockpiling, i.e. material that is mined in period t can only be processed in the same period. Also following Lamghari et al., there is only a single ore type, containing a single valuable metal. Unlike the work of Lamghari et al., the new approach decomposes the range of ore grades into two or more bands, $n_{\mathrm{G}} \geq 2$. A similar concept has already been implemented for deterministic algorithms (Whittle 2004; Whittle 2009). By balancing the ore grades, the algorithm controls the selectivity of the mineral process.

The following notation is used to parameterize the problem:

$\mathcal{B}$	: set of blocks which may be mined.
$\mathcal{B}_b^{\mathrm{DPred}}$	: the set of direct predecessors of block b.
c_{bt}	: discounted cost of mining block b in period t.
m_b	: mass of block b.
$\overline{m}_t^{\mathrm{MinPro}}$	: maximum mass of ore that can be fed into the mineral process during period t.
$\overline{m}_t^{\mathrm{Mining}}$	: maximum mass of rock that can mined during period t.
n_{G}	: number of ore grade bands.
n_{S}	: number of scenarios.
n_{T}	: number of time periods.
v_{bts}	: discounted value of processing the content of b, during period t, under scenario s.
w_{bs}	: grade of block b, under scenario s, expressed as a weight fraction.
$(\underline{w}_g,\ \overline{w}_g]$	: definition of grade band g.

Thus $\overline{m}_t^{\mathrm{Mining}}$ and $\overline{m}_t^{\mathrm{MinPro}}$ are the mining and mineral processing capacities, respectively, for period t. The solution is described algebraically as $x = \{\mathcal{B}_t\}_{t=1}^{n_{\mathrm{T}}}$ in which $\mathcal{B}_t$ is the set of blocks that is extracted during period t.

For all $b \in B$, the set of direct predecessors $\mathcal{B}_b^{\mathrm{DPred}} \subset \mathcal{B}$ consists of the blocks that must be mined during or prior to the period in which block b is mined, and which are on the level immediately above b. These sets define the block precedence constraints, and are related to the mechanical stability of open-pit. For each block b, the complete set of predecessors is denoted $\mathcal{B}_b^{\mathrm{Pred}} \subset \mathcal{B}$, which includes the direct predecessors of b, as well as their direct predecessors, and so on, recursively. The direct predecessor sets are related to the direct successor sets,

$$\mathcal{B}_b^{\mathrm{DSucc}} = \left\{ b' \in \mathcal{B} \mid b \in \mathcal{B}_{b'}^{\mathrm{DPred}} \right\}$$

and the complete set of successors $\mathcal{B}_b^{\mathrm{Succ}}$ recursively includes all successors of b.

The ore grade bands are contiguous, meaning that $\overline{w}_{g-1} = \underline{w}_g$, for $g = 2$ to n_{G} (Figure 2). Thus $g = n_{\mathrm{G}}$ corresponds to the richest class of ore. For a given solution $x = \{\mathcal{B}_t\}_{t=1}^{n_{\mathrm{T}}}$, the set of blocks belonging to grade

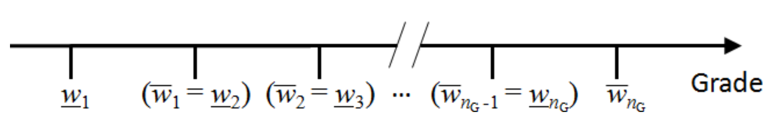

Figure 2: The range of ore grades is decomposed into $n_{\mathrm{G}} \geq 2$ bands, defined by intervals such that the upper limit of one interval corresponds to the lower limit of the next interval.

band g that are extracted in period t, under scenario s, is given by

$$\mathcal{B}_{gts} = \left\{ b \in \mathcal{B}_t \mid \underline{w}_g < w_{bs} \leq \overline{w}_g \right\}$$

A block b may be regarded as waste rock for a given scenario s if $w_{bs} \leq \underline{w}_1$. As a default, the upper limit of the richest class can be set as $\overline{w}_{n_{\mathrm{G}}} = 1$, automatically including the highest grade blocks. (One reason to choose a lower value for $\overline{w}_{n_{\mathrm{G}}}$ could be if the highest grade blocks undergo a different downstream process. However, the current study considers only a single mineral process applicable to all of the ore).

The objective function may be stated as

$$\mathbb{E}[f(x)] = -\sum_{t=1}^{n_{\mathrm{T}}} \sum_{b \in \mathcal{B}_t} c_{bt} + \frac{1}{n_{\mathrm{S}}} \sum_{t=1}^{n_{\mathrm{T}}} \sum_{s=1}^{n_{\mathrm{S}}} \sum_{g=1}^{n_{\mathrm{G}}} \sum_{b \in \mathcal{B}_{gts}} v_{bts} \phi_{gts}(x) \tag{4}$$

in which $\phi_{gts}(x) \in [0, 1]$ is the ore selectivity function; it is the fraction of ore from grade band g that is mined and processed in time t, under scenario s. If $\phi_{gts} < 1$ then some of the ore is turn away from the mineral process, which is effectively a recourse penalty.

$\phi_{gts}(x)$ is computed in a greedy fashion, to process as much of grade interval n_{G} as can be supported by the downstream capacity $\overline{m}_t^{\mathrm{MinPro}}$. If there is enough capacity to process all of class n_{G} then $\phi_{n_{\mathrm{G}}ts}$ is set to 1, and the remaining capacity is used to process as much of class $(n_{\mathrm{G}} - 1)$ as possible. If there is then enough capacity to process all of class n_{G} and all of $(n_{\mathrm{G}} - 1)$, then $\phi_{(n_{\mathrm{G}}-1)ts}$ is set to 1, and the additional capacity is used for class $(n_{\mathrm{G}} - 2)$, and so on. There may eventually be a class g^* for which there is sufficient capacity to process only a fraction of the ore ($\phi_{g^*ts}(x) < 1$); the lower grades are hence turned away, $\phi_{gts} = 0$ for all $g < g^*$. Otherwise, if there is enough capacity for all of the ore, then ϕ_{gts} is set to 1 for all grade intervals.

The computer implementation to obtain $\phi_{gts}(x)$ corresponds to a Continuous Knapsack (CK) problem (Cormen et al. 2001), in which $\overline{m}_t^{\mathrm{MinPro}}$ represents the capacity of the knapsack (Figure 3). Each ore interval represents an item that may be completely or partially passed into the knapsack, listed in increasing order of value, from 1 to n_{G}. The problem is to fill the knapsack with the items, ultimately to maximize the value of the content. To the authors' knowledge, this is the first time that such an adaptation has been used instead of a linear recourse penalty (Equation 2).

Formulaically, the ore selectivity function is given by

$$\phi_{gts}(x) = \begin{cases} 1 & \text{if } \overline{m}^{\mathrm{MinPro}} \geq \sum_{g'=1}^{g} \sum_{b \in \mathcal{B}_{g'ts}} m_b \\ \left(\frac{\overline{m}^{\mathrm{MinPro}} - \sum_{g'=1}^{g-1} \sum_{b \in \mathcal{B}_{g'ts}} m_b}{\sum_{b \in \mathcal{B}_{gts}} m_b} \right) & \text{if } \sum_{g'=1}^{g-1} \sum_{b \in \mathcal{B}_{g'ts}} m_b \leq \overline{m}^{\mathrm{MinPro}} \leq \sum_{g'=1}^{g} \sum_{b \in \mathcal{B}_{gts}} m_b \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

thereby enforcing a strict adherence to the downstream capacity.

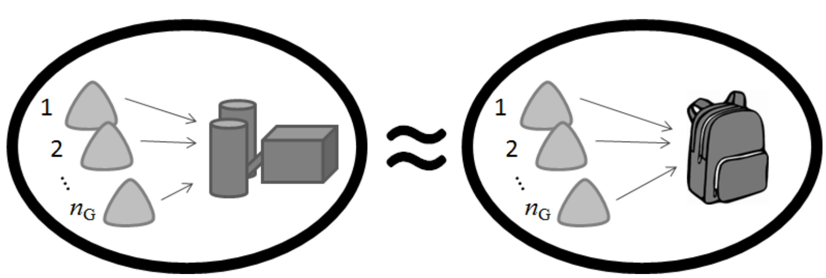

Figure 3: The ore selectivity computation is equivalent to a Continuous Knapsack problem.

The remaining constraints are identical to those described by Lamghari et al. (2013, 2014). Firstly, each block can only be mined within a single period,

$$\mathcal{B}_t \bigcap \mathcal{B}_{t'} = \emptyset \tag{6}$$

for all periods $t, t' \in \{1, \ldots, n_{\mathrm{T}}\}$, such that $t \neq t'$. The precedence constraints are expressed as

$$\bigcup_{b \in \mathcal{B}_t} \mathcal{B}_b^{\mathrm{DPred}} \subseteq \bigcup_{t'=1}^{t} \mathcal{B}_{b'} \tag{7}$$

for all periods $t \in \{1, \ldots, n_{\mathrm{T}}\}$. The mining capacity constraints are expressed as

$$\sum_{b \in \mathcal{B}_t} m_b \leq \overline{m}_t^{\mathrm{Mining}} \tag{8}$$

for all periods $t \in \{1, \ldots, n_{\mathrm{T}}\}$. Equations 6–8 can be expressed using binary variables x_{bt} (Lamghari et al. 2013, 2014), but this does not reflect the actual implementation. With appropriately designed data structures, Equation 6 is automatically enforced, while Equations 7 and 8 are efficiently maintained.

The VND algorithm searches for improvements in the extraction sequence, as it alternates between three different approaches (Lamghari et al. 2013, 2014), hence three different neighborhood structures. These approaches are called *Exchange*, *Shift-After* and *Shift-Before*. The *Exchange* method simply swaps pairs of blocks that are scheduled in consecutive periods, t and $(t+1)$, but only considering cases that satisfy Equations 7 and 8. The *Shift-After* method transfers a block $b \in \mathcal{B}_t$ and all of its contemporaneous successors $\mathcal{B}_b^{\mathrm{Succ}} \bigcap \mathcal{B}_t$ into the following period $(t+1)$. Similarly, the *Shift-Before* method transfers a block $b \in \mathcal{B}_t$ and all of its contemporaneous predecessors $\mathcal{B}_b^{\mathrm{Pred}} \bigcap \mathcal{B}_t$ into the previous period $(t-1)$. Details about how the algorithm alternates between the three methods, and iterates through the various time periods is discussed by Lamghari et al. (2013, 2014).

The main difference between the new implementation and that of Lamghari et al. is the representation of the mineral processing operations (Equations 4 and 5). By considering at least two grade bands, the CK is able to modify the grade entering the mineral process, instead of relying on a linear recourse penalty. In principle, more intervals can provide better selectivity control, but this implies a higher computational cost.

3 Sample computations

The VND approach described in the previous section has been applied to a copper sulfide deposit consisting of 9953 blocks that are excavated over 9 one-year periods. Conditional simulation was used to generate 20 geological scenarios (Dimitrakopoulos 2011), four of which are depicted in Figure 4.

Each of the blocks has identical dimensions, 20×20×10 meters, and an identical weight of 10,000 T. The set of direct predecessors for a block b is given by the nine nearest blocks in the level immediately above b (Figure 5), which corresponds to a 45° maximum slope angle. Operational parameters are given in Table 1, which assist in obtaining the remaining model data. In particular, the block recoverable values v_{bts} have been calculated according to,

$$v_{bts} = \frac{(0.8)(4400)w_{bs}m_b - 6m_b}{(1+0.08)^{t-1}} \tag{9}$$

taking into account the copper recovery (0.8), the copper price (\$4400/T of Cu), the milling cost (\$6/T of Ore), and the discount rate (0.08). Similarly, the block mining costs c_{bt} are given by,

$$c_{bt} = \frac{1.5m_b}{(1+0.08)^{t-1}} \tag{10}$$

considering the mining cost (\$1.5/T of Rock) and the discount rate (0.08).

The current computations considered 3 grade brands. Firstly, $\underline{w}_1$ was taken as the minimum cutoff grade below which v_{bts} would be negative, $\underline{w}_1 = 0.17\%$. Other values were obtained by considering the 33.3 and

Table 1: Operational data

Copper price	\$4400 /T of Cu
Mining cost	\$1.5 /T of Rock
Milling cost	\$6.0 /T of Ore
Discount rate	8%
Mining capacity	15 MT/year
Milling capacity	8 MT/year
Copper recovery from milling	80%

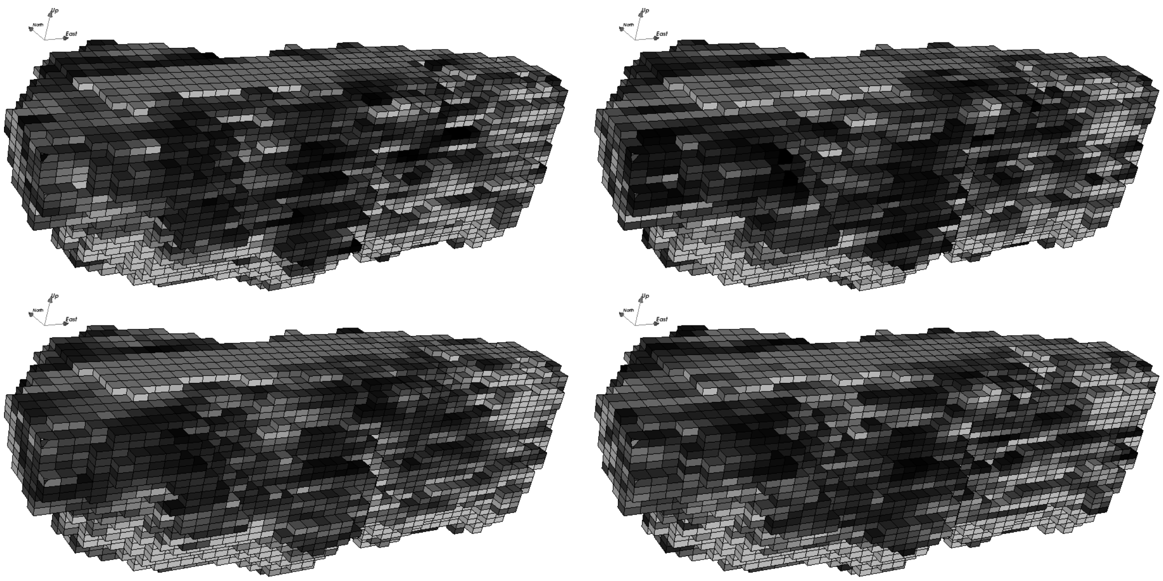

Figure 4: Four geological scenarios of the orebody.

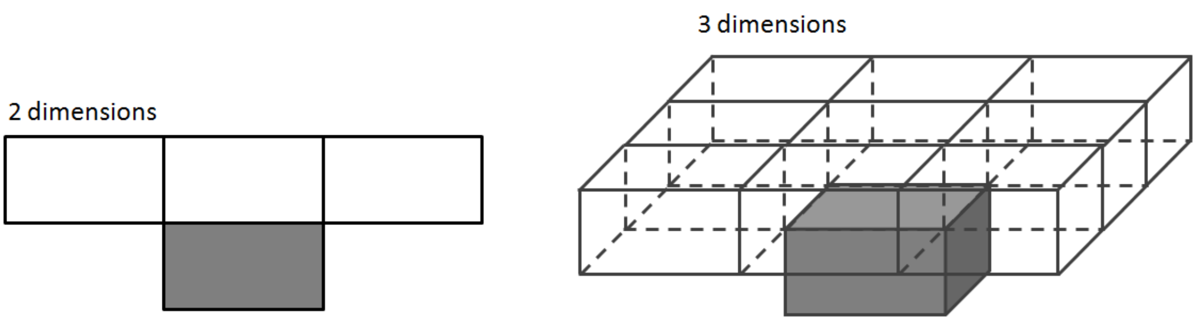

Figure 5: An interior block b may only be excavated once the nine overlying blocks have been excavated.

66.7 percentiles of the ore grade, such that $\overline{w}_1 = \underline{w}_2 = 0.31\%$, and $\overline{w}_2 = \underline{w}_3 = 0.54\%$, respectively. Lastly, the upper bound was set to its default value $\overline{w}_3 = 100\%$.

The initial solution was obtained by applying the Milawa algorithm that is implemented within the Whittle software (Whittle 2004; Whittle 2009). In itself, this algorithm does not capture the stochastic variability of orebodies, as it considers only one geological scenario at a time. Thus the software was applied independently to each of the 20 scenarios, producing one extraction plan per scenario. Any of these mine plans could be used as the starting point for the VND algorithm. It is nonetheless reasonable to select the mine plan that is expected to give the highest NPV, as per Equation 4. A similar approach was taken to provide an initial solution for a Simulated Annealing technique (Montiel and Dimitrakopoulos 2013).

In the current study, the best solution provided by Whittle was found to have an expected NPV of \$553.974 million. After an additional 321 seconds of computation on a standard Hewlett-Packard Pavilion laptop (1.4 GHz microprocessor with 6 GB of RAM), the VND algorithm obtained a final value of \$583.262 million. This corresponds to a difference of \$29.288 million, hence a 5.3% increase. Figure 6 depicts the cumulative NPV for the final schedule.

Figure 7 depicts the mining and processing tonnages. The mined tonnage reaches full capacity (15 MT/year) in the second period, before steadily declining. The mill capacity (8 MT/year) is fully utilized for the first three periods. Starting in the fourth period, there are some scenarios which do not provide enough ore to fully occupy the mill, so there is no basis to turn away any of this incoming ore. There is thus a visible separation between the tenth percentile (P10) and the ninetieth percentile (P90) in the right side of Figure 7.

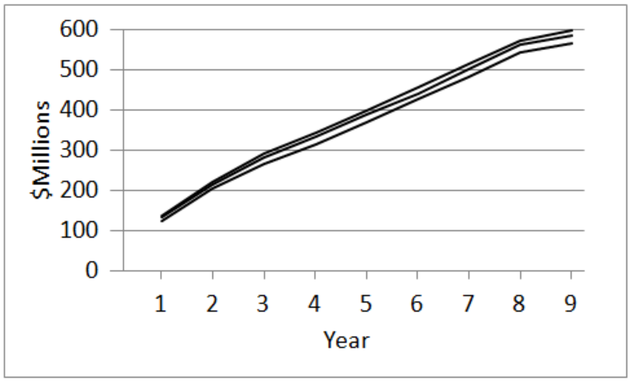

Figure 6: Cumulative NPV (10^{th}, 50^{th} and 90^{th} percentile).

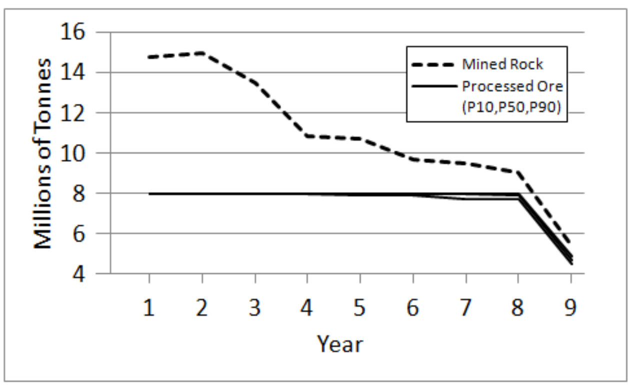

Figure 7: Mining and processing tonnage.

This observation is corroborated by Figure 8, which plots the ore selectivity function for the lowest grade ore class, ϕ_{1ts}. From period 4 onward, there are some scenarios for which $\phi_{1ts} = 1$, meaning that all of the low grade ore is accepted, as there is ample mill capacity. In period 9, the orebody approaches depletion, hence none of the scenarios fully utilize the mill (Figure 7), and ϕ_{1ts} converges to unity (Figure 8). For the higher grade classes, the ore selectivity functions were found to be unity for all periods and scenarios ($\phi_{2ts} = \phi_{3ts} = 1$), meaning that none of the higher grade ore ($> 0.31\%$) was turned away from the mill.

An ore selectivity function having a value below one ($\phi_{its} < 1$) can be interpreted as a penalty for excess ore production or, equivalently, for insufficient processing capacity. In periods 1 to 3, Figure 8 shows that most of the low grade ore is turned away, as $\phi_{1ts} \leq 0.5$. The mill capacity is strictly respected in all scenarios (Figure 8), but the feed grade is allowed to vary as depicted in Figure 9. The process is especially selective in the first period, leading to a high feed grade. (High feed grades are also observed in periods 7 and 8, as the richer regions of the orebody are exploited). Figure 10 describes the monetary value that is lost due to lack of processing capacity. Particularly in the first three periods, it may be worthwhile to develop alternative processing routes, hence to mitigate these losses.

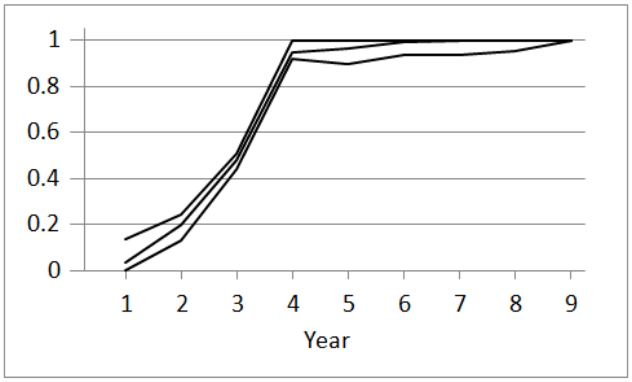

Figure 8: Ore selectivity function for the lowest grade band, ϕ_{1ts} (10^{th}, 50^{th} and 90^{th} percentile).

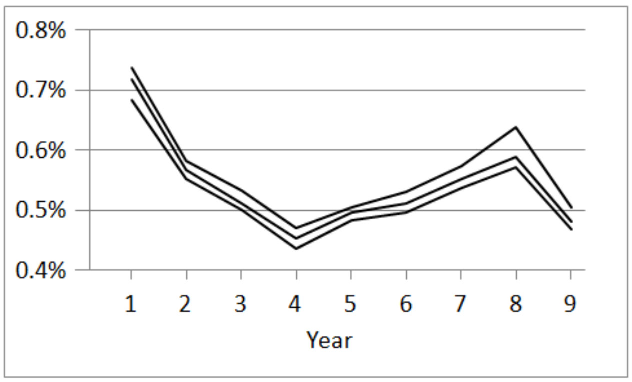

Figure 9: Mill feed grade (10^{th}, 50^{th} and 90^{th} percentile).

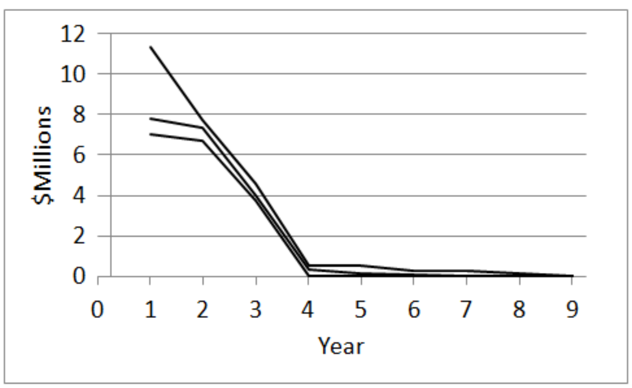

Figure 10: Lost present value due to lack of processing capacity (10^{th}, 50^{th} and 90^{th} percentile).

4 Conclusions and future work

This paper demonstrates an alternative approach to assessing production schedules that may produce excess ore. The new approach does not rely on a linear penalty term (Equation 2). Major investment decisions could thus be justified by describing the amount and the richness of the ore that would be turned away for lack of downstream capacity.

The current formulation (Equations 4–8) successfully represents a most basic recourse to excess ore, which is to increase the selectivity of the mineral processes. In addition to the ore selectivity recourse, the formulation could be extended to represent secondary processing options (e.g. stockpiling and outsourcing) that apply to the ore that is turned away from the main process. One promising avenue is to replace the CK with a network flow problem, which could describe several ore types that must share mineral processing resources (Navarra 2015; Chanda 2004).

Concepts that are described by the current framework can be extended and incorporated into mine feasibility studies, and development plans. Such methodologies may determine the installation of mineral processing resources, and whether or not a processing option is worth the capital investment.

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