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of retirement age and
Social Security benefits**

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Abstract

Aging population has put a stress on the public pension systems in most developed countries. To refrain Social Security deficits to increase the stock of public debt without bound we focus on two instruments: a reduction in the generosity of pension benefits, determined by the government, and a postponement of the effective retirement age, which we allow individuals to decide. An atomistic decision-maker would disregard the effect of his retirement decision on the public debt, and would retire as soon as possible. Conversely, an ideal farsighted agency who would decide for all current and future employees, would postpone retirement alleviating the pressure on the public debt and allowing for a greater pension in the long run. The research question is how the government may design the proper incentive strategy to induce myopic agents to act non-myopically. This strategy is defined as a two-part incentive non-linearly dependent on the stock of public debt. It is credible as long as the stock of public debt is large enough.

Résumé

Le vieillissement de la population a mis l'accent sur les systèmes publics de retraite dans la plupart des pays développés. Pour éviter que les déficits de la Sécurité Sociale augmentent le stock de la dette publique sans limite, on va se concentrer sur deux instruments: une réduction de la générosité des prestations de retraite, déterminée par le gouvernement, et un report de l'âge de retraite effective, qui sera une décision des individus. Un décideur représentant un grand groupe ne tiendrait pas compte de l'effet de sa décision sur la dette publique, et prendrait sa retraite dès que possible.

Inversement, une agence non-myope idéale qui décide pour tous ses actuels et futurs employés, reporterait la retraite en réduisant la pression sur la dette publique et en permettant une plus grande prestation de retraite dans le long terme.

La question qui se pose est de savoir quelle est la meilleure stratégie du gouvernement pour encourager les agents myopes à agir non-myope. Cette stratégie consiste en un mécanisme incitatif qui dépend de manière non-linéaire du stock de dette public. Elle est crédible pourvu que le stock de dette publique soit suffisamment grand.

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Social Security systems in many developed countries are currently in a process of change. Ageing population in these industrialized countries has put their Social Security systems under stress. The percentage of employees reaching the retirement age has been growing, meanwhile the life expectancy at retirement age has got higher. Furthermore, population growth has slowed down. Therefore, the dependency ratio defined as the number of retirees over the total labor force, has followed an upward sloping tendency.

There exists a wide variety of proposed reforms. The literature on Social Security reforms has greatly debated structural reforms as moving from a pay-as-you-go system (PAYG) to an entirely or partially funded system, switching from defined benefit to defined contribution plans, or even changing to Social Security systems privately provided (see, for example, Banks and Emmerson (2000)). Another important focus of attention has been the offering of a complementary private pension. We do not however enter this debate regarding the design of the pension system. Conversely, keeping unchanged the framework of a PAYG purely public pension system with defined benefits, we studied what Banks and Emmerson (2000) called a “parametric” reform. To reduce the pressure associated with ageing population on the Social Security systems we center our attention in two policies: a delay in the retirement age and a reduction in the generosity of pension benefits.

Most industrialized countries are already embarked in an attempt to lower public pension benefits¹ and encourage labor participation at advanced ages. Flexibility in actual retirement age, both early and late retirement, is an already common practice in many OECD countries. Further, as pointed out in Casey *et al.* (2003) in some countries early retirement is becoming more costly, while in others late retirement is being more strongly encouraged.² We must be precise here and distinguish between the legal and the effective retirement age. In this paper we will focus on the latter and the mechanism or incentives that might push employees to extend it beyond the legal retirement age. Hence, we will study the decision process of the employees regarding their effective retirement age.

The optimal determination of the effective retirement age by individual agents has been analyzed in the literature (see, for example, Heijdra and Romp (2009) or Sánchez Martín (2010) and references therein). In these models individual agents maximize their lifetime utility subject to their lifetime budget constraint. They choose a lifetime path of consumption and their optimal retirement age. However, the public pension, is determined by a given formula known by these agents. As for the interaction between the individual agents decisions on retirement and the government decision on pension generosity, to the best of our knowledge, has not been addressed whatsoever.

A dynamic game between the government and a representative consumer in the provision of a public good is analyzed, for example, in Xie (1997), Karp and Lee (2003) or Cellini and Lambertini (2007). In these papers the amount of public good depends on the tax chosen by the government and the actions of the entire population, while the evolution of the state (agent’s wealth) is dependent on each individual decisions. Contrarily, in the present paper, each agent’s public pension will depend on the government generosity and the retirement age chosen particularly by this individual, while this atomistic decision-maker has no influence in the state of the system, defined as the public debt generated by Social Security system. This state variable will depend on the retirement decisions globally considered.

The research question in this paper is: assuming that the government cannot determine the effective retirement age, but this is optimally chosen by employees, could the public pension be settled as an incentive strategy inducing the workforce to behave in a certain way? Or more precisely, could the government induce myopic employees to behave non-myopically? That is, could the government encourage myopic employees, for whom retirement age has a negligible effect on the public debt, to act as a non-atomistic decision-maker who is aware of how retirement age may alleviate the problem of a high stock of public debt.

The mechanism considered here is the use of incentive strategies. An incentive strategy can be regarded as one player’s announcement that he will stick to a desirable solution (usually the cooperative solution) as

¹Often presented as an increment in the length of the averaging period used to compute the regulatory base used to calculate the public pension. Since salaries grow with age/expertise, hence longer averaging period implies a lower pension.

²Although for many OECD countries the effective retirement age lies below the legal retirement age, some others have succeeded in pressing their employees to remain working beyond the legal retirement age (see, Whiteford and Whitehouse (2006)).

long as the other player does not depart from his part in this desired solution. The use of a linear incentive strategy to enforce cooperation in a dynamic game is first introduced by Ehtamo and Hämäläinen (1993, 1995), and later applied, for example, in Jørgensen and Zaccour (2001a,b). The important issue of the credibility of such incentive strategies is analyzed in Martín-Herrán and Zaccour (2005, 2009) for linear-state and linear-quadratic differential games. In the present paper, the incentive strategy does not seek cooperation but to induce myopic atomistic individuals to act non-myopically. Further, because the incentive strategy is non-linear in the stock of public debt, the punishment to employees who deviate from the non-myopic solution will be dependent on this stock.

The paper proceeds as follows: first we analyze the scenario of a myopic representative worker who disregards the effect of his effective retirement decision on the dynamics of the public debt. With no further incentive from the government he will choose not to postpone retirement. Second, a benchmark ideal scenario is studied, where the decision on effective retirement age does no longer correspond to individual agents, but to an agency who decides on their behalf, and not only concerned on all current workers' welfare but also on future employees'. Thus, this agency acts non-myopically, and a delay in the retirement age, which would help reduce the public debt, might arise. In a third scenario, we built an incentive strategy through which the government can induce myopic atomistic decision-makers to behave as the non-myopic agency which internalizes the effect of effective retirement age on the dynamics of the public debt. A numerical analysis is carried out to study the credibility of such incentive strategy.

1 The model

Our starting point is a situation where, because of the ageing population, current contributions to the Social Security system are not enough to maintain unaltered the previous level of generosity of the public pensions. The public pension system has switched from a situation of balanced budget to a situation where pension expenses for retirees exceed contributions from current workers. Everything else remaining equal, the deficit of the Social Security would fuel an unbounded public debt.

Here we consider two policies that may help to adjust contributions and pension expenses: a reduction in the generosity of the public pension and a postponement of the retirement age. The interaction between these two decision variables is analyzed here as a game of infinite duration between a continuum succession of identical private agents and the government. Agents belonging to the cohort that reaches its legal retirement age at time t , decide how much longer to postpone effective retirement. As for the government, it chooses the generosity of the public pension.

We abstract from any other government expending but the public pension cost. Thus, the evolution of the public debt depends on the gap between current contributions to the Social Security system (exogenous in our model) and public pension expenses. These expenses depend on the public pension, that the government agrees to pay to every retiree, and the size of this collective.³

Public debt dynamics.

Let's define \hat{p} as the public pension per capita which, assuming all workers get retired at the legal retirement age, could be financed by current contributions. Now if we define the public pension as $P(t) = \hat{p} + p(t)$, variable $p(t)$ is the excess or supplementary pension above \hat{p} , henceforth called excess-pension.⁴ If the government wishes to maintain the previous level of generosity, it should fix a positive $p(t)$, and if workers have no flexibility to decide their effective retirement age, the induced government deficit would increase the public debt.

³The number of retirees is dependent on demographic and economic aspects. Here we consider that these two factors have led the economy from a previous situation with a ratio of workers above and below the legal retirement age which maintains the public budget balanced, to a new situation with a much higher ratio (which puts pressure on the pension system). Once in the new situation, we assume the ratio remains constant and unaffected by these two forces. However, if retirement age is flexible, the number of agents above the legal retirement age does not necessarily match the size of the collective of retirees (above the effective retirement age).

⁴To reduce notation we may cancel out current contributions and \hat{p} , and focus on the excess-pension above contributions.

When workers are allowed to postpone retirement beyond the legal retirement age, the delay, $x(t)$, would reduce government spendings along the period from time t to $t + x(t)$. For tractability, we discount all the ongoing savings along the $x(t)$ -length period to time t . One may argue that the government could borrow the total amount at time t and repay it with the ongoing $x(t)$ -length period savings.

The primary deficit (which does not take into account the interests of the debt) can be defined as a function of both, the excess-pension, and the delay of the effective retirement age beyond the legal retirement age: $f(p(t), x(t))$, with $f'_{p(t)} > 0$ and $f'_{x(t)} < 0$. The effect of a longer delay in retirement can be thought of as non-linear. As $x(t)$ increases, the employee's survival probability decreases and so does the expected government savings, $f''_{x^2(t)} < 0$. One might also argue the existence of multiplicative terms: the higher the excess-pension the stronger the reduction in the public deficit with a delay in the retirement age. However, we will consider a linear function for tractability: $f(p(t), x(t)) = p(t) - \beta x(t)$. Further, the existing debt is financed issuing public bonds which yield ⁵ an interest rate, i . Thus, the dynamics of the public debt can be written as:

$$\dot{D}(t) = p(t) - \beta x(t) + iD(t), \quad \beta > 0, D(0) = D_0 > 0. \quad (1)$$

Employee's objective function.

We assume no interaction terms between the public debt and the action variables: $F_e(p(t), x(t), D(t)) = g_e(p(t), x(t)) - h_e(D(t))$. The representative employee who reaches his legal retirement age at time t values positively a supplementary pension, $(g_e)'_p > 0$, and negatively a delay in the retirement age, $(g_e)'_x < 0$. We further assume a constant marginal benefit of the excess-pension, $(g_e)''_{p^2} = 0$, but an increasing marginal cost on the delay in retirement, $(g_e)''_{x^2} < 0$. This latter is explained because late retirement does not only implies more "linear" years of labor. The longer the employee delays retirement, the more cumbersome becomes labor, while at retirement, life expectancy shortens and presumably quality of life worsens. For simplicity, we are assuming also an increasing marginal cost on early retirement. In many countries early retirement, which is only allowed for a limited period, implies a linear reduction in the final pension, as well as additional income losses from the effective till the legal retirement age.

Furthermore, an employee at his legal retirement age does not exclusively value the extra-pension paid by the government at that time. The level reached by the public debt is also valued because it constitutes an indicator of how likely his pension will be maintained, increased or reduced in the forthcoming years, hence we assume $h'(D(t)) > 0$.

For tractability we will assume a linear quadratic structure for the employee's objective function of the form:

$$F_e(p(t), x(t), D(t)) = p(t) - \frac{c}{2}x^2(t) - hD(t), \quad c, h > 0. \quad (2)$$

Government's objective function:

An additively separable function between the public debt and the players actions is assumed, $F_g(p(t), D(t)) = g_g(p(t)) - h_g(D(t))$.

The government welfare is not directly affected by the effective retirement age, $x(t)$. On the other hand, although an excess-pension enhances retirees welfare, it also boosts the public deficit, which might be regarded by younger employees as a threat to their future pensions. Thus we assume that the expected vote, and hence the government welfare, initially increases with the excess-pension, but at a decreasing rate which eventually becomes negative, $\lim_{p \rightarrow 0} (g_g)' > 0$, $(g_g)'' < 0, \forall p \geq 0$ and $(g_g)' < 0, \forall p \geq \hat{p} > 0$.

The government also wishes to maintain the public debt within certain limits. Youngsters might perceive high values of the public debt as a threat to their future pensions, even stronger than the current deficit. Likewise, a highly negative debt can also be perceived by voters as a symptom of mismanagement. For simplicity we fix the desired level of public debt equal to zero.

⁵A negative public debt, called a Social Security trust fund, would yield the same return as the interest paid by public bonds.

For tractability a linear quadratic structure is also assumed:

$$F_g(p(t), D(t)) = p(t) - \frac{q}{2}p^2(t) - \frac{\sigma}{2}D^2(t), \quad q, \sigma > 0. \quad (3)$$

2 Myopic employees

A static representative employee who reaches legal retirement age at time t , myopically chooses the delay in his effective retirement age. Even an agent with intergenerational altruism (redefined as a continuum of representative employees successively reaching retirement age) would still act myopically. Because he is a representative of a large number of identical agents, he would determine the optimal delay not taking into account the effect of his decision on the dynamics of the stock of public debt.

The myopic employee maximizes his static welfare,⁶

$$\max_x p - \frac{c}{2}x^2 - hD, \quad (4)$$

while the government solves the optimal control problem:

$$\begin{aligned} \max_p \left\{ W_g = \int_0^\infty \left(p - \frac{q}{2}p^2 - \frac{\sigma}{2}D^2 \right) e^{-\rho t} dt \right\}. \\ \text{s.t.: } \dot{D} = p - \beta x + iD, \quad D(0) = D_0 > 0. \end{aligned} \quad (5)$$

For comparison purposes, the government problem is solved using the Hamilton-Jacobi-Bellman (HJB) equation for an autonomous game:

$$\rho V_g^m(D) = \max_p \left\{ p - \frac{q}{2}p^2 - \frac{\sigma}{2}D^2 + (V_g^m)'(D)[p - \beta x + iD] \right\}$$

where $V_g^m(D)$ denotes the value function of the government when the employer behaves myopically. If we conjecture a quadratic value function, using the HJB equation and standard maximization techniques the solution of the problem can be easily found.

Proposition 1 *The government value function and the equilibrium strategies of the government and the myopic employee are given by:*

$$\begin{aligned} V_g^m(D) &= a_g^m + b_g^m D + c_g^m D^2, \\ x^m &= 0, \quad p^m(D) = \frac{1 + b_g^m + 2c_g^m D}{q}, \end{aligned}$$

with

$$\begin{aligned} c_g^m &= -\frac{(2i - \rho)q + \sqrt{(2i - \rho)^2 q^2 + 4\sigma q}}{4} < 0, \\ b_g^m &= -\frac{2c_g^m}{2c_g^m + q(i - \rho)} < 0, \quad a_g^m = \frac{(1 + b_g^m)^2}{2q\rho} > 0. \end{aligned}$$

And the optimal trajectory of the public debt reads:

$$D_m(t) = (D_0 - D_m^*)e^{\gamma_m t} + D_m^*, \quad (6)$$

with

$$D_m^* = -\frac{i - \rho}{iq(i - \rho) + \sigma}, \quad \gamma_m = \frac{q\rho - \sqrt{(2i - \rho)^2 q^2 + 4\sigma q}}{2q}.$$

⁶Henceforth the time argument is omitted when no confusion can arise.

Corollary 2 *The public debt will not converge to its steady-state equilibrium (i.e. $\gamma_m > 0$), iff:*

$$\rho > 2\sqrt{\frac{\sigma}{q}} \quad \wedge \quad i \in (r^-, r^+),$$

with $r^- \in (0, \rho/2)$ and $r^+ \in (\rho/2, \rho)$ the roots of the second order polynomial $i^2q - iq\rho + \sigma$.

Depending on the discount rate versus the rate of return on public bonds, the public debt may converge to a positive or a negative steady state value, or it may diverge.

Because there are no incentives to postpone retirement, the representative employee will retire exactly at his legal retirement age. The government will fix a public excess-pension dependent on the stock of the public debt:

$$p^m(D) = p_0^m + p_1^m D, \quad p_0^m = -\frac{2(i - \rho)}{\rho q + \sqrt{(2i - \rho)^2 q^2 + 4\sigma q}},$$

$$p_1^m = -\frac{q(2i - \rho) + \sqrt{(2i - \rho)^2 q^2 + 4\sigma q}}{2q} < 0,$$

where D evolves according to (6).

With a zero public debt the government would be willing to supplement the pension that can be paid by current contribution (i.e. $p_0^m > 0$) only if the interests of the bonds are lower than the discount rate, and viceversa. Further, because p_1^m is negative, the excess-pension would be smaller the higher the stock of public debt.

3 Non-myopic employees. Markovian Nash Equilibrium

This benchmark scenario assumes that the effective retirement age is not determined by atomistic decision-makers, but there exists an agency which fixes the optimal retirement age to maximize employees welfare. This agency is no longer myopic first, because it maximizes not only the welfare of employees that currently reach legal retirement age, but also those who will reach this age in the future. And second, because it fixes the effective retirement age of all agents belonging to a particular cohort. Hence it is aware of the effect of its decisions on the dynamics of the public debt. Denoting $V_g^N(D)$, and $V_e^N(D)$ the value functions of the government and the employee, the HJB equations for this game read:

$$\rho V_e^N(D) = \max_x \left\{ p - \frac{c}{2} x^2 - hD + (V_e^N)'(D)[p - \beta x + iD] \right\},$$

$$\rho V_g^N(D) = \max_p \left\{ p - \frac{q}{2} p^2 - \frac{\sigma}{2} D^2 + (V_g^N)'(D)[p - \beta x + iD] \right\}.$$

Because of the linear quadratic structure of the game, quadratic value functions are conjectured, and the game can be solved.

Proposition 3 *The value functions and the equilibrium strategies for the government and the non-myopic agency who represents all employees are:*

$$V_e^N(D) = a_e^N + b_e^N D + c_e^N D^2, \quad V_g^N(D) = a_g^N + b_g^N D + c_g^N D^2,$$

$$x^N(D) = -\beta \frac{b_e^N + 2c_e^N D}{c}, \quad p^N(D) = \frac{1 + b_g^N + 2c_g^N D}{q},$$

with

$$c_e^N = -\frac{c \left[2q(2i - \rho) + \sqrt{\Delta} \right]}{6q\beta^2} < 0, \quad c_g^N = \frac{-q(2i - \rho) + \sqrt{\Delta}}{12} < 0,$$

$$\begin{aligned}
b_e^N &= \frac{-2c\Theta_e + q\beta^2\Xi_e}{6q\beta^2[i(i+\rho)q + i\sqrt{\Delta} - 3\sigma]}, & b_g^N &= \frac{\{c\Theta_g - q\beta^2\Xi_g\} [(2i-\rho)q - \sqrt{\Delta}]}{12cq[i(i+\rho)q + i\sqrt{\Delta} - 3\sigma]}, \\
a_e^N &= \frac{2(1+b_e^N)(1+b_g^N)c + (b_e^N)^2q\beta^2}{2cq\rho}, & a_g^N &= \frac{(1+b_g^N)^2c + 2b_g^N b_e^N q\beta^2}{2cq\rho}, \\
\Delta &= (2i-\rho)^2q^2 - 12\sigma q, \\
\Theta_e &= \rho \left[(2i-\rho)q - \sqrt{\Delta} \right] + 4i(2i-\rho)q + 5i\sqrt{\Delta} - 12\sigma, \\
\Xi_e &= -(i+\rho)[(2i-\rho)q - \sqrt{\Delta}] - 3h[(4i+\rho)q + \sqrt{\Delta}] - 6\sigma, \\
\Theta_g &= (4i-5\rho)q + \sqrt{\Delta}, & \Xi_g &= -6hq - (2i-\rho)q + \sqrt{\Delta}.
\end{aligned}$$

And the optimal trajectory of the public debt reads:

$$D_N(t) = (D_0 - D_N^*)e^{\gamma_N t} + D_N^*, \quad (7)$$

with

$$D_N^* = -\frac{c(1+b_g^N) + qb_e^N\beta^2}{2cc_g^N + ciq + 2qc_e^N\beta^2}, \quad \gamma_N = -\frac{q(4i-5\rho) + \sqrt{\Delta}}{6q}. \quad (8)$$

Proof. Performing the maximization of the right-hand sides of these equations, the optimal excess-pension and effective retirement are obtained as affine functions of the stock of public debt. Plugging this equilibrium strategies into the HJB equations, the coefficients of the value functions can be computed by identification. Two sets of solutions satisfy conditions $c_i^N < 0$, $i \in \{e, g\}$, and therefore imply concave value functions. We choose the solution with a faster rate of convergence, and a greater set of parameters values for which the steady state equilibrium is stable. \square

The discriminant, Δ , is positive if either $i < \rho/2 - \sqrt{3\sigma/q}$ or $i > \rho/2 + \sqrt{3\sigma/q}$. Because we need $i > \rho/2$ to guarantee $c_i^N < 0$, $i \in \{e, g\}$, henceforth we will assume condition:

$$i > \frac{\rho}{2} + \sqrt{3\frac{\sigma}{q}}. \quad (9)$$

Corollary 4 *The public debt will not converge to its steady-state equilibrium (i.e. $\gamma_N > 0$), iff:*

$$\left(\rho < 4\sqrt{\frac{\sigma}{3q}} \quad \wedge \quad i < \frac{5\rho}{4} \right), \quad \text{or} \quad \left(\rho > 4\sqrt{\frac{\sigma}{3q}} \quad \wedge \quad i < r_N^- \right), \quad (10)$$

with $r_N^- \equiv 3\rho/2 - \sqrt{\rho^2 - 4\sigma/q}/2 \in (\rho, 3\rho/2)$ root of the second order polynomial $i^2 - 3i\rho + 2\rho^2 + \sigma/q$.

Stability requires the rate of return to be either greater than $5\rho/4$ or r_N^- . Both figures are greater than ρ . In consequence, a necessary condition for stability, assumed henceforth is:

$$i > \rho. \quad (11)$$

Because condition $\rho > 4\sqrt{\sigma/(3q)}$ is equivalent to $r_N^- < 5\rho/4$, then a sufficient condition for stability is given by an interest rate sufficiently above the discount rate:

$$i > \frac{5\rho}{4}. \quad (12)$$

Lemma 5 *From condition (9) which guarantees concave value functions, and the stability conditions in (10), it follows that $b_e^N < 0$ and $b_g^N > 0$.*

The sign of the public debt at the steady state cannot be fully characterized. It will be negative, defining a Social Security trust fund ($D_N^* < 0$), under condition $i \in (5\rho/4, 2\rho)$. That is, an interest rate sufficiently

large above the discount rate (so that sufficient condition for stability of the public debt (12), is fulfilled) but not too large above, $i < 2\rho$.

In a Markovian Nash equilibrium, the optimal delay in the effective retirement age, and the optimal excess-pension above contributions are both expressed as affine functions of the stock of public debt.

$$x^N(D) = x_0^N + x_1^N D, \quad p^N(D) = p_0^N + p_1^N D.$$

Further, from assumption (9) and Lemma 5 it follows that:

$$x_0^N = -\frac{\beta b_e^N}{c} > 0, \quad x_1^N = -\frac{2\beta c_e^N}{c} > 0, \quad p_0^N = \frac{1 + b_g^N}{q} > 0, \quad p_1^N = \frac{2c_g^N}{q} < 0.$$

When effective retirement age is not chosen by myopic atomistic decision-makers, but by a farsighted agency, this agency would postpone effective retirement beyond legal retirement age if the public debt were zero, $x_0^N > 0$. Further positive stocks of public debt would imply longer delays, $x_1^N > 0$. As for the government, with no public debt, it would pay a public pension exceeding current employees' contributions, $p_0^N > 0$, while in the myopic case (since we are assuming $i > \rho$) it chooses a pension below contributions, $p_0^m < 0$. Furthermore, as the problem of the public debt worsens (the stock increases), the government would reduce its generosity, although the marginal reduction is smaller than in the case of myopic atomistic employees, $p_1^m < p_1^N < 0$. Thus, for any given level of the public debt, the Markovian Nash equilibrium implies greater pensions for employees than in the myopic scenario.

As for the public debt, the convergence to its steady-state value is less faster under the assumption that the retirement age is determined by a farsighted agency, $\gamma_m < \gamma_N$. However, although at a lower speed, the public debt reached in the long run is lower than the approached when myopic employees takes retirement decisions. Because steady-state values of the public debt cannot be analytically compared, we have relied on numerical simulations to prove that $D_N^* < D_m^*$.

We have computed 10^5 random points with parameters values following a uniform distribution function within intervals: $i, \rho \in (0, .02)$, $c, q \in (0, 10)$, $\sigma \in (0, 10^{-5})$, $h \in (0, 10^5)$, and $\beta \in (.1, 5)$. As long as conditions (9) and (12) are satisfied, it always follows that $D_N^* < D_m^*$.

Furthermore, if decisions on the effective retirement age were taken by an agency which represents the continuum of current and future employees, the government welfare would be higher, $V_g^N(D) > V_g^m(D)$. This has also been numerically proved for given parameters values, and a grid of initial values for the stock of public debt.⁷

Contrary to the myopic case, where employees quit at exactly the legal retirement age, a farsighted agency would delay retirement when a positive public debt exists, and even also when the public debt is equal to zero. Correspondingly, the government may provide a greater public pension, and although more slowly, the public debt would converge toward a lower stock at the steady-state equilibrium. Without this farsighted-agency collecting current and future individual sovereignty, next section addresses the question of how the government may induce individual agents to behave as farsighted or non-myopic agents.

4 Government incentive to act non-myopically

In this section, the government announces an incentive strategy, aiming to induce myopic employees to postpone retirement in the same way as the farsighted-agency does in a Markovian Nash Equilibrium. Because the government is already a farsighted agent, he will be at equilibrium with no need of an equivalent incentive strategy from the employees.

We consider a linear-quadratic incentive strategy of the form:

$$\psi(x, D) = p^N(D) - \phi(D)(x^N(D) - x), \quad \phi(D) = \phi_0 + \phi_1 D. \quad (13)$$

⁷ $i = .01$, $\rho = .006$, $q = c = 1$, $\sigma = 10^{-5}$, $h = 10$, $\beta = .5$, $D_0 \in [-10^6, -10^6]$.

The government sticks to his Markovian Nash equilibrium as long as the employee behaves according to this equilibrium. However, if the employee gets retired before $x^N(D)$, the government would punish him. The punishment is dependent on the size of this deviation, $x^N(D) - x$, but also on the current stock of public debt $\phi(D)$. This allows the government to tighten the punishment as the problem of the public debt becomes more cumbersome (at the optimum ϕ_1 will be positive).

For the incentive strategy in (13), it follows that:

$$W_g(x^N(D), p^N(D)) \geq W_g(x^N(D), \psi(x, D)), \quad \forall x \in \mathbb{R},$$

$$p^N(D) = \psi(x^N(D), D).$$

Because the government is a farsighted agent, $p^N(D)$ is his best response to $x^N(D)$. Thus, if employees act non-myopically the government cannot do better under the incentive strategy than under his Markovian Nash equilibrium. Further, the government incentive strategy when employees act as a farsighted agent equates his Markovian Nash equilibrium.

To characterize the incentive strategy equilibrium, employees solve the static problem, assuming that the government will behave according to the incentive strategy in (13):

$$\max_x \psi(x, D) - \frac{c}{2}x^2 - hD,$$

while the government solves his standard optimal control problem, in (5) subject to (1).

Proposition 6 *The government value function and the equilibrium strategies of the government and the employee for the incentive strategy equilibrium are:*

$$\begin{aligned} V_g^i(D) &= a_g^i + b_g^i D + c_g^i D^2, \\ x^i(D) &= \frac{\phi_0 + \phi_1 D}{c}, \quad p^i(D) = \frac{1 + b_g^i + 2c_g^i D}{q}, \end{aligned}$$

with

$$\begin{aligned} c_g^i &= \frac{-cq(2i - \rho) + 2q\beta\phi_1 - \sqrt{((2i - \rho)c - 2\beta\phi_1)^2 q^2 + 4c^2\sigma q}}{4c}, \\ b_g^i &= \frac{2c_g^i(c - q\beta\phi_0)}{q\beta\phi_1 - c(2c_g^i + q(i - \rho))}, \quad a_g^i = \frac{c(1 + b_g^i)^2 - 2b_g^i q\beta\phi_0}{2cq\rho}. \end{aligned}$$

The incentive equilibrium is determined by equating $x^i(D)$ and $x^N(D)$. Hence, by identification it follows:

$$\phi(D) = \phi_0 + \phi_1 D, \quad \phi_0 = -b_e^N \beta > 0, \quad \phi_1 = -2c_e^N \beta > 0. \quad (14)$$

Corollary 7 *Given this incentive equilibrium strategy it can be proved that $c_g^i = c_g^N$, $b_g^i = b_g^N$ and $a_g^i = a_g^N$. In consequence, the public pension and the government value function coincide with the Markovian Nash equilibrium, $p^i(D) = p^N(D)$ and $V_g^i(D) = V_g^N(D)$. Further, the dynamic of the public debt matches the Markovian Nash equilibrium as described in (7) and (8), $D_i(t) = D_N(t)$.*

Employees regard the incentive strategy in (13) as a government commitment on the public pension, $p^N(D)$, as long as they stick to the non-myopic retirement age, $x^N(D)$. Alternatively, it can be rewritten as a two-part incentive:

$$\psi(x, D) = p^{x=0}(D) + \phi(D)x, \quad (15)$$

where $p^{x=0}(D) = p_0^N - c(x_0^N)^2 + [p_1^N - 2cx_0^N x_1^N]D - c(x_1^N)^2 D^2$ (a second order polynomial in the public debt) would be the excess-pension if the employee retires at exactly the legal retirement age (i.e. $x = 0$).

Further, the government rewards one year postponement of retirement with an increment of $\phi(D)$ in the public pension. This incentive is stronger the greater the stock of public debt, $\phi'(D) > 0$. The two-part incentive as expressed in (15) is more clear for employees than the same incentive but expressed as in (13). The former is the form in which some governments already expose their incentive strategy.⁸

An incentive strategy must fulfill one last property in order to be effective: credibility. The incentive equilibrium announced by the government has to be credible. We must proof that for any deviation of the employee from his farsighted solution it is in the government best interest to implement the announced incentive strategy rather than to play his non-cooperative Markovian equilibrium. That is, the incentive equilibrium is credible within the set \mathbf{X} if:

$$W_g(\hat{x}, p^N(D)) \leq W_g(\hat{x}, \psi(\hat{x}, D)), \quad \forall \hat{x} \in \mathbf{X}.$$

Usually this analysis is done assuming that the player who deviates, here the employee, still follows an affine control. In particular, we focus on strategies within the set:

$$\mathbf{X} = \{ \hat{x}(D) = \hat{x}_0 + \hat{x}_1 D, \mid \hat{x}_0 \in [0, x_0^N], \hat{x}_1 \in [\underline{x}_1, x_1^N] \}.$$

with $\underline{x}_1 = (p_1^N + i)/\beta > 0$, the minimum value of \hat{x}_1 which guarantees convergence.

Within this set the retirement age is not postponed longer than in the non-myopic Nash equilibrium.

Proposition 8 *The incentive strategy announced by the government is credible in \mathbf{X} if:*

$$W_g(\hat{x}(D), p^N(D)) \leq W_g(\hat{x}(D), \psi(\hat{x}(D), D)), \quad \forall \hat{x}(D) \in \mathbf{X},$$

and $\psi(x, D)$ given by (13) and (14).

Equivalently:

$$\int_0^\infty \left\{ p_0^N - \frac{q}{2}(p_0^N)^2 + D(t)[p_1^N - 2p_0^N p_1^N] + D^2(t) \left[-\frac{q}{2}(p_1^N)^2 - \frac{\sigma}{2} \right] \right\} e^{-\rho t}, \quad (16)$$

with

$$D(t) = \left(D_0 + \frac{p_0^N - \beta \hat{x}_0}{p_1^N - \beta \hat{x}_1 + i} \right) e^{(p_1^N - \beta \hat{x}_1 + i)t} - \frac{p_0^N - \beta \hat{x}_0}{p_1^N - \beta \hat{x}_1 + i} \quad (17)$$

is not greater than

$$\int_0^\infty \left\{ \Lambda_0 - \frac{q}{2}\Lambda_0^2 + \Lambda_1 + D(t) [\Lambda_1 - q\Lambda_0\Lambda_1] + D^2(t) \left[\Lambda_2 - q \left(\Lambda_0\Lambda_2 - \frac{\Lambda_1^2}{2} \right) \right] - D^3(t)q\Lambda_1\Lambda_2 - D^4(t)\frac{q}{2}\Lambda_2^2 \right\} e^{-\rho t}, \quad (18)$$

with

$$D(t) = \frac{\pi_2 e^{(\pi_1 - \pi_2)t} (D_0 - \pi_1) - (D_0 - \pi_2) \pi_1}{e^{(\pi_1 - \pi_2)t} (D_0 - \pi_1) - (D_0 - \pi_2)}. \quad (19)$$

where the expressions for Λ_0 , Λ_1 , Λ_2 , π_1 and π_2 (in Appendix) depend on parameters and the magnitude of the deviation from the non-myopic Nash equilibrium.

Proof. See Appendix. □

Because of its complexity, usually credibility cannot be proved analytically but numerically for a specific range of parameter values. We analyze numerically a scenario where the employees depart from the non-myopic strategy and follow an affine strategy within the set \mathbf{X} . Then the government can reply by either

⁸Here early retirement is punished in the same amount as late retirement is rewarded. In the real world, some countries although punishing early retirement do not allow for late retirement, while some others do unevenly treat early and late retirement.

sticking to its announced incentive strategy, or by changing to its Markovian Nash strategy. We compute and compare the government welfare under each of these two alternative replies. The analysis is carried out for all points in a grid in \mathbf{X} .

Tables 1 and 2 show the sign of the gap in the government's welfare between following the announced incentive strategy or switching to his Markovian-Nash equilibrium.⁹ A positive (resp. negative) sign would mean that the incentive is (resp. is not) credible for this particular deviation (C is a complex number). Tables display, in rows: the effective retirement age with zero public debt, \hat{x}_0 ; and in columns: the marginal increment in effective retirement age with increments in the public debt, \hat{x}_1 . Within this range, employees' delay in retirement is shorter than under the non-myopic benchmark case, although deviations from the Markovian-Nash equilibrium are not so large to impede convergence ($\hat{x}_1 > \underline{x}_1$).

Tables 1 and 2 show that credibility decreases with deviations of x_0 , the autonomous effective retirement age or the effective retirement age with zero public debt. Conversely, credibility is more likely the greater the reduction in x_1 , that is the softer the reaction of the effective retirement age with changes in the public debt. Further, comparing Tables 1 to 3, it follows that the incentive strategy is more credible the greater the initial value of the public debt. In fact, for a sufficiently large initial stock of the public debt, the incentive strategy is credible for any deviations within the set \mathbf{X} . The opposite would be true for an initially low stock of public debt.

5 Conclusions

This paper studies the interaction between two instruments at the center of the current debate on public retirement pensions: a reduction in the public pension versus a delay in the retirement age. Here, the effective retirement age is not considered as a rigid number enforced by the government. Conversely, individuals are allowed to retire anytime no sooner than a legal, or minimum, retirement age, which is exogenous to the model. Thus, we analyze the optimal determination, by employees, of the effective retirement age beyond the legal retirement age.

For an employee who reaches his legal retirement age and has to decide whether to get retired or to delay retirement for some time, myopia is a two-sided concept. He may not be concerned with future employees when taking his decisions. But even if he does, because he is an atomistic decision maker, delaying his retirement will have a negligible effect on the aggregate government accounts: current public deficit or stock of public debt. Myopic individuals with no further incentive would choose to get retired as soon as they reach the legal retirement age. Thus myopia puts stress upon public finances, pushing the government to reduce public pensions.

Conversely, non-myopic employees would refrain from immediate retirement, which would reduce the pressure on the public deficit alleviating the public debt and allowing for higher pensions in the long-run. Thus they are willing to trade years of labor in exchange for higher long-run retirement pensions. For a government compelled to maintaining low levels of public debt, giving employees the possibility to decide their effective retirement age might turn helpful if they would behave non-myopically.

Based on the literature on incentive strategies (which seeks players cooperation through linear incentive strategies), the paper defines non-linear strategies as a successful incentive to induce myopic employees to act non-myopically. This strategy can be regarded as an announcement, made by the government, that it will pay a high pension (the Nash equilibrium with non-myopic employees) to employees that behave non-myopically. However, it also implies a punishment, in the form of a shorter pension, to employees that get retired before a non-myopic employee would do. The reduction in the retirement pension will be stronger, the sooner the employees get retired, but also the greater the stock of the public debt. Alternatively, this strategy can be regarded as a two-part incentive: one defines the public pension at legal retirement age, and the second part raises the pension in a given percentage with additional years at work. Both parts are considered dependent on the stock of public debt. Thus, a higher stock of public debt has a double effect: it reduces the pension to

⁹Parameters' values are: $i = .01$, $\rho = .006$, $q = c = 1$, $\sigma = 10^{-5}$, $h = 10$, $\beta = .5$, $\varepsilon = 10^{-5}$.

those employees who get retired at exactly the legal retirement age; and it also reduces the reward for each year delay in retirement.

If the initial stock of public debt is sufficiently large, this incentive strategy announced by the government is proven to be credible. Thus, myopic employees, fearing the punishment announced by the government, will delay retirement in the same length as if they were non-myopic agents. And further, because the farsighted government is in equilibrium, it will stick to its announced strategy.

Appendix

Proof of Proposition 8:

Expressions (16) and (17) where the employer deviates but the government follows his Markov Nash strategy obtained in Proposition 3, immediately follows from (1) and (5).

In the second case, the employer deviates and the government sticks to the linear-quadratic incentive strategy in (13) and his optimal value determined in Proposition 6 and equation (14). From (5), expression (18) immediately follows, with $\Lambda_1 = p_1^N - c [x_0^N(x_1^N - \hat{x}_1) + x_1^N(x_0^N - \hat{x}_0)]$, $\Lambda_0 = p_0^N - cx_0^N(x_0^N - \hat{x}_0)$, and $\Lambda_2 = -cx_1^N(x_1^N - \hat{x}_1) < 0$.

The dynamics of the public debt is now defined by a second order polynomial in D :

$$\dot{D} = \Lambda_0 + \Lambda_1 D + \Lambda_2 D^2 - \beta(\hat{x}_0 + \hat{x}_1 D) + iD,$$

Or equivalently:

$$\dot{D} = Y_0 + Y_1 D + Y_2 D^2,$$

with $Y_0 = \Lambda_0 - \beta\hat{x}_0$, $Y_1 = \Lambda_1 - \beta\hat{x}_1 + i$, and $Y_2 = \Lambda_2$. The time path of the public debt in (19) can be obtained from this equation, where π_1 and π_2 are the roots of this second order polynomial (note that these roots would be real umbers under the sufficient condition of a small deviation of \hat{x}_0 from x_0^N , which guarantees $\Lambda_0 > 0$ while $\Lambda_2 < 0$).

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