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Abstract

This paper shows that the important result obtained in a static marketing channel, namely, that the manufacturer can coordinate the channel through a two-part wholesale tariff, does not extend to a dynamic setting. The existence of such a coordinated two-part tariff is shown under the restrictive assumptions of *(i)* a full commitment by the manufacturer to the vertically integrated solution; and *(ii)* a retailer that cannot influence the evolution of the state (brand equity). However, the conclusion is that it is not in the manufacturer's best interest to commit.

Key Words: Two-Part Tariff, Marketing Channels, Coordination, Differential Games.

Résumé

Un résultat important dans les canaux de distribution du type monopole bilatéral est que le manufacturier peut réaliser, d'une manière décentralisée, la solution d'intégration verticale en ayant recours à une tarification en deux parties. Cet article montre que ce résultat ne se généralise pas à un contexte dynamique. L'existence d'un tel mécanisme de coordination est montrée sous les hypothèses restrictives suivantes : *(i)* le manufacturier se commet à la solution coopérative et *(ii)* le détaillant ne peut pas influencer l'évolution de l'état du système (le capital de marque). Néanmoins, la conclusion est que le manufacturier n'a pas d'avantage à le faire.

Mots clés : Tarification en parties, canaux de distribution, coordination, jeux différentiels.

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1 Introduction

Research on conflict and cooperation in marketing channels has been continuously active during the last two decades. The marketing science approach typically adopted a static game theory paradigm with parsimonious models to analyze the interactions between the manufacturer(s) and the retailer(s). One important research question in this stream is whether or not one can induce the different players to coordinate their marketing policies. The motivation here is crystal clear: the lack of coordination in pricing and/or other marketing instruments damages the profitability of the channel and is detrimental to the consumer's welfare. The optimality of a series of mechanisms, e.g. two-part wholesale tariff, leadership, implicit understanding, cooperative advertising, etc., has been assessed in different institutional and competitive settings (for a survey, see, e.g., Ingene and Parry (2004) and Taboubi and Zaccour (2005)).

One important result, achieved by Jeuland and Shugan (1983) and Moorthy (1987), is that by using a two-part pricing scheme, the manufacturer can coordinate the bilateral-monopoly channel. This means that, with the right pricing policy, one can reproduce, in a decentralized way, the optimal collective results of the vertically integrated channel. These instances, in which the players' total payoffs are the same under cooperation and noncooperation, are so rare that this result is, by any measure, remarkable. This naturally leads us to question the generality of this result. Ingene and Parry (1995a) showed that it does not extend to the case of one manufacturer serving multiple retailers, because channel coordination is no longer optimal for the manufacturer acting as a Stackelberg leader. Ingene and Parry (1995b, 2000) further extended their conclusion to a channel formed of asymmetric competing retailers who are treated comparably. Recently, Raju and Zhang (2005) considered a channel with one manufacturer serving a dominant retailer and a fringe. They showed, contrary to Ingene and Parry (1995a), that coordination of the channel through a two-part tariff can be beneficial to the manufacturer.

One common feature of the above-cited papers is that they use a static game model. Although some studies have adopted a differential game formalism to study coordination in marketing channels, they have considered, with the exception of Jørgensen and Zaccour (1999), non-price variables (see, e.g., Chintagunta and Jain (1992) on marketing efforts; Jørgensen, Sigue and Zaccour (2000) and Jørgensen, Taboubi and Zaccour (2001, 2003) on cooperative advertising; Jørgensen, Sigue and Zaccour (2001) on leadership; Jørgensen and Zaccour (2003) and Jørgensen, Taboubi and Zaccour (2006) on incentive strategies; and Jørgensen and Zaccour (2004) for a comprehensive survey). Therefore, the idea of a two-part tariff has not, to the best of my knowledge, been considered in a dynamic game context.

Can the result in Jeuland and Shugan (1983) and Moorthy (1987) be extended to a dynamic setting? This is basically the research question I wish to tackle in this paper. The main motivation for using a dynamic setting is that some marketing variables have carry-over effects and that, therefore, a static model cannot capture the whole picture. For instance, advertising not only affects current sales (a flow), but also feeds the brand goodwill or brand equity (a stock), which in turn has an influence on sales, pricing, etc.

Another argument is that partners in marketing channels tend to develop a long term and evolving relationship. In their seminal paper, Jeuland and Shugan (1983) state that the channel's actors necessarily face some trade-offs between short- and long-term objectives when attempting to coordinate their efforts, and thus, that a dynamic approach is needed to understand this duality.

Since I will be adding two features to the models in Jeuland and Shugan (1983) and Moorthy (1987), namely retailer advertising (as well as manufacturer advertising) and dynamics, it is methodologically (or experimentally) important to be able to separate the two possible effects. Therefore, I first have to settle the question of whether or not their result can be generalized to static models where the retailer controls - on the top of the retail price - other marketing instruments, e.g., advertising. To achieve this, I verify the following:

Claim 1 *Assume that the retailer controls for advertising and retail price. The manufacturer can still coordinate the channel through a two-part tariff.*

Proof. See Appendix. ■

The above result, albeit generated with a simple example, indicates that the result in Jeuland and Shugan (1983) and Moorthy (1987) remains valid when the retailer is allowed to decide on more than one marketing instrument. That being said, I will focus from now on the dynamic generalizability.

Based on what the static games literature has shown, I state and test the following conjectures:

Conjecture 1 *The manufacturer can coordinate the dynamic bilateral-monopoly channel through a two-part wholesale tariff.*

Conjecture 2 *If there exists a two-part wholesale tariff that leads to the same retail price as in the vertically integrated channel, then the retailer's other marketing variables will be set at their coordinated levels.*

Conjecture 3 *It is profitable to the manufacturer to implement a two-part wholesale tariff.*

It is worth noting that Conjecture 1 and Conjecture 3 are related. For the sake of clarity, I am separating the *existence* of a coordinating two-part-tariff mechanism from its *profitability* to the manufacturer. Clearly, however, the manufacturer will implement such a pricing device, assuming it exists, only if it is profitable.

The rest of the paper is organized as follows. In Section 2, I introduce a simple model of the marketing channel. In Section 3, I consider different scenarios and solve the resulting games. In Section 4, I compare the profits, and in Section 5, I briefly conclude.

2 The Model

Consider a marketing channel consisting of one manufacturer, denoted M, and one retailer, denoted R. Suppose that the channel members have an infinite planning horizon, and let t denote time, $t \in [0, \infty)$. Let $a_M(t)$ represent the manufacturer's advertising rate, which influences the brand equity or goodwill, denoted $B(t)$. The retailer invests in promotional (or advertising) activities at rate $a_R(t)$. By the latter, I mean any non-price promotion action that a retailer typically undertakes, e.g., local advertising, flyers, displays, etc. I assume that the cost of advertising or promotion is quadratic and given by

$$C_j(a_j(t)) = \frac{(a_j(t))^2}{2}, \quad j = M, R.$$

This functional form is deliberately simple. The results would remain qualitatively the same if a general cost function $C_j(a_j)$ were adopted, with $C_j' > 0$, $C_j'' > 0$, $C_j(0) = 0$.

The brand equity is assumed to evolve à la Nerlove-Arrow (1962), i.e.,

$$\dot{B}(t) = a_M(t) + \gamma a_R(t) - \delta B(t), \quad B(0) = B_0 > 0 \tag{1}$$

where $\delta > 0$ is a decay rate, and $\gamma \geq 0$ is a parameter capturing the impact of the retailer's promotional activities on the brand equity. If $\gamma = 0$, then the retailer's local marketing activities do not affect the evolution of $B(t)$.

Denote by $p(t)$ the price to consumer, controlled by the retailer, and by $w(t)$ the manufacturer's wholesale (or transfer) price. Let the demand function be given by

$$Q(t) = B(t) - \alpha p(t) + a_R(t), \tag{2}$$

where α is a positive parameter.[1] In the above sales function, the brand equity corresponds to potential sales when the price and retailer's advertising are set equal to zero. The functional form is in line with the literature, which is replete with linear demand functions. Note that the two players' advertising efforts are treated differently. Indeed, the assumption here is that the retailer's (mainly local) advertising has an instantaneous effect on demand, while the manufacturer's (national) advertising only indirectly affects the current demand, i.e., through a process of building up the brand equity.

Denote by $c > 0$ the manufacturer's constant unit production cost. Letting ρ be the constant and positive discount rate, the objective functionals of the manufacturer and the retailer, respectively, are then as follows:

$$J_M = \int_0^\infty e^{-\rho t}\left((w(t) - c)\left(B(t) - \alpha p(t) + a_R(t)\right) - \frac{a_M(t)^2}{2}\right) dt, \tag{3}$$

$$J_R = \int_0^\infty e^{-\rho t}\left((p(t) - w(t))\left(B(t) - \alpha p(t) + a_R(t)\right) - \frac{a_R(t)^2}{2}\right) dt. \tag{4}$$

[1] I suppose $\alpha \neq 0.5$ to exclude dividing by zero in some formulas in the sequel.

When the manufacturer adopts a two-part wholesale tariff, then $w(t)$ will be given by

$$w(B) = c + \frac{k}{Q}, \tag{5}$$

where k is a positive constant. Note that this tariff is state-dependent, through Q, and is a quantity-discount pricing scheme. Indeed, for a given k, the higher the quantity ordered by the retailer, the lower the unit price she pays.

To summarize, by (1) and (3)-(4), I have defined a two-player infinite-horizon differential game with one state variable, $B(t)$, and four control variables: two for the manufacturer $(w(t) \geq 0, a_M(t) \geq 0)$ and two for the retailer $(p(t) \geq 0, a_R(t) \geq 0)$. Henceforth, I will skip the time argument when no ambiguity may arise.

3 Scenarios and Equilibria

To test the conjectures, I will characterize pricing and advertising strategies for the following scenarios:

1. *The vertically integrated channel game.* This is the benchmark scenario, where the game is actually converted into a dynamic optimization problem in which the players maximize the sum of their profits. The optimal policies will be superscripted by I (for integration).
2. *The noncooperative game with two-part tariff.* The game is played noncooperatively, with the wholesale price policy given by the two-part tariff in (5). The feedback Nash equilibrium is characterized and the equilibrium strategies are superscripted by N (for Nash).[2]
3. *The commitment game.* The assumption here is that the manufacturer commits to her part of the cooperative solution, i.e., she implements the strategy $\left(w(B), a_M^I(B)\right)$, where the transfer price $w(B)$ is given by (5) and the advertising strategy $a_M^I(B)$ is the optimal advertising policy in the vertically integrated channel scenario. The retailer determines her optimal price and advertising level, taken into account the manufacturer's announcement. The results are superscripted by C (for commitment).

Comparing the strategies obtained in the first two scenarios will enable me to test the first conjecture. Indeed, if the results coincide, this will verify the conjecture stating that full coordination can be reached in a decentralized way with a two-part tariff. Further, if the results coincide, in terms of the retail price and the retailer's advertising strategy, then the second conjecture would be verified. Comparing the manufacturer's outcomes will allow to testing the last conjecture. The third scenario is of interest only if coordination cannot be obtained as an equilibrium, as will actually be the case.

[2]It will be shown later that feedback Nash equilibrium and feedback Stackelberg equilibrium coincide. Therefore, it does not matter which mode of play is selected.

It will become apparent that the parameter γ plays an important role. Indeed, some of the qualitative conclusions are different depending on whether or not γ is strictly positive, meaning that the retailer's advertising affects the evolution of the state.

3.1 Results in the General Case

In this subsection, the assumption is that $\gamma > 0$. I first characterize the optimal solution for the vertically integrated channel.

Proposition 1 *Assuming an interior solution, if the channel is vertically integrated, then the optimal pricing and advertising policies are as follows:*

$$\begin{aligned} p^I(B) &= \frac{B + c(\alpha - 1) + \gamma\left(\varphi_1 B + \varphi_2\right)}{(2\alpha - 1)}, \\ a_R^I(B) &= \frac{B - \alpha c + 2\alpha\gamma\left(\varphi_1 B + \varphi_2\right)}{(2\alpha - 1)}, \\ a_M^I(B) &= \varphi_1 B + \varphi_2, \end{aligned}$$

where the constants φ_1, φ_2, *and* φ_3 *are given by*

$$\begin{aligned} \varphi_1 &= \frac{(2\delta + \rho)(2\alpha - 1) - 2\gamma \pm \sqrt{\left((2\delta + \rho)(2\alpha - 1) - 2\gamma\right)^2 - 4\left(2\alpha\gamma^2 + 2\alpha - 1\right)}}{2\left(2\alpha\gamma^2 + 2\alpha - 1\right)}, \\ \varphi_2 &= \frac{\alpha c\left(\gamma\varphi_1 + 1\right)}{\left(2\alpha\gamma^2 + 2\alpha - 1\right)\varphi_1 - (\delta + \rho)(2\alpha - 1) + \gamma}, \\ \varphi_3 &= \frac{\varphi_2^2\left(2\gamma^2 + 2\alpha - 1\right) - 2\alpha c\gamma\varphi_2 + \alpha^2 c^2}{2(2\alpha - 1)\rho}. \end{aligned}$$

The channel's value function is given by

$$V(B) = \frac{1}{2}\varphi_1 B^2 + \varphi_2 B + \varphi_3.$$

Proof. See Appendix. ■

To interpret the optimal advertising policies, I first recall that the first-order optimality conditions are given by

$$a_R = \frac{B - \alpha c + 2\alpha\gamma V'}{(2\alpha - 1)}, \; a_M = V'.$$

The condition for optimal a_R can be rewritten as

$$a_R = \frac{B - \alpha c + \gamma V'}{(2\alpha - 1)} + \gamma V' = p - c + \gamma V'.$$

This says that optimal advertising is determined by the familiar rule of marginal cost, given here by a_R, equals marginal revenue, which is the sum of the instantaneous direct marginal revenue $(p - c)$ and the indirect one $(\gamma V')$. The last term corresponds to the shadow price V' multiplied by the marginal variation of the state dynamics with respect to a_R. The optimal level of the manufacturer's advertising is determined by equating the marginal cost (a_M) to the marginal benefit, given by V'. Recall that the demand function does not directly depend on a_M and that, therefore, the marginal benefit results only from variations in the brand equity.

Given the optimal policies in the vertically integrated channel, the next step is to check if these results can be replicated in a decentralized way. The following proposition shows that the answer is no.

Proposition 2 *A feedback Nash equilibrium with the manufacturer implementing a two-part tariff does not coincide with the vertically integrated channel solution.*

Proof. See Appendix. ■

Before commenting on this result, I would like to verify if a manufacturer dedicated to cooperation could change the result, i.e., coordinate the channel.

Proposition 3 *Assume that the manufacturer commits to her part of the vertically integrated channel solution, i.e., announces and implements* $\left(w(B), a_M^V(B)\right) = (c + \frac{k}{Q}, \varphi_1 B + \varphi_2)$*, and that the retailer optimizes her own payoff. The resulting solution does not coincide with the vertically integrated channel solution.*

Proof. See Appendix. ■

Table 1 compiles the strategies in the three scenarios.

The above two propositions and the strategies in Table 1 allow the following conclusions:

Table 1: Strategies in the different scenarios ($\gamma \neq 0$)

	Vertical integration (VI)	Nash with two-part tariff	M commits to VI
p	$\frac{B+c(\alpha-1)+\gamma V'}{(2\alpha-1)}$	$\frac{B+c(\alpha-1)+\gamma N_R'}{(2\alpha-1)}$	$\frac{B+c(\alpha-1)+\gamma C_R'}{(2\alpha-1)}$
w	$-$	$c+\frac{(2\alpha-1)k}{\alpha(B-\alpha c)}$	$c+\frac{(2\alpha-1)k}{\alpha(B-\alpha c)}$
a_M	V'	N_M'	V'
a_R	$\frac{B-\alpha c+2\alpha\gamma V'}{(2\alpha-1)}$	$\frac{B-\alpha c+2\alpha\gamma N_R'}{(2\alpha-1)}$	$\frac{B-\alpha c+2\alpha\gamma C_R'}{(2\alpha-1)}$

1. A two-part wholesale tariff does not lead to the vertically integrated solution. This means (i) that full coordination cannot be achieved in a decentralized way; and (ii) that the result obtained by Jeuland and Shugan (1983) and Moorthy (1987) for static marketing channels does not extend to dynamic setting. This clearly indicates that Conjecture 1 is not verified.
2. Given that a two-part wholesale tariff policy does not lead to the vertically integrated retail price, the second and third conjectures are immaterial. However, they will become relevant in the special case dealt with in the next section.
3. Note that replacing the symmetric information structure assumption, which is inherent to the Nash equilibrium, by an asymmetric one, and thus adopting a feedback Stackelberg equilibrium with the manufacturer as leader, does not change the conclusion of Proposition 2. Indeed, given the structure of the model, i.e., the fact that the manufacturer's advertising affects the retailer's payoff only indirectly (through the brand equity) and given the linear form of the state equation, it can be shown that feedback Nash and feedback Stackelberg equilibria coincide.[3]

3.2 Specific Case Results

Given the impossibility of achieving coordination through a two-part tariff in a general setting, one may wonder if there exist circumstances that would make this feasible. A good potential case is one in which the retailer's advertising does not have an impact on the evolution of the brand equity, i.e., $\gamma = 0$. In this subsection, I analyze this case. I first state the following corollary to Proposition 1. (All results are *tilded* to distinguish them from those obtained in the case of $\gamma > 0$).

Corollary 1 *For $\gamma = 0$, the optimal policies in the vertically integrated channel become*

$$\tilde{p}^I(B) = \frac{B + c(\alpha - 1)}{(2\alpha - 1)}, \quad \tilde{a}_R^I(B) = \frac{B - \alpha c}{(2\alpha - 1)}, \; \tilde{a}_M^I(B) = \tilde{\varphi}_1 B + \tilde{\varphi}_2,$$

[3] To see it, assume that the manufacturer is the leader and announces the strategy $(c + \frac{k}{Q}, a_M(B))$. The retailer takes this into account in her optimization. Denote by $W_R(B)$ the retailer's HJB equation in this game. It reads as follows

$$\rho W_R = \max_{p \geq 0, a_R \geq 0} \left(-k + (p - c)(B - \alpha p + a_R) - \frac{a_R^2}{2} + W_R' (a_M(B) + \gamma a_R - \delta B) \right).$$

Differentiating the right-hand side with respect to p and a_R and equating to zero yields

$$\begin{aligned} p &= \frac{B + c(\alpha - 1) + \gamma W_R'}{(2\alpha - 1)}, \\ a_R &= \frac{B - \alpha c + 2\alpha\gamma W_R'}{(2\alpha - 1)}. \end{aligned}$$

Clearly, the leader's control does not appear in the above reaction functions, and hence, Nash and Stackelberg equilibria will coincide. Therefore, one should not worry here about the particular choice of information structure.

where $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ are given by

$$\begin{aligned}\tilde{\varphi}_1 &= \frac{(2\delta+\rho)}{2} \pm \sqrt{\frac{(2\delta+\rho)^2}{4} - \frac{1}{(2\alpha-1)}},\\ \tilde{\varphi}_2 &= \frac{\alpha c}{(2\alpha-1)\left(\tilde{\varphi}_1 - (\delta+\rho)\right)}.\end{aligned}$$

Proof. It suffices to set $\gamma = 0$ in Proposition 1 to get the result. ∎

The following two propositions are counterparts to Propositions 2 and 3 for the particular case of $\gamma = 0$.

Proposition 4 *Let $\gamma = 0$ and assume that the manufacturer uses the two-part wholesale tariff $w(B) = c + \frac{k}{Q}$. The feedback Nash equilibrium is given by*

$$\begin{aligned}\tilde{p}^N(B) &= \frac{B + c(\alpha-1)}{(2\alpha-1)}, \quad \tilde{a}_R^N(B) = \frac{B-\alpha c}{(2\alpha-1)},\\ w^N(B) &= c + \frac{(2\alpha-1)k}{\alpha(B-\alpha c)}, \quad \tilde{a}_M^N(B) = (2\delta+\rho)B,\end{aligned}$$

and it does not coincide with the vertically integrated channel solution. The manufacturer's value function is given by

$$N_M(B) = \frac{(2\delta+\rho)}{4}B^2 + \frac{k}{\rho}. \tag{6}$$

Proof. See Appendix. ∎

Proposition 5 *Let $\gamma = 0$ and assume that the manufacturer commits to her part of the vertically integrated channel solution, i.e., implements $\left(w(B), a_M^I(B)\right) = (c + \frac{k}{Q}, \tilde{\varphi}_1 B + \tilde{\varphi}_2)$, and that the retailer optimizes her own payoff. The resulting solution coincides with the vertically integrated channel solution.*

Proof. See Appendix. ∎

These propositions show that only in a very specific context can coordination be reached in a decentralized way. Indeed, it takes (i) a manufacturer dedicated to cooperation; and (ii) a retailer who cannot affect the evolution of the brand equity. Requirement (i) can be dealt with in a simple manner. Since by definition the players are rational, it seems reasonable to believe that the manufacturer would be willing to commit to coordination if it pays to do so. I shall deal with this in the next section. Condition (ii) is an empirical matter. An intuitive conjecture here is that one can find examples of products for which the retailer's commercial policies do have an impact on the equity of the manufacturer's brand, and other products for which they do not. Further, when the retail price is coordinated

Table 2: Strategies in the different scenarios ($\gamma = 0$)

	Vertical integration (VI)	Nash with two-part tariff	M commits to VI
p	$\frac{B+c(\alpha-1)}{(2\alpha-1)}$	$\frac{B+c(\alpha-1)}{(2\alpha-1)}$	$\frac{B+c(\alpha-1)}{(2\alpha-1)}$
w	$-$	$c+\frac{(2\alpha-1)k}{\alpha(B-\alpha c)}$	$c+\frac{(2\alpha-1)k}{\alpha(B-\alpha c)}$
a_M	$\tilde{\varphi}_1 B + \tilde{\varphi}_2$	$(2\delta+\rho)\,B$	$\tilde{\varphi}_1 B + \tilde{\varphi}_2$
a_R	$\frac{B-\alpha c}{(2\alpha-1)}$	$\frac{B-\alpha c}{(2\alpha-1)}$	$\frac{B-\alpha c}{(2\alpha-1)}$

through a two-part tariff, the retailer also selects the coordinated strategy for advertising (see Table 2). Therefore, if the retailer cannot influence the dynamics of the system, then Conjecture 2 is verified, and the result for static games, stated in the introduction, holds true for a dynamic one.

4 Profit Comparison

I turn now to the issue of the profitability of the two-part tariff policy for the manufacturer (and respond to the only remaining question, i.e., the testing of Conjecture 3).

The following proposition prescribes to a manufacturer using a two-part wholesale pricing policy not to stick to her part of the cooperative solution when it comes to deciding on the advertising strategy. The proof below provides the reason for this.

Proposition 6 *When the manufacturer implements a two-part wholesale tariff, she is better off advertising at the noncooperative level than adopting the channel-coordinated advertising strategy.*

Proof. The strategies in the second and third columns in Table 1 show that the only difference is in the advertising policy. In the third scenario, the manufacturer is not an optimizer: she simply implements her part of the coordinated strategy. In the second scenario, she optimizes her payoff. If playing the coordinated advertising policy $a_M(B) = \tilde{\varphi}_1 B + \tilde{\varphi}_2$ were optimal to her, then it would have been part of the Nash equilibrium (which coincides with the Stackelberg equilibrium). Hence, the result. ■

The total profit collected by the manufacturer in the Nash with two-part tariff scenario is given by

$$N_M(B_0) = \frac{(2\delta+\rho)}{4} B_0^2 + \frac{k}{\rho},$$

that is, her value function evaluated at initial state B_0. Can the manufacturer do better by using a one-part wholesale tariff? This question is of interest for two reasons. First, there is no rationale for implementing a two-part tariff if one can do better by using a plain one-part price. Second, a two-part tariff is a price discrimination device that may not be easily implementable in practice.

To conduct this exercise, I will assume that the benchmark payoff is the one corresponding to a Stackelberg feedback equilibrium with the manufacturer as leader. Therefore, the latter announces the strategy pair $(w(B), a_M(B))$, i.e., a one-part wholesale tariff and an advertising strategy, and the retailer reacts by optimizing her profit. Note that this information structure has often been adopted, implicitly or explicitly, in the marketing science literature. The following proposition characterizes the Stackelberg equilibrium strategies (superscripted by S).

Proposition 7 *Let $\gamma = 0$ and assume that the game is played à la Stackelberg with the manufacturer as leader. Then, the feedback equilibrium pricing and advertising strategies are as follows:*

$$\begin{aligned}
\tilde{p}^S(B) &= \frac{B(3\alpha-1)+\alpha c(\alpha-1)}{2\alpha(2\alpha-1)}, \quad \tilde{a}_R^S(B) = \frac{B-\alpha c}{2(2\alpha-1)}, \\
\tilde{a}_M^S(B) &= \left(\tilde{\lambda}_1 B + \tilde{\lambda}_2\right), \qquad \tilde{w}^S(B) = \frac{B+\alpha c}{2\alpha}.
\end{aligned}$$

The manufacturer's value function is given by

$$\tilde{S}_M(B) = \frac{1}{2}\tilde{\lambda}_1 B^2 + \tilde{\lambda}_2 B + \tilde{\lambda}_3, \tag{7}$$

where

$$\begin{aligned}
\tilde{\lambda}_1 &= \frac{(2\delta+\rho)}{2} \pm \sqrt{\frac{(2\delta+\rho)^2}{4} - \frac{1}{2\,(2\alpha-1)}}, \\
\tilde{\lambda}_2 &= \frac{\alpha c}{2\,(2\alpha-1)\left(\tilde{\lambda}_1 - (\delta+\rho)\right)}, \quad \tilde{\lambda}_3 = \frac{\alpha^2 c^2}{4\rho\,(2\alpha-1)} + \frac{\tilde{\lambda}_2^2}{2\rho}.
\end{aligned}$$

Proof. See the Appendix. ■

When we compare these strategies to their two-part-tariff counterparts (second column of Table 2), we see that they are different in all respects. To compare the total payoffs secured by the manufacturer in these two games, it suffices to compute the difference in the two value functions in (6) and (7) evaluated at initial state B_0, that is,

$$f(B_0) = \tilde{S}_M(B_0) - \tilde{N}_M(B_0) = -\frac{x}{2}B_0^2 - \frac{\alpha c}{(2\alpha-1)(\rho+2x)}B_0 + \frac{1}{\rho}(y-k),$$

where

$$x = \sqrt{\frac{(2\delta+\rho)^2}{4} - \frac{1}{2\,(2\alpha-1)}},$$

$$y = \frac{\alpha^2 c^2}{2\left(2\alpha-1\right)^2}\left(\frac{\left(2\alpha-1\right)\left(\rho+2x\right)^2+2}{\left(\rho+2x\right)^2}\right).$$

Given that x is positive (otherwise the advertising strategy would not be real numbers-see the expression of $\tilde{\lambda}_1$ in Proposition 6), the coefficient of B_0^2 is negative. The sign of B_0 depends on the value of α. It is positive for $\alpha > 1/2$ and negative for $\alpha < 1/2$. The following cases are thus possible:

1. If $\alpha > 1/2$ and $(y-k) \leq 0$, then $f(B_0) \leq 0$ for all $B_0 \geq 0$.
2. If $\alpha > 1/2$ and $(y-k) > 0$, then

$$f(B_0)\begin{cases} \geq 0, \text{ for } B_0 \in \left[0, \bar{B}_0\right] \\ \leq 0, \text{ for } B_0 \geq \bar{B}_0 \end{cases},$$

where $\bar{B}_0$ is the positive root of $f(B_0)$ and is given by

$$\bar{B}_0 = -\frac{\alpha c}{x\left(2\alpha-1\right)\left(\rho+2x\right)} + \frac{1}{x}\sqrt{\left(\frac{\alpha c}{\left(2\alpha-1\right)\left(\rho+2x\right)}\right)^2 + \frac{2x}{\rho}\left(y-k\right)}.$$

3. If $\alpha < 1/2$ and $(y-k) \geq 0$, then

$$f(B_0)\begin{cases} \geq 0, \text{ for } B_0 \in \left[0, B_0'\right] \\ \leq 0, \text{ for } B_0 \geq B_0' \end{cases},$$

where

$$B_0' = -\frac{\alpha c}{x\left(2\alpha-1\right)\left(\rho+2x\right)} + \frac{1}{x}\sqrt{\left(\frac{\alpha c}{\left(2\alpha-1\right)\left(\rho+2x\right)}\right)^2 + \frac{2x}{\rho}\left(y-k\right)}.$$

4. If $\alpha < 1/2$ and $(y-k) < 0$, then

$$f(B_0)\begin{cases} \geq 0, \text{ for } B_0 \in \left[0, B_0''\right] \\ \leq 0, \text{ for } B_0 \geq B_0'' \end{cases},$$

where

$$B_0'' = \frac{\alpha c}{x\left(2\alpha-1\right)\left(\rho+2x\right)} + \frac{1}{x}\sqrt{\left(\frac{\alpha c}{\left(2\alpha-1\right)\left(\rho+2x\right)}\right)^2 + \frac{2x}{\rho}\left(y-k\right)}.$$

Therefore, if $\alpha > 1/2$ and $(y-k) \leq 0$, then the manufacturer is better off using a two-part wholesale tariff than using a one-part tariff. In all other cases, the one-part-tariff policy is optimal for the manufacturer only if the initial brand equity is "low". In any event, the qualitative message is that a two-part-tariff policy is not *necessarily* better for the manufacturer than its one-part-tariff counterpart.

5 Concluding Remarks

The main conclusions of this paper are:

1. There is no two-part-tariff mechanism that leads, in equilibrium, to channel coordination.
2. The only context - which is however not an equilibrium - in which the existence of such a mechanism can be shown to exist is one where the manufacturer fully commits to the vertically integrated solution and the retailer cannot influence the dynamics of the system.
3. Even when the manufacturer can induce the retailer to implement, through a two-part tariff, the optimal retail price and the optimal advertising policy, it is not beneficial for the manufacturer to fully coordinate the channel, i.e., it is profitable to the manufacturer not to implement her coordinating advertising policy.
4. The manufacturer is not necessarily better off with a two-part-tariff policy than with a one-part tariff. The outcome of the comparison of the payoffs depends on the initial value of the brand equity and on the values of the model's parameters.

The main implication of these results is methodological; one should not take it for granted, qualitatively speaking, that a result obtained in a static game framework is generalizable to a dynamic game one. The lack of generalizability of results obtained in a dyad to multiple retailers and/or multiple manufacturers, and the lack of generalizability along the static-dynamic line seem to indicate that the coordination of the marketing channel is a real puzzle.

Appendix A

Proof of Claim 1. To illustrate, I will consider a very simple model. Let the demand Q be given by the following multiplicative function:

$$Q = (\sigma - \xi p)\, a_R, \quad \sigma > 0, \xi > 0,$$

where p is the retail price and a_R the retailer's marketing effort (e.g., advertising).

The advertising cost is quadratic and given by $1/2ga_R^2$, $g > 0$. The payoffs of the players are as follows:

$$\begin{aligned} \text{Manufacturer:} \quad & \pi_M = (w - c)\,(\sigma - \xi p)\, a_R, \\ \text{Retailer:} \quad & \pi_R = (p - w)\,(\sigma - \xi p)\, a_R - 1/2ga_R^2, \end{aligned}$$

where c is the unit production cost.

It is easy to verify that the vertically integrated channel solution is given by

$$p = \frac{\sigma + \xi c}{\xi}, \quad a_R = \frac{(\sigma - \xi c)^2}{4\xi g}. \tag{8}$$

Now, assume that the manufacturer announces the two-part wholesale tariff

$$w = c + \frac{k}{Q}, \quad k > 0.$$

The retailer's payoff is then given by

$$\pi_R = (p - c)(\sigma - \xi p) a_R - k - 1/2 g a_R^2.$$

Differentiating with respect to p and a_R, equating to zero, and solving yields the same solution as in (8). Hence the claim. ∎

Proof of Proposition 1. Denote by $V(B)$ the channel's value function. The Hamilton-Jacobi-Bellman equation is given by

$$\rho V = \max_{p\geq 0, a_M \geq 0, a_R \geq 0} \left[(p - c)(B - \alpha p + a_R) - \frac{a_M^2}{2} - \frac{a_R^2}{2} + V' (a_M + \gamma a_R - \delta B) \right]. \quad (9)$$

Differentiating the right-hand side with respect to the three control variables and equating to zero gives

$$\begin{aligned} p &= \frac{B + c(\alpha - 1) + \gamma V'}{(2\alpha - 1)}, \\ a_R &= \frac{B - \alpha c + 2\alpha\gamma V'}{(2\alpha - 1)}, \\ a_M &= V'. \end{aligned}$$

Substituting in (9) and rearranging terms leads to

$$\rho V = \frac{(B - \alpha c)^2}{2(2\alpha - 1)} + \frac{\gamma V'}{(2\alpha - 1)} \left(\alpha \gamma V' + B - \alpha c \right) + 1/2 \left(V' \right)^2 - \delta B V'. \quad (10)$$

I conjecture a quadratic value function given by

$$V(B) = \frac{1}{2} \varphi_1 B^2 + \varphi_2 B + \varphi_3.$$

Substituting V and V' by their values in (10) yields

$$\begin{aligned} \rho \left(\frac{1}{2} \varphi_1 B^2 + \varphi_2 B + \varphi_3 \right) &= \frac{\gamma (\varphi_1 B + \varphi_2)}{(2\alpha - 1)} (\alpha\gamma (\varphi_1 B + \varphi_2) + B - \alpha c)^2 \\ &\quad + \frac{(B - \alpha c)^2}{2(2\alpha - 1)} + 1/2 (\varphi_1 B + \varphi_2) - \delta B (\varphi_1 B + \varphi_2). \end{aligned}$$

By identification, and after rearranging terms, I get the following system:

$$\varphi_1^2 \left(2\alpha\gamma^2 + 2\alpha - 1 \right) - \varphi_1 \left((2\delta + \rho)(2\alpha - 1) - 2\gamma \right) + 1 = 0,$$

$$\begin{aligned}
\varphi_1\varphi_2\left(2\alpha\gamma^2+2\alpha-1\right)-\varphi_2\left((\delta+\rho)(2\alpha-1)-\gamma\right)-\alpha c\left(\gamma\varphi_1+1\right) &= 0,\\
2(2\alpha-1)\rho\varphi_3-\varphi_2^2\left(2\alpha\gamma^2+2\alpha-1\right)+2\alpha c\gamma\varphi_2-\alpha^2c^2 &= 0.
\end{aligned}$$

Solving the first equation gives the following for φ_1:

$$\varphi_1=\frac{((2\delta+\rho)(2\alpha-1)-2\gamma)\pm\sqrt{((2\delta+\rho)(2\alpha-1)-2\gamma)^2-4\left(2\alpha\gamma^2+2\alpha-1\right)}}{2\left(2\alpha\gamma^2+2\alpha-1\right)},$$

provided that $\left(2\alpha\gamma^2+2\alpha-1\right)\neq 0$. By substitution, one gets φ_2 and φ_3.

Note that one must usually conduct a stability analysis to choose the right root for φ_1. Since the objective here is to see if a two-part tariff can lead to the coordinated tariff, there is no need to do so here, since I can always impose restrictions on the parameters in order to get stability. ■

Proof of Proposition 2. Denote by $N_M(B)$ and $N_R(B)$ the HJB equation of the manufacturer and the retailer, respectively. They are given by

$$\begin{aligned}
\rho N_M &= \max_{a_M\geq 0}\left(k-\frac{a_M^2}{2}+N_M'\left(a_M+\gamma a_R-\delta B\right)\right),\\
\rho N_R &= \max_{p\geq 0,a_R\geq 0}\left(-k+(p-c)\left(B-\alpha p+a_R\right)-\frac{a_R^2}{2}+N_R'\left(a_M+\gamma a_R-\delta B\right)\right).
\end{aligned}$$

Maximizing the right-hand side of the manufacturer's HJB with respect to a_M and the retailer's HJB with respect to p and a_R gives

$$\begin{aligned}
a_M &= N_M',\\
p &= \frac{B+c(\alpha-1)+\gamma N_R'}{(2\alpha-1)},\\
a_R &= \frac{B-\alpha c+2\alpha\gamma N_R'}{(2\alpha-1)}.
\end{aligned}$$

If this solution is to coincide with the integrated-channel one, then it must hold that

$$N'=\varphi_1 B+\varphi_2=N_M'=N_R'.$$

Substitute in the HJB function for $a_M=N_M'$ and $a_R=\frac{B-\alpha c+2\alpha\gamma N_R'}{(2\alpha-1)}$ to get

$$\rho N_M=k+\frac{\left(N_M'\right)^2}{2}+N_M'\left(\gamma\left(\frac{B-\alpha c+2\alpha\gamma N_R'}{(2\alpha-1)}\right)-\delta B\right).$$

Assuming that $N_M'=\varphi_1 B+\varphi_2, N_M=\frac{1}{2}\varphi_1 B^2+\varphi_2 B+z_3$, where z_3 is a constant, and $N_R'=\varphi_1 B+\varphi_2$ and substituting in the above equation yields

$$\rho\left(\frac{1}{2}\varphi_1 B^2+\varphi_2 B+z_3\right) = k+\frac{\left(\varphi_1 B+\varphi_2\right)^2}{2}-\delta B\left(\varphi_1 B+\varphi_2\right)$$

$$+\frac{\gamma(\varphi_1 B+\varphi_2)}{(2\alpha-1)}\left(B-\alpha c+2\alpha\gamma(\varphi_1 B+\varphi_2)\right).$$

By identification and after rearranging terms, I get

$$\varphi_1^2\left(4\alpha\gamma^2+2\alpha-1\right)-\varphi_1\left((2\delta+\rho)(2\alpha-1)-2\gamma\right)=0.$$

The two possible solutions for φ_1 are

$$\varphi_1=0 \quad \text{or} \quad \varphi_1=\frac{(2\delta+\rho)(2\alpha-1)-2\gamma}{(4\alpha\gamma^2+2\alpha-1)}.$$

Since neither corresponds to the value obtained for φ_1 in the vertically integrated case, one concludes that the two solutions do not coincide. ■

Proof of Proposition 3. Denote by $C_R(B)$ the retailer's value function. The manufacturer announces the pair $(w(B), a_M(B)) = (c+\frac{k}{Q}, \varphi_1 B+\varphi_2)$. Taking this into account, the retailer's HJB equation then reads as follows:

$$\rho C_R=\max_{p\geq 0, a_R\geq 0}\left(-k+(p-c)(B-\alpha p+a_R)-\frac{a_R^2}{2}+C_R'(\varphi_1 B+\varphi_2+\gamma a_R-\delta B)\right). \tag{11}$$

Differentiating the right-hand side with respect to p and a_R and equating to zero yields

$$\begin{aligned} p &= \frac{B+c(\alpha-1)+\gamma C_R'}{(2\alpha-1)}, \\ a_R &= \frac{B-\alpha c+2\alpha\gamma C_R'}{(2\alpha-1)}. \end{aligned}$$

Inserting in (11) gives

$$\begin{aligned} \rho C_R &= \frac{1}{2(2\alpha-1)}\left((B-\alpha c)^2+2\alpha\gamma^2\left(C_R'\right)^2+2\gamma(B-\alpha c)C_R'\right) \\ &\quad +C_R'(\varphi_1 B+\varphi_2)-\delta B C_R'-k. \end{aligned} \tag{12}$$

I conjecture the following quadratic value function:

$$C_R(B)=\frac{1}{2}\theta_1 B^2+\theta_2 B+\theta_3.$$

Substituting for C_R and C_R' in (12), I get

$$\begin{aligned} \rho\left(\frac{1}{2}\theta_1 B^2+\theta_2 B+\theta_3\right) &= \frac{\left[2\alpha\gamma^2(\theta_1 B+\theta_2)^2+(B-\alpha c)^2+2\gamma(B-\alpha c)(\theta_1 B+\theta_2)\right]}{2(2\alpha-1)} \\ &\quad +(\theta_1 B+\theta_2)(\varphi_1 B+\varphi_2)-\delta B(\theta_1 B+\theta_2)-k. \end{aligned}$$

By identification, and after rearranging terms, I obtain the following system:

$$2\alpha\gamma^2\theta_1^2 - \theta_1\left(\left(2\alpha-1\right)\left(2\delta+\rho-2\varphi_1\right)-2\gamma\right)+1=0, \tag{13}$$

$$2\theta_1\theta_2\alpha\gamma^2 + \theta_1\left(\varphi_2\left(2\alpha-1\right)-\alpha c\gamma\right) - \theta_2\left(\left(2\alpha-1\right)\left(\delta+\rho-\varphi_1\right)-\gamma\right)=0, \tag{14}$$

$$2\left(2\alpha-1\right)\left(\rho\theta_3-\theta_2\varphi_2+k\right)-2\alpha\gamma^2\theta_2^2-\alpha^2c^2+2\alpha c\gamma\theta_2=0. \tag{15}$$

Solving the first equation gives the following for θ_1:

$$\theta_1 = \frac{\left(2\delta+\rho-2\varphi_1\right)\left(2\alpha-1\right)-2\gamma \pm \sqrt{\left(\left(2\delta+\rho-2\varphi_1\right)\left(2\alpha-1\right)-2\gamma\right)^2-8\alpha\gamma^2}}{4\alpha\gamma^2}.$$

By substitution, I obtain the values of θ_2 and θ_3 :

$$\begin{aligned}
\theta_2 &= \frac{\theta_1\left(\varphi_2\left(2\alpha-1\right)-\alpha c\gamma\right)}{\left(2\alpha-1\right)\left(\delta+\rho-\varphi_1\right)-2\theta_1\alpha\gamma^2-\gamma}, \\
\theta_3 &= \frac{2\left(2\alpha-1\right)\left(\theta_2\varphi_2-k\right)+2\alpha\gamma^2\theta_2^2+\alpha^2c^2-2\alpha c\gamma\theta_2}{2\rho\left(2\alpha-1\right)}.
\end{aligned}$$

Now, the retail price and the retailer's advertising policies will be identical in both scenarios if and only if $\varphi_1 = \theta_1$ and $\varphi_2 = \theta_2$. Indeed,

$$\begin{aligned}
p^v &= p \Leftrightarrow \frac{B+c(\alpha-1)+\gamma\left(\varphi_1 B+\varphi_2\right)}{\left(2\alpha-1\right)} = \frac{B+c(\alpha-1)+\gamma\left(\theta_1 B+\theta_2\right)}{\left(2\alpha-1\right)} \\
&\Leftrightarrow \left(\varphi_1 B+\varphi_2\right) = \left(\theta_1 B+\theta_2\right) \\
a_R^v &= a_R \Leftrightarrow \frac{B-\alpha c+2\alpha\gamma\left(\varphi_1 B+\varphi_2\right)}{\left(2\alpha-1\right)} = \frac{B-\alpha c+2\alpha\gamma\left(\theta_1 B+\theta_2\right)}{\left(2\alpha-1\right)} \\
&\Leftrightarrow \left(\varphi_1 B+\varphi_2\right) = \left(\theta_1 B+\theta_2\right)
\end{aligned}$$

It suffices to show that $\theta_1 \neq \varphi_1$ to get the result. Recall that these two value functions' parameters are the solutions to the following equations:

$$\varphi_1^2\left(2\alpha\gamma^2+2\alpha-1\right)-\varphi_1\left(\left(2\delta+\rho\right)\left(2\alpha-1\right)-2\gamma\right)+1 = 0 \tag{16}$$

$$2\alpha\gamma^2\theta_1^2-\theta_1\left(\left(2\alpha-1\right)\left(2\delta+\rho-2\varphi_1\right)-2\gamma\right)+1 = 0 \tag{17}$$

Assume that $\theta_1 = \varphi_1$ and compute the difference between the two equations. This gives

$$(16)-(17) = \varphi_1^2\left(2\alpha-1\right) \neq 0.$$

Recalling that $\alpha \neq 0.5$, therefore, pricing and advertising strategies are not the same in the two scenarios. Hence the result. ■

Proof of Proposition 4. Denote by $\tilde{N}_M(B)$ and $\tilde{N}_R(B)$ the HJB equation of the manufacturer and the retailer, respectively. They are given by

$$\begin{aligned}
\rho\tilde{N}_M &= \max_{a_M \geq 0}\left(k - \frac{a_M^2}{2} + \tilde{N}_M'(a_M - \delta B)\right), \\
\rho\tilde{N}_R &= \max_{p\geq 0, a_R \geq 0}\left(-k + (p-c)(B - \alpha p + a_R) - \frac{a_R^2}{2} + \tilde{N}_R'(a_M - \delta B)\right).
\end{aligned}$$

Maximizing the right-hand side of the manufacturer's HJB with respect to a_M and the retailer's HJB with respect to p and a_R gives

$$\begin{aligned}
a_M &= \tilde{N}_M', \\
p &= \frac{B + c(\alpha - 1)}{(2\alpha - 1)}, \\
a_R &= \frac{B - \alpha c}{(2\alpha - 1)}.
\end{aligned}$$

Substituting for a_M from above in the manufacturer's HJB leads to

$$\rho\tilde{N}_M = \left(k + \frac{\left(\tilde{N}_M'\right)^2}{2} - \delta B\tilde{N}_M'\right).$$

I conjecture the quadratic value function

$$\tilde{N}_M = \frac{1}{2}\tilde{v}_1 B^2 + \tilde{v}_2 B + \tilde{v}_3.$$

Substituting for $\tilde{N}_M$ and $\tilde{N}_M'$ in the manufacturer's HJB equation and identifying the parameters yields the following system:

$$\begin{aligned}
\frac{\rho}{2}\tilde{v}_1 &= \frac{1}{2}\tilde{v}_1^2 - \delta\tilde{v}_1, \\
\rho\tilde{v}_2 &= \tilde{v}_1\tilde{v}_2 - \delta\tilde{v}_2, \\
\rho\tilde{v}_3 &= k + \frac{1}{2}\tilde{v}_2^2.
\end{aligned}$$

It is easy to check that the above system has two solutions: $(\tilde{v}_1, \tilde{v}_2, \tilde{v}_3) = (0, 0, k/\rho)$ and $(\tilde{v}_1, \tilde{v}_2, \tilde{v}_3) = (2\delta + \rho, 0, k/\rho)$. I retain the second solution because it provides a higher total payoff to the manufacturer. The total payoff is given by the value function evaluated at the initial state B_0, that is,

$$\tilde{N}_M = \frac{1}{2}(2\delta + \rho) B_0^2 + k/\rho.$$

The advertising and the wholesale tariff are given by

$$\begin{aligned} a_M &= (2\delta+\rho)\,B, \\ w &= c+\frac{(2\alpha+1)\,k}{\alpha\,(B-\alpha c)}. \end{aligned}$$

Comparing the Nash equilibrium to the vertically integrated solution in Corollary 1 shows that they are not the same. ■

Proof of Proposition 5. Setting $\gamma = 0$ in Proposition 3 yields

$$\tilde{p}^C(B) = \frac{B+c(\alpha-1)}{(2\alpha-1)}, \quad \tilde{a}_R^C(B) = \frac{B-\alpha c}{(2\alpha-1)}, \quad \tilde{a}_M^C(B) = \tilde{\varphi}_1 B + \tilde{\varphi}_2,$$

with the constants $\tilde{\varphi}_1, \tilde{\varphi}_2$, and $\tilde{\varphi}_3$ given by

$$\begin{aligned} \tilde{\varphi}_1 &= \frac{(2\delta+\rho)}{2} \pm \sqrt{\frac{(2\delta+\rho)^2}{4} - \frac{1}{(2\alpha-1)}}, \\ \tilde{\varphi}_2 &= \frac{\alpha c}{(2\alpha-1)\,(\tilde{\varphi}_1-(\delta+\rho))}, \quad \tilde{\varphi}_3 = \frac{\tilde{\varphi}_2^2\,(2\alpha-1)+\alpha^2 c^2}{2\,(2\alpha-1)\,\rho}. \end{aligned}$$

Clearly, the above strategies are the same as the ones in Corollary 1. The wholesale two-part tariff is given by

$$w^C(B) = c + \frac{k}{Q} = c + \frac{(2\alpha+1)\,k}{\alpha\,(B-\alpha c)}.$$

■

Proof of Proposition 6. Denote by $\tilde{S}_R(B)$ the retailer's value function. Her HJB equation is given by

$$\rho\tilde{S}_R = \max_{p\geq 0, a_R\geq 0}\left((p-w)\,(B-\alpha p+a_R) - \frac{a_R^2}{2} + \tilde{S}_R'\,(a_M-\delta B)\right). \tag{18}$$

Differentiating the right-hand side and equating to zero provides the following reaction functions:

$$p(w,a_M) \equiv p(w) = \frac{B+w(\alpha-1)}{(2\alpha-1)}, \tag{19}$$

$$a_R(w,a_M) \equiv a_R(w) = \frac{B-\alpha w}{(2\alpha-1)}. \tag{20}$$

The manufacturer's HJB equation is given by

$$\rho\tilde{S}_M = \max_{w\geq 0, a_M\geq 0}\left[(w-c)\,(B-\alpha p+a_R) - \frac{a_M^2}{2} + \tilde{S}_M'\,(a_M-\delta B)\right].$$

Substituting from (19)-(20) for the values of p and a_R leads to

$$\rho\tilde{S}_M = \max_{w\geq 0, a_M\geq 0}\left[\frac{\alpha\,(w-c)\,(B-\alpha w)}{(2\alpha-1)} - \frac{a_M^2}{2} + \tilde{S}'_M\,(a_M-\delta B)\right]. \tag{21}$$

Differentiating the right-hand side with respect to w and a_M and equating to zero gives

$$\begin{aligned} w &= \frac{B+\alpha c}{2\alpha}, \\ a_M &= \tilde{S}'_M. \end{aligned}$$

Substituting in (21) leads to

$$\rho\tilde{S}_M = \frac{(B-\alpha c)^2}{4\,(2\alpha-1)} + \frac{\left(\tilde{S}'_M\right)^2}{2} - \delta B\tilde{S}'_M. \tag{22}$$

I conjecture a quadratic value function of the form

$$\tilde{S}_M = \frac{1}{2}\tilde{\lambda}_1 B^2 + \tilde{\lambda}_2 B + \tilde{\lambda}_3.$$

Then (22) can be written

$$\rho\left(\frac{1}{2}\tilde{\lambda}_1 B^2 + \tilde{\lambda}_2 B + \tilde{\lambda}_3\right) = \frac{(B-\alpha c)^2}{4\,(2\alpha-1)} + \frac{1}{2}\left(\tilde{\lambda}_1 B + \tilde{\lambda}_2\right)^2 - \delta B\left(\tilde{\lambda}_1 B + \tilde{\lambda}_2\right).$$

By identification, I get the following system:

$$\frac{\rho}{2}\tilde{\lambda}_1 = \frac{1}{4\,(2\alpha-1)} + \frac{1}{2}\tilde{\lambda}_1^2 - \delta\tilde{\lambda}_1, \tag{23}$$

$$\rho\tilde{\lambda}_2 = \frac{-\alpha c}{2\,(2\alpha-1)} + \tilde{\lambda}_1\tilde{\lambda}_2 - \delta\tilde{\lambda}_2, \tag{24}$$

$$\rho\tilde{\lambda}_3 = \frac{\alpha^2 c^2}{4\,(2\alpha-1)} + \frac{1}{2}\tilde{\lambda}_2^2. \tag{25}$$

Solving the first equation gives

$$\tilde{\lambda}_1 = \frac{(2\delta+\rho)}{2} \pm \sqrt{\frac{(2\delta+\rho)^2}{4} - \frac{1}{2\,(2\alpha-1)}}.$$

It suffices to proceed by successive substitution to get $\tilde{\lambda}_2$ and $\tilde{\lambda}_3$. ∎

References

[1] Chintagunta, P.K. and Jain, D. (1992), "A Dynamic Model of Channel Member Strategies for Marketing Expenditures", *Marketing Science*, 11(2), 168–188.

[2] Ingene, C.A. and Parry, M.E. (1995a), "Coordination and Manufacturer Profit Maximization: The Multiple Retailer Channel", *Journal of Retailing*, 72(2), 129–151.

[3] Ingene, C.A. and Parry, M.E. (1995b), "Channel Coordination when Retailers Compete", *Marketing Science*, 14(4), 360–377.

[4] Ingene, C.A. and Parry, M.E. (2000), "Is Channel Coordination All It Is Cracked Up to Be?", *Journal of Retailing*, 76(4), 511–547.

[5] Ingene, C.A. and Parry, M.E. (2004), *Mathematical Models of Distribution Channels* , International Series in Quantitative Marketing, Kluwer Academic Publishers, Boston.

[6] Jeuland, A.P. and Shugan, S.M. (1983), "Managing Channel Profits", *Marketing Science*, 2(3), 239–272.

[7] Jørgensen, S., Sigué, S. P. and Zaccour, G. (2000), "Dynamic Cooperative Advertising in a Channel", *Journal of Retailing*, 76(1), 71–92.

[8] Jørgensen, S., Sigué, S. P. and Zaccour, G. (2001), "Stackelberg Leadership in a Marketing Channel", *International Game Theory Review*, 3(1), 13–26.

[9] Jørgensen, S., Taboubi, S. and Zaccour, G. (2001), "Cooperative Advertising in a Marketing Channel", *Journal of Optimization Theory and Applications*, 110(1), 145–158.

[10] Jørgensen, S., Taboubi, S. and Zaccour, G. (2003), "Retail Promotions with Negative Brand Image Effects: Is Cooperation Possible?", *European Journal of Operational Research*, 150(2), 395–405.

[11] Jørgensen, S., Taboubi, S. and Zaccour, G. (2006), "Incentives for Retailer Promotion in a Marketing Channel", *Annals of the International Society of Dynamic Games*, Vol. 8.

[12] Jørgensen, S. and Zaccour, G. (1999), "Equilibrium Pricing and Advertising Strategies in a Marketing Channel", *Journal of Optimization Theory and Applications*, 102(1), 111–125.

[13] Jørgensen, S. and Zaccour, G. (2003), "Channel Coordination over Time: Incentive Equilibria and Credibility", *Journal of Economic Dynamics and Control*, 27, 801–822.

[14] Jørgensen, S. and Zaccour, G. (2004), *Differential Games in Marketing*, International Series in Quantitative Marketing, Kluwer Academic Publishers, Boston.

[15] Moorthy S. (1987), "Managing Channel Profits: Comments", *Marketing Science*, 6(4), 375–379.

[16] Nerlove, M. and Arrow, K.J. (1962), "Optimal Advertising Policy Under Dynamic Conditions", *Economica*, 39(114), 129–142.

[17] Raju, R. and Zhang, Z.J., (2005), "Channel Coordination in the Presence of a Dominant Retailer", *Marketing Science*, 24(2), 254–262.

[18] Taboubi, S. and Zaccour, G. (2005), "Coordination Mechanisms in Marketing Channels: A Survey of Game Theory Models", *Les Cahiers du GERAD*, G–2005–36, HEC Montréal.