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Abstract

In this paper, we model a two-period electricity market with interrelated demand, where oligopolistic generators make investments in peak- and base-load capacities. Different prices are obtained in the two periods, and residential consumers can react to prices across demand periods. We characterize the Cournot equilibrium obtained as a function of price and cross-price effects and present a numerical illustration based on the Ontario (Canada) electricity market.

Key Words: Interdependent Demand, Electricity, Nash Equilibrium, Oligopoly, Ontario (Canada).

Résumé

Cet article propose un modèle du marché de l'électricité caractérisé par deux types d'interactions : (1) interaction oligopolistique entre les générateurs et (2) interaction entre les demandes résidentielles d'électricité durant les périodes de pointe et de base. Cette seconde interaction est liée à l'élasticité croisée entre les demandes de pointe et de base. L'équilibre unique de Cournot est étudié, représentant les capacités de pointe et de base que les générateurs mettent sur le marché. L'application numérique basée sur le marché ontarien de l'électricité illustre le réalisme du modèle, ainsi que les impacts sur les capacités et les prix que des changements d'élasticité prix et d'élasticité croisée peuvent avoir.

Mots clés : Demandes interdépendantes, électricité, équilibre de Nash, oligopole, Ontario (Canada).

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1 Introduction

The absence of price signals in retail markets has been identified as one of the main determinants of the crises that occurred in some deregulated markets such as those of California or Ontario (see, e.g., Sweeney (2002) for the California case and Trebilcock and Hrab (2005) for the Ontario one, as well as for a comparative perspective[1]). The necessity of developing pricing schemes that could provide incentive for consumers, in order to manage demand efficiently is by no means a new topic. Indeed, Boiteux developed the ideas of marginal-cost and peak-load pricing long ago[2] (Boiteux (1949)). Since then, other significant contributions have been made, notably by Steiner (1957), Williamson (1966) and Turvey (1968). For a review of more recent contributions, the interested reader may consult Crew et al. (1995). The institutional framework in this literature is, unsurprisingly, one of a regulated, vertically integrated monopoly utility. It is probably not an overstatement to say that the recommendations made in such context are of marginal relevance to players engaged in a competitive electricity industry. In the latter context, the level of investment in production capacity, as well as the prices are expected to follow a market logic, i.e., to be endogenous to consumer behavior and to the degree of competition.

The economics and operations research literatures have long traditions modelling interdependent firms' choices of production capacities and outputs (or prices) in oligopolistic industries. In the electricity context, the change of competitive structure, from a regulated monopoly to a (de facto) oligopoly, triggered literature on the new problems that appeared and helped to design the market mechanisms that were put in place. Competition in spot markets has undoubtedly been an important issue. Contributions in this area include, among many others, those of Bohn et al. (1984), Green and Newbery (1992) and Bolle (1992). Given the nature of the object under investigation, their models ignored investment decisions, which are actually relevant only in a long-term perspective. Murphy and Smeers (2002) and Pineau and Murto (2003) are examples of the long-term perspective, where the models deal with both production and investment decisions. In these papers, the demands in different market segments, i.e., base-load and peak-load segments, are independent. This amounts to saying that the cross-price elasticity is zero. Such an assumption is also made in some recent models of the long-term impacts of real-time pricing (e.g., Boreinstein (2005)).

In this paper, we wish to study output (also to be interpreted as production capacities) decisions in an oligopolistic electricity industry, in which players serve two market segments, namely, base-load and peak-load segments, with two different production technologies characterized by different cost structures. Our main objective is to shed light on production strategies when the demands in both segments are interdependent, and more specifically, when they exhibit positive cross-price elasticities. This assumption has

[1] Lafferty et al. (2001) present the regulator's views on the importance of sending correct price signals to consumers to promote efficient investment.

[2] This theory had been presented in a half-century-old report of the Organisation for European Economic Co-operation (OEEC (1958)).

been documented in the econometric literature dealing with time-of-use tariffs. Indeed, Lawrence and Aigner (1979), Manning et al. (1979), Tishler and Ye (1993), Aigner et al. (1994), Filippini (1995), Mountain and Lawson (1995) and Matsukawa (2001) all depicted significant positive cross-price elasticities between price periods (segments).

The reaction of consumption patterns to price clearly has an impact on technology choices and on investments, even if this is not yet fully understood in the new deregulated environment. These issues are of strategic importance to electricity firms in terms of the technology mix of base-load and peak-load capacities. They are also relevant to governments facing environmental constraints and politically sensitive to price levels.

In this paper, we present a model of an electricity market, characterized by n investors-suppliers, two technologies (base- and peak-load technologies) and two interdependent market segments (base- and peak-load) with different own- and cross-price parameters. Using this parsimonious model, we will characterize equilibrium output strategies and investigate their sensitivity with respect to key parameters. Our main contribution lies in the study of this equilibirum with respect to price and cross-price parameters. We show that positive cross-price elasticities reduce capacity in both market segments and lead to opposite price changes in each segment. Variations in price elasticities have opposite impacts in each market segment, but result in overall greater (lower) capacity when price elasticity increases (decreases). An empirical application illustrates these results in a realistic setting.

The rest of the paper is organized as follows: In Section 2, we introduce the model. In Section 3, we derive the unique Nash equilibrium in output strategies and present some comparative statics results. In Section 4, we apply the model to the Ontario (Canada) market. In Section 5, we briefly conclude.

2 The Model

2.1 Demand Characterization

Consider n electricity producers (players) competing à la Cournot in a given market. Each player has at her disposal two types of production technology, to which we shall generically refer as base-load and peak-load capacities. Denote by $q_{ib}, i = 1, \ldots, n$, the hourly quantity produced by the base-load capacity and similarly by q_{ip} its peak-load counterpart. Let $Q_b = \sum_{i=1}^{n} q_{ib}$ and $Q_p = \sum_{i=1}^{n} q_{ip}$ be the total quantity put on the market by all producers during the base-load and peak-load periods, respectively.

Denote by T_b the tariff (or price) of a kilowatthour (kWh) during the base-load period and by T_p the tariff during the peak-load period. Both tariffs are endogenous and are assumed to be given by the following inverse demands

$$T_b = \alpha_b - \beta_b Q_b - \gamma_b Q_p, \tag{1}$$

$$T_p = \alpha_p - \beta_p Q_p - \gamma_p Q_b, \tag{2}$$

where α_j, β_j and $\gamma_j, j \in \{base, peak\}$, are positive parameters. This linear specification is rather standard in economics and implicitly assumes that the (nonnegative) quantities and parameters' values are such that prices in equilibrium are nonnegative. It is easy to verify that the above system of inverse demands can be equivalently written as

$$Q_b = \left(\frac{\alpha_b\beta_p - \alpha_p\gamma_b}{\beta_b\beta_p - \gamma_b\gamma_p}\right) - \left(\frac{\beta_p}{\beta_b\beta_p - \gamma_b\gamma_p}\right) T_b + \left(\frac{\gamma_b}{\beta_b\beta_p - \gamma_b\gamma_p}\right) T_p, \quad (3)$$

$$Q_p = \left(\frac{\alpha_p\beta_b - \alpha_b\gamma_p}{\beta_b\beta_p - \gamma_b\gamma_p}\right) - \left(\frac{\beta_b}{\beta_b\beta_p - \gamma_b\gamma_p}\right) T_p + \left(\frac{\gamma_p}{\beta_b\beta_p - \gamma_b\gamma_p}\right) T_b. \quad (4)$$

We make the following assumptions:

A1: $\beta_j > \max\{\gamma_b, \gamma_p\}, \quad j \in \{b, p\}$,

A2: $\alpha_p > \alpha_b$,

A3: $\dfrac{\gamma_b}{\beta_p} < \dfrac{\alpha_b}{\alpha_p}$

A4: $\gamma_b > \gamma_p$.

Assumption *A1* implies that $\beta_b\beta_p - \gamma_b\gamma_p > 0$ and hence, that the total quantities in (1) and (2) are, as they should be, decreasing in own price and increasing in the price of the substitute. This assumption also says that the direct-price effect is greater than the cross-price effect, i.e., $\beta_b > \gamma_b$ and $\beta_p > \gamma_p$. In terms of inverse demand, *A1* states that the price in one period is more sensitive to the quantity supplied during that period than to the quantity supplied in the other period. The parameter α_b, respectively α_p, can be interpreted as the maximum price consumers are willing to pay for a unit Q_b (respectively Q_p). By the very nature of the peak-load demand, it is intuitive to assume that α_p is higher than α_b. These constant terms in (1) and (2) represent the levels of demand when both prices are zero. Hence, they must be positive. By *A1* and *A2,* the constant term in (4) is clearly positive. By assumption *A3,* we ensure that the constant term in (3) is also positive. Finally, assumption *A4* states that base-load demand is more sensitive to the peak-load tariff than the other way around.

2.2 Cost Characterization

Each type of production capacity is characterized by an operating cost and a capital (or acquisition) cost. To keep things simple, we assume, not unrealistically, that all players are using the same base-load technology and the same peak-load technology, and hence face the same cost structure. Let the marginal operating cost be denoted by v_j and the annualized capital cost be denoted by $K_j, j = b, p$. We assume that the base-load capacity is in operation all the time. Let t be the given duration of the base-load reference period, i.e., $t = 8,760$ hours in a year (and $8,784$ in leap years such as 2004). The total cost of the base-load capacity per unit of energy (kilowatthour or kWh) is therefore given by

$$c_b = v_b + k_b,$$

where $k_b = \frac{K_b}{t}$. We assume, naturally, that the unit cost is less than the consumer's maximum willingness-to-pay price, i.e., $c_j < \alpha_j, j = b, p$.

Let τ represent the proportion of the reference period in which the peak-load capacity is needed. Thus, τt is the number of hours per year of operation at the peak-load capacity, i.e., the duration of the period during which the annualized capacity cost K_b is recovered. The total cost per unit of energy (kWh) is therefore given by

$$c_p = v_p + k_p,$$

where $k_p = \frac{K_p}{\tau t}$.

It is well known (see, e.g., Crew et al. (1995)) that the marginal operation cost of the base-load technology is lower than its peak-load counterpart, and that it is the other way around for capital cost, i.e.,

$$v_b < v_p, \quad K_b > K_p.$$

The choice between base- and peak-load technologies depends on how much a technology is used. This is measured by the capacity factor cf, which varies between 0 and 1. In a given period (a year, for instance), if $cf = 0$, only the capital cost K_j has to be paid because there is no production. When $cf = 1$, production happens all the time, so in addition to K_j, c_j times the total number of hours in the period (t) has to be paid. Figure 1 is a screening curve (see Stoft (2002)), depicting the total cost of using the base- and peak-load technologies when the capacity factor varies from 0 to 1. The screening curve in Figure 1 is based on cost parameters presented in Table 1 and used later in the numerical illustration (costs are based on Ayres et al. (2004)).

The screening curve should be interpreted as follows. The two curves intersect at a capacity factor $cf^* = 0.4377 = \tau$, where both technologies have the same total cost. As long as a power plant is used less than 43.77% of the time, the peak technology is cheaper. If a plant is used more than 43.77% of the time, then the base technology is cheaper. The threshold cf^* where technologies have the same cost is

$$cf^* = \frac{(K_p - K_b)}{(v_b - v_p)\, t} = \tau.$$

Finally, we assume profit maximization behavior. Player i's, $i = 1, \ldots, n$, objective function reads as follows:

$$\pi_i = (\alpha_b - \beta_b Q_b - \gamma_b Q_p)\, q_{ib} + (\alpha_p - \beta_p Q_p - \gamma_p Q_b)\, q_{ip} - c_b q_{ib} - c_p q_{ip}.$$

Table 1: Technology Costs

Technology	Investment cost (\$/MW)	K_j	v_j
Base-load	2,000,000	179,420	10
Peak-load	500,000	44,860	45
Values of K_j are for a 25-year amortization period and a 7.5% discount rate.			

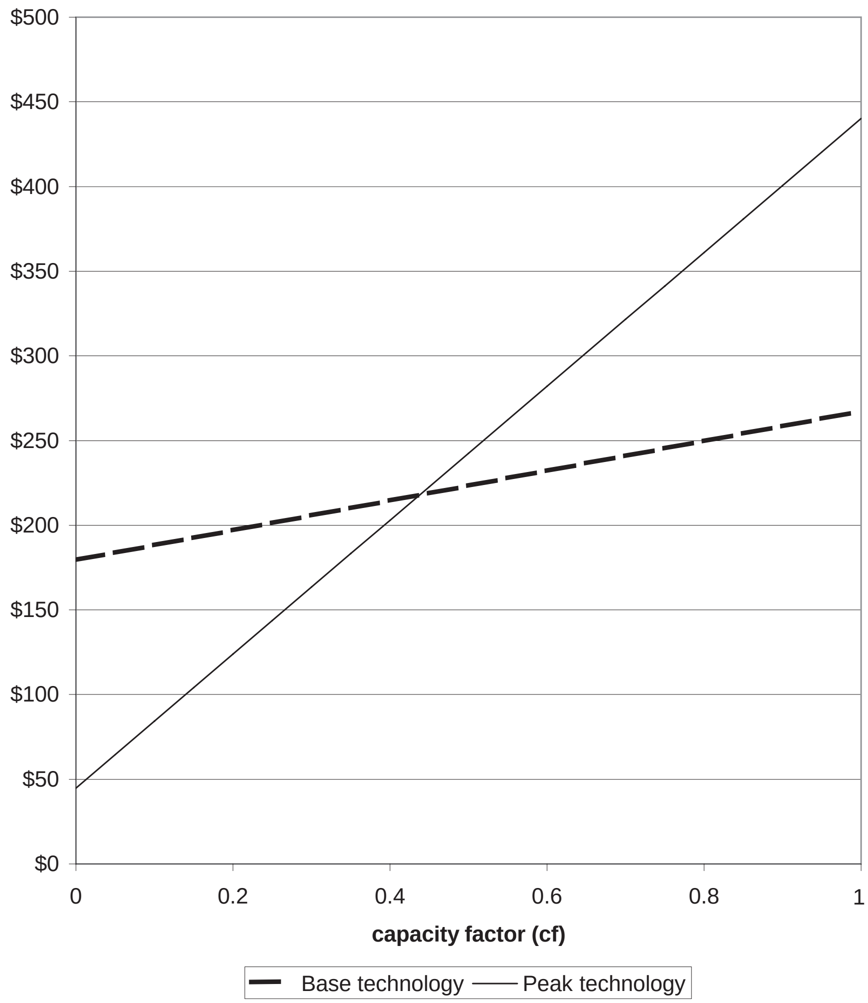

Figure 1: Screening Curve

Although the model just introduced is simple and parsimonuous, it allows us to capture the main features of interest in our context, namely the interdependency between base-load and peak-load demands, and the strategic interaction between players in term of production decisions or capacity choices[3].

[3]As the duration of base and peak loads are exogenous to the model (and given by cost characteristics), quantity and capacity choices are equivalent. The following relationship links quantity and capacity: $Capacity_{base} = \frac{Q_b}{T}$ and $Capacity_{peak} = \frac{Q_p}{t}$.

3 Equilibrium Results

To focus on the interplay between the two types of demand and the impact of the number of players on output (or production capacity), we shall confine our interest to the analysis of symmetric equilibria. The following proposition characterizes the unique Nash equilibrium.

Proposition 1 *Assuming an interior solution, the unique Nash equilibrium is given by*

$$q_b^* = \frac{(n+1)\,\beta_p\,[\alpha_b - c_b] - (\gamma_b n + \gamma_p)\,[\alpha_p - c_p]}{(n+1)^2\beta_p\beta_b - (\gamma_p n + \gamma_b)\,(\gamma_b n + \gamma_p)}, \tag{5}$$

$$q_p^* = \frac{(n+1)\,\beta_b\,[\alpha_p - c_p] - (\gamma_p n + \gamma_b)\,[\alpha_b - c_b]}{(n+1)^2\beta_p\beta_b - (\gamma_p n + \gamma_b)\,(\gamma_b n + \gamma_p)}. \tag{6}$$

Proof. The first-order conditions for a Nash equilibrium are given by

$$\frac{\partial \pi}{\partial q_{ib}} = (\alpha_b - \beta_b Q_b - \gamma_b Q_p) - \beta_b q_{ib} - \gamma_p q_{ip} - c_b = 0,$$

$$\frac{\partial \pi}{\partial q_{ip}} = (\alpha_p - \beta_p Q_p - \gamma_p Q_b) - \beta_p q_{ip} - \gamma_b q_{ib} - c_p = 0.$$

Under the symmetry assumption where $q_{ib} = q_b$ and $q_{ip} = q_p$ for $i = 1, \ldots, n$, the above conditions become

$$\frac{\partial \pi}{\partial q_b} = \alpha_b - (n+1)\,\beta_b q_b - (\gamma_b n + \gamma_p)\,q_p - c_b = 0$$

$$\frac{\partial \pi}{\partial q_p} = \alpha_p - (n+1)\beta_p q_p - (\gamma_p n + \gamma_b)\,q_b - c_p = 0$$

Which leads to (5)-(6).

Finally, it is easy to verify that each player's optimization problem is strictly concave in her decision variables. Therefore the optimality conditions are necessary and sufficient. □

The equilibrium quantities q_b^* and q_p^* depend on all of the model's parameters, namely, on the number of players, the demand and the cost parameters. The derivation of this equilibrium is based on some (implicit) conditions that induce restrictions on the parameters' values. First, we have assumed that the solution is interior. It is easy to verify that, under assumption *A1,* the denominators of q_b^* and q_p^* are strictly positive. Hence, if the numerators in (5) and (6) were strictly positive, then the solution would be interior. Positiveness of these numerators translates into the following condition:

$$\frac{(n+1)\,\beta_p}{(\gamma_b n + \gamma_p)} > \frac{[\alpha_p - c_p]}{[\alpha_b - c_b]} > \frac{(\gamma_p n + \gamma_b)}{(n+1)\,\beta_b}. \tag{7}$$

Given the definition of base-load and peak-load demands, the total equilibrium quantities must be such that $Q_b^* > Q_p^*$. In terms of the model's parameters, this condition is equivalent to

$$\frac{[\alpha_p - c_p]}{[\alpha_b - c_b]} < \frac{(n+1)\,\beta_p + (\gamma_p n + \gamma_b)}{(n+1)\,\beta_b + (\gamma_b n + \gamma_p)}. \tag{8}$$

Further, we require the equilibrium tariffs to be strictly positive, and the peak-load tariff to be higher than its base-load counterpart. We shall verify that the empirical results do indeed satisfy this, and the above, condition. To simplify the notation, we denote the denominator of the equilibrium quantities by

$$D = (n+1)^2\,\beta_p\beta_b - (\gamma_p n + \gamma_b)\,(\gamma_b n + \gamma_p)\,.$$

The following propositions provide some sensitivity analysis with respect to the model's parameters.

Proposition 2 *Each equilibrium quantity is decreasing in its production cost and increasing in the cost of the alternative technology.*

Proof. It suffices to differentiate (5) and (6) to get

$$\begin{aligned}\frac{\partial q_b^*}{\partial c_b} &= \frac{-\beta_p\,(n+1)}{D} < 0, \quad \frac{\partial q_p^*}{\partial c_p} = \frac{-\beta_b\,(n+1)}{D} < 0,\\ \frac{\partial q_b^*}{\partial c_p} &= \frac{\gamma_b n + \gamma_p}{D} > 0, \qquad \frac{\partial q_p^*}{\partial c_b} = \frac{\gamma_p n + \gamma_b}{D} > 0.\end{aligned}$$

□

The proposition says that increasing the cost of a technology leads to a decrease in production (or capacity to be installed of this technology). Increasing the cost of the alternative technology leads to an increase in the production of the technology under consideration. These results are rather intuitive. Indeed, increasing the cost leads to an increase in the price, which translates into less demand and hence less production.

Proposition 3 *Equilibrium quantities satisfy*

$$\begin{aligned}\frac{\partial q_i^*}{\partial \alpha_i} &> 0, \quad \frac{\partial q_i^*}{\partial \alpha_j} < 0, \quad i,j = b,p, i \neq j.\\ \frac{\partial q_i^*}{\partial \beta_i} &< 0, \quad \frac{\partial q_i^*}{\partial \beta_j} > 0, \quad i,j = b,p, i \neq j.\end{aligned}$$

Proof. Derivations of (5) and (6), and taking into account the condition for interior solution in (7) lead straightforwardly to:

$$\frac{\partial q_b^*}{\partial \alpha_b} = \frac{\beta_p\,(n+1)}{D} > 0, \quad \frac{\partial q_p^*}{\partial \alpha_p} = \frac{\beta_b\,(n+1)}{D} > 0,$$

$$
\begin{aligned}
\frac{\partial q_b^*}{\partial \alpha_p} &= -\frac{\gamma_b n+\gamma_p}{D}<0, \quad \frac{\partial q_p^*}{\partial \alpha_b}=-\frac{\gamma_p n+\gamma_b}{D}<0,\\
\frac{\partial q_b^*}{\partial \beta_b} &= \frac{\beta_p (n+1)^2}{D^2}\left[(\alpha_p-c_p)(\gamma_b n+\gamma_p)-\beta_p (n+1)(\alpha_b-c_b)\right]<0\\
\frac{\partial q_b^*}{\partial \beta_p} &= -\frac{(n+1)(\gamma_b n+\gamma_p)}{D^2}\left[(\alpha_b-c_b)(\gamma_p n+\gamma_b)-\beta_b (n+1)(\alpha_p-c_p)\right]>0,\\
\frac{\partial q_p^*}{\partial \beta_p} &= \frac{\beta_b (n+1)^2}{D^2}\left[(\alpha_b-c_b)(\gamma_p n+\gamma_b)-\beta_b (n+1)(\alpha_p-c_p)\right]<0,\\
\frac{\partial q_p^*}{\partial \beta_b} &= -\frac{(n+1)(\gamma_p n+\gamma_b)}{D^2}\left[(\alpha_p-c_p)(\gamma_b n+\gamma_p)-\beta_p (n+1)(\alpha_b-c_b)\right]>0.
\end{aligned}
$$

□

The results show that increasing the parameter $\alpha_i, i=b,p$, leads to an increase in the supply in segment i and to a decrease in the alternative segment. The conclusions with respect to variations of $\beta_i, i=b,p$, are the other way around. Given that the parameter α_i represents the consumer's willingness to pay, and β_i the consumer's price sensitivity, the results are not surprising. In terms of the magnitude of the effects in the two segments, note that, by assumption *A1*, the following relationships hold true

$$
\left|\frac{\partial q_i^*}{\partial \alpha_i}\right| > \left|\frac{\partial q_j^*}{\partial \alpha_i}\right|, \qquad \left|\frac{\partial q_i^*}{\partial \beta_i}\right| > \left|\frac{\partial q_j^*}{\partial \beta_i}\right|, \quad i,j=b,p, \quad i\neq j.
$$

The above inequalities say that the direct impact is higher, in absolute terms, than the indirect one for both parameters. This leads, among other things, to the following observation. If the consumer is willing to pay more for electricity in the peak-load market, then the demand would increase in this segment without decreasing in the same magnitude in the alternative base-load market. Hence a higher total production capacity would be required to meet total demand. If the latter cannot be increased instantaneously (in our model it can), then an excess demand can result. This is one possible reading of the California crisis.

The important item in our model is the introduction of the cross-elasticity terms in the demand functions. Performing a sensitivity analysis on the equilibrium quantities with respect to $\gamma_i, i=b,p$, leads to large expressions without a clear sign (unless we introduce additional restrictions on the parameters). An interesting result can nevertheless be obtained by comparing the polar cases of positive and zero cross-price elasticities. Indeed, computing the differences in equilibrium quantities gives

$$
\begin{aligned}
q_b^*(\gamma>0)-q_b^*(\gamma=0) &= \frac{(\gamma_b n+\gamma_p)\left[(\gamma_p n+\gamma_b)(\alpha_b-c_b)-(n+1)\beta_b(\alpha_p-c_p)\right]}{D(n+1)\beta_b},\\
q_p^*(\gamma>0)-q_p^*(\gamma=0) &= \frac{(\gamma_p n+\gamma_b)\left[(\gamma_b n+\gamma_p)(\alpha_p-c_p)-(n+1)\beta_p(\alpha_b-c_b)\right]}{D(n+1)\beta_p},
\end{aligned}
$$

where $\gamma = (\gamma_b, \gamma_p)$. Under the assumption of interior solution in (7), it is easy to see that both differences are negative. Thus, if the two market segments were connected, then in both segments the required capacity would be lower than in they were isolated. Governments facing environmental constraints and interested in reducing the total capacity of electricity production (or at least, reducing the rate of increase of this capacity) should encourage the fluidity between the two segments by encouraging consumers to become switchers. This behavior can occur if tariffs are lower. Recall that the tariffs are given by (1) and (2). Again comparing the two polar situations, we get

$$\begin{aligned} T_b^*(\gamma > 0) - T_b^*(\gamma = 0) &= \frac{(n+1)\beta_b(\alpha_p - c_p) - (\gamma_p n + \gamma_b)(\alpha_b - c_b)}{D}\left(\frac{n(\gamma_p - \gamma_b)}{n+1}\right), \\ T_p^*(\gamma > 0) - T_p^*(\gamma = 0) &= \frac{(n+1)\beta_b(\alpha_p - c_p) - (\gamma_p n + \gamma_b)(\alpha_b - c_b)}{D}\left(\frac{n(\gamma_b - \gamma_p)}{n+1}\right). \end{aligned}$$

Noting that $\gamma_b > \gamma_p$ (by assumption A4), clearly, the first difference is negative and the second one is positive. This means that if the demands in the two segments were connected, then the tariff in the base-load segment would decrease and the tariff in the peak-load segment would increase. These results are illustrated and further discussed in the next section.

4 Empirical Application

We shall illustrate our model and the type of insight it can provide with, the case of Ontario, Canada.

4.1 Model Calibration

In September 2004, Ontario had 30,922 MW of available generation capacity (IESO, 2005a). The load duration curve in Figure 2 shows how much of this capacity was used in 2004 (a leap year with 366 days or $t = 8,784$ hours). About 12,000 MW were used all the time, while about 6,000 MW were never used. More precisely, the lowest load was 11,983 MW (which took place at 4:00 a.m. on May 24) and the highest load was 24,979 MW (at 6:00 p.m. on December 20), see IESO (2005b).

Although loads in Figure 2 were supplied by various technologies using different fuels (see Table 2), we use the simplifying assumption than only two similar technologies are used, as presented in Table 1. This assumption is a close approximation of reality because nuclear, hydro and coal have low production cost and high capacity costs, while the other technologies have (relatively) high production costs and low capacity costs. See Ayres et al. (2004) or Royal Academy of Engineering (2004) for recent empirical studies on electricity production costs with various technologies.

Using the value $cf^* = 0.4377$ on the actual Ontario 2004 load duration curve, we find that it corresponds to a capacity of 18,115 MW. This means that, in Ontario, 18,115 MW

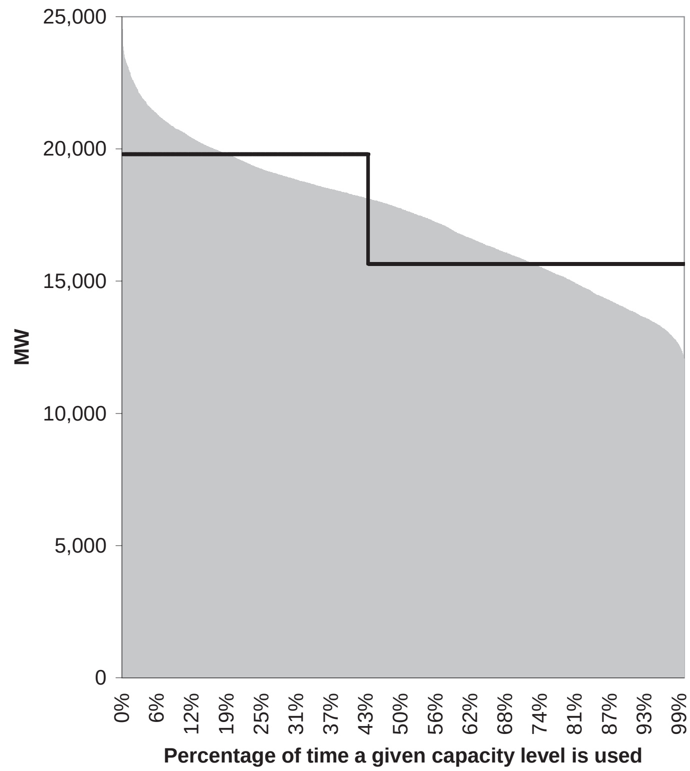

Figure 2: Ontario 2004 Load Duration Curve

were used more than 43.77% of the time in 2004, while the remaining capacity was used less than 43.77% of the time. In Figure 2, 18,115 MW is represented by the points where the vertical bold line intersects with the load duration curve.

If actual technology choices were to be made based on the technologies presented in Table 1 and on the load duration curve in Figure 2, then there should be 18,115 MW of base-load capacity and 6,864 MW of peak-load capacity.[4] However, for modelling purposes, as we only define two demand curves-one for each period-we assume that the average capacity is used all the time in its respective period. This corresponds to 15,652.66 MW during the base-load period and 19,799.35 MW during the peak-load period, so that 15,652.66 MW of base technology is required and used all the time, while 4,146.69 MW of peak technology is only used 43.77% of the time. Using these average capacities and load durations (the bold line in Figure 2), exactly the same amount of energy is obtained for the 2004 Ontario market: 153.47 TWh (see Tables 3 and 4).

[4]This is obtained by using the maximum load of 24,979 MW and planning for no reserve capacity (another simplifying assumption, less realistic this time).

Table 2: Generation Capacities in Ontario, by Fuel Type

Fuel Type	Capacity (MW)
Nuclear	10,823.0
Hydro	7,894.9
Coal	7,205.0
Natural Gas	2,674.4
Oil/Gas	2,100.0
Wood Waste	224.6
Total	30,921.9

Table 3: Capacity, Energy Used and Price per Period

Period	Number of Hours	Average Capacity (MW)	Energy Used (MWh)	Average Price ($ per MWh)
Base	4,939	15,652.66	77,308,469	40.08
Peak	3,845	4,146.69	76,128,501	64.58
Total	8,784	19,799.35	153,436,970	52.21

Table 4: Energy Generated by Type of Technology

Technology	Percentage of time	Number of Hours	Capacity (MW)	Energy Generated (MWh)
Base	100.00%	8,784	15,652.66	137,492,932.11
Peak	43.77%	3,845	4,146.69	15,944,037.89
Total			19,799.35	153,436,970

The spot market prices of electricity ranged from $5.25/MWh (at 6:00 a.m. on July 6) to $340.45/MWh (at 6:00 p.m. on January 14) with an average of $52.21/MWh.[5] During the peak hours, the average price was $64.52/MWh and only $40.08/MWh during the base-load hours.

In the following, we consider that the demand for electricity is based on the actual energy used during the base- and peak-load periods. We assume that this energy is used uniformly in each period, over a length of t hours during the base-load period, and for the peak-load period, over a total of cf^*t hours. Table 4 illustrates this by providing the total energy generated by each technology.

Given the technologies described in Table 1 and the value of cf^*, the cost parameters can be assessed and are given in Table 5.

[5] This is the weighted average price, or the price paid on average, for each MWh of electricity sold and bought on the spot market. The unweighted average price, not taking quantity into account, was $49.95/MWh in 2004.

Table 5: Cost parameters

Technology	Number of Hours	Costs			
	t	K_j	k_j	v_j	c_j
Base	8,784	\$179,420	\$20.42	\$10	\$30.42
Peak	3,845	\$44,860	\$11.66	\$45	\$56.66

Table 6: Residential Price Elasticities

	Short Run		Long Run	
	Base ε_b	Peak ε_p	Base ε_b	Peak ε_p
Taylor and Schwarz (1990)			−0.26 to −0.29	−1.02 to −1.93
Filippini (1995)	−2.3 to −2.36	−1.25 to −1.29		
Mountain and Lawson (1995)	−1.01 to −1.49	−0.61 to −0.99		
Stevens and Lerner (1996)	−0.06 to −0.49		−0.51 to −1.82	

4.1.1 Demand Parameters In order to estimate the parameters α_j, β_j and γ_j of inverse demand curves in (1) and (2), we shall essentially rely on published studies. Table 6 reports different estimates for short- as well as long-term price elasticities in the residential market. As we can see, the estimates differ, sometimes greatly, from one study to another. This is not really surprising and is generally attributed to the following factors. First, climate and type of appliances have a significant impact on price (and cross-price) elasticities (Caves et al. (1984)). Second, different methodologies and experimental designs lead to different contexts, not necessarily comparable. In any event, we shall conduct some sensitivity analyses to see the impact of varying this on the output (or capacity) strategies. Note, however, that the estimates in Table 6 share the following features: (i) as expected, long-run elasticities are greater than shorter ones; and (ii) base-load demand is more elastic than peak-load demand, especially in the short run (Taylor and Schwarz (1990), however, find that, in the long run, peak demand is more elastic). See Stevens and Lerner (1996) for a more general discussion on the price elasticities of electricity.

The interrelationship between the quantities and prices of peak and base loads can be studied through the cross-price elasticity and the elasticity of substitution. The cross-price elasticity of base-load demand with respect to peak-load electricity is given by:

$$\varepsilon_{bp} = \frac{\partial Q_b}{\partial T_p} \cdot \frac{T_p}{Q_b}.$$

The cross-price elasticity of peak- to base-load electricity is symmetrically defined as ε_{pb}. The elasticity of substitution in our context "measures the percentage reduction in the [peak] to [base] usage ratio for each one percent [increase] in the [peak] to [base] price ratio" (Baladi et al. 1998:238-239). See Stern (2004) for a general and thorough survey of various types of elasticities of substitution. Table 7 presents various estimates of cross-price elasticities and elasticities of substitution for peak- and base-load electricity from residential time-of-use experiments.

The estimates given in Table 7 show that demand for electricity across periods is interdependent, and that the main impact is a substitution of base-load demand to peak-load demand as price in the peak-load period increases. A shift to peak-load period also happens (cross-price elasticity of peak- to base-load period, ε_{pb}) but to a much lower extent, as shown in Mountain and Lawson (1995) and Filippini (1995).

Based on the values for price and cross-price elasticities shown above, on actual quantities Q_b and Q_p purchased in Ontario in 2004 (see Table 4) and on the uniform price of \$52.21/MWh in both periods (as most consumers faced in Ontario in 2004), we can estimate the parameters for demand curves (3) and (4). Table 8 presents the assumptions of price and cross-price elasticities, and Table 9, the resulting inverse demand function parameters. It can easily be verified that these values satisfy assumptions *A1-A4*.

Table 7: Cross-Price Elasticities and elasticities of Substitution

	Cross-Price elasticity		Elasticity of Substitution
Lawrence and Braithwait (1979)	ε_{bp}	0.08 to 0.15	
Caves, et al. (1984)			0.07 to 0.194
Filippini (1995)	ε_{pb}	0.34	2.56
	ε_{bp}	0.97	
Mountain and Lawson (1995)	ε_{pb}	0.009 to 0.088	
	ε_{bp}	0.03 to 0.141	
Baladi et al. (1998)			0.127 to 0.173

Table 8: Direct- and Cross-Price Elasticities (Reference Case)

Period	Price Elasticity	Cross-Price Elasticity	$\left(\frac{\alpha_b\beta_p-\alpha_i\gamma_j}{\beta_b\beta_p-\gamma_b\gamma_p}\right)$	$-\left(\frac{\beta_i}{\beta_b\beta_p-\gamma_b\gamma_p}\right)$	$\left(\frac{\gamma_j}{\beta_b\beta_p-\gamma_b\gamma_p}\right)$
Base	-1.2	0.05	295.61	-3.1602	0.1317
Peak	-0.8	0.01	28.54	-0.2443	0.0031

Table 9: Inverse Demand Functions Parameters (Reference Case)

Period	α_j	β_j	γ_j
Base	98.46	0.3166	0.1706
Peak	118. 05	4.0953	0.0039

Table 10: Capacities, Prices and Profits for Different Number of Players

		Capacity MW	Price ($)	Total Profit (10^6 $)
2004 Values	Base	15,652	40.08	
(Average Regulated Price)	Peak	4,147	64.58	
$n = 1$	Base	12,068	64.00	3,767
	Peak	1,361	96.19	
$n = 2$	Base	15,981	52.65	3,346
	Peak	2,068	84.92	
$n = 3$	Base	17,909	47.03	2,821
	Peak	2,468	78.56	
$n = 4$	Base	19,056	43.67	2,406
	Peak	2,722	74.51	
$n = 7$	Base	20,760	38.67	1,643
	Peak	3,124	68.13	
$n = 100$	Base	23,339	31.06	146
	Peak	3,787	57.60	

4.2 Simulation Results

Using the retained parameters' values in our model, we obtain the equilibrium results presented in Table 10 for capacity, prices and profits. These results are the reference case results for different numbers of players. These are results with cross-price elasticities, a characteristic that is not relevant for actual 2004 values, as consumers faced a uniform average price (despite the existence of a hourly spot price). With three players and positive cross-price elasticities, the total production capacity is comparable to the 2004 one (20,337 MW against 19,799 MW), but with more base-load and less peak-load capacity, as consumption adjustments have been made by consumers. If both cross-price elastcities were set equal to zero in the three-player case, then peak-load capacity would increase to 2,923 MW and base-load capacity would also increase to 18,349 MW (see Table 11), for a total of 21,272 MW. This illustrates the impact on peak-load capacity of positive cross-price elasticity, as formally established in Section 3.

It can be interesting to note that at the "competitive" level of $n = 100$, prices are almost at the cost level c_j, with levels of production much higher for base-load demand, but still below current peak-load demand.

Finally, if keeping price levels at the 2004 level were a (political) concern, it would require seven symmetric players under the model's parameters. With seven players, base-load price is slightly lower than the current one (at $38.67 per MWh) and peak-load price, slightly higher (at $68.13). However, base-load consumption would be much higher (requiring more than 5,000 MW of additional base-load capacity).

Table 11: Sensitivity Analysis for n=3

		Capacity MW	Price ($)	Total Profit (10^6 $)
Reference Case	$\varepsilon_b = -1.2; \varepsilon_{bp} = 0.05$	17,909	47.03	2,821
	$\varepsilon_p = -0.8; \varepsilon_{pb} = 0.01$	2,468	78.56	
No Cross-Price	$\varepsilon_b = -1.2; \varepsilon_{bp} = 0$	18,349	47.43	2,914
Elasticity	$\varepsilon_p = -0.8; \varepsilon_{pb} = 0$	2,923	72.01	
Elastic	$\varepsilon_b = -1.2; \varepsilon_{bp} = 0.05$	17,743	46.88	2,851
Demand	$\varepsilon_p = -1.1; \varepsilon_{pb} = 0.01$	3,400	78.50	
Inelastic	$\varepsilon_b = -0.9; \varepsilon_{bp} = 0.05$	13,414	47.01	2,156
Demand	$\varepsilon_p = -0.8; \varepsilon_{pb} = 0.01$	2,582	76.92	

These numerical results illustrate the complexity of simultaneously (1) introducing effective competition; (2) providing time-sensitive price signals to consumers; and (3) limiting capacity requirements. Indeed, effective competition requires a very large number of players. Competitive price signals (under realistic parameter values) quickly result in additional base-load capacity, along with a peak-load capacity reduction. However, the overall effect may require more base-load than under a system with no incentives to adjust consumption between demand segments. In our simulation, with limited competition (only three players) and therefore high prices, total capacity was already higher than the 2004 situation.

Table 11 presents numerical results for various elasticity scenarios. It illustrates proposition 3, showing how a change in parameters β_i or β_j(and consequently in elasticities) affects quantity q_i. In a case were both segments would be elastic (illustrated here by a change of ε_p from -0.8 to -1.1), capacity in peak periods increases by almost 1,000 MW while price goes down in both segments (in comparison to the reference case). Base-load capacity decreases by a much smaller quantity (less than 200 MW). This illustrates the somehow paradoxical situation where increasing cross-price elasticities leads to an overall reduction of capacity, while an increase in price-elasticity leads to increased capacity.

The reverse situation is illustrated by the last case considered, where price-elasticity decreases (in the base-load segment, from -1.2 to -0.9, with all other parameters remaining constant). This leads to a drastic reduction of base-load capacity (more than 4,000 MW), while only slightly increasing peak-load capacity (by a little more than 100 MW), and leaving prices almost unchanged compared to the reference case.

The policy advice for governments interested in reducing overall capacity (or in limiting capacity growth) would therefore be to reduce price elasticity and to increase the cross-price elasticity of demand. This could be done by creating disincentives for energy substitutions away from electricity, while making inter-temporal substitution of electricity consumption easier. As natural gas and oil prices increase and as technology allows a better control over the time of use of many appliances (dryers, water heaters, etc.), this set of disincentives and incentives could naturally lead to changes in market outcomes. Players' profits, however, are directly in opposition with these changes. Profits decrease with the development of cross-price elasticities and increase with a more elastic demand. (See Table 11 for an illustration of this).

5 Conclusion

The contribution of this paper is to study the oligopolistic equilibrium in an electricity market characterized by two interdependent demands. Despite the fact that the fluctuation of tariffs over time is an important characteristic of liberalized electricity markets, it is the first time such a question is studied in the literature. We establish the unique equilibrium under a realistic set of assumptions and study its sensitivity with respect to its parameters. Our main result, besides characterizing the equilibrium, is to show that an increased cross-price elasticity leads to an overall decrease of total capacity. Price-elasticity changes, reflected by a variation in parameter β_i's value, change quantities in both market segments, but more so in the segment where the elasticity changes. Our empirical application using the Ontario market shows how realistic the model is and illustrates how relatively small variations in elasticity and cross-price elasticity can affect the market equilibrium.

These results illustrate the significant implications of changing competition and tariff policies in electricity markets. This is especially relevant for governments trying to manage the growth of their electricity industry. Initiatives favoring cross-price elasticities but reducing price-elasticity could be pursued, but the difficult political implications related to price levels and producers' profits may make these changes more complex to implement in practice.

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