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Abstract

Installment options are a generalization of compound options, where the holder periodically decides whether to keep an option alive or not by paying the installment. We propose a numerical procedure, based on dynamic programming coupled with piecewise polynomial approximations, to price installment options when the underlying asset price follows a GARCH process. Numerical experiments are carried out using data from the Australian Stock Exchange. Computed option prices under GARCH and Black-Scholes models are compared to traded prices.

Key Words: Dynamic Programming, Option pricing, Installment Option.

Résumé

Les options à paiement différé sont une généralisation des options composées, où le détenteur doit décider à dates fixes s'il conserve l'option en vie en versant ou non un paiement. Dans cet article, nous proposons une procédure numérique, basée sur la programmation dynamique et l'approximation polynomiale par morceaux, pour la tarification d'options à paiement différé lorsque l'évolution du prix du sous-jacent est décrite par un processus GARCH. Nous présentons des expériences numériques utilisant des données de la bourse Australienne. Les prix d'options calculés à partir de modèles GARCH et Black-Scholes sont comparés aux prix de marché.

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1 Introduction

Installment derivatives have two important features differentiating them from other types of derivatives: the premium is paid periodically, at pre-specified dates (the installment schedule), and the holder has the right to stop making the payments, thereby terminating the contract. For instance, the holder of an American installment option may

1. At any date until maturity: exercise the option, which puts an end to the contract;
2. At any pre-specified date of the installment schedule: decide between paying the installment, which keeps the option alive until the next decision date, or not paying the installment, which puts an end to the contract.

Installment options (IO) introduce flexibility in the liquidity management of portfolio strategies. Instead of paying a lump sum for a derivative instrument, the holder of the IO will pay the installments as long as the need for being long in the option is present. In addition, the non-payment of an installment suffices to close the position at no transaction cost. This reduces the liquidity risk typically associated with other over-the-counter derivatives. Installment options can also be used as an analogy for real options (Davis et al. 2003) or for the valuation of callable corporate debt (Bounab 2005).

The analogy with compound options can be used for pricing European IOs. In that setting, Davis et al. (2001, 2002) derive no-arbitrage bounds for the price of the IO and study static versus dynamic hedging strategies within a Black-Scholes framework with stochastic volatility. In the constant volatility Black-Scholes (1973) setting, Wystup et al. (2004) use the compound options analogy to obtain a closed-form solution, which nonetheless requires the numerical evaluation of multivariate normal integrals.

A numerical algorithm to price American IOs under the Black-Scholes assumptions has been proposed by Ben-Ameur et al. (2004). They use a Dynamic Programming approach combined with a piecewise linear approximation of the option value. Ciurlia and Roko (2004) propose a closed-from formula for American options with continuous installments, again in the Black-Scholes, constant volatility setting.

These papers rely on the constant volatility assumption; an exception is Davis et al. (2001, 2002), who do not propose a pricing algorithm, but bounds based on hedging portfolios. However, empirical evidence supports time-varying volatility in financial time series. In particular, the *Generalized Autoregressive Conditional Heteroscedasticity* (GARCH) class of models is prominently used to estimate such series. No-arbitrage option pricing in a GARCH context was proposed by Duan (1995) and pricing methods for American-style options in the GARCH framework include lattice-based approaches (Ritchken and Trevor 1999, Duan et al. 2003), Markov chain approximation (Duan and Simonato 2001) and Dynamic Programming (Ben-Ameur et al. 2005).

The aim of this paper is to propose a numerical pricing algorithm for American Installment options in the GARCH setting. Similarly to Ben-Ameur et al. (2005), we use a piecewise polynomial approximation of the option value at discrete estimation times. Numerical experiments using data from the Australian Stock Exchange (ASX) are performed in order to investigate the properties of IOs in the GARCH setting and to compare the prices obtained using the GARCH and Black-Scholes (BS) specifications. We also examine the installment warrant contract traded on the ASX.

The rest of the paper is organized as follows. In Section 2, we briefly present the GARCH family of models. Section 3 is devoted to the model, the DP recursion and the approximation scheme. Section 4 discusses the Installment contract traded on the ASX and reports on numerical experiments. Section 5 concludes.

2 The GARCH Option Pricing Models

The GARCH specifications have been suggested as a response to a first generation of asset pricing models that assume a constant return volatility for underlying financial assets. In addition to accommodating changes in the volatility of asset returns, the large GARCH family allows for the modeling of the asymmetrical over reaction of the market to “bad” news.

We briefly present the discrete-time GARCH family of models. Let $(\Omega, \mathcal{F}, P)$ a filtered probability space, $\{S_t, \text{ for } t \geq 0\}$ an $\mathcal{F}-$adapted process as for a basic asset price (hereafter referred to as the stock), and $\{H_t, \text{ for } t \geq 1\}$ an $\mathcal{F}-$predictable process as for the volatility of the stock return. Here, $\mathcal{F}_t$ indicates the set of available information to investors from all relevant variables up to time t. The general form of a GARCH(p, q) model, as proposed by Bollerslev (1986), is

$$H_t = \mu + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j H_{t-j},$$

where p and $q \in \mathbb{N}$ are model-selection integers and $\mu > 0$, α_i and $\beta_j \geq 0$, for all i and j, are real parameters. The collection of random variables $\varepsilon_t/\sqrt{H_t} \mid \mathcal{F}_{t-1}$, for $t \geq 1$, are independent and play the role of an innovation process. Since then, various GARCH specifications were suggested, the most popular using $p = q = 1$. The difference between the various specifications mainly resides in the use of different, possibly asymmetrical, effects of the innovation process on the variable H_t. In the sequel, we use the specification of Duan and Simonato (2001), but the method can be easily modified to accommodate the others. Under that specification, the dynamic of the stock return and its conditional volatility under the data generating probability measure P is described by

$$\ln \frac{S_{t+1}}{S_t} = r + \lambda \sqrt{H_{t+1}} - \frac{1}{2} H_{t+1} + \sqrt{H_{t+1}} \varepsilon_{t+1} \quad (1)$$

$$\begin{aligned} H_{t+1} &= \beta_0 + \beta_1 H_t + \beta_2 H_t (\varepsilon_t - \theta)^2 \\ \varepsilon_{t+1} \mid \mathcal{F}_t &\sim \mathcal{N}(0,1), \end{aligned}$$

where r is the *risk-free interest-rate of return* over one period (continuously compounded), λ is a constant *risk premium*, and θ is a *leverage* parameter. The parameters β_0, β_1, and β_2 are assumed to satisfy $\beta_0 > 0$, $\beta_1 \geq 0$, $\beta_2 \geq 0$, and the *stationary* condition $\beta_1 + \beta_2 < 1$. In this setting, the vector process $\{(S_t, H_{t+1}), \text{ for } t \geq 0\}$ verifies the Markov property. The main result achieved by Duan (1995) is the existence of a $P-$equivalent probability measure Q, called *locally risk-neutral*, under which the process of the stock price discounted by the risk-free interest-rate of return is a martingale, and no-arbitrage pricing is valid. Under Q, the dynamic of the asset return and its conditional volatility is

$$\begin{aligned} \ln \frac{S_{t+1}}{S_t} &= r - \frac{1}{2} H_{t+1} + \sqrt{H_{t+1}} \epsilon_{t+1} \\ H_{t+1} &= \beta_0 + \beta_1 H_t + \beta_2 H_t (\epsilon_t - \theta - \lambda)^2 \\ \epsilon_{t+1} \mid \mathcal{F}_t &\sim \mathcal{N}(0,1) \end{aligned} \tag{2}$$

and the no-arbitrage price at t of a claim that promises an $\mathcal{F}_T-$measurable random payoff Y_T at T is given by:

$$\begin{aligned} v_t(s,h) &= E^Q \left[e^{-r(T-t)} Y_T \mid S_t = s \text{ and } H_{t+1} = h \right] \\ &\equiv E_{tsh} \left[e^{-r(T-t)} Y_T \right], \end{aligned} \tag{3}$$

under the condition that the right-hand side of (3) exists.

3 The Installment Option Pricing Model

3.1 Dynamic Programming Recursion

Consider an American-style installment option written on an underlying asset described by the GARCH process (1) with maturity T and exercise payoff $\kappa(t,s)$ when at time t the underlying asset's price is s. The no-arbitrage pricing equation (3) makes explicit the fact that the value of the option depends on the current date, denoted t, the observed price of the underlying asset, denoted s, and the measured volatility level at $t+1$, denoted h. Here, the time steps $t = 0, 1, \ldots, T$ correspond to the (equally spaced) time steps of the GARCH process.

Let $m_0 = 0$ be the installment option (IO) inception date and $\{m_1, \ldots, m_n\}$, $0 < m_1 < m_2 < \ldots < m_n < T$, a collection of n installment dates scheduled in the contract. We assume for simplicity that installment dates coincide with time steps (but not all time steps are installment dates). An installment design is characterized by the vector of premia $c = (c_1, ..., c_n)$ that are to be paid by the holder at dates $m_1, ..., m_n$ to keep the IO alive. The price of the IO is the up-front payment required to enter the contract at $t = 0$.

The value of the IO at time t is thus given recursively by:

$$v_t(s,h) = \max\left\{\kappa(t,s); E_{tsh}\left[e^{-r} v_{t+1}(S_{t+1}, H_{t+2})\right] - C_t; 0\right\}, \quad t = 0, \dots, T-1 \tag{4}$$

$$v_T(s,h) = \kappa(s,T) \tag{5}$$

where

$$C_t = \begin{cases} c_j \text{ if } t = m_j \\ 0 \text{ otherwise,} \end{cases}$$

and where $s = S_t$ is the price of the underlying asset at t, $h = H_{t+1}$ is the measured volatility at $t+1$, and $E_{tsh}\left[e^{-r} v_{t+1}(S_{t+1}, H_{t+2})\right]$ is defined by (3).

The three elements in the right hand side of (4) are respectively the exercise value, the (net) holding value, and the (null) abandonment value. Equation (4) thus models the choices that are available to the option holder: at an installment date, he will pay the installment and hold the option as long as the (net) holding value is positive and larger than the exercise value. Otherwise, according to the exercise payoff, he will either exercise the option (when positive) or abandon the contract (when null). At all other dates, the option holder will exercise the option if the exercise payoff is larger than the holding value. Notice that European installment options are special cases with $\kappa(t,s) = 0$ for $0 \leq t < T$.

One way of pricing this IO and obtain the value and up-front payment $v_0(S_0, H_1)$ is via backward induction using (4)-(5) from the known function $v_T(s,\cdot) = \kappa(T,s)$ and, simultaneously, to identify the optimal exercise and payment strategy. However, the value function v_t, for $t = 0, \dots, T$, cannot be obtained in closed-form and must be approximated in some way. We propose an approximation on a finite grid on the product space of asset prices and volatilities.

3.2 Approximation

We approximate the value of the installment option at some pricing date t by a piecewise quadratic-linear interpolation, denoted $\widehat{v}_t$, over a grid of size $p \times q$, where p is odd. Let $0 = a_0 < a_1 < \ldots < a_p < a_{p+1} = \infty$, $0 = b_0 < b_1 < \ldots < b_q < b_{q+1} = \infty$, and define the evaluation grid points by

$$\mathcal{G} = \{(a_k, b_l) \mid k = 1, 2, \dots, p \text{ and } l = 1, 2, \dots, q\}.$$

Define also $I = \{1, 3, 5, ..., p-2\}$, $\overline{I} = I \cup \{-1, p\}$, $J = \{1, 2, 3, ..., q-1\}$, and $\overline{J} = J \cup \{0, q\}$ with the convention that $a_{-1} = 0$ and $a_{p+2} = \infty$. Thus, the rectangles $[a_i, a_{i+2}] \times [b_j, b_{j+1}]$, for $i \in \overline{I}$ and $j \in \overline{J}$, cover the state space $(0, \infty) \times (0, \infty)$.

Suppose that an approximation $\tilde{v}_t$ of v_t is available at t on $\mathcal{G}$.

For each $j = 1, \ldots, q$, we use a second-degree Newton polynomial interpolation:

$$\begin{aligned} P_{ij}^t(s) &= \omega_{ij0}^t + \omega_{ij1}^t(s - a_i) + \omega_{ij2}^t(s - a_i)(s - a_{i+1}) , \\ & \quad \text{for } s \in [a_i, a_{i+2}] \text{ and } i \in I . \end{aligned} \tag{6}$$

The coefficients ω_{ij0}^t, ω_{ij1}^t, and ω_{ij2}^t are obtained so that $P_{ij}^t(s) = \tilde{v}_t(s, b_j)$, for $s \in \{a_i, a_{i+1}, a_{i+2}\}$, which result in

$$\begin{aligned} \omega_{ij0}^t &= \tilde{v}_t(a_i, b_j) \\ \omega_{ij1}^t &= (\tilde{v}_t(a_{i+1}, b_j) - \tilde{v}_t(a_i, b_j)) \,/\, (a_{i+1} - a_i) \\ \omega_{ij2}^t &= (\tilde{v}_t(a_i, b_j)\,(a_{i+2} - a_{i+1}) - \tilde{v}_t(a_{i+1}, b_j)\,(a_{i+2} - a_i) + \\ & \quad \tilde{v}_t(a_{i+2}, b_j)\,(a_{i+1} - a_i)) \,/\, ((a_{i+1} - a_i)\,(a_{i+2} - a_{i+1})\,(a_{i+2} - a_i)) . \end{aligned}$$

Outside I, we use the interpolation coefficients of the adjacent interval:

$$\begin{aligned} P_{-1,j}^t(s) &= P_{0j}^t(s) \\ P_{pj}^t(s) &= P_{p-2,j}^t(s). \end{aligned}$$

Then, for $s \in [a_i, a_{i+2}]$ and $i \in \overline{I}$, we interpolate $\tilde{v}_t\,(s, .)$ to each $[b_j, b_{j+1}]$, for $j \in J$, or extrapolate to $[0, b_1]$ or $[b_q, \infty]$ linearly. The quadratic-linear interpolation $\widehat{v}_t$ of v_t to $R = [a_0, a_{p+1}] \times [b_0, b_{q+1}]$ is then

$$\begin{aligned} \hat{v}_t(s, h) &= \frac{b_{j+1} - h}{b_{j+1} - b_j} P_{ij}^t(s) + \frac{h - b_j}{b_{j+1} - b_j} P_{i,j+1}^t(s), \\ & \quad \text{for } (s, h) \in [a_i, a_{i+2}] \times [b_j, b_{j+1}] \text{ and } (i, j) \in \overline{I} \times \overline{J} . \end{aligned} \tag{7}$$

3.3 Evaluation

Assume that $\hat{v}_{t+1}$ is known. Using (4) and (7), an approximation of the holding value at t and $(a_k, b_l) \in \mathcal{G}$ takes the form:

$$\begin{aligned} & E_{ta_kb_l}[e^{-r}\hat{v}_{t+1}(S_{t+1}, H_{t+2})] \\ = e^{-r} \textstyle\sum_{i\in\overline{I}} \sum_{j\in\overline{J}} \quad & \alpha_{ij0}^{t+1} E_{ta_kb_l}\left[\mathbb{I}\left(R_{ij}\right)\right] + \alpha_{ij1}^{t+1} E_{ta_kb_l}\left[S_{t+1}\mathbb{I}\left(R_{ij}\right)\right] \\ & + \alpha_{ij2}^{t+1} E_{ta_kb_l}\left[H_{t+2}\mathbb{I}\left(R_{ij}\right)\right] + \alpha_{ij3}^{t+1} E_{ta_kb_l}\left[S_{t+1}H_{t+2}\mathbb{I}\left(R_{ij}\right)\right] \\ & + \alpha_{ij4}^{t+1} E_{ta_kb_l}\left[S_{t+1}^2\mathbb{I}\left(R_{ij}\right)\right] + \alpha_{ij5}^{t+1} E_{ta_kb_l}\left[S_{t+1}^2 H_{t+2}\mathbb{I}\left(R_{ij}\right)\right] , \end{aligned} \tag{8}$$

where $\mathbb{I}$ is the indicator function,

$$R_{ij} = \{S_{t+1} \in [a_i, a_{i+2}] \text{ and } H_{t+2} \in [b_j, b_{j+1}]\} ,$$

and the $\alpha_{ij0}^{t+1}, \ldots, \alpha_{ij5}^{t+1}$ are obtained directly from (6)-(7). The inputs of the evaluation formula (8) are thus the known coefficients of the interpolation (7) at date $t+1$ and the GARCH *transition tables*

$$\begin{aligned} T_{klij}^{0} &= E_{ta_kb_l}\left[\mathbb{I}\left(R_{ij}\right)\right], \quad T_{klij}^{1} = E_{ta_kb_l}\left[S_{t+1}\mathbb{I}\left(R_{ij}\right)\right], \\ T_{klij}^{2} &= E_{ta_kb_l}\left[H_{t+2}\mathbb{I}\left(R_{ij}\right)\right], \quad T_{klij}^{3} = E_{ta_kb_l}\left[S_{t+1}H_{t+2}\mathbb{I}\left(R_{ij}\right)\right], \\ T_{klij}^{4} &= E_{ta_kb_l}\left[S_{t+1}^{2}\mathbb{I}\left(R_{ij}\right)\right], \quad \text{and } T_{klij}^{5} = E_{ta_kb_l}\left[S_{t+1}^{2}H_{t+2}\mathbb{I}\left(R_{ij}\right)\right]. \end{aligned} \tag{9}$$

For example, T_{klij}^{0} is the probability that the state vector visits the rectangle $[a_i, a_{i+2}] \times [b_j, b_{j+1}]$ at $t+1$, starting from (a_k, b_l) at t. The matrices $T_{klij}^{0}, \ldots, T_{klij}^{5}$ can be written in closed-form (see Ben-Ameur et al. (2005)). Notice that these matrices exhibit special structure, due to the fact that the random variables S_{t+1} and H_{t+2} are functions of the single normally distributed random variable ε_{t+1}. The exploitation of this special structure is important with regards to the efficiency of the numerical procedure (storage cost and computational burden).

3.4 Implementation

The DP algorithm may be summarized as follows:

1. Precompute the transition tables (see Appendix 1);
2. Set $\tilde{v}_T(a_k, b_l) = \kappa(a_k)$, for $(a_k, b_l) \in \mathcal{G}$, and set $t = T-1$;
3. By (7), interpolate $\tilde{v}_{t+1}$ to $\widehat{v}_{t+1}$;
4. By (8), compute $E_{ta_kb_l}[\hat{v}_{t+1}(S_{t+1}, H_{t+2})]$, for $(a_k, b_l) \in \mathcal{G}$;
5. By (4), compute $\widetilde{v}_t\left(a_k, b_l\right) = \max\left\{\kappa(a_k); E_{tsh}\left[e^{-r}v_{t+1}\left(S_{t+1}, H_{t+2}\right)\right] - C_t; 0\right\}$, for $(a_k, b_l) \in \mathcal{G}$;
6. Record the optimal decision at t and $(a_k, b_l))$, for $(a_k, b_l) \in \mathcal{G}$;
7. If $t = 0$, stop; else set $t \leftarrow t-1$ and go to step 3.

4 Numerical Experiments

4.1 ASX Installment Warrants

Installment options are mostly traded over the counter. However, Installment warrants have been traded on the Australian Stock Exchange (ASX) since January 1997, where both the number of listed Installment Warrants (IW) and the trading volume have been growing exponentially. The following is an excerpt from the ASX web site.

"Installment warrants give holders the right to buy the underlying shares or instrument by payment of several instalments (usually two) during the life of the warrant. The Final

payment is usually between 40% and 60% of the price of the underlying instrument at the time of issue. Installment warrants are often covered warrants with the underlying asset being held in trust / custody for the benefit of the holder. A common feature of installment warrants is that the holder is entitled to any dividends or distributions and possibly franking credits paid by the underlying asset during the life of the warrant. An interest component is usually part of the payments due."

A plain IW, with two installments, is thus very much similar to a deeply in the money call option, except for the fact that the holder is entitled to dividends and other credits, and except for fiscal considerations (part of the payments are considered interest). The first installment is akin to the option premium, and the second one to the strike price. Because the underlying is held in trust, there is no dilution effect.

4.2 An Example

We report here on the valuation of two Installment Warrants (IW) written by ABN AMRO. The first is written on Qanta Airways Limited (QAN) and the second on The Australian Gas Light Company (AGL). In both cases, we use the daily prices of the underlying stock from 20/03/2002 to 18/11/2004 (675 trading days) to estimate the GARCH parameters using a maximum likelihood procedure. We also estimate the daily volatility of stock return, denoted σ_{BS}, assumed to be constant under Black-Scholes. The results are presented in Table 1.

We use both Black-Scholes and GARCH model to price the IWs on November 18, 2004 ($t = 0$). The risk-free rate is 0.000137, which is consistent with data available from the Australian bond market. Dividends are paid on both securities and are taken into account in the price of the IW (recall that the owner of an IW receives the dividends). Table 2 below gives the relevant data regarding the information available and computed prices under the two models on November 18, 2004, where AGLIZO and QANIZO are the codes of ABN AMRO IWs on AGL and QAN respectively. Both models seem to perform rather well with regards to estimating the value of the IW.

4.3 ASX Rolling Installment Warrants

Several IW include the possibility to be rolled over. In the case of the Rolling IW traded on the ASX, the premium to be paid by the owner is called the *rollover amount*. This payment allows the holder to defer the last installment payment of the first IW by rolling into a second installment series. This rollover amount is calculated at maturity as the difference between the capital component of the second IW and the last installment payment to be paid on the first IW. Therefore, the rollover amount is not known in advance. Notice that the it may be negative, resulting in a cash payment from the issuer to the owner of the IW.

Table 1: Fitting results

Parameter	AGL	QAN
λ	0.045100	0.031551
β_0	0.00001	0.00022
β_1	0.808300	0.174259
β_2	0.05790000	0.00000639
θ	0	0.406517
σ_{BS}	0.01091399	0.02156000

Table 2: IW contracts data and prices

	AGLIZO	QANIZO
dividend	0.61 at $t = 126$	0.10 at $t = 138$
maturity	207	207
2^{nd} installment	6	2
$H_1 \times 10^5$	6.879487	26.64369
S_0	13.15	3.54
Market price	7.56	1.62
BS price	7.3933	1.6211
GARCH price	7.3187	1.5994

In a second experiment, we apply our IO algorithm to compute the rollover amount implied by the market price of the IW under both the BS and GARCH models. More precisely, we find the value of the installment at date $t = 207$ such that the value of an IO with maturity $207+T'$ is equal to the market price of the plain IW maturing at date $T = 207$, for increasing values of T'.

The results are presented in Table 3 for AGLIZO and Table 4 for QANIZO for various maturities of the second installment series.

The value of the implied installment is increasing with the maturity of the second contract, which is consistent with what is expected of the rollover amount, since the price of the second IW should be increasing with maturity, all other parameters being equal. Notice that for short maturities, the implied installment is negative, and that, under the BS model, the implied installment is consistently higher than under the GARCH model, which is consistent with the fact that the BS price is higher than the GARCH price.

The results of this second numerical experiment may be interpreted in the following way: we find an IO contract, with known installment to occur at date $t = 207$, and with

Table 3: Implied rollover amount for AGLIZO

Maturity	BS	GARCH
207+60	-0.2001	-0.20986
207+120	-0.1514	-0.16116
207+180	-0.10308	-0.11285
207+240	-0.05515	-0.06494
207+300	-0.00764	-0.01741
207+360	0.03952	0.02973
207+420	0.08631	0.07649
207+480	0.13271	0.12284
207+540	0.17877	0.16898
207+600	0.22447	0.21464
207+660	0.26983	0.25994
207+720	0.31483	0,30488

Table 4: Implied rollover amount for QANIZO

Maturity	BS	GARCH
207+30	-0.00996	-0.01577
207+60	0.00018	-0.0064
207+90	0.01055	0.00315
207+120	0.02110	0.01283
207+150	0.03179	0.02264
207+180	0.04256	0.03262
207+210	0.05344	0.04256
207+240	0.06434	0.05266
207+270	0.07625	0.06275
207+300	0.08907	0.07276
207+330	0.10215	0.08300
207+360	0.11534	0.09317

maturity 207+T', which is equivalent to the rolling IW traded on the ASX in the sense that its theoretical price is equal to the market price of the IW. This procedure allows the investor to extract the market forecast on the rollover amount to be paid. This is of special interest since rolling IW are traded on the ASX and can also be purchased via the underwriter. Recovering the “implied” rollover amount from market prices therefore serves as a benchmark to set over-the-counter prices, which helps preclude arbitrage.

5 Conclusion

In this paper, we propose a numerical procedure based on dynamic programming and piecewise polynomial approximation for pricing installment options in the GARCH framework. We illustrate the procedure using data from the Australian Stock Exchange. We estimate the parameters of the GARCH process followed by two stocks and then price Installment Warrants written on these stocks under both GARCH and Black-Scholes assumptions. We compare the theoretical prices obtained under these two different models for the underlying asset process to the market price of these Installment Warrants. Finally, we extract the market forecast on the rollover amount implied by market prices for rolling Installment Warrants of various maturities.

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