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On Column Generation Formulations for the RWA Problem

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Abstract

We present a review of column generation formulations for the Routing and Wavelength Assignment (RWA) problem with the objective of minimizing the blocking rate. Several improvements are proposed together with a comparison of the different formulations with respect to the quality of their continuous relaxation bounds and their computing solution ease.

Key Words: WDM network, network dimensioning, RWA problem, column generation, optimal solution.

Résumé

Nous présentons une synthèse des formulations de génération de colonnes pour le problème de routage et affectation de longueurs d'onde (RWA) avec l'objectif de minimiser le taux de blocage. Plusieurs améliorations sont proposées avec une comparaison des différentes formulations relativement à la qualité de leurs bornes de relaxation continue et de leur facilité de résolution en pratique.

Mots clés : Réseau WDM, dimensionnement de réseau, problème RWA, génération de colonnes, résolution exacte.

1 Introduction

Many papers have already appeared on the RWA problem, i.e., the routing and wavelength assignment problem, one of the central problem in the dimensioning of optical WDM networks. As it is a highly combinatorial problem, various heuristic scheme solutions have been proposed under different assumptions on the static or dynamic traffic patterns, with single or multi hops, and for various objectives, cf. the surveys of Dutta and Rouskas [2] and Zang, Jue and Mukherjee [12] for a summary of the works until 2000, and Jaumard, Meyer and Thiongane [5] for a recent survey on symmetrical systems under various objectives.

Several compact ILP formulations have been proposed for the RWA problem: see [3] and [5] for recent surveys in the asymmetrical and symmetrical cases respectively. They all share the drawback to be highly symmetrical with respect to wavelength permutations. As a consequence, even problems of moderate size can hardly be solved to optimality. In an attempt to overcome this drawback, column generation like formulations have been proposed (Ramaswami and Sivarajan [11], Lee *et al.* [8]). We review these formulations, compare them and propose a new one.

The paper is organized as follows. In the next section, we present a more formal statement of the RWA problem and define notation that will be used throughout the paper. The following sections are each devoted to a specific column generation formulation of the RWA problem: Section 3 to the maximal independent set formulation of Ramaswami and Sivarajan [11], Section 4 to the independent routing configuration formulation of Lee *et al.* [8], Section 5 to a new maximal independent routing configuration formulation. Conclusions are drawn in the last section.

2 Statement of the max-RWA problem

We assume that the optical network is represented by a multigraph $G = (V, E)$ with a node set $V = \{v_1, v_2, \ldots, v_n\}$ where each node is associated with a node of the physical network, and with an arc set $E = \{e_1, e_2, \ldots, e_m\}$ where each arc is associated with a fiber link of the physical network: the number of arcs from v_i to v_j is equal to the number of fibers supporting traffic from v_i to v_j. Connections and fiber links are assumed to be directional, and the traffic to be asymmetrical. The set of available wavelengths is denoted by $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_W\}$ with $W = |\Lambda|$. The traffic is defined by a $n \times n$ matrix T where T_{sd} defines the number of requested connections from v_s to v_d. All wavelengths are assumed to have the same capacity. Let $\mathcal{SD} = \{(v_s, v_d) \in V \times V : T_{sd} > 0\}$. Denote by $\mathcal{P}_{sd}$ the set of elementary paths from v_s to v_d for $(v_s, v_d) \in \mathcal{SD}$ and by $\mathcal{P}$ the overall collection of paths, i.e., $\mathcal{P} = \bigcup_{(v_s,v_d) \in \mathcal{SD}} \mathcal{P}_{sd}$. Let $\omega^+(v_i)$ (resp. $\omega^-(v_i)$) be the set of outgoing (resp. incoming) fiber links at node v_i.

We consider only single-hop connections, i.e., the same wavelength is used from the source to the destination for all connection requests.

The RWA problem can then be formally stated as follows: given a multigraph G corresponding to a WDM optical network, and a set of requested connections, find a suitable lightpath (p, λ) for each (accepted) connection where p is a routing path and λ a wavelength, so that no two paths sharing an arc of G are assigned the same wavelength. We study the objective of minimizing the blocking rate, that is equivalent to maximizing the number of accepted connections, leading to the so-called max-RWA problem.

3 Maximal Independent Set Modeling

A first column generation formulation, i.e., a formulation with an exponential number of variables, was proposed by Ramaswami and Sivarajan [11]. In order to express it, let us first define the wavelength clash (or conflict) graph $G_W = (V_W, E_W)$. The set of nodes is a union of node sets

$$V_W = \bigcup_{(v_s,v_d)\in\mathcal{SD}} V_W^{sd},$$

where $V_W^{sd} = \{r_p : p \in \mathcal{P}_{sd}\}$ is a set of route nodes, i.e., of nodes associated with potential routes for connections from v_s to v_d for all $(v_s, v_d) \in \mathcal{SD}$, and $E_W = \{\{r_p, r_{p'}\} \in V_W \times V_W :$ paths p and p' have at least one common fiber link$\}$. For each path p, let s_p and d_p denote its source and destination nodes. Let $\mathcal{I}_{\max}$ be the overall set of maximal independent sets of G_W, and let w_I be the number of wavelengths associated with I for each $I \in \mathcal{I}_{\max}$.

3.1 MAX_IS Mathematical Formulation

Let us define the following set of coefficients:

$$\delta_{pI} = |\{r_p\} \cap I| = \begin{cases} 1 & \text{if path } p \text{ is such that } r_p \text{ belongs to independent set } I \\ 0 & \text{otherwise} \end{cases}$$

and observe that

$$\sum_{p\in\mathcal{P}_{sd}} \delta_{pI} = |I \cap V_W^{sd}| \qquad I \in \mathcal{I}_{\max}, (v_s, v_d) \in \mathcal{SD}. \tag{1}$$

The Ramaswami and Sivarajan [11] formulation amounts to find a set of $q \le W$ maximal independent sets subject to some constraints. It is formally expressed as follows :

$$\max \quad \sum_{(v_s,v_d)\in\mathcal{SD}} y_{sd}$$

subject to:

$$\sum_{I\in\mathcal{I}_{\max}} w_I \le W \tag{2}$$

$$x_p \leq \sum_{I \in \mathcal{I}_{\max}} w_I \delta_{pI} \qquad p \in \mathcal{P} \tag{3}$$

$$y_{sd} \leq \sum_{p \in \mathcal{P}_{sd}} x_p \qquad (v_s, v_d) \in \mathcal{SD} \tag{4}$$

$$0 \leq y_{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{5}$$

$$x_p \geq 0 \qquad p \in \mathcal{P} \tag{6}$$

$$w_I \in \mathbb{N} \qquad I \in \mathcal{I}_{\max}. \tag{7}$$

Note that variables x_p that express the number of times a given path is selected for a lightpath, may be eliminated by combining constraints (3) and (4). Using (1), we obtain the following MAX_IS formulation:

$$\max \qquad z_{\text{MAX\_IS}}(w, y) = \sum_{(v_s, v_d) \in \mathcal{SD}} y_{sd}$$

subject to:

$$\sum_{I \in \mathcal{I}_{\max}} w_I \leq W \tag{2}$$

$$y_{sd} - \sum_{I \in \mathcal{I}_{\max}} w_I |I \cap V_W^{sd}| \leq 0 \qquad (v_s, v_d) \in \mathcal{SD} \tag{8}$$

$$0 \leq y_{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{9}$$

$$w_I \in \mathbb{N} \qquad I \in \mathcal{I}_{\max}. \tag{10}$$

The most important feature of the MAX_IS formulation lies in the fact that wavelengths are assigned only once an optimal solution has been found, therefore eliminating the symmetry problem arising from equivalent solutions up to a wavelength permutation in the classical ILP formulations, see, e.g., Jaumard, Meyer and Thiongane [5]. Let w^* be an optimal solution of the MAX_IS formulation and let $I_1, I_2, \ldots, I_q, q \leq W$ be the independent sets such that $w_I^* \geq 1$. Distribute the wavelengths over the independent sets as follows: assign $\lambda_t, t = 1 + \sum_{i=1}^{\tau-1} w_{I_i}^*, ..., \sum_{i=1}^{\tau} w_{I_i}^*$ to the independent set I_τ for $\tau = 1, 2, \ldots, q$ with the convention that $w_{I_0}^* = 0$.

3.2 Solution of the LP Relaxation of MAX_IS

The LP relaxation, denoted by LP_MAX_IS, is obtained by replacing the integer constraints (10) by $w_I \geq 0$ for all $I \in \mathcal{I}_{\max}$. As the number of maximal independent sets can be exponential, let us consider the LP_MAX_IS formulation with all variables y_{sd} such that $(v_s, v_d) \in \mathcal{SD}$ and a variable subset of $\{w_I : I \in \mathcal{I}_{\max}\}$, leading to the so-called *restricted master problem*. To check whether the optimal solution of the restricted master problem is also optimal for the original LP_MAX_IS, we need to verify whether there exists a variable with a positive reduced cost that could be added to the restricted master problem, see, e.g., Nemhauser and Wolsey [10] for an introduction to column generation. If such a variable exists, it is added to the variable subset of the restricted master problem that is solved again, and we iterate until no variable with a positive cost reduced can be found: the LP_MAX_IS has then been solved optimally.

Let u^0 be the dual value associated with constraint (2) and u^1_{sd} the dual value associated with constraint (8) in an optimal solution of the current restricted master problem. Then the reduced cost for variable w_I is $\bar{c}(w_I) = -u^0 + \sum_{(v_s,v_d)\in\mathcal{SD}} |I \cap V_W^{sd}| u^1_{sd}$. The existence of a variable with positive reduced cost is then checked by solving the following auxiliary problem, called AUX_MAX_IS:

$$\max \left\{ -u^0 + \sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{p:r_p\in V_W^{sd}} u^1_{sd}\,\alpha_p : \alpha_p + \alpha_{p'} \le 1 \text{ for } (r_p, r_{p'}) \in E_W; \alpha_p \in \{0,1\} \text{ for } r_p \in V_W \right\}$$

where $\alpha_p = 1$ if path p is selected and 0 otherwise. The auxiliary problem corresponds to a weighted independent set problem for which many exact methods have been proposed, see, e.g., Mehrotra and Trick [9], Balas and Xue [1].

Observe that the AUX_MAX_IS auxiliary problem is solved on the wavelength clash graph that may involve a large number of vertices as each vertex is associated with an elementary path for a given source and destination pair of nodes.

3.3 Comparison with the LP Relaxations of the Compact ILP Formulations

Ramaswami and Sivarajan [11] have shown that the LP relaxation upper bound obtained with this formulation is never worse than the LP relaxation bound obtained with the classical compact ILP formulations, and can even be better for some instance, see, e.g., the example in Section III.A in [11]. We recall that all known compact ILP formulations have the same LP relaxation bound, see [3].

4 Independent Routing Configuration Modeling

Lee *et al.* [8] (see also [6, 7]) have introduced the concept of independent routing configuration where an independent routing configuration is implicitly associated with a set of paths, not necessarily unique, that can be used for satisfying a given fraction of the connections with the same wavelength. An independent routing configuration C is represented by a non-negative vector a^C such that

$a^C_{sd} =$ number of connection requests from v_s to v_d that are supported by configuration C

$a^C_{sd} \le T_{sd} \qquad (v_s, v_d) \in \mathcal{SD}$

We denote by $\mathcal{C}$ the set of all possible independent routing configurations.

4.1 IRC Mathematical Formulation

We define the variables w_C that indicates how many occurrences of a given independent routing configuration can be used simultaneously, each occurrence with a different wavelength. The routing configuration formulation, named IRC, can then be expressed as follows for the max-RWA :

$$\max \qquad z_{\text{IRC}}(w) = \sum_{C \in \mathcal{C}} \sum_{(v_s,v_d) \in \mathcal{SD}} a_{sd}^C w_C$$

subject to:

$$\sum_{C \in \mathcal{C}} a_{sd}^C w_C \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{11}$$

$$\sum_{C \in \mathcal{C}} w_C \leq W \tag{12}$$

$$w_C \in \mathbb{N} \qquad C \in \mathcal{C}. \tag{13}$$

4.2 Solution of the LP relaxation of the IRC Formulation

The LP relaxation, denoted by LP-IRC, is obtained by replacing the integrality constraints (13) in IRC by $w_C \geq 0$ for all $C \in \mathcal{C}$. As the number of independent routing configurations can be exponential, we consider again a so-called restricted master problem on a subset of the variables and examine the reduced cost to determine whether or not we have reached the optimal solution of LP-IRC. Let (u^0, u^1_{sd}) be an optimal solution of the dual of the current restricted master problem. Then the reduced cost $\bar{c}(w_C)$ of column w_C can be written

$$\bar{c}(w_C) = \sum_{(v_s,v_d) \in \mathcal{SD}} (1 - u^1_{sd}) a^C_{sd} - u^0.$$

To find whether there exists a configuration with a positive reduced cost, Lee *et al.* [7] consider the following auxiliary problem:

$$\max \quad \bar{c}_{\text{AUX1-IRC}}(\alpha) = -u^0 + \sum_{(v_s,v_d) \in \mathcal{SD}} \sum_{p \in \mathcal{P}_{sd}} \alpha_p (1 - u^1_{sd})$$

subject to:

$$\sum_{p \in \mathcal{P}} \delta_e^p \alpha_p \leq 1 \qquad e \in E \tag{14}$$

$$\sum_{p \in \mathcal{P}_{sd}} \alpha_p \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{15}$$

$$\alpha_p \in \{0, 1\} \qquad p \in \mathcal{P} \tag{16}$$

where $\alpha_p = 1$ if path p is selected and 0 otherwise, and $\delta_e^p = 1$ if fiber link e belongs to path p and 0 otherwise. The auxiliary problem corresponds here again to a weighted independent set problem, but with some cardinality constraints. Lee *et al.* [7] solve it using column generation and a branch-and-price algorithm, or in other words they have a column generation algorithm for solving the auxiliary problems embedded in the column generation (heuristic) algorithm for solving the master problem.

An alternative is to formulate the auxiliary problem as a multi-flow problem:

$$\max \quad \bar{c}_{\text{AUX2-IRC}}(\alpha) = -u^0 + \sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{e\in\omega^+(v_s)} \alpha_e^{sd}(1 - u^1_{sd})$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}} \alpha_e^{sd} \leq 1 \qquad e \in E \tag{17}$$

$$\sum_{e\in\omega^+(v_i)} \alpha_e^{sd} = \sum_{e\in\omega^-(v_i)} \alpha_e^{sd} \qquad (v_s, v_d) \in \mathcal{SD}, v_i \in V \setminus \{v_s, v_d\} \tag{18}$$

$$\sum_{e\in\omega^+(v_s)} \alpha_e^{sd} - \sum_{e\in\omega^-(v_s)} \alpha_e^{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{19}$$

$$\alpha_e^{sd} \in \{0, 1\} \qquad (v_s, v_d) \in \mathcal{SD}, \quad e \in E \tag{20}$$

where $\alpha_e^{sd} = 1$ if a path from v_s to v_d goes through fiber link e, and 0 otherwise. Constraints (17) and (18) define a set of disjoint paths, i.e., a configuration. If $\bar{c}_{\text{AUX2-IRC}}(\alpha) \leq 0$ then LP-IRC has been solved to optimality. Otherwise the routing configuration C induced by $(\alpha_e^{sd})_{e\in E,(v_s,v_d)\in\mathcal{SD}}$ is added to the restricted master problem, which is solved again.

5 Maximal Independent Routing Configuration Modeling

By combining the ideas of the formulations of the two previous Sections, we obtain a new formulation that requires only maximal independent routing configurations where an independent routing configuration C is maximal if there does not exist another independent routing configuration C' such that $a^{C'} \geq a^C$. Observe that each independent routing configuration is implicitly associated with sets of independent sets of the wavelength clash graph G_W such that each set of independent sets is supporting the same fraction of connections.

5.1 MAX_IRC Mathematical Formulation

Let $\mathcal{C}_{\max}$ be the set of all maximal independent routing configurations and let again w_C the number of occurrences of the independent routing configuration C that can be used, each with a different wavelength. Then MAX_IRC can be formulated as follows:

$$\max \qquad z_{\text{MAX\_IRC}}(w, y) = \sum_{(v_s,v_d)\in\mathcal{SD}} y_{sd}$$

subject to:

$$
\begin{aligned}
& y_{sd} \le \sum_{C \in \mathcal{C}_{\max}} a^C_{sd} w_C && (v_s, v_d) \in \mathcal{SD} && (21)\\
& y_{sd} \le T_{sd} && (v_s, v_d) \in \mathcal{SD} && (22)\\
& \sum_{C \in \mathcal{C}_{\max}} w_C \le W && && (23)\\
& w_C \in \mathbb{N} && C \in \mathcal{C}_{\max} && (24)\\
& y_{sd} \ge 0 && (v_s, v_d) \in \mathcal{SD}. && (25)
\end{aligned}
$$

5.2 Solution of the LP Relaxation of MAX_IRC

Let u^0 be the dual value associated with constraint (23) and u^1_{sd} the dual value associated with constraint (21) in the optimal solution of the restricted master problem. The reduced cost for variable w_C is $-u^0 + \sum_{(v_s,v_d) \in \mathcal{SD}} a^C_{sd} u^1_{sd}$. The auxiliary problem is then defined by:

$$
\max \quad \bar{c}_{\text{AUX\_MAX\_IRC}}(\alpha) = -u^0 + \sum_{(v_s,v_d) \in \mathcal{SD}} \sum_{e \in \omega^+(v_s)} \alpha^{sd}_e u^1_{sd}
$$

subject to:

$$
\begin{aligned}
& \sum_{(v_s,v_d) \in \mathcal{SD}} \alpha^{sd}_e \le 1 && e \in E && (26)\\
& \sum_{e \in \omega^+(v_i)} \alpha^{sd}_e = \sum_{e \in \omega^-(v_i)} \alpha^{sd}_e && (v_s, v_d) \in \mathcal{SD}, v_i \in V \setminus \{v_s, v_d\} && (27)\\
& \sum_{e \in \omega^+(v_s)} \alpha^{sd}_e - \sum_{e \in \omega^-(v_s)} \alpha^{sd}_e \le T_{sd} && (v_s, v_d) \in \mathcal{SD} && (28)\\
& \alpha^{sd}_e \in \{0, 1\} && (v_s, v_d) \in \mathcal{SD}, e \in E. && (29)
\end{aligned}
$$

5.3 Comparison of Formulations IRC and MAX_IRC

Let $z^{\text{LP}}_{\text{IRC}}$ and $z^{\text{LP}}_{\text{MAX\_IRC}}$ be the optimal values of the LP relaxation of formulation IRC and MAX_IRC respectively. Then we have the following result:

Proposition 1 $z^{\text{LP}}_{\text{IRC}} = z^{\text{LP}}_{\text{MAX\_IRC}}$. See [4] for the details of the proof.

One of the advantages of the MAX_IRC formulation over IRC is that the former generally requires less columns. Indeed consider a network with 4 nodes v_1, v_2, v_3, v_4 such that there exists a pair of fiber links between the following pair of nodes: (v_1, v_2), (v_1, v_3) and (v_1, v_4). Assume that the traffic matrix is $T_{12} = 3$, $T_{13} = 2$ and $T_{14} = 1$, and that 3 wavelengths are available. There is only one possible maximal independent routing configuration C such that $a^C_{v_1 v_2} = a^C_{v_1 v_3} = a^C_{v_1 v_4} = 1$. An optimal solution of the MAX_IRC formulation is therefore defined by this configuration with weight $w^*_C = 3$. In contrast, an optimal solution of the IRC configuration will require at least 2 columns: a first column C defined by $a^C_{v_1v_2} = a^C_{v_1v_3} = 1$ with weight $w^*_C = 2$ and a second column C' defined by $a^{C'}_{v_1v_2} = a^{C'}_{v_1v_4} = 1$ with weight $w^*_{C'} = 1$. Note however that the MAX_IRC formulation requires the additional variables y_{sd}.

Observe that there exists another optimal solution for formulation IRC with 3 columns, each with a weight equal to 1: the first column C is characterized by $a^C_{v_1v_2} = a^C_{v_1v_3} = a^C_{v_1v_4} = 1$, the second one by $a^{C'}_{v_1v_2} = a^{C'}_{v_1v_3} = 1$ and the third one by $a^{C''}_{v_1v_2} = 1$. As already observed for the compact formulations, it is usually not desirable that there exist too many solutions with the same value.

5.4 Comparison of formulations MAX_IRC and MAX_IS

Note that in formulation MAX_IS, we can restrict the independent sets to those satisfying the cardinality constraints:

$$\sum_{p: r_p \in V^{sd}_W} \alpha_p \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD}.$$

The definition of maximal independent sets is modified accordingly. We denote by MAX_IS$'$ the resulting new independent set formulation. We then have the following relation between the optimal values of the LP relaxations of MAX_IS and MAX_IS$'$:

Proposition 2 $z^{\text{LP}}_{\text{MAX\_IS}'} \leq z^{\text{LP}}_{\text{MAX\_IS}}$. See again [4] for the proof.

We now compare the formulations MAX_IS$'$ and MAX_IRC. The MAX_IS$'$ auxiliary problem can be written

$$\max \quad \sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{p: r_p \in V^{sd}_W} \alpha_p (1 - u^1_{sd})$$

$$\text{s.t.} \quad \begin{cases} \alpha_p + \alpha_{p'} \leq 1 & (r_p, r_{p'}) \in E_W & (30) \\ \sum_{p: r_p \in V^{sd}_W} \alpha_p \leq T_{sd} & (v_s, v_d) \in \mathcal{SD} & (31) \\ \alpha_p \in \{0, 1\} & r_p \in V_W. \end{cases}$$

By definition of the wavelength clash graph G_W defined in Section 3, $(r_p, r_{p'}) \in E_W$ if and only if the paths p and p' share at least one edge $e \in E$. Hence inequalities (30) can be written

$$\alpha_p + \alpha_{p'} \leq 1 \qquad r_p, r_{p'} \in V_W : p \cap p' \supseteq \{e\} \in E$$

which in turn can be written

$$\sum_{r_p \in V_W : e \in p} \alpha_p \leq 1 \qquad e \in E. \qquad (32)$$

Using path notation, the auxiliary problem can then be rewritten:

$$\max \sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{p\in\mathcal{P}_{sd}} \alpha_p(1-u^1_{sd})$$

$$\text{s.t.} \quad \begin{cases} \sum\limits_{p\in\mathcal{P}:e\in p} \alpha_p \leq 1 & e \in E \quad (33) \\ \sum\limits_{p\in\mathcal{P}_{sd}} \alpha_p \leq T_{sd} & (v_s, v_d) \in \mathcal{SD} \quad (34) \\ \alpha_p \in \{0,1\} & p \in \mathcal{P}. \quad (35) \end{cases}$$

Since $\mathcal{P}_{sd}$ is the set of all elementary paths from v_s to v_d, the constraints (33)-(35) can be replaced by flow constraints: we then obtain the AUX_MAX_IRC auxiliary problem of formulation MAX_IRC.

Note that although the auxiliary problems are identical for the 2 formulations, it does not imply that the set of columns in the master problem are identical. Indeed, a column corresponding to a maximal independent routing configuration in the MAX_IRC formulation identifies the number of disjoint paths for each pair of sources/destination. It does not explicitly provide the paths ; the only information we have is that there exist paths that can support the maximal independent routing configuration. In contrast, a column corresponding to an independent set in formulation MAX_IS′ identifies a set of disjoint paths. Since a given maximal independent routing configuration may be associated with several different sets of disjoint paths, it follows that for one column of MAX_IRC, we may have many corresponding columns of MAX_IS′. In other words, MAX_IRC eliminates a second type of symmetry.

The following result is a rather direct consequence of the relation between the 2 formulations:

Proposition 3 $z^{\text{LP}}_{\text{MAX\_IS}'} = z^{\text{LP}}_{\text{MAX\_IRC}}$. See again [4] for the proof.

6 Conclusions

We have described and compare three column generation formulations for the max-RWA problem. The next steps are on one hand to adapt them to different objectives such as the minimization of the congestion or of the network load and, on the other hand, to study how to implement efficiently the last one, the MAX_IRC formulation, and compare its performance with those of the compact ILP formulations.

References

[1] E. Balas and J. Xue. Minimum Weighted Coloring of Triangulated Graphs, with Application to Maximum Weight Vertex Packing and Clique Finding in Arbitrary Graphs. *SIAM Journal on Computing*, 20(2):209–221, 1991.

[2] R. Dutta and G.N. Rouskas. A survey of virtual topology design algorithms for wavelength routed optical networks. *Optical Networks Magazine*, 1(1):73–89, January 2000.

[3] B. Jaumard, C. Meyer, and B. Thiongane. Comparison of ILP formulations for the RWA problem. Submitted for Publication.

[4] B. Jaumard, C. Meyer, and B. Thiongane. Decomposition methods for the RWA problem. In Preparation.

[5] B. Jaumard, C. Meyer, and B. Thiongane. ILP formulations for the RWA problem for symmetrical systems. Submitted for Publication.

[6] K. Lee. *Routing and Capacity Assignment Models and Algorithms for the Design of Telecommunication Networks*. PhD thesis, KAIST, Daejeon, Korea, 1998.

[7] K. Lee, K.C. Kang, T. Lee, and S. Park. An optimization approach to routing and wavelength assignment in WDM all-optical mesh networks without wavelength conversion. *ETRI Journal*, 24(2):131–141, April 2002.

[8] T. Lee, K. Lee, and S. Park. Optimal routing and wavelength assignment in WDM ring networks. *IEEE Journal on Selected Areas in Communications*, 18(10):2146–2154, October 2000.

[9] A. Mehrotra and M.A. Trick. A column generation approach for graph coloring. *INFORMS Journal on Computing*, 8(4):344–354, 1996.

[10] G.L. Nemhauser and L.A. Wolsey. *Integer and Combinatorial Optimization*. Wiley, 1999, reprint of the 1988 edition.

[11] R. Ramaswami and K.N. Sivarajan. Routing and wavelength assignment in all-optical networks. *IEEE/ACM Transactions on Networking*, 5(3):489–501, October 1995.

[12] H. Zang, J. P. Jue, and B. Mukherjee. A review of routing and wavelength assignment approaches for wavelength-routed optical WDM networks. *Optical Networks Magazine*, pages 47–60, January 2000.