Les Cahiers du GERAD ISSN: 0711–2440

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G–2004–78

November 2004

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Abstract

This paper deals with a class of continuous-time singular linear systems with Markovian jump parameters and time delays. Sufficient conditions on stochastic stability and stochastic stabilizability are developed. A design algorithm for a state feedback controller which guarantees that the closed-loop dynamics will be regular, impulse free and stochastically stable is proposed in terms of the solutions to linear matrix inequalities (LMIs).

Key Words: Singular systems, Jump linear systems, Linear matrix inequality, Stochastic stability, Stochastic stabilizability, state feedback.

Résumé

Cet article traite de la classe des systèmes singuliers à sauts markoviens et à retard. Des conditions suffisantes de stabilité et de stabilisabilité sont developpées. Un algorithme de design d'un contrôleur en forme de retour d'état, qui garantie que le système en boucle fermée est régulier, sans impulsion et stable dans le sens stochastique, est proposé. Tous les résultats de cet article sont en forme d'inégalités matricielles linéaires ce qui facilite leur résolution.

1 Introduction

In the past decades, there have been considerable research efforts on the study of singular systems. This is due to the extensive applications of singular systems in many practical systems, such as circuits boundary control systems, chemical processes, and other areas (Refs. 1, 2, 3, 4). Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems (Refs. 1, 3). A great number of fundamental notions and results in control and systems theory based on state-space systems have been successfully extended to singular systems; see, e.g., Refs. 5–14, and the references therein.

Recently, a class of stochastic systems driven by continuous-time Markov chains has been used to model many practical systems, where random failures and repairs and sudden environment changes may occur. For more detail, we refer the reader to Refs. 15, 16, and the references therein. This motivates the study of Markovian jump systems. For example, sufficient conditions on stochastic stability and stabilization for such systems were reported in Refs. 17–21 via different approaches. The $\mathscr{H}_\infty$ control problem was investigated in Refs. 22 and 23, where sufficient conditions for the solvability of this problem was proposed. When time delays appear in a Markovian jump system, the results on stability analysis and $\mathscr{H}_\infty$ control were reported in Refs. 24, 25, and 26 for different types of time delays. For more detail on Markovian jumping systems with time delay, we refer the reader to Ref. 15 and the references therein. However, up to date singular systems with Markovian jump parameters and time delays has not yet been fully investigated.

This paper is concerned with the problems of stability analysis and stabilization for singular Markovian jump systems with time delays. In terms of a set of linear matrix inequalities (LMIs), we present first a sufficient condition, which guarantees regularity, absence of impulses and stochastic stability of such systems. Based on this, a sufficient condition for the existence state feedback controller ensuring regularity, absence of impulses and stochastic stability is proposed. Finally, a numerical example is provide to demonstrate the effectiveness of the proposed methods.

The rest of this paper is organized as follows. In Section 2, the problem is stated and the goal of the paper is clarified. In Section 3, the main results are given and these include results on stochastic stability, stochastic stabilizability. A memoryless controller is used in this paper and a design algorithm in terms of the solutions to linear matrix inequalities is proposed to synthesize the controller gains we are using.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$^\top$" denotes matrix transposition and the notation $X \geq Y$(respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{I}$ is the identity matrices with compatible dimensions. $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respective to some probability measure $\mathcal{P}$. L_2 is the space of integral vector over $[0, \infty)$. $\|\cdot\|$ will refer to the Euclidean vector norm whereas $\|\cdot\|$ denotes the L_2-norm over $[0, \infty)$ defined as $\|f\|^2 = \int_0^\infty f^T(t)f(t)\ dt$.

2 Problem statement

Consider a stochastic hybrid system with N modes, i.e., $\mathscr{S} = \{1, 2, \ldots, N\}$. The mode switching is assumed to be governed by a continuous-time Markov process $\{r_t, t \geq 0\}$ taking values in the state space $\mathscr{S}$ and having the following infinitesimal generator

$$\Lambda = (\lambda_{ij}), i, j \in \mathscr{S},$$

where $\lambda_{ij} \geq 0, \forall j \neq i, \lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$.

The mode transition probabilities are described as follows:

$$P[r_{t+\Delta} = j | r_t = i] = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & j \neq i \\ 1 + \lambda_{ii}\Delta + o(\Delta), & j = i \end{cases} \tag{1}$$

where $\lim_{\Delta \to 0} o(\Delta)/\Delta = 0$.

Let $x(t) \in \mathbb{R}^n$ be the physical state of the system, which satisfies the following dynamics:

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + A_1(r_t)x(t-h) + B(r_t)u(t), \\ x(s) = \phi(s), -h \leq s \leq 0 \end{cases} \tag{2}$$

where $u(t) \in \mathbb{R}^m$ is the control input system, $A(r_t)$, $A_1(r_t)$, and $B(r_t)$ are known real matrices with appropriate dimensions for each $r_t \in \mathscr{S}$, the matrix E may be singular, and we assume $0 \leqslant \text{rank}(E) = n_E \leqslant n$, $h > 0$ represents the system delay, $\phi(t)$ is a smooth vector-valued initial function in $[-h, 0]$. The initial condition of the system is specified as $(r_0, \phi(\cdot))$ with r_0 is the initial mode and $\phi(.)$ is the initial functional such that $x(s) = \phi(s) \in L_2[-h, 0] \stackrel{\Delta}{=} \{f(\cdot) | \int_0^\infty f^\top(t)f(t)dt < \infty\}$.

Definition 2.1. (Ref. 1)

i. System (2) is said to be regular if the characteristic polynomial, $\det(sE - A(i))$ is not identically zero for each mode $i \in \mathscr{S}$.

ii. System (2) is said to be impulse free, i.e. the $\deg(\det(sE - A(i))) = \text{rank}(E)$ for each mode $i \in \mathscr{S}$.

For more details on other properties and the existence of the solution of system (2), we refer the reader to Ref. 7, and the references therein. In general, the regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

For system (2) with $u(.) \stackrel{\Delta}{=} 0$ for $t \geq 0$, we have the following definitions:

Definition 2.2. System (2) with $u(.) \stackrel{\Delta}{=} 0, \forall t \geq 0$, is said to be stochastically stable (SS) if there exists a constant $T(r_0, \phi(\cdot))$ such that

$$\mathbb{E}\left[\int_0^\infty \|x(t)\|^2 dt \bigg| r_0, x(s) = \phi(s), s \in [-h, 0]\right] \leq T(r_0, \phi(\cdot)); \tag{3}$$

In this paper we are interested in the design of a stabilizing controller of the following form:

$$u(t) = K(r_t)x(t) \tag{4}$$

where $K(i)$ is a design parameter that has to be determined for every $i \in \mathscr{S}$.
Definition 2.3. System (2) is said to be stabilizable in the stochastic sense if there exists a control law of the form (4) such that the closed-loop system is stochastically stable.

This paper studies the stochastic stability and the stochastic stabilizability of the class of systems (2). Our goal in this paper is to design a state feedback controller guaranteing that the closed-loop is regular, impulse free and stochastically stable. In the rest of this paper, we will assume that all the required assumptions are satisfied, i.e. the complete access to the system mode and state. Our methodology in this paper will be mainly based on the Lyapunov theory and some algebraic results. The conditions we will develop here will be in terms of the solutions to linear matrix inequalities that can be easily obtained using LMI control toolbox.

3 Main results

In this section, we will develop results that assure that the free system (2) (i.e. $u(t) = 0$ for all $t \geq 0$) is regular, impulse free and stochastically stable. We will also design a state feedback controller of the form (4) that guarantees the same goal.

Let us now consider the free system and see under which condition the corresponding dynamics will be regular, impulse free and stochastically stable. The following theorem gives such results.
Theorem 3.1. The free singular Markovian jump system (2) is regular, impulse-free and stochastically stable if there exist a set of matrices $P = (P(1), \cdots, P(N))$ and a symmetric and positive-definite matrix $Q > 0$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$:

$$E^\top P(i) = P^\top(i)E \geq 0 \tag{5}$$

$$\begin{bmatrix} P^\top(i)A(i) + A^\top(i)P(i) + Q + \sum_{j=1}^N \lambda_{ij} E^\top P(j) & P^\top(i)A_1(i) \\ A_1^\top(i)P(i) & -Q \end{bmatrix} < 0. \tag{6}$$

Proof. Under the condition of the theorem, we will first show the regularity and absence of impulses of system (2). By (6), it is easy to see that the following holds for each $i \in \mathscr{S}$:

$$P^\top(i)A(i) + A^\top(i)P(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top P(j) < 0 \tag{7}$$

Now, choose two nonsingular matrices $\widehat{M}$ and $\widehat{N}$ such that

$$\widehat{M}E\widehat{N} = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{bmatrix}$$

and write

$$\widehat{M}A(i)\widehat{N} = \begin{bmatrix} \widehat{A}_1(i) & \widehat{A}_2(i) \\ \widehat{A}_3(i) & \widehat{A}_4(i) \end{bmatrix}, \quad \widehat{M}^{-\top}P(i)\widehat{N} = \begin{bmatrix} \widehat{P}_1(i) & \widehat{P}_2(i) \\ \widehat{P}_3(i) & \widehat{P}_4(i) \end{bmatrix}.$$

Then, by (5), it can be shown that $\widehat{P}_2(i) = 0$. Pre- and post-multiplying (7) by $\widehat{N}^\top$ and $\widehat{N}$, respectively, we have

$$\begin{bmatrix} * & * \\ * & \widehat{A}_4^\top(i)\widehat{P}_4(i) + \widehat{P}_4^\top(i)\widehat{A}_4(i) \end{bmatrix} < 0,$$

where $*$ will not be used in the following development. Then, by (8), we have

$$\widehat{A}_4^\top(i)\widehat{P}_4(i) + \widehat{P}_4^\top(i)\widehat{A}_4(i) < 0$$

which implies that $\widehat{A}_4(i)$ is nonsingular. Therefore, system (2) is regular and impulse-free.

Next, we will show the stochastic stability. Since (2) is regular and impulse-free, for any $i \in \mathscr{S}$, we can choose nonsingular matrices $\widetilde{M}(i)$ and $\widetilde{N}(i)$ such that

$$\widetilde{M}(i)E\widetilde{N}(i) = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{bmatrix}, \quad \widetilde{M}(i)A(i)\widetilde{N}(i) = \begin{bmatrix} \widetilde{A}(i) & 0 \\ 0 & \mathbb{I} \end{bmatrix}.$$

Write

$$\begin{aligned} \widetilde{P}(i) = \widetilde{M}^{-\top}(i)P(i)\widetilde{N}(i) &= \begin{bmatrix} \widetilde{P}_1(i) & \widetilde{P}_2(i) \\ \widetilde{P}_3(i) & \widetilde{P}_4(i) \end{bmatrix}, \\ \widetilde{Q}(i) = \widetilde{N}^\top(i)Q\widetilde{N}(i) &= \begin{bmatrix} \widetilde{Q}_1(i) & \widetilde{Q}_2(i) \\ \widetilde{Q}_2^\top(i) & \widetilde{Q}_4(i) \end{bmatrix}, \\ \widetilde{M}(i)A_1(i)\widetilde{N}(i) &= \begin{bmatrix} \widetilde{A}_{11}(i) & \widetilde{A}_{12}(i) \\ \widetilde{A}_{13}(i) & \widetilde{A}_{14}(i) \end{bmatrix}. \end{aligned}$$

Then, for any $i \in \mathscr{S}$, system (2) becomes equivalent to the following one:

$$\dot{\xi}_1(t) = A(i)\xi_1(t) + \widetilde{A}_{11}(i)\xi_1(t-h) + \widetilde{A}_{12}(i)\xi_2(t-h), \tag{8}$$

$$0 = \xi_2(t) + \widetilde{A}_{13}(i)\xi_1(t-h) + \widetilde{A}_{14}(i)\xi_2(t-h). \tag{9}$$

where

$$\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \widetilde{N}^{-1}(i)x(t).$$

Now, pre- and post-multiplying (6) by diag $\left(\widetilde{N}^\top(i), \widetilde{N}^\top(i)\right)$ and its transpose, we have

$$\begin{bmatrix} * & * & * & * \\ * & \widetilde{P}_4(i) + \widetilde{P}_4^\top(i) + \widetilde{Q}_4(i) & * & \widetilde{P}_4^\top(i)\widetilde{A}_{14}(i) \\ * & * & * & * \\ * & \widetilde{A}_{14}(i)\widetilde{P}_4(i) & * & -\widetilde{Q}_4(i) \end{bmatrix} < 0$$

where $P_2(i) = 0$ is used. It follows from this that

$$\begin{bmatrix} \widetilde{P}_4(i) + \widetilde{P}_4^\top(i) + \widetilde{Q}_4(i) & \widetilde{P}_4^\top(i)\widetilde{A}_{14}(i) \\ \widetilde{A}_{14}^\top(i)\widetilde{P}_4(i) & -\widetilde{Q}_4(i) \end{bmatrix} < 0.$$

Pre- and post-multiply this by $\begin{bmatrix} -\widetilde{A}_{14}^\top(i) & \mathbb{I} \end{bmatrix}$ and $\begin{bmatrix} -\widetilde{A}_{14}(i) \\ \mathbb{I} \end{bmatrix}$ respectively, we get:

$$\widetilde{A}_{14}^\top(i)Q_4(i)\widetilde{A}_{14}(i) - Q_4(i) < 0$$

Therefore

$$\rho\left(\widetilde{A}_{14}(i)\right) < 1. \tag{10}$$

where $\rho(\widetilde{A}_{14}(i)$ is the spectral radius of the matrix $\widetilde{A}_{14}(i)$.

Now, let us choose the following Lyapunov functional:

$$V(x(t), r_t) = x^\top(t)E^\top P(r_t)x(t) + \int_{t-h}^{t} x^\top(\alpha)Qx(\alpha)d\alpha,$$

Let $\mathscr{L}$ be the weak infinitesimal generator of the random process $\{(\mathbf{x}(t),\ r_t), t \geq 0\}$, with $\mathbf{x}(t)$ taking values in $\mathscr{C}[-h, 0]$ and defined by $\mathbf{x}(t) = x(s+t), t-h \leq s \leq t$. Then, for each $r_t = i$, $i \in \mathscr{S}$, we have

$$\begin{aligned} \mathscr{L}V(\mathbf{x}(t), i) = {} & 2x^\top(t)P^\top(i)\left[A(i)x(t) + A_1(i)x(t-h)\right] + x^\top(t)\left(\sum_{j=1}^{N}\lambda_{ij}E^\top P(j)\right)x(t) \\ & + x^\top(t)Qx(t) - x^\top(t-h)Qx(t-h) \\ = {} & \chi^\top(t)\Psi(i)\chi(t) \end{aligned} \tag{11}$$

where

$$\chi(t) = \begin{bmatrix} x^\top(t) & x^\top(t-h) \end{bmatrix}^\top,$$
$$\Psi(i) = \begin{bmatrix} P^\top(i)A(i) + A^\top(i)P(i) + Q + \sum_{j=1}^N \lambda_{ij} E^\top P(j) & P^\top(i)A_1(i) \\ A_1^\top(i)P(i) & -Q \end{bmatrix}. \quad (12)$$

Noting (6), (10), (11), and following a similar line as in the proof of Theorem 1 in Refs. 7 and 27, we can deduce that system (2) is stochastically stable. This completes the proof. □

Let us now concentrate on the design a state feedback controller of the form (4) that guarantees that the closed-loop will be regular, impulse free and stochastically stable. For this purpose, plugging controller (4) in the dynamics (2) gives:

$$\begin{cases} \dot{x}(t) = A_{cl}(r_t)x(t) + A_1(r_t,t)x(t-h) \\ x(s) = \phi(s), -h \le s \le 0 \end{cases} \quad (13)$$

where $A_{cl}(r_t) = A(r_t) + B(r_t)K(r_t)$.

Based on the results of Theorem 3.1, this dynamics will be regular, impulse free and stochastically stable if there exist a set of matrices $P = (P(1), \cdots, P(N))$ and a symmetric and positive-definite matrix $Q > 0$ such the following set of coupled LMIs holds for each $i \in \mathscr{S}$:

$$E^\top P(i) = P^\top(i)E \ge 0$$
$$\begin{bmatrix} J(i) & P^\top(i)A_1(i) \\ A_1^\top(i)P(i) & -Q \end{bmatrix} < 0,$$

where $J(i) = P^\top(i)A_{cl}(i) + A_{cl}^\top(i)P(i) + Q + \sum_{j=1}^N \lambda_{ij} E^\top P(j)$.

Now pre- and post-multiplying the second LMI respectively by $\mathrm{diag}\left(P^{-\top}(i), \mathbb{I}\right)$ and $\mathrm{diag}\left(P^{-1}(i), \mathbb{I}\right)$, we get

$$\begin{bmatrix} \tilde{J}(i) & A_1(i) \\ A_1^\top(i) & -Q \end{bmatrix} < 0, \quad (14)$$

where

$$\begin{aligned} \tilde{J}(i) &= A_{cl}(i)P^{-1}(i) + P^{-\top}(i)A_{cl}^\top(i) + P^{-\top}(i)QP^{-1}(i) \\ &+ \sum_{\substack{j=1 \\ j\neq i}}^N \lambda_{ij} P^{-\top}(i)E^\top P(j)P^{-1}(i) + \lambda_{ii}P^{-\top}(i)E^\top \end{aligned}$$

If the following holds for each $i \in \mathscr{S}$:

$$E^\top P(j) \leq \epsilon_j P^\top(j)P(j), \epsilon_j > 0 \tag{15}$$

then, a sufficient condition for (14) is

$$\begin{aligned} &A_{cl}(i)P^{-1}(i) + P^{-\top}(i)A_{cl}^\top(i) + P^{-\top}(i)QP^{-1}(i) + A_1(i)Q^{-1}A_1^\top(i) \\ &\quad + \sum_{\substack{j=1\\ j\neq i}}^{N} \lambda_{ij}\epsilon_j P^{-\top}(i)P^\top(j)P(j)P^{-1}(i) + \lambda_{ii}P^{-\top}(i)E^\top < 0 \end{aligned}$$

If we define:

$$\begin{aligned} G_i &= \left[\sqrt{\lambda_{i1}}P^{-\top}(i), \cdots, \sqrt{\lambda_{ii-1}}P^{-\top}(i), \sqrt{\lambda_{ii+1}}P^{-\top}(i), \cdots, \sqrt{\lambda_{iN}}P^{-\top}(i)\right] \\ J_i &= \mathrm{diag}\left[\epsilon_1^{-1}P^{-1}(1)P^{-\top}(1), \cdots, \epsilon_{i-1}^{-1}P^{-1}(i-1)P^{-\top}(i-1), \epsilon_{i+1}^{-1}P^{-1}(i+1)P^{-\top}(i+1),\right. \\ &\qquad \left.\cdots, \epsilon_N^{-1}P^{-1}(N)P^{-\top}(N)\right] \end{aligned}$$

then we obtain

$$\sum_{\substack{j=1\\ j\neq i}}^{N} \lambda_{ij}\epsilon_j P^{-\top}(i)P^\top(j)P(j)P^{-1}(i) = G_i J_i^{-1} G_i^\top$$

Using this we have

$$\begin{bmatrix} \tilde{J}_0(i) & P^{-\top}(i) & G_i \\ P^{-1}(i) & -Q^{-1} & 0 \\ G_i^\top & 0 & -J_i \end{bmatrix} < 0, \tag{16}$$

where

$$\tilde{J}_0(i) = A_{cl}(i)P^{-1}(i) + P^{-\top}(i)A_{cl}^\top(i) + A_1(i)Q^{-1}A_1^\top(i) + \lambda_{ii}P^{-\top}(i)E^\top$$

Noticing that the following holds for each $i \in \mathscr{S}$:

$$\epsilon_i^{-1}P^{-1}(i)P^{-\top}(i) \geq P^{-1}(i) + P^{-\top}(i) - \epsilon_i\mathbb{I}$$

we get the following sufficient condition:

$$\begin{bmatrix} \tilde{J}_0(i) & P^{-\top}(i) & G_i \\ P^{-1}(i) & -Q^{-1} & 0 \\ G_i^\top & 0 & -W_i \end{bmatrix} < 0, \tag{17}$$

where

$$W_i = \text{diag}\Big[P^{-1}(1) + P^{-\top}(1) - \epsilon_1\mathbb{I}, \cdots, P^{-1}(i-1) + P^{-\top}(i-1) - \epsilon_{i-1}\mathbb{I},$$
$$P^{-1}(i+1) + P^{-\top}(i+1) - \epsilon_{i+1}\mathbb{I}, \cdots, P^{-1}(N) + P^{-\top}(N) - \epsilon_N\mathbb{I}\Big]$$

Using now the expression of $A_{cl}(i)$, letting $Z = Q^{-1}$, $X(i) = P^{-1}(i)$, and $Y(i) = K(i)X(i)$ and noting that (15) can be rewritten as:

$$X^\top(i)E^\top \leq \epsilon_i\mathbb{I}$$

we get the following results for the stabilization.

Theorem 3.2. There exists a state feedback controller of the form (4) such that the closed-loop system (2) is regular, impulse-free and stochastically stable if there exist a set of matrices $X = (X(1), \cdots, X(N))$, $Y = (Y(1), \cdots, Y(N))$, a symmetric and positive-definite matrix $Z > 0$ and a set of positive scalars $\epsilon = (\epsilon_1, \cdots, \epsilon_N)$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$:

$$\varepsilon_i\mathbb{I} \geq X^\top(i)E^\top = EX(i) \geq 0 \tag{18}$$
$$\begin{bmatrix} \widehat{J}(i) & X^\top(i) & \mathcal{S}_i(X) \\ X(i) & -Z & 0 \\ \mathcal{S}_i^\top(X) & 0 & -\mathcal{X}_i(X) \end{bmatrix} < 0, \tag{19}$$

where

$$\widehat{J}(i) = A(i)X(i) + X^\top(i)A^\top(i) + B(i)Y(i) + B^\top(i)Y^\top(i) + A_1(i)ZA_1^\top(i) + \lambda_{ii}X^\top E^\top$$
$$\mathcal{X}_i(X) = \text{diag}\Big[X(1) + X^\top(1) - \epsilon_1\mathbb{I}, \cdots, X(i-1) + X^\top(i-1) - \epsilon_{i-1}\mathbb{I},$$
$$X(i+1) + X^\top(i+1) - \epsilon_{i+1}\mathbb{I}, \cdots, X(N) + X^\top(N) - \epsilon_N\mathbb{I}\Big]$$
$$\mathcal{S}_i(X) = \Big[\sqrt{\lambda_{i1}}X^\top(i), \cdots, \sqrt{\lambda_{ii-1}}X^\top(i), \sqrt{\lambda_{ii+1}}X^\top(i), \cdots, \sqrt{\lambda_{iN}}X^\top(i)\Big]$$

The stabilizing controller gain is given by $K(i) = Y(i)X^{-1}(i)$, $i \in \mathscr{S}$.

The results we developed in this paper extend those developed for the deterministic case on stability and stabilizability. In fact, if we fix in our condition the number of modes to one, i.e. $N = 1$, we get those of deterministic singular linear systems with time delay.

4 Numerical example

To show the validness of our results, let us consider a numerical example of a two-mode singular system with state space in $\mathbb{R}^3$. This example is borrowed from Xu et al. (Ref. 7) and modified to satisfied our case. The data of this system are as follows:

- mode # 1:

$$A(1) = \begin{bmatrix} 1 & 0.5 & 1 \\ -0.2 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \ A_1(1) = \begin{bmatrix} -0.5 & 1 & 0 \\ 0.2 & 0 & 0.5 \\ 0.3 & 0.5 & -0.6 \end{bmatrix}, \ B(1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & -3 \end{bmatrix}$$

- mode # 2:

$$A(2) = \begin{bmatrix} 1.2 & 0.3 & 1 \\ -1.2 & 1 & 0 \\ 0 & 0.4 & 0 \end{bmatrix}, \ A_1(2) = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 1 & 0 & 0.4 \\ 0.5 & 0.2 & 0 \end{bmatrix}, \ B(2) = \begin{bmatrix} 0.5 & 2 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix},$$

The transition matrix rates, Λ, and the singular matrix, E, are given by the following expressions:

$$\Lambda = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Solving the LMIs (18)-(19), we get:

$$Z = \begin{bmatrix} 7.6458 & 0.0000 & -0.0000 \\ 0.0000 & 7.6458 & -0.0000 \\ -0.0000 & -0.0000 & 7.8220 \end{bmatrix}, \ \epsilon_1 = 0.4800, \epsilon_2 = 0.5382$$

$$X(1) = \begin{bmatrix} 0.4209 & 0.0000 & 0.0 \\ 0.0000 & 0.4209 & 0.0 \\ -0.0000 & -0.0000 & 0.5306 \end{bmatrix}, \ X(2) = \begin{bmatrix} 0.3977 & -0.0000 & 0.0 \\ -0.0000 & 0.3977 & 0.0 \\ 0.0000 & 0.0000 & 0.4574 \end{bmatrix},$$

$$Y(1) = \begin{bmatrix} -1.6036 & 0.7925 & -0.0000 \\ -0.7931 & 1.8805 & -0.0000 \\ 0.8376 & 0.0000 & 0.3673 \end{bmatrix}, \ Y(2) = \begin{bmatrix} -0.1557 & -1.4352 & -0.0000 \\ 0.0603 & -0.0000 & -0.0000 \\ 0.0344 & -0.7351 & 1.0056 \end{bmatrix}$$

which gives the following gains:

$$K(1) = \begin{bmatrix} -0.3316 & 0.1397 & -0.0000 \\ -0.1398 & 0.2827 & -0.0000 \\ 0.7561 & -0.3186 & 1.4446 \end{bmatrix}, \ K(2) = \begin{bmatrix} -0.0000 & 6.5958 & 0.0000 \\ -0.2771 & -0.7156 & -0.0000 \\ -0.2026 & -0.7490 & 0.4548 \end{bmatrix}.$$

5 Conclusion

This paper dealt with a class of continuous-time singular linear systems with Markovian jumps and time-delay in the state vector. Results on stochastic stability and its robustness, and stochastic stabilizability and its robustness are developed. The LMI framework is used to establish the different results on stability and stabilizability. The conditions we developed are delay independent. The results we developed can easily be solved using any LMI toolbox like the one of Matlab or the one of Scilab.

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