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Constant Gain Stabilization for Stochastic Systems with Multiplicative Noise

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Abstract

This paper deals with the stabilization problem of the class of uncertain stochastic hybrid systems with multiplicative noise. The uncertainties are norm bounded type. Under the complete access to the system mode a constant gain static output feedback controller that stochastically stabilizes this class of systems is designed. The gain of this controller is the solution of some linear matrix inequalities (LMIs). A numerical example is provided to show the usefulness of the developed results.

Key Words: Stochastic hybrid systems, Markovian jumping parameters, Stabilization, Static output feedback control, singular systems, Linear matrix inequalities.

Résumé

Cet article traite de la stabilisation des systèmes stochastiques hybrides incertains avec bruit Brownien. Les incertitudes sont de types bornés en norme. Sous l'hypothèse de l'accès au mode du système, un correcteur par retour de sortie à gain constant est conçu. Le gain de ce correcteur est la solution d'un ensemble d'inégalités matricielles. Un exemple numérique est proposé pour montrer la validité des résultats développés.

1 Introduction

The linear time-invariant system has been extensively used to analyze and design practical systems. But it is well known that some industrial systems can't be represented by this class of linear time-invariant model since the behavior of the dynamics of these systems is random with some special features. As an example of such systems, we mention those with abrupt changes, breakdowns of components, etc. Such class of dynamical systems can be adequately described by the class of stochastic hybrid systems or the class of piecewise deterministic systems which is the subject of this paper.

Stochastic stability and stabilizability problems, $\mathscr{H}_\infty$ control problem and filtering problem of the class of stochastic hybrid systems has attracted a lot of researchers and many problems have been tackled and solved. For more details on what it has been done on this class of systems, we refer the reader to the recent books by Boukas and Liu [5] and Boukas [3] and the references therein. These two books present a good literature review on the subject up to 2004.

Particularly, the stabilization problem has attracted many researchers from the control community and many results have been reported in the literature. Most of the techniques have been used to stabilize this class of systems. For more details on this subject, we refer the reader to ([5, 3, 12, 4, 6, 11, 8, 7]). To the best of our knowledge the case of stabilization of continuous-time singular systems with Markovian jumps and multiplicative noise using a static output feedback controller has never tackled and our objective in this paper is to study this problem.

Our aim in this paper consists of designing a static output feedback controller that stochastically stabilizes the class of systems we are studying. Under the assumption of the complete access to the system mode a stabilizing static output feedback controller is designed. The gains of such controller will be determined by solving a set of LMIs.

The rest of the paper is organized as follows. In Section 2, the problem we are considering is stated and some useful definitions are given. Section 3 gives the main results of the paper that determines the static output feedback controller that stochastically stabilizes the stochastic hybrid systems. In Section 4, a numerical example is provided to show the validness of the proposed results.

Throughout this paper, the following notations will be used. The superscript "T" denotes matrix transposition and for symmetric matrices X and Y, the notation $X > Y$ (respectively $X < Y$) means that $(X - Y)$ is positive-definite (resp. negative-definite). $\mathbb{I}$ denotes the identity matrix with the appropriate dimension that may be understood from the context. $\mathbb{E}[.]$ stands for the mathematical expectation operator with respect to the given probability.

2 Problem Statement

Let us consider a dynamical singular system defined in a fundamental probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and assume that its dynamics is described by the following differential systems:

$$\begin{cases} Edx(t) = A(r_t, t)x(t)dt + B(r_t, t)u(t)dt + \mathbb{W}(r_t)x(t)dw(t), x(0) = x_0 \\ y(t) = C(r_t)x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $x_0 \in \mathbb{R}^n$ is the initial state, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$, $w(t) \in \mathbb{R}^l$ is an external Wiener process that we assume to be independent of the continuous-time Markov process $\{r_t, t \geq 0\}$, which is taking values in a finite space $\mathscr{S} = \{1, 2, \cdots, N\}$ and describes the evolution of the mode at time t, E is a known singular matrix with rank $(E) = n_E < n$, $A(r_t, t) \in \mathbb{R}^{n \times n}$ and $B(r_t, t) \in \mathbb{R}^{n \times m}$ are matrices with the following forms for every $i \in \mathscr{S}$:

$$\begin{aligned} A(i, t) &= A(i) + D_A(i)F_A(i, t)E_A(i) \\ B(i, t) &= B(i) + D_B(i)F_B(i, t)E_B(i) \end{aligned}$$

with $A(i) \in \mathbb{R}^{n \times n}$, $D_A(i)$, $E_A(i)$, $B(i) \in \mathbb{R}^{n \times m}$, $D_B(i)$, $E_B(i)$, $C(i) \in \mathbb{R}^{p \times n}$, and $\mathbb{W}(r_t) \in \mathbb{R}^{n \times l}$ are real known matrices with appropriate dimensions, and $F_A(i, t)$ and $F_B(i, t)$ are unknown real matrices that satisfy the following:

$$\begin{cases} F_A^\top(i, t)F_A(i, t) \leq \mathbb{I} \\ F_B^\top(i, t)F_B(i, t) \leq \mathbb{I} \end{cases} \tag{2}$$

The Markov process $\{r_t, t \geq 0\}$ beside taking values in the finite set $\mathscr{S}$, represents the switching between the different modes and its dynamics is described by the following probability transitions:

$$\mathbb{P}\left[r_{t+h} = j | r_t = i\right] = \begin{cases} \lambda_{ij}h + o(h) & \text{when } r_t \text{ jumps from } i \text{ to } j \\ 1 + \lambda_{ij}h + o(h) & \text{otherwise} \end{cases} \tag{3}$$

where λ_{ij} is the transition rate from mode i to mode j with $\lambda_{ij} \geq 0$ when $i \neq j$ and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$ and $o(h)$ is such that $\lim_{h\to 0} \frac{o(h)}{h} = 0$.

Remark 2.1 *Notice that when E is not singular, (1) can be transformed easily to the class of Markov jump linear systems and the results developed in the literature can be used to check the stochastic stability, to design the appropriate controller that stochastically stabilizes this class of systems (see Boukas [3]).*

Definition 2.1 *System (1) with $u(t) \equiv 0$ is said to be stochastically stable if there exists a constant $M(x_0, r_0) > 0$ such that the following holds for any initial conditions (x_0, r_0):*

$$\mathbb{E}\left[\int_0^\infty x^\top(t)x(t) | x_0, r_0\right] \leq M(x_0, r_0). \tag{4}$$

Definition 2.2 *System (1) is said to be stochastically stabilizable if there exists a control*

$$u(t) = Fy(t) \tag{5}$$

with F a constant matrix such that the closed-loop system is stochastically stable.

The robust stochastic stability and the robust stochastic stabilizability are defined in a similar manner.

The aim of this paper is to develop LMI-based stability conditions for nominal and uncertain systems (1) with $u(t) \equiv 0$; and design a static output feedback controller of the form (5) that stochastically stabilizes the class of nominal and uncertain systems under study.

Before closing this section let us give some lemmas that we will use in the rest of the paper.

Lemma 2.1 *[10] Let H, F and G be real matrices of appropriate dimensions then, for any scalar $\varepsilon > 0$ for all matrices F satisfying $F^T F \leq \mathbb{I}$, we have:*

$$HFG + G^\top F^\top H^\top \leq \varepsilon HH^\top + \varepsilon^{-1} G^\top G \tag{6}$$

Lemma 2.2 *The linear matrix inequality*

$$\begin{bmatrix} H & S^\top \\ S & R \end{bmatrix} > 0$$

is equivalent to

$$R > 0, H - S^\top R^{-1} S > 0$$

where $H = H^\top$, $R = R^\top$ and S is a matrix with appropriate dimension.

3 Main Results

Before developing the design approach for the static output feedback controller, let us assume that $u(t) = 0$, for $t \geq 0$ and study the stochastic stability of the nominal system (1). Our concern is to establish LMI conditions that can be used to check if a given dynamical system belonging to the class of systems we are considering in this paper is stochastically stable. The following theorem states the first result on stability of such class of systems.

Theorem 3.1 *System (1) is stochastically stable if and only if there exists a symmetric and positive-definite matrix $P > 0$, such that the following LMIs hold for every $i \in \mathscr{S}$:*

$$E^\top PA(i) + A^\top(i)PE + \mathbb{W}^\top(i)E^\top PE\mathbb{W}(i) < 0 \tag{7}$$

Proof: Since we are only interested by sufficient conditions in this paper, the necessary condition will not be proven here and the interested readers can consult the proof in Boukas [3]. To prove the sufficient condition. Let $r_t = i$ and consider the following Lyapunov function:

$$V(x(t), i) = x^\top(t) E^\top P E x(t)$$

where P is a solution of (7).

Let $\mathscr{L}$ denote the weak infinitesimal generator of the Markov process $\{(x(t), i)\}$. Then, a direct computation gives (see Boukas [3]):

$$\begin{aligned}\mathscr{L}V(x(t), i) = x^\top(t) \Big[A^\top(i) P E + E^\top P A(i) + & \mathbb{W}^\top(i) E^\top P E \mathbb{W}(i) \\ & + \sum_{j=1}^{N} \lambda_{ij} E^\top P E \Big] x(t)\end{aligned}$$

Using the fact $\sum_{j=1}^{N} \lambda_{ij} = 0$ and since (7) holds, it results that:

$$\mathscr{L}V(x(t), i) \leq -\alpha \|x(t)\|^2$$

where α is given by:

$$\alpha = -\min_{i \in \mathscr{S}} \left[A^\top(i) P E + E^\top P A(i) + \mathbb{W}^\top(i) E^\top P E \mathbb{W}(i) \right]$$

Using Dynkin's formula, we obtain

$$\begin{aligned}\mathbb{E}[V(x(t), i)] - \mathbb{E}[V(x_0, r_0)] &= \mathbb{E} \left[\int_0^t \left[\mathscr{L}V(x_s, r_s) \right] ds | x_0, r_0 \right] \\ &\leq -\alpha \mathbb{E} \left[\int_0^t \|x_s\|^2 ds | x_0, r_0 \right]\end{aligned}$$

Since $\mathbb{E}[V(x(t), i)] \geq 0$, the last equation implies

$$\begin{aligned}\alpha \mathbb{E} \left[\int_0^t \|x_s\|^2 ds | x_0, r_0 \right] &\leq \mathbb{E}[V(x(t), i)] + \alpha \mathbb{E} \left[\int_0^t \|x_s\|^2 ds | x_0, r_0 \right] \\ &\leq \mathbb{E}[V(x_0, r_0)], \forall t > 0\end{aligned}$$

This proves that the system under study is stochastically stable and this completes the proof of Theorem 3.1. □

In the absence of the Wiener process, the following results can be established following the same lines:

Corollary 3.1 *System (1) is stochastically stable if and only if there exists a symmetric and positive-definite matrix $P > 0$, such that the following LMIs hold for every $i \in \mathscr{S}$:*

$$E^\top PA(i) + A^\top(i)PE < 0 \tag{8}$$

Let us now concentrate on the design of the static output feedback controller. Plugging the controller (5) in the system dynamics (1) gives:

$$Edx(t) = [A(i) + B(i)FC(i)]\, x(t)dt + \mathbb{W}(i)x(t)dw(t) = A_{cl}(i)x(i)dt + \mathbb{W}(i)x(t)dw(t)$$

with $A_{cl}(i) = A(i) + B(i)FC(i)$.

Based on Theorem 3.1, the closed-loop system is stochastically stable if there exists a set of symmetric and positive-definite matrix such that the following holds:

$$E^\top PA_{cl}(i) + A_{cl}^\top(i)PE + \mathbb{W}^\top(i)E^\top PE\mathbb{W}(i) < 0$$

which gives

$$\begin{aligned} E^\top PA(i) + A^\top(i)PE + E^\top PB(i)FC(i) + [PB(i)FC(i)]^\top E \\ + \mathbb{W}^\top(i)E^\top PE\mathbb{W}(i) < 0 \end{aligned}$$

This inequality matrix is nonlinear in the design parameters P and F. To put it into the LMI form let $X = P^{-1}$. Pre- and post-multiply this inequality by X give:

$$\begin{aligned} XE^\top X^{-1}A(i)X + XA^\top(i)X^{-1}EX + XE^\top X^{-1}B(i)FC(i)X \\ + X\left[X^{-1}B(i)FC(i)\right]^\top EX + X\mathbb{W}^\top(i)E^\top X^{-1}E\mathbb{W}(i)X < 0 \end{aligned}$$

Now let us assume that $EX = XE^\top$ holds, which implies:

$$\begin{aligned} EA(i)X + XA^\top(i)E^\top + EB(i)FC(i)X + XC^\top(i)F^\top B^\top(i)E^\top \\ + X\mathbb{W}^\top(i)E^\top X^{-1}E\mathbb{W}(i)X < 0 \end{aligned}$$

Now if we let $F = ZY^{-1}$ and $YC(i) = C(i)X$ hold for every $i \in \mathscr{S}$ for some appropriate matrices that we have to determine, we get:

$$\begin{aligned} EA(i)X + XA^\top(i)E^\top + EB(i)ZC(i) + C^\top(i)Z^\top B^\top(i)E^\top \\ + X\mathbb{W}^\top(i)E^\top X^{-1}E\mathbb{W}(i)X < 0 \end{aligned}$$

Finally using Schur complement gives:

$$\begin{bmatrix} J(i) & X\mathbb{W}^\top(i)E^\top \\ E\mathbb{W}(i)X & -X \end{bmatrix} < 0$$

with $J(i) = EA(i)X + XA^\top(i)E^\top + EB(i)ZC(i) + C^\top(i)Z^\top B^\top(i)E^\top$

The following theorem summarizes the results of this development.

Theorem 3.2 *If there exist symmetric and positive-definite matrices $X > 0$, and $Y > 0$ and a matrix Z, such that the following holds for each $i \in \mathscr{S}$:*

$$\begin{cases} \begin{bmatrix} J(i) & X\mathbb{W}^\top(i)E^\top \\ E\mathbb{W}(i)X & -X \end{bmatrix} < 0 \\ EX = XE^\top \\ YC(i) = C(i)X \end{cases} \tag{9}$$

where

$$J(i) = EA(i)X + XA^\top(i)E^\top + EB(i)ZC(i) + C^\top(i)Z^\top B^\top(i)E^\top$$

then system (1) is stochastically stable and the controller gain is given by $F = ZY^{-1}$.

In a similar way, we can design a static gain output feedback controller that stochastically stabilizes the class of systems we are considering when the Wiener process is not not acting. These results are given by the following corollary:

Corollary 3.2 *If there exist symmetric and positive-definite matrices $X > 0$, and $Y > 0$ and a matrix Z, such that the following holds for each $i \in \mathscr{S}$:*

$$\begin{cases} EA(i)X + XA^\top(i)E^\top + EB(i)ZC(i) + C^\top(i)Z^\top B^\top(i)E^\top < 0 \\ EX = XE^\top \\ YC(i) = C(i)X \end{cases} \tag{10}$$

then system (1) is stochastically stable and the controller gain is given by $F = ZY^{-1}$.

Let us now consider the effect of the uncertainties. Following the steps as before, we get:

$$\begin{aligned} \mathscr{L}V(x(t),i) &= x^\top(t)\left[A^\top(i)PE + E^\top PA(i) + \mathbb{W}^\top(i)E^\top PE\mathbb{W}(i)\right]x(t) \\ &+x^\top(t)\left[E^\top PD_A(i)F_A(i,t)E_A(i) + E_A^\top(i)F_A^\top(i,t)D_A^\top(i)PE\right]x(t) \end{aligned}$$

Using Lemma 2.1, he have:

$$\begin{aligned} &E^\top PD_A(i)F_A(i,t)E_A(i) + E_A^\top(i)F_A^\top(i,t)D_A^\top(i)PE \\ &\leq \varepsilon_A^{-1}(i)E^\top PD_A(i)D^\top(i)PE + \varepsilon_A(i)E_A^\top(i)E_A(i) \end{aligned}$$

Based on this inequality and the Schur complement 2.2, we can establish the following result.

Corollary 3.3 *System (1) is robust stochastically stable if there exist a symmetric and positive-definite matrix* $P > 0$*, and a set of positive scalars* $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$*, such that the following LMIs hold for every* $i \in \mathscr{S}$*:*

$$\begin{bmatrix} J_u(i) & E^\top P D_A(i) \\ D_A^\top(i) P E & -\varepsilon_A(i)\mathbb{I} \end{bmatrix} < 0 \tag{11}$$

with $J_u(i) = E^\top P A(i) + A^\top(i) P E + \varepsilon_A(i) E_A^\top(i) E_A(i) + \mathbb{W}^\top(i) E^\top P E \mathbb{W}(i)$*.*

The robust stability result when the Wiener process is not acting is given by the following result.

Corollary 3.4 *System (1) is robust stochastically stable if there exist a symmetric and positive-definite matrix* $P > 0$*, and a set of positive scalars* $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$*, such that the following LMIs hold for every* $i \in \mathscr{S}$*:*

$$\begin{bmatrix} J_u(i) & E^\top P D_A(i) \\ D_A^\top(i) P E & -\varepsilon_A(i)\mathbb{I} \end{bmatrix} < 0 \tag{12}$$

with $J_u(i) = E^\top P A(i) + A^\top(i) P E + \varepsilon_A(i) E_A^\top(i) E_A(i)$*.*

For the design of the robust stabilizing static output feedback controller, we can follow the same steps and establish the following result for uncertain system using Lemma 2.1 and Schur complement Lemma 2.2.

Corollary 3.5 *If there exist symmetric and positive-definite matrices* $X > 0$*, and* $Y > 0$ *and a matrix* Z*, and sets of positive scalars* $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$*, and* $\varepsilon_B = (\varepsilon_B(1), \cdots, \varepsilon_B(N))$*, such that the following holds for each* $i \in \mathscr{S}$*:*

$$\begin{cases} \begin{bmatrix} J_X & X E_A^\top(i) & C^\top(i) Z^\top E_B^\top(i) & X \mathbb{W}^\top(i) E^\top \\ E_A(i) X & -\varepsilon_A(i)\mathbb{I} & 0 & 0 \\ E_B^\top(i) Z C(i) & 0 & -\varepsilon_B(i)\mathbb{I} & 0 \\ E \mathbb{W}(i) X & 0 & 0 & -X \end{bmatrix} < 0 \\ EX = XE^\top \\ YC(i) = C(i)X \end{cases} \tag{13}$$

where

$$\begin{aligned} J_X(i) = {} & EA(i)X + XA^\top(i)E^\top + EB(i)ZC(i) + C^\top(i)Z^\top B^\top(i)E^\top \\ & + \varepsilon_A(i) E D_A(i) D_A^\top(i) E^\top + \varepsilon_B(i) E D_B(i) D_B^\top(i) E^\top \end{aligned}$$

then system (1) is robust stochastically stable and the controller gain is given by $F = ZY^{-1}$*.*

When the Wiener process is not acting on the dynamics, the following result gives the procedure to design a constant gain static output feedback controller that robust stochastically stabilizes the class of systems we are studying:

Corollary 3.6 *If there exist symmetric and positive-definite matrices $X > 0$, and $Y > 0$ and a matrix Z, and sets of positive scalars $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$, and $\varepsilon_B = (\varepsilon_B(1), \cdots, \varepsilon_B(N))$, such that the following holds for each $i \in \mathscr{S}$:*

$$\begin{cases} \begin{bmatrix} J_X(i) & XE_A^\top(i) & C^\top(i)Z^\top E_B^\top(i) \\ E_A(i)X & -\varepsilon_A(i)\mathbb{I} & 0 \\ E_B^\top(i)ZC(i) & 0 & -\varepsilon_B(i)\mathbb{I} \end{bmatrix} < 0 \\ EX = XE^\top \\ YC(i) = C(i)X \end{cases} \tag{14}$$

where

$$\begin{aligned} J_X(i) = {} & EA(i)X + XA^\top(i)E^\top + EB(i)ZC(i) + C^\top(i)Z^\top B^\top(i)E^\top \\ & + \varepsilon_A(i)ED_A(i)D_A^\top(i)E^\top + \varepsilon_B(i)ED_B(i)D_B^\top(i)E^\top \end{aligned}$$

then system (1) is robust stochastically stable and the controller gain is given by $F = ZY^{-1}$

4 Numerical Examples

To show the validness of our results, let us consider a two modes system with the following data:

- mode # 1:

$$A(1) = \begin{bmatrix} 0.0 & 1.0 & 1.0 \\ -1.0 & 0.3 & 0.0 \\ 0.0 & 0.0 & -1.0 \end{bmatrix}, \quad B(1) = \begin{bmatrix} 0.0 & 2.0 \\ 1.0 & 0.0 \\ 1.0 & 1.0 \end{bmatrix},$$

$$C(1) = \begin{bmatrix} 100 & 0.0 & 0.0 \\ 0.0 & 0.0 & 100 \end{bmatrix}, \qquad \mathbb{W}(1) = \begin{bmatrix} 0.1 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.1 \end{bmatrix}$$

- mode # 2:

$$A(2) = \begin{bmatrix} 0.0 & 1.5 & 1.5 \\ -1.0 & 0.3 & 0.0 \\ 1.0 & 0.0 & 0.0 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 0.0 & 1.0 \\ 1.2 & 0.0 \\ 2.0 & 1.0 \end{bmatrix},$$

$$C(2) = \begin{bmatrix} 100 & 0.0 & 0.0 \\ 0.0 & 0.0 & 100 \end{bmatrix}, \qquad \mathbb{W}(2) = \begin{bmatrix} 0.2 & 0.0 & 0.0 \\ 0.0 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.2 \end{bmatrix}$$

The matrix E is given by:

$$E = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

The switching between the two modes is described by:

$$\Lambda = \begin{bmatrix} -2.0 & 2.0 \\ 1.0 & 1.0 \end{bmatrix}$$

Solving LMI (9), gives:

$$X = \begin{bmatrix} 0.0029 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0004 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.0029 & 0.0000 \\ 0.0000 & 0.0004 \end{bmatrix},$$

$$Z = \begin{bmatrix} 0.0000 & -0.0000 \\ -2.7563 & -0.0000 \end{bmatrix}$$

This gives the following gains:

$$F = \begin{bmatrix} 0.0089 & -0.0000 \\ -936.9289 & -0.0086 \end{bmatrix}$$

Remark 4.1 *Notice that we wrote by purpose* -0.0000 *instead of* 0.0000 *to give the chance to the reader to compare the results given the LMI toolbox of Matlab.*

5 Conclusion

This paper dealt with the class of singular uncertain stochastic hybrid systems. Under the assumption that the state vector is not available for feedback a static output feedback controller is designed to robust stochastically stabilize this class of systems. The controller gains are determined by solving a set of LMIs either for the nominal or the uncertain system.

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