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Stochastic Hybrid Systems:
Stability and Stabilization

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Abstract

This paper deals with the control of the class of uncertain hybrid stochastic systems. The uncertainties we are considering are of norm bounded type. The stochastic stabilization and robust stabilization problems are treated. LMIs based conditions are developed to design the state feedback controller with constant gain that stochastically (robust stochastically) stabilizes the studied class of systems. Numerical examples are given to show the usefulness of the proposed results.

Résumé

Cet article traite de la commande des systèmes incertains stochastiques à sauts markoviens. Les incertitudes considérées dans ce travail sont du type bornées en norme. Les problèmes de stabilité et de stabilisation de cette classe de systèmes sont considérés ainsi que leur robustesse. Des conditions en forme d'inegalités matricielles linéaires pour le design d'un correcteur par retour d'état à gain constant (indépdent du mode du système) sont développées. Des exemples numériques pour montrer l'éfficacité des résultats développés sont présentés.

1 Introduction

In practice there exist some systems that unfortunately the linear invariant system is unable to model. Among these systems, we quote those with abrupt changes in their dynamics that results from causes like connections or disconnections of some components, failures in the components etc. Examples of such systems are manufacturing systems, power systems, telecommunications systems, etc. The occurrence of these events is random in more cases. These practical systems have been modelled by the class of linear systems with Markovian jumps that we will term in this paper as stochastic hybrid systems. It has two components in the state vector. The first component of this state vector takes values in $\mathbb{R}^n$ and evolves continuously in time and it represents the classical state vector that is usually used in the modern control theory. The second one takes values in a finite set and switches in a random manner between a finite number of states (see Mariton [9], Boukas and Liu [5] and Boukas [3] and the references therein. This component is represented by a continuous-time Markov process. Usually the state vector of the class of stochastic hybrid systems is denoted by $(x(t), r_t)$.

This class of systems has attracted a lot researchers and many problems have been tackled and solved. Among these problems, we quote those of stability, stabilizability, $\mathcal{H}_\infty$ control problem and filtering problem. For more details on what it has been done on this class of systems, we refer the reader to the recent books by Boukas and Liu [5] and Boukas [3] and the references therein. These two books present a good review of the literature of the subject up to 2004.

Regarding the stabilization problem all what have been reported in the literature consider the case of controllers with mode dependent gains which requires the knowledge of the mode at each time we want to switch the controller gain. For more details on this, we refer the reader to ([5, 3, 12, 4, 6, 11, 8, 7]). But practically, this is not always possible which restricts the use of such controllers. It is possible to estimate the mode as it was done by Zhang [13] and therefore continue to use the controller. But if we are interested by real time applications this is not possible unless the size of the system is small.

Our goal in this paper consists of dealing with this limitation and develop a design procedure for state feedback controllers with constant gain that do not require the knowledge of the system mode. We will deal with the stability and the stabilizability problems. It is also question to deal with the robust stabilization.

The rest of the paper is organized as follows. In Section 2, the problem we are considering is stated and some useful definitions are given. Section 3 gives the main results of the paper. In Section 4, some numerical examples are provided to show the usefulness of the proposed results.

2 Problem Statement

Let us consider a dynamical system defined in a fundamental probability space $(\Omega, \mathscr{F}, \mathscr{P})$ and assume that its dynamics is described by the following differential equations:

$$\begin{cases} \dot{x}(t) = A(r_t,t)x(t) + B(r_t,t)u(t), \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $x_0 \in \mathbb{R}^n$ is the initial state, $u(t) \in \mathbb{R}^m$ is the control input, $\{r_t, t \geq 0\}$ is the continuous-time Markov process taking values in a finite space $\mathscr{S}$ and that describes the evolution of the mode at time t, when the mode r_t takes the value $i \in \mathscr{S}$, $A(i,t) \in \mathbb{R}^{n\times n}$ and $B(i,t) \in \mathbb{R}^{n\times m}$ are matrices with the following forms:

$$\begin{aligned} A(i,t) &= A(i) + D_A(i)F_A(i,t)E_A(i) \\ B(i,t) &= B(i) + D_B(i)F_B(i,t)E_B(i) \end{aligned}$$

with $A(i)$, $D_A(i)$, $E_A(i)$, $B(i)$, $D_B(i)$ and $E_B(i)$ are real known matrices with appropriate dimensions, and $F_A(i,t)$ and $F_B(i,t)$ are unknown real matrices that satisfy the following:

$$\begin{cases} F_A^\top(i,t)F_A(i,t) \leq \mathbb{I} \\ F_B^\top(i,t)F_B(i,t) \leq \mathbb{I} \end{cases} \tag{2}$$

The Markov process $\{r_t, t \geq 0\}$ beside taking values in the finite set $\mathscr{S}$, the switching between the different modes is described by the following probability transitions:

$$\mathbb{P}\left[r_{t+h} = j | r_t = i\right] = \begin{cases} \lambda_{ij}h + o(h) & \text{when } r_t \text{ jumps from } i \text{ to } j \\ 1 + \lambda_{ij}h + o(h) & \text{otherwise} \end{cases} \tag{3}$$

where λ_{ij} is the transition rate from mode i to mode j with $\lambda_{ij} \geq 0$ when $i \neq j$ and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$ and $o(h)$ is such that $\lim_{h\to 0} \frac{o(h)}{h} = 0$.

For system (1), when $u(t) \equiv 0$ for all $t \geq 0$, we have the following definitions.

Definition 2.1 *System (1) with $F_A(i,t) = F_B(i,t) = 0$ for each $i \in \mathscr{S}$ and for all $t \geq 0$, is said to be*

(i) stochastically stable (SS) if there exists a finite positive constant $T(x_0, r_0)$ such that the following holds for any initial conditions (x_0, r_0):

$$\mathbb{E}\left[\int_0^\infty \|x(t)\|^2 dt | x_0, r_0\right] \leq T(x_0, r_0); \tag{4}$$

(ii) mean square stable (MSS) if

$$\lim_{t\to\infty} \mathbb{E}\|x(t)\|^2 = 0 \tag{5}$$

holds for any initial condition (x_0, r_0);

(iii) mean exponentially stable (MES) if there exist positive constants α and β such that the following holds for any initial conditions (x_0, r_0):

$$\mathbb{E}\left[\|x(t)\|^2 | x_0, r_0\right] \le \alpha \|x_0\| e^{-\beta t}. \tag{6}$$

Definition 2.2 *System (1) with $F_A(i,t) = F_B(i,t) = 0$ for all modes and for $t \ge 0$, is said to be stabilizable in the SS (MES, MSQS) sense if there exists a controller such that the closed-loop system is SS (MES, MSQS) for every initial conditions (x_0, r_0).*

When the uncertainties are not equal to zero, the different types of stability and the stabilizability are defined in the same way and termed respectively as robust stability and robust stabilizability.

The goal of this paper is to design a state feedback controller with constant gain that stochastically (robust stochastically) stabilizes the class of systems we are considering in this paper. The structure of the controller we will be using here is given by the following expression:

$$u = \mathscr{K} x(t) \tag{7}$$

where $\mathscr{K}$ is a constant gain that we have to determine.

We are mainly concerned with the design of such controller. LMI based conditions are searched since the design becomes easier and the gain can be obtained by solving the appropriate LMIs using the existing developed algorithms.

Before closing this section let us give some lemmas that we will use in the rest of the paper.

Lemma 2.1 *[10] Let H, F and G be real matrices of appropriate dimensions then, for any scalar $\varepsilon > 0$ and for all matrices F satisfying $F^T F \le I$, we have:*

$$HFG + G^\top F^\top H^\top \le \varepsilon H H^\top + \varepsilon^{-1} G^\top G \tag{8}$$

Lemma 2.2 *The linear matrix inequality*

$$\begin{bmatrix} H & S^\top \\ S & R \end{bmatrix} > 0$$

is equivalent to

$$R > 0, H - S^\top R^{-1} S > 0$$

where $H = H^\top$, $R = R^\top$ and S is a constant matrix.

3 Main Results

Our goal in this paper consists of designing a state feedback controller with constant gain that stochastically (robust stochastically) stabilizes the class of systems we are considering. The rest of this section will develop results for the nominal systems and then extend the obtained ones to handle the case of stochastic hybrid systems with norm bounded uncertainties.

Let all the uncertainties of the system be equal to zero and pose $u(t) = 0$ for all $t \geq 0$. In this case the dynamics becomes:

$$\begin{cases} \dot{x}(t) = A(r_t)x(t) \\ x(0) = x_0 \end{cases} \tag{9}$$

The stability of this system is given by the following theorem.

Theorem 3.1 *If there exists a symmetric and positive-definite matrix, P, such that the following holds for each $i \in \mathscr{S}$:*

$$A^\top(i)P + PA(i) < 0 \tag{10}$$

then, system (9) is stochastically stable under the state feedback controller with constant gain.

Proof: The proof of this theorem is similar to the one of the next Theorem. Therefore we will skip its proof. The interested reader by the details of this proof can adapt the one of the next theorem. □

Remark 3.1 *Since the results developed in this paper are only sufficient conditions for robust stochastic stability, we will restrict ourselves to the proofs of the sufficient condition only.*

Let us start with the nominal system. Using the system dynamics and the expression of the controller (7), we get the following expression for the closed-loop:

$$\dot{x}(t) = [A(i) + B(i)\mathscr{K}]\, x(t) \tag{11}$$

The following theorem states the first results on stochastic stabilizability of the nominal system.

Theorem 3.2 *Let $\mathscr{K}$ be a given gain matrix. If there exists a symmetric and positive-definite matrix, P, such that the following holds for each $i \in \mathscr{S}$:*

$$A^\top(i)P + \mathscr{K}^\top B^\top(i)P + PA(i) + PB(i)\mathscr{K} < 0 \tag{12}$$

then system (1) is stochastically stable under the state feedback controller with constant gain.

Proof: Let P be a symmetric and positive-definite matrix and definite the Lyapunov candidate function as follows:

$$V(x_t, i) = x^\top(t)Px(t) \tag{13}$$

Let $\mathscr{L}$ denote the infinitesimal generator of the Markov process $(x(t), i)$. Its expression is given by:

$$\mathscr{L}V(x(t), i) = \dot{x}^\top(t)Px(t) + x^\top(t)P\dot{x}(t) + \sum_{j=1}^{N} \lambda_{ij} x^\top(t)Px(t)$$

Using the fact that $\sum_{j=1}^{N} \lambda_{ij} = 0$ we get:

$$\begin{aligned}
\mathscr{L}V(x(t), i) &= \dot{x}^\top(t)Px(t) + x^\top(t)P\dot{x}(t) \\
&= [[A(i) + B(i)\mathscr{K}]\, x(t)]^\top Px(t) + x^\top P\, [A(i) + B(i)\mathscr{K}] \\
&= x^\top(t) \left[A^\top(i)P + \mathscr{K}^\top B^\top(i)P + PA(i) + P\mathscr{K}B(i)\right] x(t) \\
&= x^\top(t)\Lambda(i)x(t)
\end{aligned}$$

with $\Lambda(i) = A^\top(i)P + \mathscr{K}^\top B^\top(i)P + PA(i) + PB(i)\mathscr{K}$.

Using condition (12) we get:

$$\mathscr{L}V(x(t), i) \leq -\min_{i \in \mathscr{S}} \{\lambda_{min}(-\Lambda(i))\}\, x^\top(t)x(t)$$

Combining this again with Dynkin's formula (see Arnold [1]) yields:

$$\begin{aligned}
\mathbb{E}\,[V(x(t), i)] - \mathbb{E}\,[V(x(0), r_0)] &= \mathbb{E}\left[\int_0^t \mathscr{L}V(x(s), r_s)ds|(x_0, r_0)\right] \\
&\leq -\min_{i \in \mathscr{S}}\{\lambda_{min}(-\Lambda(i))\}\mathbb{E}\left[\int_0^t x^\top(s)x(s)ds|(x_0, r_0)\right],
\end{aligned}$$

implying, in turn,

$$\begin{aligned}
&\min_{i \in \mathscr{S}}\{\lambda_{min}(-\Lambda(i))\}\mathbb{E}\left[\int_0^t x^\top(s)x(s)ds|(x_0, r_0)\right] \\
&\qquad \leq \mathbb{E}\,[V(x(0), r_0)] - \mathbb{E}\,[V(x(t), r_t)] \\
&\qquad \leq \mathbb{E}\,[V(x(0), r_0)]\,.
\end{aligned}$$

This yields that

$$\mathbb{E}\left[\int_0^t x^\top(s)x(s)ds|(x_0, r_0)\right] \leq \frac{\mathbb{E}\,[V(x(0), r_0)]}{\min_{i \in \mathscr{S}}\{\lambda_{min}(-\Lambda(i))\}}$$

holds for any $t > 0$. Letting t goes to infinity implies that

$$\mathbb{E}\left[\int_0^t x^\top(s)x(s)ds|(x_0, r_0)\right]$$

is bounded by a constant $T(x_0, r_0)$ given by:

$$T(x_0, r_0) = \frac{\mathbb{E}\left[V(x(0), r_0)\right]}{\min_{i\in\mathscr{S}}\{\lambda_{min}(-\Lambda(i))\}}$$

which ends the proof of Theorem 3.2. □

From this theorem, it is possible to stochastically stabilize the class of systems we are considering by a constant gain state feedback controller of the form (7) if the condition (12) is satisfied. The P we are searching is constant.

To determine the gain $\mathscr{K}$, let us transform the condition (12). For this purpose, let $X = P^{-1}$ and pre- and post-multiply this inequality by X gives the following inequality:

$$XA^\top(i) + X\mathscr{K}^\top B^\top(i) + A(i)X + B(i)\mathscr{K}X < 0$$

Letting $K = \mathscr{K}X$, we get:

$$XA^\top(i) + K^\top B^\top(i) + A(i)X + B(i)K < 0$$

If this LMI is feasible, the controller gain is then given by:

$$\mathscr{K} = KX^{-1} = KP$$

The controller gain can be determined by solving the LMI of the following theorem.

Theorem 3.3 *If there exist a symmetric and positive-definite matrix $X > 0$ and a constant gain K such that following LMI holds for each $i \in \mathscr{S}$:*

$$XA^\top(i) + A(i)X + K^\top B^\top(i) + B(i)K < 0 \tag{14}$$

then, the state feedback controller with the gain KX^{-1} stochastically stabilizes the system.

If the uncertainties are acting on the dynamics, the previous results can be extended to handle this case. In fact, if we replace $A(i)$ and $B(i)$ by $A(i)+\Delta A(i,t)$ and $B(i)+\Delta B(i,t)$ respectively in the previous condition that determines the controller gain, we get:

$$\begin{aligned}XA^\top(i) + A(i)X + K^\top B^\top(i) + B(i)K + X\Delta A^\top(i) + \Delta A(i)X \\ +K^\top \Delta B^\top(i) + \Delta B(i)K < 0\end{aligned}$$

Using now lemma 2, we get:

$$\begin{aligned}X\Delta A^\top(i) + \Delta A(i)X &\leq \varepsilon_A(i)D_A(i)D_A^\top(i) + \varepsilon_A^{-1}(i)XE_A^\top(i)E_A(i)X \\ K^\top \Delta B^\top(i) + \Delta B(i)K &\leq \varepsilon_B(i)D_B(i)D_B^\top(i) + \varepsilon_B^{-1}(i)K^\top E_B^\top(i)E_B(i)K\end{aligned}$$

Based on these inequalities, to guarantee the stochastic stability of the closed-loop, we need to have the following:

$$XA^\top(i) + A(i)X + K^\top B^\top(i) + B(i)K + \varepsilon_A(i)D_A(i)D_A^\top(i) + \varepsilon_B(i)D_B(i)D_B^\top(i) \\ +\varepsilon_A^{-1}(i)XE_A^\top(i)E_A(i)X + \varepsilon_B^{-1}(i)K^\top E_B^\top(i)E_B(i)K < 0$$

Using Schur complement, we get:

$$\begin{bmatrix} \mathscr{J}(i) & XE_A^\top(i) & K^\top E_B^\top(i) \\ E_A(i)X & -\varepsilon_A(i)\mathbb{I} & 0 \\ E_B(i)K & 0 & -\varepsilon_B(i)\mathbb{I} \end{bmatrix} < 0$$

with

$$\mathscr{J}(i) = XA^\top(i) + A(i)X + K^\top B^\top(i) + B(i)K \\ +\varepsilon_A(i)D_A(i)D_A^\top(i) + \varepsilon_B(i)D_B(i)D_B^\top(i)$$

The following theorem gives the results that determines the controller with constant gain that robustly stochastically stabilizes the class of systems with norm bounded uncertainties we are studying.

Theorem 3.4 *If there exist a symmetric and positive-definite matrix $X > 0$ and sets of positive constants $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$ and $\varepsilon_B = (\varepsilon_B(1), \cdots, \varepsilon_B(N))$ and a constant gain K such the following LMI holds for each $i \in \mathscr{S}$:*

$$\begin{bmatrix} \mathscr{J}(i) & XE_A^\top(i) & K^\top E_B^\top(i) \\ E_A(i)X & -\varepsilon_A(i)\mathbb{I} & 0 \\ E_B(i)K & 0 & -\varepsilon_B(i)\mathbb{I} \end{bmatrix} < 0 \tag{15}$$

with

$$\mathscr{J}(i) = XA^\top(i) + A(i)X + K^\top B^\top(i) + B(i)K \\ +\varepsilon_A(i)D_A(i)D_A^\top(i) + \varepsilon_B(i)D_B(i)D_B^\top(i)$$

then, the stabilizing controller gain is KX^{-1}.

The advantage of the developed results in this paper is that the expression of the controller does not depend of the mode and therefore has the power to be implemented for systems with abrupt changes in the dynamics that can be modelled by the class of systems we are treating here in real-time. No estimation for the mode is needed.

4 Numerical Examples

In the previous section, we developed results that determine the constant gain that stochastically (robust stochastically) stabilizes the class of systems we are treating in this paper.

The advantage of the results proposed in this paper is that the controller expression does not require the knowledge the system mode. It require only the accessibility of the state vector. The conditions we developed are in the LMI form which makes their resolution easy. In the rest of this section we will give some numerical examples to show the usefulness of our results. Two numerical examples are presented.

Example 4.1 *Let us consider a system with two modes with the following data:*

- *transition probability rates matrix:*

$$\Lambda = \begin{bmatrix} -2.0 & 2.0 \\ 3.0 & -3.0 \end{bmatrix}$$

- *mode 1:*

$$A(1) = \begin{bmatrix} 1.0 & -0.5 \\ 0.1 & 1.0 \end{bmatrix}, B(1) = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

- *mode 2:*

$$A(1) = \begin{bmatrix} -0.2 & -0.5 \\ 0.5 & -0.25 \end{bmatrix}, B(1) = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

First of all notice the system is instable in mode 1 and it is stochastically instable. Solving the LMI (12), we get:

$$X = \begin{bmatrix} 0.4791 & 0.1129 \\ 0.1129 & 0.4698 \end{bmatrix},$$
$$K = \begin{bmatrix} -1.0478 & -22.8520 \\ 22.8557 & -1.1226 \end{bmatrix},$$

which are both symmetric and positive-definite matrices. Using (7) gives the following constant gain:

$$\mathscr{K} = \begin{bmatrix} 9.8277 & -51.0015 \\ 51.1597 & -14.6804 \end{bmatrix}.$$

With this controller, the closed-loop becomes:

$$\dot{x}(t) = A_{cl}(r_t)x(t)$$

with

$$A_{cl}(i) = \begin{cases} \begin{bmatrix} 10.8277 & -51.5015 \\ 51.2597 & -13.6804 \end{bmatrix} & \text{when } i = 1 \\ \begin{bmatrix} 9.6277 & -51.5015 \\ 51.6597 & -14.9304 \end{bmatrix} & \text{otherwise} \end{cases} \tag{16}$$

The standard conditions for stochastic stability, which can be summarized as follows says: If there exists a set of symmetric and positive-definite matrices $P = (P(1), P(2))$ such that the following holds for each $i \in \mathscr{S}$:

$$A^\top(i)P(i) + P(i)A(i) + \sum_{j=1}^{N} \lambda_{ij} P(j) < 0$$

then the closed-loop is stochastically stable.

Using now these conditions, we get the following matrices:

$$P(1) = \begin{bmatrix} 8.5974 & -2.0276 \\ -2.0276 & 8.4410 \end{bmatrix}, P(2) = \begin{bmatrix} 5.5621 & -0.4028 \\ -0.4028 & 5.6338 \end{bmatrix}$$

which are both symmetric and positive-definite matrices and therefore the closed-loop system is stochastically stable under the constant gain state feedback controller.

Example 4.2 *To design a robust stabilizing controller with constant gain, let us consider again a system with two modes and the following data:*

- *mode 1:*

$$\begin{aligned} A(1) &= \begin{bmatrix} 1.0 & -0.5 \\ 0.1 & 1.0 \end{bmatrix}, D_A(1) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, E_A(1) = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix} \\ B(1) &= \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} D_B(1) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, E_B(1) = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix} \end{aligned}$$

- *mode 2:*

$$\begin{aligned} A(2) &= \begin{bmatrix} -0.2 & 0.5 \\ 0.0 & -0.25 \end{bmatrix}, D_A(2) = \begin{bmatrix} 0.13 \\ 0.1 \end{bmatrix} E_A(2) = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix} \\ B(2) &= \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}, D_B(2) = \begin{bmatrix} 0.13 \\ 0.1 \end{bmatrix}, E_B(2) = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix} \end{aligned}$$

The transition probability rates matrix between the two modes is given by:

$$\Lambda = \begin{bmatrix} -2.0 & 2.0 \\ 3.0 & -3.0 \end{bmatrix}.$$

Letting $\varepsilon_A(1) = \varepsilon_A(2) = 0.5$ and $\varepsilon_B(1) = \varepsilon_B(2) = 0.1$ and solving the LMIs (15), we get:

$$\begin{aligned} X &= \begin{bmatrix} 0.2641 & 0.0897 \\ 0.0897 & 0.2001 \end{bmatrix}, \\ K &= \begin{bmatrix} -0.5365 & -0.0496 \\ -0.0315 & -0.5139 \end{bmatrix}. \end{aligned}$$

which gives the following gain:

$$\mathscr{K} = \begin{bmatrix} -2.2975 & 0.7827 \\ 0.8891 & -2.9678 \end{bmatrix}.$$

Using the results of this theorem, it results that the system of this example is stochastically stable under the state feedback controller with the computed constant gain. We can also show this by following the steps of the previous example.

5 Conclusion

This paper dealt with the class of hybrid stochastic systems. Both the stability and the stabilizability problems are treated. A state feedback controller with constant gain is proposed to stochastically (robust stochastically) stabilize the class of hybrid systems. The uncertainties we considered in this paper are of norm bounded type. The conditions we developed are in LMI form which makes the resolution easier using the existing tools. The advantage of the proposed results in this paper is that the controller expression does not require the knowledge the system mode. It require only the accessibility of the state vector.

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