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S. Ibrir
E.K. Boukas

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S. Ibrir

Mechanical Engineering Department
École Polytechnique de Montréal
P.O. Box 6079, station "Centre-ville"
Montréal, Québec, Canada H3C 3A7
ibrir.salim@polymtl.ca

E.K. Boukas

Mechanical Engineering Department
GERAD and École Polytechnique de Montréal
P.O. Box 6079, station "Centre-ville"
Montréal, Québec, Canada H3C 3A7
el-kebir.boukas@polymtl.ca

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Abstract

This paper considers the stability and stabilizability problems for the class of linear switching singular (LSS) systems. The stabilizing class of controllers we will be considering in this work is a constant-gain one. First, we begin by giving sufficient LMI-based conditions for stability and stabilizability of regular switching systems. Subsequently, we extend the theory to linear singular switching systems (LSS) that can be regularized by output derivative injection. Numerical examples are given to show the effectiveness of the control strategy.

Résumé

Ce papier traite de la stabilité et de la stabilisabilité de la classe des systèmes singuliers à sauts. Un contrôleur à gain constant est proposé pour stabiliser la classe de systèmes considérée. Dans un premier temps, nous étudions la classe des systèmes réguliers. Ensuite, après avoir régularisé la classe de systèmes considérée dans cet article par un contrôleur spécial, les résultats développés précdemment seront étendus à cette classe. Les résultats développés sont en forme de LMI. La méthodologie repose sur la méthode de Lyapunov. Plusieurs exemples numériques sont présentés pour montrer l'importance des résulats.

1 Introduction

Switching systems are an important class of systems that can be used to model a variety of physical systems. This fact has been the catalyst of the development of this class. Typical examples of such systems include transmission and stepper motors, computer disk drives, robotic systems under constrained environments, and other types of hybrid systems, see for instance [6], [7]. The switches are caused by causes such as connections and disconnections of mechanical components, switches in electrical circuits, flow-control valves, and so forth. In these systems, which we call switching systems, discrete switches of their continuous dynamics often have great influence on their performance. Considerable efforts have been made to give a systematic framework for the stabilization of such systems. However, many control strategies are discontinuous and are based on the assumption that the switching mode is well detected by an appropriate mechanism.

The idea of switching between different stabilizing controllers is well-known in modern control theory. However, this requires the knowledge of the mode at each time the systems switch which makes the control a little bit harder when the mode is difficult to estimate. We refer the reader to the references [3], [4] to see what have been done in this area. An alternate consists of using a controller with fixed gain that ensures the stability of the closed-loop of the switching system whatever the changes in the nominal matrices are. This control strategy permits us to achieve the desired stabilization objective without any knowledge of both switching instants and switching modes for the class of systems under consideration. This paper considers the stability and stabilizability problems for the class of linear switching singular (LSS) systems. The stabilizing class of controllers we will be considering in this work is a constant-gain one. First, we begin by giving sufficient LMI-based conditions for stability and stabilizability of regular switching systems. Subsequently, we extend the theory to linear singular switching systems (LSS) that can be regularized by output derivative injection. Numerical examples are given to show the effectiveness of the control strategy.The rest of this paper is organized as follows. In Section 2, the problem formulation is stated and some preliminary results are established. In Section 3, the main results of this paper are given. Section 4 will be the subject of the control of LSS system with prescribed degree of stability. In Section 5, an illustrative example is provided to show the usefulness of the proposed results.

2 Problem Statement and Preliminary Results

Let us consider the following class of systems:

$$\left.\begin{array}{l} E\dot{x}(t) = A(\xi)x(t) + B(\xi)u(t), \\ y(t) = C(\xi)x(t), \end{array}\right\} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector at time t, $y(t) \in \mathbb{R}^p$ is the system output and $u(t) \in \mathbb{R}^m$ is the control input, the matrices $A(\xi)$, $B(\xi)$ and $C(\xi)$ with $\xi = 1, 2, \cdots, s$ are supposed to belong to the known sets $\mathcal{A} = \{A_1, A_2, \cdots, A_s\}$, $\mathcal{B} = \{B_1, B_2, \cdots, B_s\}$ and

$\mathcal{C} = \{C_1, C_2, \cdots, C_s\}$. The matrix E is a known matrix that is supposed to be singular with $rank(E) = r \leq n$.

During the functioning of the system, the parameters ξ will switch between s known values which makes the state matrix and the control matrix change with time. The switching time is not defined by an appropriate deterministic or stochastic rule. To control such class of systems, we need to know at which time the system switches and which mode the system occupies at this switch. Notice that this is a difficult problem that requires the estimation of the mode after the switch. To overcome this problem our goal here consists of designing a controller with constant gain that doesn't require the knowledge of the mode at the switch. Our methodology comprises two steps. The first one consists of transforming the singular class of systems we are considering to a regular one by choosing an appropriate transformation and then compute the controller that stabilizes the obtained class of systems by a controller with a fixed gain.

Before tackling the problem we are considering in this paper, let us consider the continuous-time linear system

$$\left.\begin{array}{l} \dot{x}(t) = A(\xi)\, x(t) + B(\xi)\, u(t), \\ y(t) = C(\xi)\, x(t), \end{array}\right\} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $y(t) \in \mathbb{R}^p$ is the system output. The matrices $A(\xi) \in \mathbb{R}^{n\times n}$, $B(\xi) \in \mathbb{R}^{n\times m}$ and $C(\xi) \in \mathbb{R}^{p\times n}$ switch between the matrices $(A_i)_{1\leq i\leq s}$, $(B_i)_{1\leq i\leq s}$, and $(C_i)_{1\leq i\leq s}$, respectively. We shall refer to this as the linear switching system (LS). Let

$$\widetilde{A}(\xi) = \begin{bmatrix} A(\xi) & B(\xi) \\ 0_{m\times n} & 0_{m\times m} \end{bmatrix}, \widetilde{B} = \begin{bmatrix} 0_{n\times m} \\ I_{m\times m} \end{bmatrix}. \tag{3}$$

Then system (2) is stabilizable by a fixed-gain state feedback if the pairs

$$\big(\widetilde{A}(\xi), \widetilde{B}\big), \quad 1 \leq \xi \leq s, \tag{4}$$

are controllable. This is due to the fact that the switch in matrix $B(\xi)$ can be translated to the state matrix by adding m integrators to system (2). If we put $\dot{u} = v$, we have

$$\left.\begin{array}{l} \begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} = \begin{bmatrix} A(\xi) & B(\xi) \\ 0_{m\times n} & 0_{m\times m} \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} 0_{n\times m} \\ I_{m\times m} \end{bmatrix} v(t), \\ y(t) = \begin{bmatrix} C(\xi) & 0_{p\times m} \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}. \end{array}\right\} \tag{5}$$

The fact that the switch is regrouped in the state matrix only, this will simplify enormously the design of a constant-gain state feedback that stabilizes system (2) at the origin.

Based on the controllability condition (4), then we are ready to give LMI-based sufficient conditions for the stability and the stabilizability of system (2) by a fixed-gain controllers. The stability result is summarized in the following dentition.

Definition 1 *System (2) is asymptotically stable* ($u(t) \equiv 0$) *if there exist a set of symmetric and positive definite matrices* $P_1, \cdots, P_s$ *such that the set of coupled LMIs*

$$A'(j)\left(\sum_{i=1}^{s} P_i\right) + \left(\sum_{i=1}^{s} P_i\right) A(j) < 0, \tag{6}$$

is feasible for all $j = 1, \cdots, s$.

The result given above is a direct consequence of the well-known Lyapunov theorem for linear systems. It is clear that if $\left(\sum_{i=1}^{s} P_i\right)$ is replaced by another positive definite matrix, the result becomes evident and translates that all modes of the switching system are stable.

Next, we shall derive LMI-based sufficient conditions for the stabilizability of system (2) with a constant-gain state feedback. Define

$$\eta(t) = \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \, \widetilde{C}(\xi) = \begin{bmatrix} C(\xi) & 0_{p\times m} \end{bmatrix},$$

then system (5) is rewritten as

$$\left.\begin{aligned} \dot{\eta}(t) &= \widetilde{A}(\xi)\, \eta(t) + \widetilde{B}\, v(t), \\ y(t) &= \widetilde{C}(\xi)\, \eta(t). \end{aligned}\right\} \tag{7}$$

Let $v(t) = K\Big(\sum_{i=1}^{s} P_i\Big)^{-1} \eta(t)$ be the proposed stabilizing controller where K is a constant-gain matrix of appropriate dimension to be determined and $\big(P_i\big)_{1\leq i\leq s}$ are symmetric positive definite matrices to be calculated. Plugging the expression of the controller $v(t)$ in the dynamics (7), we get the following one for the closed loop:

$$\dot{\eta}(t) = \left(\widetilde{A}(\xi) + \widetilde{B}K\left(\sum_{i=1}^{s} P_i\right)^{-1}\right)\eta(t). \tag{8}$$

Taking $V(\eta) = \eta'(t)\Big(\sum_{i=1}^{s} P_i\Big)^{-1} \eta(t)$ as a Lyapunov function candidate for (8), we obtain

$$\begin{aligned} \dot{V}(\eta(t)) &= \dot{\eta}'(t)\left(\sum_{i=1}^{s} P_i\right)^{-1} \eta(t) + \eta'(t)\left(\sum_{i=1}^{s} P_i\right)^{-1} \dot{\eta}(t) \\ &= \eta'(t)\left(\widetilde{A}'(\xi)\left(\sum_{i=1}^{s} P_i\right)^{-1} + \left(\sum_{i=1}^{s} P_i\right)^{-1} \widetilde{A}(\xi)\right. \end{aligned}$$

$$+ \left(\sum_{i=1}^{s} P_i\right)^{-1} K'\widetilde{B}'\left(\sum_{i=1}^{s} P_i\right)^{-1}$$
$$+ \left(\sum_{i=1}^{s} P_i\right)^{-1} \widetilde{B}K\left(\sum_{i=1}^{s} P_i\right)^{-1}\Bigg)\eta(t).$$

The switching system (7) is stable if for each mode j there exist a constant matrix K and a set of $P_i > 0$, $i = 1, \cdots, s$ such that

$$\begin{aligned} &\left(\widetilde{A}'(j)\left(\sum_{i=1}^{s} P_i\right)^{-1} + \left(\sum_{i=1}^{s} P_i\right)^{-1} \widetilde{A}(j)\right. \\ +\ &\left(\sum_{i=1}^{s} P_i\right)^{-1} K'\widetilde{B}'\left(\sum_{i=1}^{s} P_i\right)^{-1} \\ +\ &\left.\left(\sum_{i=1}^{s} P_i\right)^{-1} \widetilde{B}K\left(\sum_{i=1}^{s} P_i\right)^{-1}\right) < 0,\ 1 \le j \le s. \end{aligned} \tag{9}$$

Pre- and post-multiplying the last inequality by $\left(\sum_{i=1}^{s} P_i\right)$, then (9) is equivalent to

$$\begin{aligned} \left(\left(\sum_{i=1}^{s} P_i\right)\widetilde{A}'(j) + \widetilde{A}(j)\left(\sum_{i=1}^{s} P_i\right) + K'\widetilde{B}' + \widetilde{B}K\right) < 0, \\ 1 \le j \le s. \end{aligned} \tag{10}$$

The set of coupled LMI conditions (10) is linear with respect to $(P_i)_{1 \le i \le s}$ and K, which renders the search of a stabilizing controller quite simple by the use of any LMI package. We have proved the following theorem.

Theorem 1 *System (7) is stabilizable by the constant-gain feedback*

$$v(t) = K\left(\sum_{i=1}^{s} P_i\right)^{-1} \eta(t)$$

provided that there exist a set of $(n+m)$ *by* $(n+m)$ *symmetric and positive definite matrices* $P_1, P_2, \cdots, P_s$ *and* $K \in \mathbb{R}^{m \times n+m}$ *such that the LMI problem (10) is feasible.*

To show the usefulness of Theorem 1, let us consider the two-states switching system having the following nominal matrices

$$A(1) = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix},\ B(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A(2) = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix},\ B(2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

This implies

$$\begin{aligned}\widetilde{A}(1) &= \begin{bmatrix} -1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \widetilde{A}(2) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.\end{aligned}$$

It is easy to show that this system is controllable with a fixed-gain controller by checking the rank controllability condition of the pairs $\left(\widetilde{A}(1), \widetilde{B}\right)$ and $\left(\widetilde{A}(2), \widetilde{B}\right)$, see Eq. (4). Solving the LMI conditions (10), we obtain

$$P_1 = \begin{bmatrix} 8.6152 & -3.0889 & -8.2816 \\ -3.0889 & 1.4071 & 1.3233 \\ -8.2816 & 1.3233 & 19.7763 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 3.4725 & -1.1218 & -4.0583 \\ -1.1218 & 0.4721 & 0.7571 \\ -4.0583 & 0.7571 & 8.7987 \end{bmatrix},$$

$$K = \begin{bmatrix} -28.5748 & 5.0280 & -19.6086 \end{bmatrix}.$$

The two matrices P_1 and P_2 satisfy the conditions of Theorem 1 therefore, the closed-loop is asymptotically stable.

3 Main Result

Let us now return to the initial problem and see how we can transform the LSS system to a regular one by using appropriate transformation. For this purpose, let us consider the LSS system

$$\left.\begin{array}{l} E\dot{x}(t) = A(\xi)\, x(t) + B(\xi)u(t), \\ y(t) = C(\xi)\, x(t), \end{array}\right\} \tag{11}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the system output and $u(t) \in \mathbb{R}^m$ is the control input. The matrix $E \in \mathbb{R}^{n\times n}$ is a singular matrix, and we shall assume that rank $(E) = r \leq n$. The matrices $A(\xi)$, $B(\xi)$ and $C(\xi)$ are defined as in Section 2. For each mode ξ, system (11) is said to be regular if the pencil pair $s\,E - A(\xi)$ is regular, i.e., $det\,(s\,E - A(\xi)) \neq 0 \ \forall \xi$. Since the regularity condition implies both the existence and the uniqueness of solutions, this restricts, in turn, the family of practical singular systems. Consequently, it is advantageous to think about regularization of the singular system by some appropriate feedbacks [1]. In this paper we restrict ourselves to output-derivative controller of the form

$$u(t) = -L\dot{y}(t) + v(t), \tag{12}$$

where $v(t)$ is the new controller to be designed later, and L is some constant matrix with appropriate dimension.

The first term of controller (12) is used to regularize the singular system (11) while the second term is employed to stabilize the switching regularized system. Combining (12) and (11), we get

$$\big(E + B(\xi)LC(\xi)\big)\dot{x} = A(\xi)\,x + B(\xi)v, \tag{13}$$

If the following rank condition

$$\operatorname{rank}\big(E + B(j)LC(j)\big) = n, \qquad 1 \le j \le s, \tag{14}$$

is verified, then system (11) is equivalent to the following

$$\begin{aligned}\dot{x}(t) &= \big(E + B(\xi)LC(\xi)\big)^{-1}A(\xi)\,x(t)\\ &\quad+\big(E + B(\xi)LC(\xi)\big)^{-1}B(\xi)v(t),\end{aligned} \tag{15}$$

Let

$$\bar{A}(\xi) = \big(E + B(\xi)LC(\xi)\big)^{-1}A(\xi),$$
$$\bar{B}(\xi) = \big(E + B(\xi)LC(\xi)\big)^{-1}B(\xi),$$

be the new nominal matrices of the regularized system, then under the feedback (12), system (11) is equivalent to the following system

$$\left.\begin{array}{l}\dot{x}(t) = \bar{A}(\xi)\,x(t) + \bar{B}(\xi)v(t),\\ y(t) = C(\xi)\,x(t),\end{array}\right\} \tag{16}$$

which is in the form of the problem studied earlier. By adding a set of integrators to the last system and define

$$w(t) = \dot{v}(t),$$
$$\bar{\eta}(t) = \begin{bmatrix} x(t) & v(t) \end{bmatrix}',$$

then we can apply the result of the last section to derive the controller $w(t)$. This gives

$$\dot{\bar{\eta}}(t) = \underbrace{\begin{bmatrix} \bar{A}(\xi) & \bar{B}(\xi) \\ 0_{m\times n} & 0_{m\times m} \end{bmatrix}}_{\widehat{A}(\xi)} \bar{\eta}(t) + \underbrace{\begin{bmatrix} 0_{n\times m} \\ I_{m\times m} \end{bmatrix}}_{\widehat{B}} w(t). \tag{17}$$

The whole design of the control strategy is given by the following theorem.

Theorem 2 *Consider system (17) satisfying the controllability condition for each mode* ξ *and suppose that there exist a set of symmetric and positive definite matrices* $\big(X_i\big)_{1\leq i\leq s}$*, and two matrices* L *and* Y *of appropriate dimensions such that the following hold for each mode* j*:*

$$rank\Big(\begin{bmatrix} E & B(j) \end{bmatrix}\Big) = n, \tag{18}$$

$$\Big(E + B(j)LC(j)\Big)'\Big(E + B(j)LC(j)\Big) > 0, \tag{19}$$

and

$$\left(\Big(\sum_{i=1}^{s} X_i\Big)\widehat{A}'(j) + \widehat{A}(j)\Big(\sum_{i=1}^{s} X_i\Big) + Y'\widehat{B}' + \widehat{B}Y\right) < 0. \tag{20}$$

Then the feedbacks

$$w(t) = Y\left(\sum_{i=1}^{s} X_i\right)^{-1}\bar{\eta}(t), \tag{21}$$

and

$$u(t) = -L\dot{y}(t) + \int_0^t w(\tau)\,d\tau, \tag{22}$$

stabilize system (17) and (11), respectively.

Proof. The rank condition (14) is equivalent to the rank condition (18). Condition (18) is an extension of the normalization rank condition given by Dai (see [2] for the proof) for one mode system. The proof of this theorem is quite similar to the proof of Theorem 1 since the derivative controller $-L\dot{y}(t)$ makes system (11) in the form of system (7).

Remark 1 *If the switch in the matrix* $\widehat{B}(\xi)$ *is absent, then it is not necessary to augment the order of the system by adding integrators. The controller, in this case, will be a static one.*

Remark 2 *If we start by augmenting the order of the system, the rank condition that allows the regularization of the singular system becomes impossible. Consequently, moving the switch to the state matrix should be done after the regularization of the switching system.*

4 Prescribed Degree of Stability

The solution of the LMIs (20) does not guarantee in advance the desired stability margin. Hence, it is preferable to prescribe a certain degree of stability when applying a unique controller to stabilize all modes of the switching system. For this aim, consider again system (17) with the following change of coordinate

$$\widetilde{\eta}(t) = e^{\alpha\, t}\bar{\eta}(t). \tag{23}$$

This gives

$$\begin{aligned}\dot{\widetilde{\eta}}(t) &= \alpha e^{\alpha\, t}\bar{\eta}(t) + e^{\alpha\, t}\dot{\bar{\eta}}(t)\\ &= \big(\widehat{A}(\xi) + \alpha I\big)\bar{\eta}(t) + e^{\alpha t}\widehat{B}w(t),\\ &= \widehat{A}_{\alpha}(\xi)\widetilde{\eta}(t) + \widehat{B}\widetilde{w}(t),\end{aligned} \tag{24}$$

where $\widehat{A}_{\alpha}(\xi) = \widehat{A}(\xi) + \alpha I$, and $\widetilde{w}(t) = e^{\alpha t}w(t)$. Applying the result of Theorem 2, we conclude that the stability of the system

$$\dot{\widetilde{\eta}}(t) = \widehat{A}_{\alpha}(\xi)\widetilde{\eta}(t) + \widehat{B}\widetilde{w}(t), \tag{25}$$

by the feedback $\widetilde{w}(t) = Y\Big(\sum_{i=1}^{s} X_i\Big)^{-1}\widetilde{\eta}(t)$ requires the solvability of the following LMIs

$$\bigg(\Big(\sum_{i=1}^{s} X_i\Big)\Big(\widehat{A}(j) + \alpha I\Big)' + \Big(\widehat{A}(j) + \alpha I\Big)\Big(\sum_{i=1}^{s} X_i\Big) + Y'\widehat{B}' + \widehat{B}Y\bigg) < 0, \tag{26}$$

for each j. This result implies the following.

Theorem 3 *Consider system (17). If the following hold*

i) the pairs

$$\big[\ \widehat{A}(j) \quad \widehat{B}\ \big], \tag{27}$$

are controllable for each mode j;

ii) for each mode j

$$rank\Big(\big[\ E \quad B(j)\ \big]\Big) = n, \tag{28}$$

iii) for each mode j

$$\Big(E + B(j)LC(j)\Big)'\Big(E + B(j)LC(j)\Big) > 0, \tag{29}$$

iv) there exists a set of symmetric and positive definite matrices $\big(X_i\big)_{1\le i\le s}$ and a matrix Y of appropriate dimensions such that

$$\bigg(\Big(\sum_{i=1}^{s} X_i\Big)\Big(\widehat{A}(j) + \alpha I\Big)' + \Big(\widehat{A}(j) + \alpha I\Big)\Big(\sum_{i=1}^{s} X_i\Big) + Y'\widehat{B}' + \widehat{B}Y\bigg) < 0; \tag{30}$$

is verified for each j. Then the feedback

$$u(t) = -L\dot{y}(t) + \int_0^t Y\left(\sum_{i=1}^{s} X_i\right)^{-1} \bar{\eta}(\tau)\, d\tau,$$

ensures the stability of system (17) with a prescribed degree of stability α.

5 Illustrative Examples

Consider the LSS system

$$\left.\begin{array}{l} E\dot{x}(t) = A(\xi)\, x(t) + B(\xi)u(t), \\ y(t) = C\, x(t), \end{array}\right\} \tag{31}$$

where

$$E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix},\ A(1) = \begin{bmatrix} -1 & 1 & 1 \\ -2 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix},$$

$$A(2) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix},\ B(1) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 1 & 0 \end{bmatrix},$$

$$B(2) = \begin{bmatrix} -1 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

The rank condition (18) is verified for the two possible modes of system (31). Then there exists a derivative gain L such that $E + B(\xi)LC$ are nonsingular for each ξ. Here, the gain is chosen as

$$L = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

The regularization of system (31) by the controller $-L\dot{y}(t) + v(t)$ leads to a new LS system having

$$\bar{A}(1) = \begin{bmatrix} -1 & 0 & 0.5 \\ -2 & 1 & 1.5 \\ 0 & 0 & -1. \end{bmatrix},$$

$$\bar{A}(2) = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -3 \\ -2 & 1 & 0 \end{bmatrix},$$

$$\bar{B}(1) = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \\ 1 & 0 \end{bmatrix},\ \bar{B}(2) = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}.$$

as nominal matrices. By augmenting the obtained LS system by two integrators, then we have

$$\widehat{A}(1) = \begin{bmatrix} -1 & 0 & 0.5 & -0.5 & 0.5 \\ -2 & 1 & 1.5 & 0.5 & 0.5 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widehat{A}(2) = \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ -2 & 1 & -3 & 1 & 1 \\ -2 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \widehat{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We shall apply the result of Theorem 3 to deliver the controller gain. Solving the set of the coupled LMIs (30) for $\alpha = 2$, we obtain $X(1) =$

$$\begin{bmatrix} 0.2200 & 0.2065 & -0.0311 & 0.4463 & -0.9701 \\ 0.2065 & 0.2002 & -0.0232 & 0.2468 & -0.8968 \\ -0.0311 & -0.0232 & 0.0105 & -0.1922 & 0.0755 \\ 0.4463 & 0.2468 & -0.1922 & 34.1392 & -25.8785 \\ -0.9701 & -0.8968 & 0.0755 & -25.8785 & 31.0948 \end{bmatrix},$$

and $X(2) =$

$$\begin{bmatrix} 0.0680 & 0.0631 & -0.0096 & 0.4265 & -0.5764 \\ 0.0631 & 0.0605 & -0.0072 & 0.3083 & -0.5008 \\ -0.0096 & -0.0072 & 0.0031 & -0.1153 & 0.0758 \\ 0.4265 & 0.3083 & -0.1153 & 22.7555 & -19.5046 \\ -0.5764 & -0.5008 & 0.0758 & -19.5046 & 20.5117 \end{bmatrix},$$

$$Y' = \begin{bmatrix} 53.1391 & -37.8315 \\ -4.8363 & 3.0819 \\ -58.5033 & 41.4067 \\ -163.4882 & 69.7774 \\ 71.7782 & -126.3998 \end{bmatrix}.$$

The eigenvalues of the closed-loop system relating to the first mode are

$$\begin{aligned} s_1 &= -32.6312 + 99.9533i, \\ s_2 &= -32.6312 - 99.9533i, \\ s_3 &= -7.2153 + 8.8208i, \\ s_4 &= -7.2153 - 8.8208i, \\ s_5 &= -2.5899, \end{aligned}$$

and the eigenvalues of the closed-loop system relating to the second mode are

$$\begin{aligned} s_1 &= -29.26 + 118.31i, \\ s_2 &= -29.26 - 118.31i, \\ s_3 &= -15.94, \\ s_4 &= -2.41 + 4.97i, \\ s_5 &= -2.41 - 4.97i, \end{aligned}$$

It is clear that the maximum values of the real parts of the poles of the system in each mode are lower than -2.

Now, we consider a special case of a singular switching system described by:

$$\begin{aligned} E\dot{x}(t) &= A(\xi)\,x(t) + B(\xi)u(t), \\ y(t) &= C\,x(t), \end{aligned} \tag{32}$$

where

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A(1) = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, A(2) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix},$$
$$B(1) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, B(2) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The controller

$$u(t) = -\dot{y}(t) + v(t), \tag{33}$$

regularizes system (35). One can easily check that

$$E + B(\xi)C, \qquad 1 \leq \xi \leq s, \tag{34}$$

are full rank matrices. With the feedback (33), system (35) becomes

$$\begin{aligned} \dot{x}(t) &= \bar{A}(\xi)\,x(t) + \bar{B}(\xi)v(t), \\ y(t) &= C\,x(t), \end{aligned} \tag{35}$$

where

$$\bar{A}(1) = \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}, \bar{A}(2) = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix},$$
$$\bar{B}(1) = \bar{B}(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

This is a typical example where the switch in the matrix B is absent. In this case, we are not in need of to augment the order of the system. i.e., $\widehat{A}(j)$ is replaced by $\bar{A}(j)$ and

$\widehat{B}$ by $\bar{B}(1)$ which is equal to $\bar{B}(2)$. Applying result of Theorem 2, we obtain

$$X_1 = X_2 = \begin{bmatrix} 95.5555 & 19.7703 \\ 19.7703 & 5.4333 \end{bmatrix},$$
$$Y = \begin{bmatrix} -190.9030 & 74.5472 \end{bmatrix}.$$

Finally, we conclude that the controller

$$u(t) = -\dot{y}(t) + K(P_1 + P_2)^{-1}\, x(t), \tag{36}$$

makes the origin a stable equilibrium point for system (35).

6 Conclusion

This paper dealt with class of singular switching linear systems. Both stability and stabilizability problem have been considered. It was shown, after regularizing the singular switching system that the design of the stabilizing controller is brought to the solution of a set of coupled LMIs. The examples we presented showed the efficacy of the proposed method.

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