Stochastic approximation based approaches for remote estimation with packet drops

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The Remote Estimation Problem

\( X_t \xrightarrow{} \text{Transmitter} \xrightarrow{U_t} \text{Erasure Channel} \xrightarrow{Y_t} \text{Receiver} \xrightarrow{} \hat{X}_t \)

(Regenerative) Error Process

\[
E_{t+1} = \begin{cases} 
    aE_t + W_t, & \text{if } Y_t = \mathcal{C} \\
    W_t, & \text{if } Y_t \neq \mathcal{C}.
\end{cases}
\]

Objective Functions

\[
C^*_\beta(\lambda) := C_\beta(f^*, g^*; \lambda) = \inf_{(k)} D_\beta^{(k)}(e) + \lambda N_\beta^{(k)}(e), \quad \lambda \geq 0
\]

\[
D^*_\beta(\alpha) := D_\beta(f^*, g^*) = \inf_{(k): N_\beta^{(k)} \leq \alpha} D_\beta(k),
\]
1. Renewal Monte Carlo
2. 1000 Sample averages

1. Kiefer Wolfowitz - Costly
2. Robbins Monro - Constrained

Stochastic Approximation Algorithm

Threshold → + → Policy Evaluation → Cost

Policy Improvement

Stochastic Approximation

Policy Evaluation

Policy Improvement

Cost

SA in Remote Estimation
Results

**Figure**: The sample paths for costly and constrained cases for $p_d = 0.3$. Bold lines represent the sample means for 100 runs and shaded regions correspond to mean ± twice standard deviation.
Conclusions & Future Work

Conclusions

- We present stochastic approximation algorithms to compute optimal thresholds for remote state estimation over communication channels with packet drops.
- We verified accuracy of these methods by comparing with analytical results for no packet-drop case.
- Policy evaluation: Regenerative nature of the error process and associated renewal relations.
- Policy improvement: Structural result that threshold based strategies are optimal.

Future Work

- Extension to Gilbert-Elliott channels
- Extension to higher dimensions

Thank you.