The first passage functionals for Lévy processes with jumps of rational Laplace transforms

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Plan.

1. The Model :
   - **Lévy processes with jumps.**
2. The problem :
   - **The first passage functionals for Lévy processes**
3. Solution
   - **The Feynman-Kac formula**

The Model

A Lévy jump-diffusion process $X = \{X_t, t \geq 0\}$ is defined as

$$X_t = \underbrace{X_0 + \mu t + \sigma B_t}_{\text{diffusion}} + \underbrace{\sum_{i=1}^{N_t} Y_i}_{\text{jump}}, \qquad \mu \in \mathbb{R}, \sigma > 0. \tag{1}$$

$B = \{B_t, t \geq 0\}$: is a (standard) Brownian motion.

$N = \{N_t, t \geq 0\}$: is a homogeneous Poisson process with rate λ.

$Y = \{Y_i, i = 1, 2, \ldots\}$: i.i.d random variables

B, N, Y are mutually independent.

Merton ($\mathcal{N}(0,1)$), *Kou* ($\mathcal{E}xp$), *Kou&Cai* ($\mathcal{H}yperexponential$)

$$X_t = X_0 + \mu t + \sigma B_t + \sum_{i=1}^{N_t} Y_i, \qquad \mu \in \mathbb{R}, \sigma > 0. \tag{2}$$

We suppose that the probability density function (pdf) of Y_1 is given by (Lewis and Mordecki 2008)

$$f(y) = \sum_{j=1}^{m}\sum_{i=1}^{m_j} p_{ij}\frac{(\eta_j)^i y^{i-1}}{(i-1)!}e^{-\eta_j y}\mathbf{1}_{\{y\geq 0\}} + \sum_{j=1}^{n}\sum_{i=1}^{n_j} q_{ij}\frac{(\theta_j)^i(-y)^{i-1}}{(i-1)!}e^{\theta_j y}\mathbf{1}_{\{y<0\}}, \tag{3}$$

The problem : Let $\tau := \inf\{t \geq 0 : X_t \leq h \textbf{ or } X_t \geq H\}$.

Find explicit formulae for $\phi(x)$ with

$$\phi(x) = \mathbb{E}_x\left[e^{-\alpha\tau}g(X_\tau)\right]; \tag{4}$$

$$\phi(x) = \mathbb{E}_x \left[e^{-\alpha\tau} g(X_\tau) \right]. \qquad (5)$$

By Feynman-Kac formula we have that $\phi(x)$ must satisfy

$$\begin{cases} (\mathcal{L} - \alpha)\,\phi(x) = 0 & \text{in } (h, H), \\ \phi(x) = g(x) & \text{on } (-\infty, h] \cup [H, +\infty). \end{cases} \qquad (6)$$

The generator $\mathcal{L}$ is given by

$$\begin{aligned} \mathcal{L}h(x) &:= \lim_{t \searrow 0} \frac{\mathbb{E}\left[h(X_t) | X_0 = x\right] - h(x)}{t} \\ &= \mu h'(x) + \frac{\sigma^2}{2} h''(x) + \lambda \int_{-\infty}^{+\infty} \left(h(x-y) - h(x)\right) f(y) \mathrm{d}y, \end{aligned}$$

Solution

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- Y.Ting Chen. 3013 **A note on first passage function for hyper-exponential jump diffusion.Elec. Comm. Prob**. vol 18, 1-8.