Incomplete AHP : The Signal and The Noise

Numerical experimentation to explore the possibility of quantifying decision maker (DM) preferences with the AHP method with **much less** pairwise comparaisons (PC)

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"There are various research papers on methods for dealing with incomplete preferences ***but very few investigated the relation between the number of missing comparisons and the stability of the obtained priority vector****. One of these rare studies was by Carmone et al. [1997] and it is safe to say that there is need and space for further investigation."*

- *from* Brunelli, Matteo (2015), "*Introduction to the Analytic Hierarchy Process*", SpringerBriefs in Operations Research.

Carmone *et al* (1997), "*A Monte Carlo investigation of incomplete pairwise comparison matrices in AHP*", European Journal of Operations Research, Elsevier

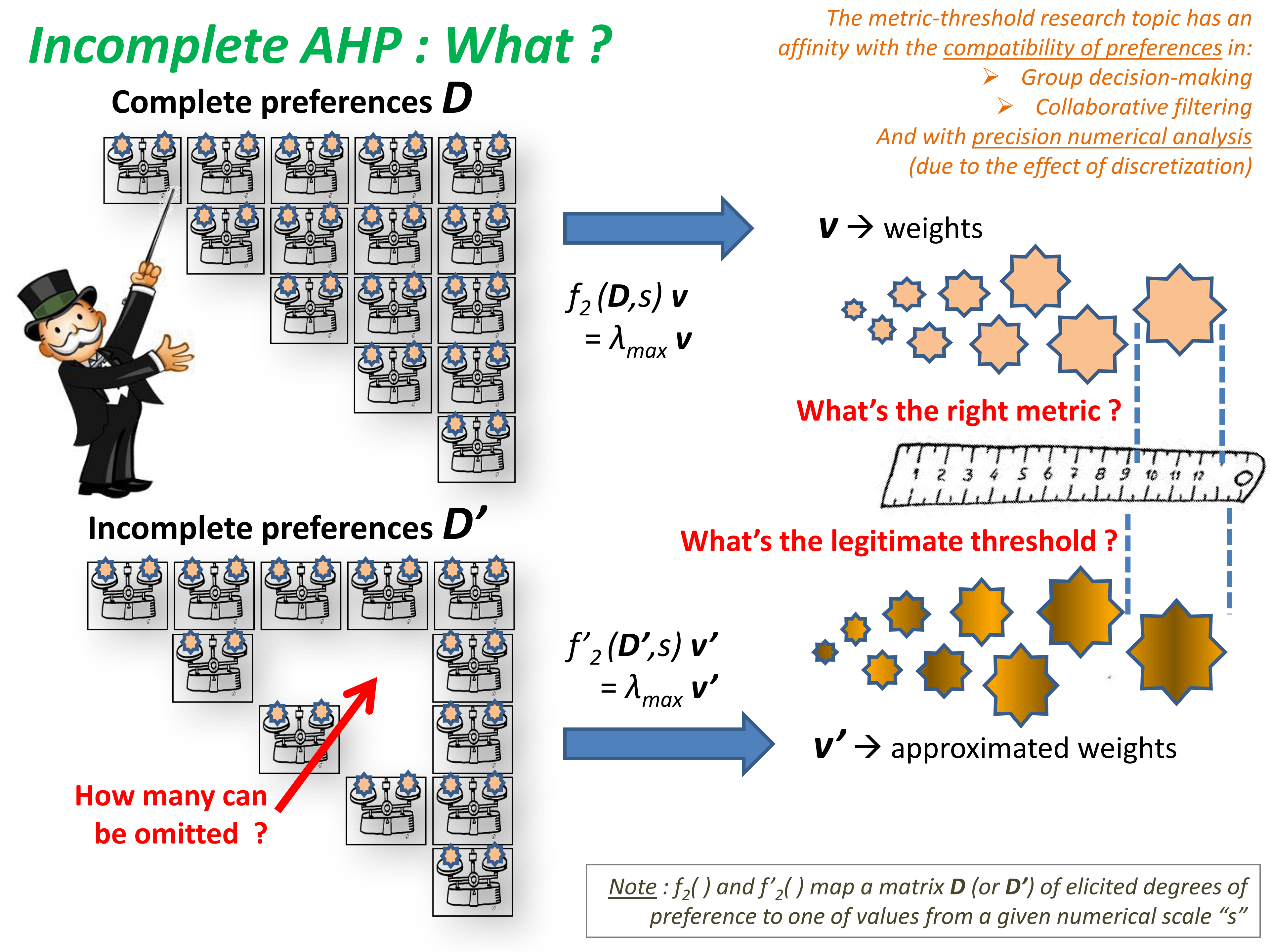
Meesariganda, B. R., & Ishizaka, A. (**2017**), "***Mapping verbal AHP scale to numerical scale*** *for cloud computing strategy selection*" , Applied Soft Computing, 53, 111-118.

As a research topic ...

The incomplete PC

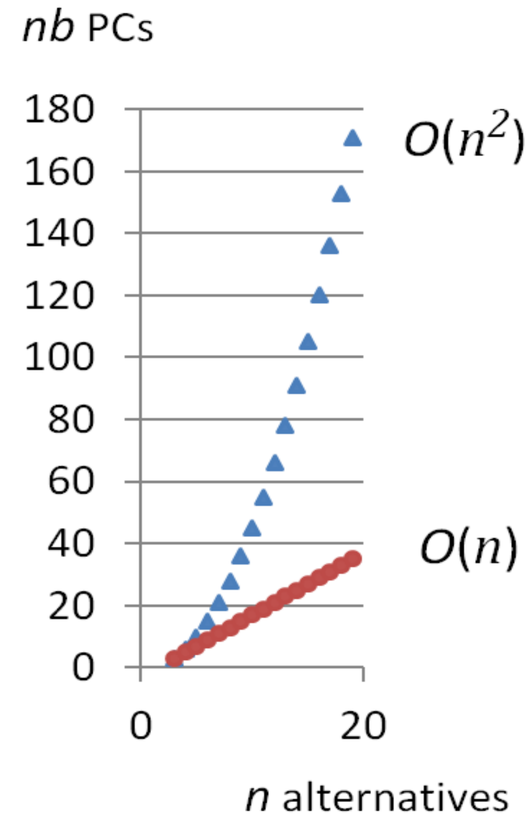
... is inseparable from these others ...

Preferences compatibility + Redundancy/Inconsistency + Sampling + Num. scale calibration


Incomplete AHP : What ?
The metric-threshold research topic has an affinity with the compatibility of preferences in:
Group decision-making
Collaborative filtering
And with precision numerical analysis
(due to the effect of discretization)
Complete preferences D
v → weights
f₂ (D,s) v = λmax v
What's the right metric ?
1 2 3 4 5 6 7 8 9 10 11 12
Incomplete preferences D'
What's the legitimate threshold ?
f'₂ (D',s) v' = λmax v'
v' → approximated weights
How many can be omitted ?
Note : f₂( ) and f'₂( ) map a matrix D (or D') of elicited degrees of preference to one of values from a given numerical scale "s"

Incomplete AHP : Why ?

- Utter motivation
  - Effort *reduction* / Time *saving*
    - $\frac{n*(n-1)}{2}$ *PC are required to completely fill a preference matrix*
  - Potential for *better* results via *sufficient* redundancy*



- This research intends to …
  - Rejuvenate treatment :
    shift from *piecemeal* and *secluded* to *holistic*
  - Replace *arbitrary* elements by *factual* ones
  - Ascertain feasibility of *reducing* number of *required* PC from $O(n^2)$ to $O(n)$ … *under what conditions* !

* Wedley, W. (2009), "*Fewer Comparisons - Efficiency via Sufficient Redundancy*", Online Proceedings of the 10th International Symposium on the Analytic Hierarchy Process, ISSN 1556-8296

Incomplete AHP : Where from and where to ?

Main approach* used since 1987 has gone stale

- **Starting rules**
- **Which one(s) next ?**
- **Stopping rules**
- Matrices in isolation
- Num. scale-specific
- Indistinct : alt. vs crit.

- Which proximity metric ? *(mixed bag of distance measures or simply left to the DM's whim)*
- **Simulation data with no relation to actual empirical observations ?**
- **Inconsistency barely addressed ?**
- Impact on overall solution ?

- Piecemeal treatment
- Only 55% would be required, but ?
- Still $O(n^2)$

Revived approach

- ~~Starting rules~~
- ~~Which one(s) next ?~~
- ~~Stopping rules~~
- ~~Matrices in isolation~~
- ~~Scale-specific~~

- **Reopen "heuristic" path (Which ones are needed) ?**
- **Consider empirical observations for sampling**
- **Subject to what consistency conditions ?**
- Considers the overall solution (vs isolated matrix)
  - Focus on alternatives (pertinence)

Needs this ...

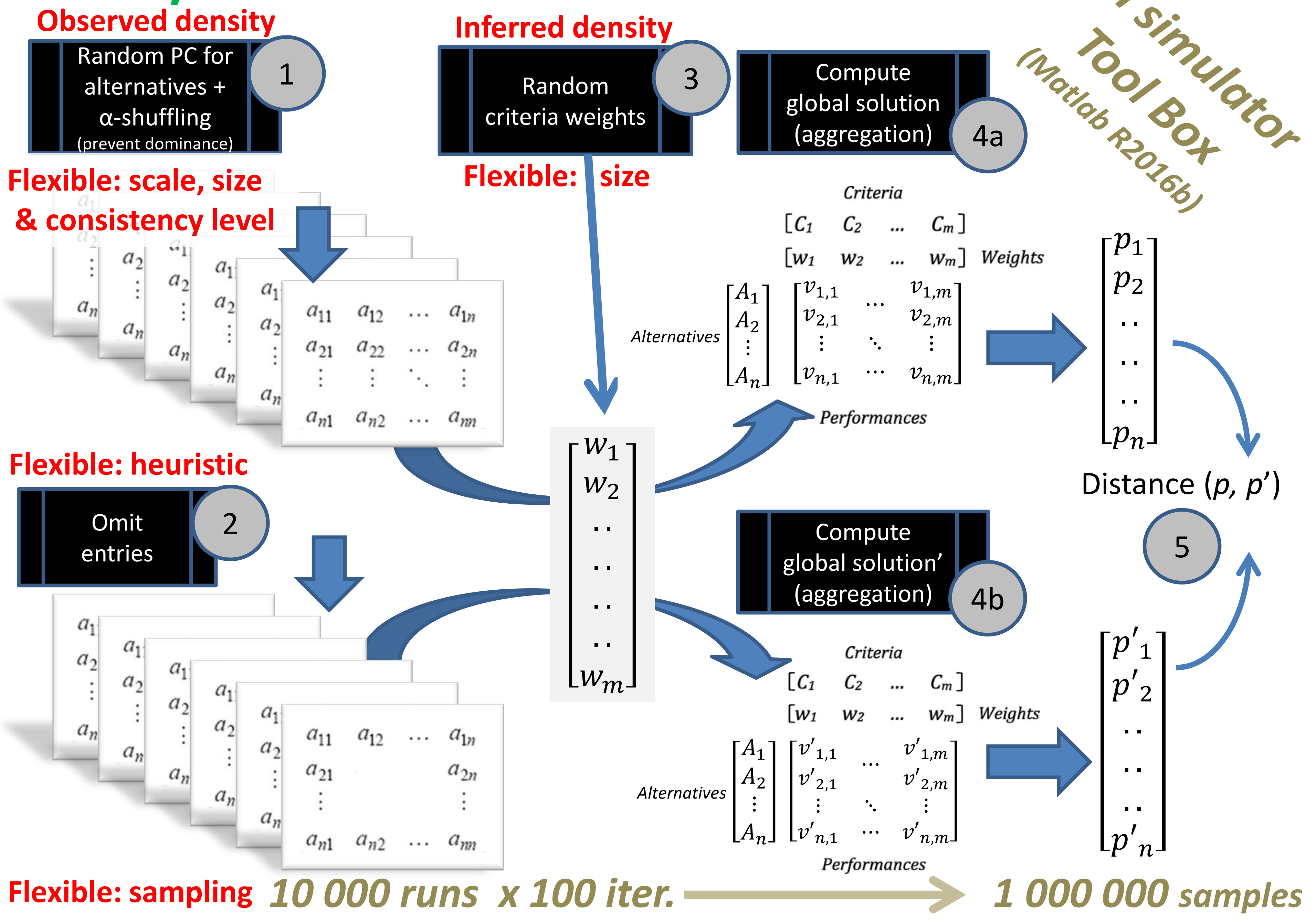
- Establishing factual tolerance for accuracy
  - Key : Impact of discretization
- Systematizing inconsistency characterization
  - Key : Layers of decreasing consistency
- Working with degrees of preference
- Generating simulation data in line with observations about empirical matrices

... to get that ...

- Reliable proximity thresholds
- Heuristics to omit entries
- Rules for collecting input
- Holistic and num. scale-agnostic
- Find if can be $O(n)$?

** Another completely distinct path aims to estimate the missing entries (not considered here as it addresses a different need)*

Incomplete AHP : How ?



Incomplete AHP : So ?

Preliminary Findings

Conditions = {linear scale, **near-consistency**, 6-14 alternatives, look for sufficient min. nb of PC}

→ **Genetic algorithm optimizer finds min. nb of entries ~ n +2, <u>but no patterns</u>**

Conditions = {linear scale, **near-consistency**, 8 alternatives x 8 criteria, fixed (2n -3) pattern*}

→ **Presence of error cancellation :**

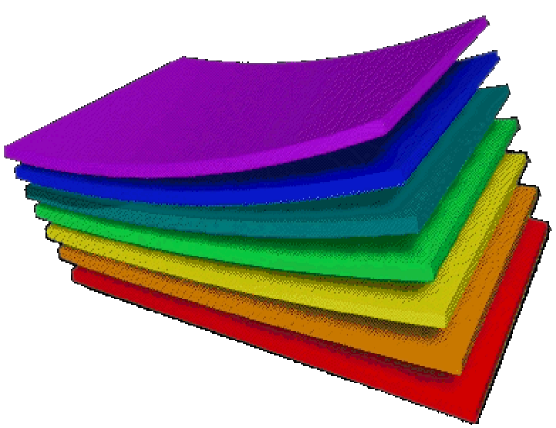
<u>Global gap ≤ max { individual gaps } ≤ threshold</u>

** all to best and all to worst*

Next

Verify **weak-consistency** in **range of usefulness** to get **usable** results
i.e. {6-14 criteria x 6-14 alternatives, various scales, various heuristics}

<u>Layers of gradually declining consistency</u>



Perfect consistency	$a_{i,j} = \frac{v_i}{v_j}$	*artificial*
Near-consistency	$a_{i,j} = round(\frac{v_i}{v_j}, s)$	
Weak-consistency	$a_{i,j} = \max\{a_{i,k}, a_{k,j}\}, \forall k$	*natural (guided)*
Row-dominance	$\forall (i,j)$, *either* $a_{i,k} \leq a_{j,k}$, $\forall k$ *or* $a_{i,k} \geq a_{j,k}$, $\forall k$	
Ordinal transitivity	$a_{i,k} \wedge a_{k,j} \geq 1 \Rightarrow a_{i,j} \geq 1, \forall k$	
Tolerated intransitivity	Consistency ratio ≤ limit	*natural (unguided)*
Random	Consistency ratio > limit	