Structure of optimal strategies for remote estimation over erasure channel with feedback

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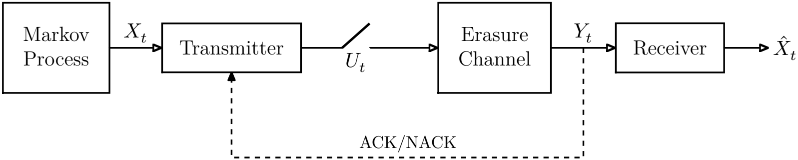
The remote estimation setup



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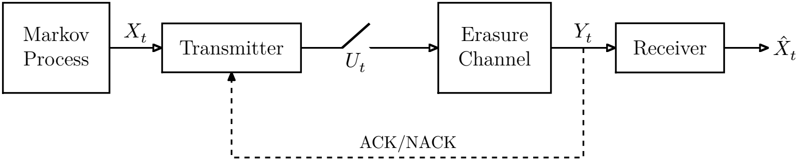
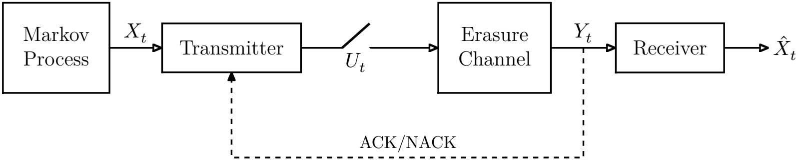


- **Source model** : $X_{t+1} = aX_t + W_t$, $a \in \mathbb{R}$ and $W_t \in \mathbb{R}$
  Unimodal, symmetric.

- **Channel model** : *Gilbert-Elliott* Channel. $S_t \in \{\text{on}, \text{off}\}$.
  For $r, s \in \{0, 1\}$, $Q_{rs} := \mathbb{P}(S_{t+1} = s | S_t = r)$.

The remote estimation setup



Motivation

- Applications in network-control systems, smart grids, internet of things...
- Sequential transmission of data; costly transmission, size of data packet not an issue

The optimization problem

Channel output: $$Y_t = \begin{cases} X_t, & \text{if } U_t = 1 \text{ and } S_t = 1 \\ \mathfrak{E}_1, & \text{if } U_t = 0 \text{ and } S_t = 1 \\ \mathfrak{E}_0, & \text{if } S_t = 0. \end{cases}$$

The optimization problem

Information structure

- **Informed receiver:** $I_t^{\mathrm{Rx}} = \{S_{0:t}, Y_{0:t}\}$.
- **One-step delayed transmitter:** $I_t^{\mathrm{Tx}} = \{X_{0:t}, U_{0:t-1}, S_{0:t-1}, Y_{0:t-1}\}$.

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Communication strategies

- **Transmission strategy:**
  $U_t = f_t(I_t^{\mathrm{Tx}}) = f_t(X_{0:t}, U_{0:t-1}, S_{0:t-1}, Y_{0:t-1})$.
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$$J(f, g) = \mathbb{E}\left[\sum_{t=0}^{T} \lambda U_t + d(X_t, \hat{X}_t)\right].$$

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Conceptual difficulty

- **Dynamic team** - the agents have access to different information
- **The information** at Tx and Rx **grows with time** - need for a *sufficient statistic* which is tractable

Theorem: Main results

1. **Optimal estimation strategy** : $\hat{X}_0 = 0$, and for $t \geq 0$,

$$\hat{X}_t = \begin{cases} a\hat{X}_{t-1}, & \text{if } Y_t \in \{\mathfrak{E}_0, \mathfrak{E}_1\} \\ Y_t, & \text{if } Y_t \in \mathbb{R} \end{cases} \tag{1}$$

2. **Optimal transmission strategy** :

$$U_t = \begin{cases} 1, & \text{if } |X_t - a\hat{X}_{t-1}| \geq k_t(S_{t-1}) \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

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Threshold functions: $k_t \colon \{0, 1\} \to \mathbb{R}_{\geq 0}$.

Key factors in the sketch of proof

- *Pre-* and *post-* transmission beliefs
- *Person-by-person* approach to find the information state at Tx
- Sufficient statistic for the *Common information* between Tx and Rx
- *Error process* - renewal relationships
- Symmetry and monotonicity of state dynamics and cost

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Future directions

- Computation of optimal thresholds and optimal performances
- Extension to higher dimensions
- Addition of noise in the erasure channel

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Thank you.