Les Cahiers du GERAD ISSN: 0711–2440

Cat-Suite: A collection of optimization problems with categorical and quantitative variables for benchmarking

E. Hallé-Hannan, C. Audet, Y. Diouane, S. Le Digabel, C. Tribes

G–2025–39

June 2025
Revised: January 2026

La collection *Les Cahiers du GERAD* est constituée des travaux de recherche menés par nos membres. La plupart de ces documents de travail a été soumis à des revues avec comité de révision. Lorsqu'un document est accepté et publié, le pdf original est retiré si c'est nécessaire et un lien vers l'article publié est ajouté.

Citation suggérée : E. Hallé-Hannan, C. Audet, Y. Diouane, S. Le Digabel, C. Tribes (Juin 2025). Cat-Suite: A collection of optimization problems with categorical and quantitative variables for benchmarking, Rapport technique, Les Cahiers du GERAD G–2025–39, GERAD, HEC Montréal, Canada. Version révisée: Janvier 2026

Avant de citer ce rapport technique, veuillez visiter notre site Web (https://www.gerad.ca/fr/papers/G-2025-39) afin de mettre à jour vos données de référence, s'il a été publié dans une revue scientifique.

The series *Les Cahiers du GERAD* consists of working papers carried out by our members. Most of these pre-prints have been submitted to peer-reviewed journals. When accepted and published, if necessary, the original pdf is removed and a link to the published article is added.

Suggested citation: E. Hallé-Hannan, C. Audet, Y. Diouane, S. Le Digabel, C. Tribes (June 2025). Cat-Suite: A collection of optimization problems with categorical and quantitative variables for benchmarking, Technical report, Les Cahiers du GERAD G–2025–39, GERAD, HEC Montréal, Canada. Revised version: January 2026

Before citing this technical report, please visit our website (https://www.gerad.ca/en/papers/G-2025-39) to update your reference data, if it has been published in a scientific journal.

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2026
– Bibliothèque et Archives Canada, 2026

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2026
– Library and Archives Canada, 2026

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Cat-Suite: A collection of optimization problems with categorical and quantitative variables for benchmarking

Edward Hallé-Hannan
Charles Audet
Youssef Diouane
Sébastien Le Digabel
Christophe Tribes

GERAD and Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal (Qc), Canada, H3C 3A7

edward.halle-hannan@polymtl.ca
charles.audet@gerad.ca
youssef.diouane@polymtl.ca
sebastien.le-digabel@polymtl.ca
christophe.tribes@polymtl.ca

June 2025
Revised: January 2026
Les Cahiers du GERAD
G–2025–39

Copyright © 2026 Hallé-Hannan, Audet, Diouane, Le Digabel, Tribes

Les textes publiés dans la série des rapports de recherche *Les Cahiers du GERAD* n'engagent que la responsabilité de leurs auteurs. Les auteurs conservent leur droit d'auteur et leurs droits moraux sur leurs publications et les utilisateurs s'engagent à reconnaître et respecter les exigences légales associées à ces droits. Ainsi, les utilisateurs:

- Peuvent télécharger et imprimer une copie de toute publication du portail public aux fins d'étude ou de recherche privée;
- Ne peuvent pas distribuer le matériel ou l'utiliser pour une activité à but lucratif ou pour un gain commercial;
- Peuvent distribuer gratuitement l'URL identifiant la publication.

Si vous pensez que ce document enfreint le droit d'auteur, contactez-nous en fournissant des détails. Nous supprimerons immédiatement l'accès au travail et enquêterons sur votre demande.

The authors are exclusively responsible for the content of their research papers published in the series *Les Cahiers du GERAD*. Copyright and moral rights for the publications are retained by the authors and the users must commit themselves to recognize and abide the legal requirements associated with these rights. Thus, users:

- May download and print one copy of any publication from the public portal for the purpose of private study or research;
- May not further distribute the material or use it for any profit-making activity or commercial gain;
- May freely distribute the URL identifying the publication.

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Abstract : Benchmarking new optimization methods on test problems is essential for assessing their performance and tuning their parameters. Yet, few problems are available in the literature for mixed-variable optimization. This manuscript introduces Cat-Suite, a collection of optimization problems involving categorical, integer, and continuous variables. Currently, Cat-Suite includes 60 analytical problems, half of which are constrained and half unconstrained. Each of these problems is either a modification of an existing one from the literature or derived from classical continuous test functions, such as Rosenbrock. Cat-Suite is an ongoing project, and additional problems will be added in the future. The goal is to facilitate benchmarking and testing of novel mixed-variable optimization methods with a diverse problem collection. The problems are implemented in Python and available at https://github.com/bbopt/Cat-Suite.

Keywords : Optimization; benchmarking; mixed-variable; categorical variables; constraints

Acknowledgements: This research is funded by a Natural Sciences and Engineering Research Council of Canada (NSERC) PhD Excellence Scholarship (PGS D), a Fonds de Recherche du Québec (FRQNT) PhD Excellence Scholarship and an Institut de l'Énergie Trottier (IET) PhD Excellence Scholarship, as well as by the NSERC discovery grants RGPIN–2020–04448 (Audet), RGPIN–2024–05093 (Diouane) and RGPIN–2024–05086 (Le Digabel).

1 Introduction

In mixed-variable optimization, categorical variables are notoriously hard to tackle due to their qualitative nature and lack of inherent structure. Such variables take values, called categories, in sets that lack well-defined metrics and may be unordered [5, 12]. Many practical applications involve these variables. Optimal engineering designs are often found through the optimization of physics-based models with variables that include material choices, such as steel or alloys. Designated methods tackling such problems form an active research field, notably in mixed-variable derivative-free optimization, where functions are frequently *expensive-to-evaluate* computer simulations without expressions [1, 3, 7, 8, 17, 20, 22, 23]. However, the current literature offers few test problems that are mixed-variable, making it difficult to thoroughly test the performance of novel solution methods. This manuscript introduces Cat-Suite, a collection of mixed-variable optimization problems.

The purpose of Cat-Suite is to facilitate *benchmarking* and *testing* of mixed-variable optimization methods. The project draws inspiration from CUTEst, a well-known collection of problems for continuous optimization [9]. Cat-Suite is an ongoing project. The collection will be expanded with additional and more diverse problems. Contributions and participation are welcome.

Currently, Cat-Suite contains 60 mixed-variable analytical problems, half of which are unconstrained and the other half constrained. Each problem contains at least one categorical and one continuous variables. Most of these problems also contains integer variables. The categorical variables influence the behavior of the functions involve via cases, tables or penalties taking continuous and integer variables. Approximately half of the problems are nonsmooth, *i.e.*, there is a least one function that is not differentiable for fixed categorical and integer variables. Nonsmoothness typically adds considerable challenge, but it is highly relevant in some optimization context, such as derivative-free or blackbox optimization where functions are often contaminated by noise or discontinuities [2, 4]. The current test problems proposed are either modifications of existing problems from the literature or adaptations of classic continuous optimization problems, such as the Rosenbrock and Rastrigin problems [13].

1.1 Problem statement

The problems in Cat-Suite are mixed-variable with inequality constraints, expressed as follows:

$$\min_{\boldsymbol{x}\in\Omega} f(\boldsymbol{x}) \tag{1}$$

where $f : \mathcal{X} \to \mathbb{R}$ is the objective function, $\mathcal{X}$ is the domain of the objective and constraint functions, $\Omega \coloneqq \{\boldsymbol{x} \in \mathcal{X} \ : \ g_j(\boldsymbol{x}) \leq 0, \ j \in \{1, 2, \ldots, m\}\}$ is the feasible set, $g_j : \mathcal{X} \to \overline{\mathbb{R}}$ is the j-th constraint function of the problem, with $j \in J$, and $m \in \mathbb{N}$ is the number of constraints.

The variables of a given type $t \in \{\text{cat}, \text{int}, \text{cont}\}$ are contained in a column vector expressed as

$$\boldsymbol{x}^t \coloneqq \left(x_1^t, x_2^t, \ldots, x_{n^t}^t\right) \in \mathcal{X}^t \coloneqq \mathcal{X}_1^t \times \mathcal{X}_2^t \times \ldots \times \mathcal{X}_{n^t}^t, \tag{2}$$

where $n^t \in \mathbb{N}$ is the number of variables of type t, $\mathcal{X}^t$ is the set of type t, $x_i^t \in \mathcal{X}_i^t$ denotes the i-th variable of type t, and $\mathcal{X}_i^t$ contains the bounds of x_i^t. The indices of the variables of type t are contained in the set $I^t \coloneqq \{1, 2, \ldots, n^t\}$. Following this, a point is formalized as a partition of these vectors by types and the domain is constructed with Cartesian products of the sets by types, such that

$$\boldsymbol{x} \coloneqq \left(\boldsymbol{x}^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) \in \mathcal{X} \coloneqq \mathcal{X}^{\text{cat}} \times \mathcal{X}^{\text{int}} \times \mathcal{X}^{\text{cont}}. \tag{3}$$

In this manuscript, the problems considered are not *hierarchical*. There are no *meta variables* that control the inclusion, bounds or admissible values of other variables, as described in [5, 10]. The number of variables is fixed, and the bounds of each variable is predefined.

The rest of this document is organized as follows. Section 2 summarizes the main characteristics of the problems, such as the number of variables or constraints, using tables. This section also discusses

the techniques used to construct the problems. Afterwards, Sections 3 and 4 details the mathematical formulations of the unconstrained and constrained problems respectively.

2 Summary and characteristics of the test problems

The characteristics of the unconstrained and constrained test problems are presented in Table 1 and Table 2, respectively. The $\star$ symbol denotes a user-specified positive integer. The implementations are publicly available at `https://github.com/bbopt/Cat-Suite`. Additional information is also provided, such as the best feasible value known for problems.

Table 1: Unconstrained test problems.

Name	Sec.	n^{cat}	ℓ	n^{int}	n^{cont}	Smooth	Ref.	Original problem	Modifications
Cat-1	3.1	2	9	2	$\star$	No	[13]	Ackley	Fct-to-cases & penalty
Cat-2	3.2	2	9	2	3	No	[13]	Beale	Cont-to-cat
Cat-3	3.3	2	4	2	2	Yes	[17]	Augmented-Branin	Added integers & removed constraints
Cat-4	3.4	2	4	2	4	No	[13]	Bukin-6	Cont-to-cat
Cat-5	3.5	1	6	1	3	Yes	[14]	EVD-52	Added integers & minimax-to-cat
Cat-6	3.6	2	9	0	2	Yes	[17]	Goldstein	Removed constraints
Cat-7	3.7	3	16	3	2	No	[13]	Goldstein-Price	Penalty
Cat-8	3.8	1	4	1	5	Yes	[14]	HS78	Added integer, added category & minimax-to-cat
Cat-9	3.9	2	9	2	$\star$	No	[13]	Rastrigin	Cont-to-cat & fct-to-cat
Cat-10	3.10	2	6	2	$\star$	No	[13]	Rosenbrock	Fct-to-cat & penalty
Cat-11	3.11	1	4	1	4	Yes	[14]	Rosen-Suzuki	Added integer & minimax-to-cat
Cat-12	3.12	1	5	$\star$	$\star$	No	[13]	Styblinski-Tang	Cont-to-cat
Cat-13	3.13	1	10	0	4	Yes	[16]	Toy	Added variables & modified functions
Cat-14	3.14	1	10	0	8	No	[16]	Toy	Added variables & modified functions
Cat-15	3.15	1	5	3	4	Yes	[14]	Wong-1	Cont-to-int & minimax-to-cat
Cat-16	3.16	2	9	2	$\star$	No	[13]	Zakharov	Fct-to-cat & cont-to-int
Cat-17	3.17	2	49	1	7	No	[21]	Ishigami	Added terms, cont-to-int & cat table
Cat-18	3.18	2	100	1	7	No	[21]	Hartmann	Added terms, cont-to-int & cat table
Cat-19	3.19	2	64	0	$\star$	No	[21]	Levy	Modified objective & cont-to-cat
Cat-20	3.20	3	80	2	4	No	[21]	Camel	Added terms & variables
Cat-21	3.21	1	61	0	4	No	[14]	Gamma	Minimax-to-cat, modified objective & table values
Cat-22	3.22	1	51	0	6	Yes	[14]	EVD-61	Minimax-to-cat & modified objective
Cat-23	3.23	3	12	0	5	No	[11]	Hal-04	No modifications
Cat-24	3.24	1	21	2	3	Yes	[14]	OET5	Cont-to-cat, modified objective & added bounds
Cat-25	3.25	1	18	10	10	Yes	[14]	Wong 3	Minimax-to-cat & cont-to-int
Cat-26	3.26	1	10	3	5	Yes	[19]	Roustant	Modified objective, modified mappins & added variables
Cat-27	3.27	2	100	1	4	Yes	[15]	Kowalik-Osborne	Modified objective, added tables & variables
Cat-28	3.28	2	36	0	2	Yes	[13]	Three-Hump	Cont-to-cat & modified objective
Cat-29	3.29	2	64	3	4	Yes	[13]	McCormick	Cont-to-cat, added terms & variables
Cat-30	3.30	4	81	4	6	Yes	[13]	Shekel	Cont-to-cat, added terms and variables

The problem statements, with the objective and constraints functions, are presented in their corresponding subsection. A problem is nonsmooth if there is a least one function that is not differentiable with respect to its continuous variables. For each problem, a reference and a summary of the modifications, when applicable, are provided.

Four techniques are employed to create mixed-variable problems from classic problems. The first technique replaces continuous variables with functions that depend on both categorical and continuous variables. For example, in Cat-12, detailed in Subsection 3.12, the function s in the summation replaces a continuous variable x_i^{cont} from the original Styblinski-Tang objective. This functions s now maps x_i^{cont} differently depending on the category taken by the categorical variable $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$. This is referred to as "con-to-cat" in the tables. The second technique, similar to the first one, transforms a

Table 2: Constrained test problems.

Name	Sec.	n^{cat}	ℓ	n^{int}	n^{cont}	m	Smooth	Ref.	Original problem	Modifications
Cat-cstrs-1	4.1	2	9	2	3	3	No	[13]	Beale	Cat-2 with added constraints
Cat-cstrs-2	4.2	2	4	2	2	1	Yes	[17]	Augmented-Branin	Added integers
Cat-cstrs-3	4.3	2	9	2	4	2	No	[13]	Bukin-6	Cat-4 with added constraints
Cat-cstrs-4	4.4	1	4	4	4	3	Yes	[14]	Dembo-5	Con-to-int & minimax-to-cat
Cat-cstrs-5	4.5	1	6	1	3	1	No	[14]	EVD-52	Added integers & added constraints
Cat-cstrs-6	4.6	2	9	2	3	4	Yes	[6]	G-09	Con-to-int, modified constraints & cat-tables
Cat-cstrs-7	4.7	2	9	0	2	1	Yes	[17]	Goldstein	None
Cat-cstrs-8	4.8	2	25	2	2	2	Yes	[13]	Himmelblau	Added constraints
Cat-cstrs-9	4.9	2	4	3	5	4	Yes	[14]	HS-114	Con-to-int & minimax-to-cat
Cat-cstrs-10	4.10	1	3	2	4	6	Yes	[14]	Pentagon	Con-to-int & minimax-to-cat
Cat-cstrs-11	4.11	1	8	2	2	3	Yes	[6]	Pressure-Vessel	Con-to-int, added integers & con-to-cat
Cat-cstrs-12	4.12	2	25	1	2	2	Yes	[6]	Reinforced-Concrete	Added continuous & simplified cat tables
Cat-cstrs-13	4.13	2	6	2	$\star$	1	No	[13]	Rosenbrock	Cat-10 with added constraints
Cat-cstrs-14	4.14	1	5	$\star$	$\star$	2	No	[13]	Styblinski–Tang	Cat-12 with added constraints
Cat-cstrs-15	4.15	1	10	0	4	2	Yes	[16]	Toy	Added variables & added constraints
Cat-cstrs-16	4.16	1	6	4	6	3	Yes	[14]	Wong-2	Con-to-int, minimax-to-cat & added cstrs
Cat-cstrs-17	4.17	2	64	0	6	11	No	[6]	Speed-reducer	Added table cat & bounds
Cat-cstrs-18	4.18	1	50	2	2	5	No	[6]	Spring	Modified objective, added variables and fct-to-cat
Cat-cstrs-19	4.19	6	64	2	2	8	No	[6]	G07	Cont-to-cat
Cat-cstrs-20	4.20	2	100	2	7	10	Yes	[6]	Car-side-impact	Modified objective, added cat table & bounds
Cat-cstrs-21	4.21	1	18	0	16	7	No	[14]	Dembo 7	Minimax-to-cat & added constraints
Cat-cstrs-22	4.22	1	12	0	12	3	No	[14]	MAD	Minimax-to-cat & combined problems
Cat-cstrs-23	4.23	1	13	10	10	4	No	[14]	Wong 3	Cat-25 with added constraints
Cat-cstrs-24	4.24	2	100	2	4	5	No	[18]	Welded-beam	Modified objective
Cat-cstrs-25	4.25	1	10	0	2	3	No	[18]	Three-bar truss	Modified objective
Cat-cstrs-26	4.26	2	36	2	2	6	No	[13]	Three-hump	Cat-28 with added constraints
Cat-cstrs-27	4.27	2	64	3	4	3	No	[13]	McCormick	Cat-29 with added constraints
Cat-cstrs-28	4.28	2	100	2	2	5	No	[9]	G06	Added constraints and cat table
Cat-cstrs-29	4.29	4	81	4	6	3	No	[13]	Shekel	Cat-30 with added constraints
Cat-cstrs-30	4.30	2	49	1	7	3	No	[21]	Ishigami	Cat-17 with added constraints

function into another one with cases, each assigned to a category. This is named "fct-to-cases". For example, in Cat-1, formulated in 3.1, the three function cases of the second term are variations of the original Ackley objective function. The last technique, called "penalty", simply adds a penalty function with cases assigned to categories. The first term in the objective function of the Cat-1 problem correspond to such penalty.

The problems modified from [14] are originally minimax problems on many functions. These problems are transformed into mixed-variable problems by assigning each function to a categorical component $\boldsymbol{x}^{\text{cat}} \in \mathcal{X}^{\text{cat}}$. This technique is taken from [16] and this is referred to as "minimax-to-cat" in the tables. Some problems are adapted from [16], with minor modifications such as introducing integer variables. Similarly, problems taken from [17] undergo small modifications. The Cat-23 constrained problem in Table 2 is the only unmodified problem taken directly from [17]. The minimax problems in [14] are strictly continuous. Some continuous variables are transformed into integer variables and this is referred to as "con-to-int" in the tables. The mixed-variable constrained problems from [6] have been modified with the different techniques discussed above.

The last technique, taken from [17], inserts categorical variables through mappings with predetermined values. These values are stored either in columns (for a single categorical variable) or in tables (for two categorical variables). For example, in Cat-17 4.17, two categorical variables are incorporated into the objective function via a mapping with image $s(x_1^{\text{cat}}, x_2^{\text{cat}}) \in \mathbb{R}$. This technique is referred to as "cat table" in the tables describing the problems.

In the constrained problems, the constraints are either directly taken from the original problem or constructed so that at least one feasible point always exists. To this end, one may start from a reference point and add constraints in such a way that this point remains feasible. If a constraint violates feasibility, its sign can be switched to restore it.

3 Description of the unconstrained problems

The following subsections details the unconstrained problems.

3.1 Cat-1: modified Ackley [13]

$$
\begin{aligned}
&f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = s \\
&+ \begin{cases}
-20\exp\left(-\frac{2}{10}\sqrt{\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\left(x_i^{\text{cont}} + x_1^{\text{int}}\right)^2}\right) - \exp\left(\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\cos\left(2\pi x_i^{\text{cont}} \times x_2^{\text{int}}\right)\right) & \text{if } x_1^{\text{cat}} = \text{A} \\
-10\exp\left(-\frac{1}{10}\sqrt{\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\left|x_i^{\text{cont}} + x_1^{\text{int}}\right|}\right) + 5\sum_{i=1}^{n^{\text{cont}}}\cos\left(2\pi x_i^{\text{cont}} - x_2^{\text{int}}\right) & \text{if } x_1^{\text{cat}} = \text{B} \\
-15\exp\left(-\sqrt{\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\left|x_i^{\text{cont}} \times x_1^{\text{int}}\right|}\right) - 5\exp\left(-\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\cos\left(2\pi x_i^{\text{cont}} + x_2^{\text{int}}\right)\right) & \text{if } x_1^{\text{cat}} = \text{C}
\end{cases}
\end{aligned}
$$

where

$$
s_2 \coloneqq s_2\left(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\right) = \begin{cases}
\left|\sum_{i=1}^{n^{\text{cont}}} x_i^{\text{cont}}\right| \times 15 & \text{if } x_2^{\text{cat}} = \text{A} \\
\left|\sum_{i=1}^{n^{\text{cont}}} \left(x_i^{\text{cont}} - 1\right)\right| \times 15 & \text{if } x_2^{\text{cat}} = \text{B} \\
\left|\sum_{i=1}^{n^{\text{cont}}} x_i^{\text{cont}}\right| \times 10 & \text{if } x_2^{\text{cat}} = \text{C}
\end{cases}
$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{1, 2, \ldots, 10\}$, $x_2^{\text{int}} \in \{-1, 0, 1\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-5, 5]$ for $i \in I^{\text{cont}}$.

3.2 Cat-2: modified Beale [13]

$$
\begin{aligned}
f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = &\left(1.5 - s_1 + x_1^{\text{int}}\left(1 - s_2\right)\right)^2 + \left(2.25 - s_1 + x_2^{\text{int}}\left(1 - s_2\right)^2\right)^2 \\
&+ \left(2.625 - s_1 + x_3^{\text{cont}}\left(1 - s_2\right)^3\right)^2
\end{aligned}
$$

where

$$
s_1 \coloneqq s_1(x_1^{\text{cat}}, x_1^{\text{cont}}) = \begin{cases}
\lfloor x_1^{\text{cont}} \rfloor & \text{if } x_1^{\text{cat}} = \text{A} \\
x_1^{\text{cont}} & \text{if } x_1^{\text{cat}} = \text{B} \\
\exp\left(\frac{x_1^{\text{cont}}}{2}\right) & \text{if } x_1^{\text{cat}} = \text{C}
\end{cases}
\quad \text{and} \quad
s_2 \coloneqq s_2(x_2^{\text{cat}}, x_2^{\text{cont}}) = \begin{cases}
\sqrt{|x_2^{\text{cont}}| + 1} & \text{if } x_2^{\text{cat}} = \text{A} \\
\left|x_2^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{B} \\
\left(x_2^{\text{cont}}\right)^2 - 2 & \text{if } x_2^{\text{cat}} = \text{C}
\end{cases}
$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-2, -1, 0, 1, 2\}$;
- $n^{\text{cont}} = 3$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}} \in [-4.5, 4.5]$.

3.3 Cat-3: modified Branin [17]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{A} \\ 0.4s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{B} \\ -0.75s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{A} \\ -0.5s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{B} \end{cases}$$

where

$$s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \frac{1}{a}\left[\left(\left(15x_2^{\text{cont}} - \frac{x_1^{\text{int}}}{4\pi^2}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)^2 + \frac{x_1^{\text{int}}}{\pi}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)\right)^2 + \left(x_1^{\text{int}} - \frac{1}{8\pi}\right)\cos\left(15x_1^{\text{cont}} - 5\right) + x_1^{\text{int}}\right) - b\right]$$

with $a = 51.9496$, $b = 54.8104$,

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{1, 2, 3, 4, 5\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 1]$.

3.4 Cat-4: modified Bukin-6 [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = 100\sqrt{|s_1 - 0.01s_2|} + 0.01\left|s_2 + x_2^{\text{int}}\right|$$

where

$$s_1 \coloneqq s_1(x_1^{\text{cat}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \sqrt{|x_2^{\text{cont}} + x_3^{\text{cont}}| + 2} & \text{if } x_1^{\text{cat}} = \text{A} \\ \left|x_2^{\text{cont}} + x_3^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{B} \\ \frac{\left(x_2^{\text{cont}} + x_3^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2 \coloneqq s_2(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}) = \begin{cases} \sqrt{|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}| + 1.5} & \text{if } x_2^{\text{cat}} = \text{A} \\ \left|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{x_1^{\text{int}} + \left(x_1^{\text{cont}} + x_4^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-5, -4, \ldots, 4, 5\}$;
- $n^{\text{cont}} = 4$ and $\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) \in [-15, 5] \times [-3, 3] \times [-15, 5] \times [-3, 3]$.

3.5 Cat-5: modified EVD-52 [14]

$$f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{A} \\ \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}} - 2\right)^2 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{B} \\ x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}} - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{C} \\ x_1^{\text{cont}} + x_2^{\text{cont}} - x_3^{\text{cont}} + 1 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{D} \\ 2\left(x_1^{\text{cont}}\right)^3 + 6\left(x_2^{\text{cont}}\right)^2 + 2\left(5x_3^{\text{cont}} - x_1^{\text{cont}} + 1\right)^2 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{E} \\ \left(x_1^{\text{cont}}\right)^2 - 9x_3^{\text{cont}} + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{F} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{-25, -24, \ldots, , 24, 25\}$;
- $n^{\text{cont}} = 3$ and $x_i^{\text{cont}} \in [-25, 25]$ for $i \in I^{\text{cont}}$.

3.6 Cat-6: modified Goldstein [17]

$$
\begin{aligned}
f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \\
& 53.3108 + 0.184901 x_1^{\text{cont}} \\
& - 5.02914 \left(x_1^{\text{cont}}\right)^3 \times 10^{-6} + 7.72522 \left(x_1^{\text{cont}}\right)^4 \times 10^{-8} \\
& - 0.0870775 x_2^{\text{cont}} - 0.106959 s_1 \\
& + 7.98772 \left(s_1\right)^3 \times 10^{-6} + 0.00242482 s_2 \\
& + 1.32851 \left(s_2\right)^3 \times 10^{-6} - 0.00146393 x_1^{\text{cont}} x_2^{\text{cont}} \\
& - 0.00301588 x_1^{\text{cont}} s_1 - 0.00272291 x_1^{\text{cont}} s_2 \\
& + 0.0017004 x_2^{\text{cont}} s_1 + 0.0038428 x_2^{\text{cont}} s_2 \\
& - 0.000198969 s_1 s_2 \\
& + 1.86025 x_1^{\text{cont}} x_2^{\text{cont}} s_1 \times 10^{-5} \\
& - 1.88719 x_1^{\text{cont}} x_2^{\text{cont}} s_2 \times 10^{-6} \\
& + 2.50923 x_1^{\text{cont}} s_1 s_2 \times 10^{-5} \\
& - 5.62199 x_2^{\text{cont}} s_1 s_2 \times 10^{-5}
\end{aligned}
$$

where

$$
s_1 \coloneqq s_1\left(x_1^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_1^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_1^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_1^{\text{cat}} = \text{C}, \end{cases} \qquad \text{and} \qquad s_2 \coloneqq s_2\left(x_2^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_2^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_2^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_2^{\text{cat}} = \text{C}. \end{cases}
$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 100]$.

3.7 Cat-7: modified Goldstein-Price [13]

$$
f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = s_1 + s_2 + s_3
$$

where

$$
\begin{aligned}
s_1 &\coloneqq s_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) \\
&= \left(1 + \left(x_1^{\text{cont}} + x_2^{\text{cont}} + 1\right)^2 \left(19 - 14 x_1^{\text{cont}} + 3\left(x_1^{\text{cont}}\right)^2 - 14 x_2^{\text{cont}} + 6 x_1^{\text{cont}} x_2^{\text{cont}} + 3\left(x_2^{\text{cont}}\right)\right)\right) \\
&\quad \left(30 + \left(2 x_1^{\text{cont}} - 3 x_2^{\text{cont}}\right)^2 \left(18 - 32 x_1^{\text{cont}} + 12\left(x_1^{\text{cont}}\right)^2 + 48 x_2^{\text{cont}} - 36 x_1^{\text{cont}} x_2^{\text{cont}} + 27\left(x_2^{\text{cont}}\right)^2\right)\right)
\end{aligned}
$$

and

$$
s_2 \coloneqq s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} 2 + \frac{\left(x_1^{\text{cont}} + x_1^{\text{int}}\right)^2 + \left(x_2^{\text{cont}} + x_2^{\text{int}}\right)^2}{2} & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{A} \\ 1.5 + \frac{\left(x_1^{\text{cont}} + x_1^{\text{int}}\right)^2 + \left|x_2^{\text{cont}} + x_2^{\text{int}}\right|}{4} & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{B} \\ 1.5 + \frac{\left|x_1^{\text{cont}} + x_1^{\text{int}}\right| + \left(x_2^{\text{cont}} + x_2^{\text{int}}\right)^2}{4} & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{A} \\ 1 + \frac{\left|x_1^{\text{cont}} + x_1^{\text{int}}\right| + \left|x_2^{\text{cont}} + x_2^{\text{int}}\right|}{1} & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{B} \end{cases}
$$

and

$$s_3 \coloneqq s_3\left(x_3^{\text{cat}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} \frac{\left|x_3^{\text{int}} + x_2^{\text{cont}}\right| + 2}{2} & \text{if } x_3^{\text{cat}} = \text{A} \\ \frac{\left|x_3^{\text{int}} - x_2^{\text{cont}}\right| + 2}{2} & \text{if } x_3^{\text{cat}} = \text{B} \\ \frac{\left|-x_3^{\text{int}} + x_2^{\text{cont}}\right| + 2}{2} & \text{if } x_3^{\text{cat}} = \text{C} \\ \frac{\left|-x_3^{\text{int}} - x_2^{\text{cont}}\right| + 2}{2} & \text{if } x_3^{\text{cat}} = \text{D} \end{cases}$$

- $n^{\text{cat}} = 3$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$ and $x_3^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}} = 3$ and $x_i^{\text{int}} \in \{-2, -1, 0, 1, 2\}$ for $i \in I^{\text{int}}$;
- $n^{\text{cont}} = 2$ and $x_j^{\text{cont}} \in [-2, 2]$ for $j \in I^{\text{cont}}$.

3.8 Cat-8: modified HS-78 [14]

$$f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}) = \prod_{i=1}^{5} x_i^{\text{cont}} + x^{\text{int}} s$$

where

$$s \coloneqq s\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}\right) = \begin{cases} \sum_{i=1}^{5} \left(x_i^{\text{cont}}\right)^2 - 10 & \text{if } x^{\text{cat}} = \text{A} \\ x_2^{\text{cont}} x_3^{\text{cont}} - 5 x_4^{\text{cont}} x_5^{\text{cont}} & \text{if } x^{\text{cat}} = \text{B} \\ \left(x_1^{\text{cont}}\right)^3 + \left(x_2^{\text{cont}}\right)^3 + 1 & \text{if } x^{\text{cat}} = \text{C} \\ \frac{1}{2}\left(x_2^{\text{cont}} x_3^{\text{cont}} - 5 x_4^{\text{cont}} x_5^{\text{cont}} + \left(x_1^{\text{cont}}\right)^3 + \left(x_2^{\text{cont}}\right)^3 + 1\right) & \text{if } x^{\text{cat}} = \text{D} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{0, 1, 2, 3, 4, 5\}$;
- $n^{\text{cont}} = 5$ and $x_i^{\text{cont}} \in [-2, 2]$ for $i \in I^{\text{cont}}$.

3.9 Cat-9: modified Rastragin [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = 10 s_1 + \sum_{i=1}^{n^{\text{cont}}} \left(\left(x_i^{\text{cont}}\right)^2 - s_2 + \cos\left(x_1^{\text{cont}}\right)\right)$$

where

$$s_1 \coloneqq s_1\left(x_1^{\text{cat}}, x_1^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \frac{1}{n^{\text{cont}}} \begin{cases} \sum_{i=1}^{n^{\text{cont}}} \max\left\{0, x_1^{\text{int}} + x_i^{\text{cont}}\right\}^2 & \text{if } x_1^{\text{cat}} = \text{A} \\ \frac{1}{2} \sum_{i=1}^{n^{\text{cont}}} \left|x_1^{\text{int}} + x_i^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{B} \\ \sum_{i=1}^{n^{\text{cont}}} \max\left\{x_1^{\text{int}} + x_i^{\text{cont}}, 0\right\} \left|x_i^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2 \coloneqq s_2\left(x_2^{\text{cat}}, x_2^{\text{int}}\right) = \begin{cases} 1 + 0.1 x_2^{\text{int}} & \text{if } x_2^{\text{cat}} = \text{A} \\ \frac{1}{1 + 0.1 x_2^{\text{int}}} & \text{if } x_2^{\text{cat}} = \text{B} \\ 1 + 0.05\left(x_2^{\text{int}}\right)^2 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{-5, -4, \ldots, 4, 5\}$, $x_2^{\text{int}} \in \{-2, -1, 0, 1, 2\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-5.12, 5.12]$ for $i \in I^{\text{cont}}$.

3.10 Cat-10: modified Rosenbrock [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \left|x_1^{\text{int}}\right| + s + \begin{cases} \sum_{i=1}^{n^{\text{cont}}-1} 100\left(x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right)^2 + x_2^{\text{int}}(x_i^{\text{cont}} - 1)^2 & \text{if } x_1^{\text{cat}} = \text{A} \\ \sum_{i=1}^{n^{\text{cont}}-1} 100\left|x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right| + 5x_2^{\text{int}}\left|x_i^{\text{cont}} - 1\right| & \text{if } x_1^{\text{cat}} = \text{B} \end{cases}$$

where

$$s := s(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}) = \begin{cases} \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} 1.1 \max\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{A} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} -0.9 \min\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} \left|x_i^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}} \in \{\text{A}, \text{B}\}$, $x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{-2, -1, 0, 1, 2\}$, $x_2^{\text{int}} \in \{-5, -4, \ldots, 4, 5\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-10, 10]$ for $i \in I^{\text{cont}}$.

3.11 Cat-11: modified Rosen-Suzuki [14]

$$f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}) = s + \begin{cases} 0 & \text{if } x^{\text{cat}} = \text{A} \\ 10\left(\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + \left(x_4^{\text{cont}}\right)^2 + x_1^{\text{cont}} - x_2^{\text{cont}} + x_3^{\text{cont}} - x_4^{\text{cont}} - x^{\text{int}} - 8\right) & \text{if } x^{\text{cat}} = \text{B} \\ 10\left(\left(x_1^{\text{cont}}\right)^2 + 2\left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + 2\left(x_4^{\text{cont}}\right)^2 - x_1^{\text{cont}} - x_4^{\text{cont}} - 2x^{\text{int}} - 10\right) & \text{if } x^{\text{cat}} = \text{C} \\ 10\left(2\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + 2\left(x_4^{\text{cont}}\right)^2 - x_1^{\text{cont}} - x_2^{\text{cont}} - 3x^{\text{int}} - 5\right) & \text{if } x^{\text{cat}} = \text{D} \end{cases}$$

where

$$s := s(x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}) = \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + 2\left(x_3^{\text{cont}}\right)^2 + \left(x_4^{\text{cont}}\right)^2 - 5x_1^{\text{cont}} - 5x_2^{\text{cont}} - 21x_3^{\text{cont}} + 7x_4^{\text{cont}} + x^{\text{int}} x_3^{\text{cont}}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{-5, -4, \ldots, 4, 5\}$;
- $n^{\text{cont}} = 4$ and $x_i^{\text{cont}} \in [-5, 5]$ for $i \in I^{\text{cont}}$.

3.12 Cat-12: modified Styblinski-Tang [13]

$$f\left(x^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \frac{1}{2} \sum_{i=1}^{n^{\text{cont}}} \left(\left(x_i^{\text{cont}}\right)^4 - 16\left(x_i^{\text{cont}}\right)^2 + 5x_i^{\text{int}} + 8s\right)$$

where $n^{\text{cont}} = n^{\text{int}}$ and

$$s := s\left(x_1^{\text{cat}}, x_i^{\text{cont}}\right) = \begin{cases} \left|x_i^{\text{cont}} - 1\right| & \text{if } x^{\text{cat}} = \text{A} \\ \frac{\left(\left(x_i^{\text{cont}}\right)^2 + 1\right)}{2} & \text{if } x^{\text{cat}} = \text{B} \\ \exp\left(\left|x_i^{\text{cont}} + 1\right|\right) - 1 & \text{if } x^{\text{cat}} = \text{C} \\ \frac{\left(x_i^{\text{cont}}\right)^2}{\left(1 + \left|x_i^{\text{cont}}\right|\right)} & \text{if } x^{\text{cat}} = \text{D} \\ 1 - \exp\left(-\left(x_i^{\text{cont}}\right)^2\right) & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$;
- $n^{\text{int}} \in \mathbb{N}$ and $x_i^{\text{int}} \in \{-5, -4, \ldots, 9, 10\}$ for $i \in I^{\text{int}}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_j^{\text{cont}} \in [-5, 10]$ for $i \in I^{\text{cont}}$.

3.13 Cat-13: modified Toy [16]

$$f\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = 2 + \begin{cases} \cos\left(3.6\pi\left(x_1^{\text{cont}} - 2\right) + x_2^{\text{cont}}\right) + x_3^{\text{cont}} - 1 + \left(x_4^{\text{cont}}\right)^2 & \text{if } x^{\text{cat}} = \text{A}, \\ 2\cos\left(1.1\pi \exp\left(x_1^{\text{cont}}\right)\right) - \frac{x_2^{\text{cont}}}{2} + \left(x_3^{\text{cont}}\right)^2 + 2\log\left(1 + \left(x_4^{\text{cont}}\right)^2\right) & \text{if } x^{\text{cat}} = \text{B}, \\ \cos\left(2\pi x_1^{\text{cont}}\right) + \frac{x_2^{\text{cont}}}{2} + x_3^{\text{cont}} x_4^{\text{cont}} & \text{if } x^{\text{cat}} = \text{C}, \\ x_1 \cos\left(3.4\pi\left(x_1^{\text{cont}} - 1\right)\right) - x_2^{\text{cont}} - 1 + x_3^{\text{cont}} + \left(x_4^{\text{cont}}\right)^3 & \text{if } x^{\text{cat}} = \text{D}, \\ -\frac{\left(x_1^{\text{cont}}\right)^2}{2} + \log\left(1 + \left(x_2^{\text{cont}}\right)^2\right) + \left(x_3^{\text{cont}}\right)^2 + x_4^{\text{cont}} & \text{if } x^{\text{cat}} = \text{E}, \\ 2\cos\left(\frac{\pi}{4}\exp\left(-\left(x_1^{\text{cont}}\right)^4\right)\right)^2 - \frac{x_2^{\text{cont}}}{2} + x_3^{\text{cont}} x_4^{\text{cont}} + 1 & \text{if } x^{\text{cat}} = \text{F}, \\ x_1 \cos\left(3.4 x_1^{\text{cont}}\right) - \frac{x_2^{\text{cont}}}{2} + x_3^{\text{cont}} + \left(x_4^{\text{cont}}\right)^3 + 1 & \text{if } x^{\text{cat}} = \text{G}, \\ x_1^{\text{cont}}\left(-\cos\left(\frac{7}{2\pi}\right)\frac{x_2^{\text{cont}}}{2}\right) + x_3^{\text{cont}} + x_4^{\text{cont}} + 2 & \text{if } x^{\text{cat}} = \text{H}, \\ -\frac{\left(x_1^{\text{cont}}\right)^3}{2} + \left(x_2^{\text{cont}}\right)^2 + x_3^{\text{cont}} x_4^{\text{cont}} + 1 & \text{if } x^{\text{cat}} = \text{I}, \\ -\cos\left(5\pi x_1^{\text{cont}}\right)^2 \sqrt{x_1^{\text{cont}}} - \frac{-\log\left(x_2^{\text{cont}} + x_3^{\text{cont}} + 0.5\right)}{2} + \left(x_4^{\text{cont}}\right)^3 - 1.3 & \text{if } x^{\text{cat}} = \text{J}. \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 4$ and $x_i^{\text{cont}} \in [0, 1]$ for $i \in I^{\text{cont}}$.

3.14 Cat-14: modified Toy [16]

$$f(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}, x_8^{\text{cont}}) = 2 + \begin{cases} \cos(3.6\pi(x_1^{\text{cont}} + x_2^{\text{cont}} - 2)) + |x_3^{\text{cont}}| + \lfloor x_4^{\text{cont}} \rfloor - 0.5 & \text{if } x^{\text{cat}} = \text{A}, \\ 2\cos(1.1\pi e^{x_1^{\text{cont}} + x_5^{\text{cont}}}) - \frac{|x_2^{\text{cont}} + x_6^{\text{cont}}|}{2} + |x_3^{\text{cont}} - x_4^{\text{cont}}| + 2 & \text{if } x^{\text{cat}} = \text{B}, \\ \cos(2\pi(x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}})) + \frac{|x_4^{\text{cont}} + x_5^{\text{cont}}|}{2} - \lfloor x_6^{\text{cont}} \rfloor & \text{if } x^{\text{cat}} = \text{C}, \\ x_1^{\text{cont}} x_2^{\text{cont}}\left(\cos(3.4\pi(x_3^{\text{cont}} - 1)) - \frac{|x_4^{\text{cont}} + x_5^{\text{cont}}|}{2}\right) & \text{if } x^{\text{cat}} = \text{D}, \\ -\frac{|x_1^{\text{cont}} x_6^{\text{cont}}|^2}{2} + x_3^{\text{cont}} + |x_4^{\text{cont}} - x_5^{\text{cont}}| & \text{if } x^{\text{cat}} = \text{E}, \\ 2\cos^2\left(\frac{\pi}{4} e^{-(x_3^{\text{cont}} x_5^{\text{cont}})^4}\right) - \frac{x_6^{\text{cont}} + x_7^{\text{cont}}}{2} + |x_8^{\text{cont}}| + 1 & \text{if } x^{\text{cat}} = \text{F}, \\ x_2 \cos(3.4\pi(x_4^{\text{cont}} + x_5^{\text{cont}})) - \frac{x_6^{\text{cont}}}{2} + |x_1^{\text{cont}} - x_7^{\text{cont}}| + 0.25 & \text{if } x^{\text{cat}} = \text{G}, \\ x_1^{\text{cont}} x_8^{\text{cont}}\left(-\cos\left(\frac{7\pi}{2}(x_2^{\text{cont}} + x_3^{\text{cont}})\right) - \frac{x_6^{\text{cont}}}{2} + 2\right) & \text{if } x^{\text{cat}} = \text{H}, \\ -\frac{|x_1^{\text{cont}} x_2^{\text{cont}} x_3^{\text{cont}}|^3}{2} + |x_4^{\text{cont}}| + |x_5^{\text{cont}}| & \text{if } x^{\text{cat}} = \text{I}, \\ -\cos^2\left(5\pi(x_1^{\text{cont}} x_2^{\text{cont}} + x_3^{\text{cont}})\right)\sqrt{|x_4^{\text{cont}}|} - \frac{\log(|x_5^{\text{cont}}| + 0.5)}{2} + |x_6^{\text{cont}}| - 0.6 & \text{if } x^{\text{cat}} = \text{J}. \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 8$ and $x_i^{\text{cont}} \in [0, 1]$ for $i \in I^{\text{cont}}$.

3.15 Cat-15: modified Wong-1 [14]

$$f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = s + \begin{cases} 0 & \text{if } x^{\text{cat}} = \text{A} \\ 10\left(2\left(x_1^{\text{cont}}\right)^2 + 3\left(x_1^{\text{int}}\right)^4 + x_2^{\text{cont}} + \left(x_2^{\text{int}}\right)^2 + 5x_3^{\text{cont}} - 127\right) & \text{if } x^{\text{cat}} = \text{B} \\ 10\left(7x_1^{\text{cont}} + 3x_1^{\text{int}} + 10\left(x_2^{\text{cont}}\right)^3 + x_2^{\text{int}} - x_3^{\text{cont}} - 282\right) & \text{if } x^{\text{cat}} = \text{C} \\ 10\left(23x_1^{\text{cont}} + \left(x_1^{\text{int}}\right)^2 + 6x_3^{\text{int}} - 8x_4^{\text{cont}} - 196\right) & \text{if } x^{\text{cat}} = \text{D} \\ 10\left(4\left(x_1^{\text{cont}}\right)^2 + \left(x_1^{\text{int}}\right) - 3x_1^{\text{cont}}x_1^{\text{int}} + 2\left(x_2^{\text{cont}}\right)^2 + 5x_3^{\text{int}} - 11x_4^{\text{cont}}\right) & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

where

$$s := s(x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}) = \left(x_1^{\text{cont}} - 10\right)^2 + 5\left(x_1^{\text{int}} - 12\right)^2 + \left(x_2^{\text{cont}}\right)^4 + 3\left(x_2^{\text{int}} - 11\right)^2 + 10\left(x_3^{\text{cont}}\right)^6 + 7\left(x_3^{\text{int}}\right)^2 + \left(x_4^{\text{cont}}\right)^4 - 4x_3^{\text{int}}x_4^{\text{cont}} - 10x_3^{\text{int}} - 8x_4^{\text{cont}}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$;
- $n^{\text{int}} = 3$ and $x_i^{\text{int}} \in \{-1, 0, 1\}$ for $i \in I^{\text{int}}$;
- $n^{\text{cont}} = 4$ and $x_j^{\text{cont}} \in [-1, 1]$ for $i \in I^{\text{cont}}$.

3.16 Cat-16: modified Zakharov [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = 1 + \sum_{i=1}^{n^{\text{cont}}} \left(x_i^{\text{cont}}\right)^2 + \left(\sum_{i=1}^{n^{\text{cont}}} 0.5i\left(x_i^{\text{cont}} + s_1\right)\right)^2 + \left(\sum_{i=1}^{n^{\text{cont}}} 0.5i\left(x_i^{\text{cont}} + s_2\right)\right)^4$$

where

$$s_1 := s_1\left(x_1^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_i^{\text{cont}}\right) = \begin{cases} 0.1\left(x_1^{\text{int}} - x_2^{\text{int}} + x_i^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A} \\ 0.05\lfloor x_1^{\text{int}} + x_2^{\text{int}} + x_i^{\text{cont}} \rfloor & \text{if } x_1^{\text{cat}} = \text{B} \\ -0.1\sqrt{x_i^{\text{cont}} + 5}\left(x_1^{\text{int}} + x_2^{\text{int}}\right) & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2 := s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_i^{\text{cont}}\right) = \begin{cases} 0.1\left(-x_1^{\text{int}} + x_2^{\text{int}} - x_i^{\text{cont}}\right) & \text{if } x_2^{\text{cat}} = \text{A} \\ 0.05\lfloor -x_1^{\text{int}} - x_2^{\text{int}} - x_i^{\text{cont}} \rfloor & \text{if } x_2^{\text{cat}} = \text{B} \\ 0.1\sqrt{x_i^{\text{cont}} + 5}\left(x_1^{\text{int}} + x_2^{\text{int}}\right) & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-3, -2, \ldots, 2, 3\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-5, 5]$ for $i \in I^{\text{cont}}$.

3.17 Cat-17: modified Ishigami [21]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = s^2 \times \left[\sin(\pi x_1^{\text{cont}} + 7\sin^2(\pi x_2^{\text{cont}}) + 0.1(\pi x_3^{\text{cont}})^4 \sin(\pi x_1^{\text{cont}})\right] + s \times \left[\left(\left(x_4^{\text{cont}}\right)^2 + \left(x_5^{\text{cont}}\right)^2 - 1/2\right) + \frac{x^{\text{int}}}{20}\sin\left(4\left(x_4^{\text{cont}} + x_5^{\text{cont}}\right)\right)\right] + 0.3\left[\left(\left(x_6^{\text{cont}}\right)^2 + \left(x_7^{\text{cont}}\right)^2 - 1\right)^2 + \frac{x^{\text{int}}}{20}\cos\left(4\left(x_6^{\text{cont}} - x_7^{\text{cont}}\right)\right)\right]$$

where the images of s are provided by Table 3

Table 3: Images for $s := s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}						
		A	B	C	D	E	F	G
x_2^{cat}	A	0.40	0.66	0.91	1.06	0.89	0.61	0.37
	B	0.56	0.81	1.07	1.26	1.09	0.78	0.52
	C	0.71	0.97	1.22	1.43	1.24	0.94	0.69
	D	0.91	1.17	1.43	1.67	1.47	1.14	0.85
	E	1.12	1.37	1.63	1.92	1.70	1.31	1.03
	F	1.36	1.61	1.94	2.23	1.96	1.57	1.26
	G	1.61	1.93	2.24	2.53	2.22	1.82	1.51

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{1, 2, \ldots, 20\}$;
- $n^{\text{cont}} = 7$ and $x_i^{\text{cont}} \in [-1, 1]$ for $i \in \{1, 2, \ldots, 7\}$.

3.18 Cat-18: modified Hartmann [21]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = s \times \left(1 + 0.02\, x^{\text{int}}\right) \left[-\sum_{i=1}^{4} \alpha_i \exp\left(-\sum_{j=1}^{6} A_{ij} \left(x_j^{\text{cont}} - P_{ij}\right)^2\right)\right] + 0.1\, s \sin\left(4\pi x_7^{\text{cont}} + \frac{2\pi x^{\text{int}}}{17}\right).$$

where

- $\alpha = \begin{bmatrix} 1.0 \\ 1.2 \\ 3.0 \\ 3.2 \end{bmatrix}$,
- $A = \begin{bmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{bmatrix}$,
- $P = \begin{bmatrix} 0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5886 \\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991 \\ 0.2348 & 0.1415 & 0.3522 & 0.2883 & 0.3047 & 0.6650 \\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381 \end{bmatrix}$.

and the images of s are provided by Table 4

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{0, 1, \ldots, 17\}$;
- $n^{\text{cont}} = 7$ and $x_j^{\text{cont}} \in [0, 1]$ for $j \in \{1, 2, \ldots, 7\}$.

Table 4: Images for $s \coloneqq s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}									
		A	B	C	D	E	F	G	H	I	J
x_2^{cat}	A	0.41	0.52	0.59	0.70	0.76	0.79	0.73	0.68	0.61	0.47
	B	0.50	0.61	0.71	0.81	0.88	0.92	0.87	0.80	0.69	0.57
	C	0.57	0.69	0.81	0.93	1.01	1.05	0.99	0.91	0.79	0.65
	D	0.64	0.77	0.89	1.04	1.13	1.18	1.11	1.03	0.91	0.73
	E	0.71	0.86	1.01	1.16	1.26	1.31	1.24	1.14	0.99	0.81
	F	0.78	0.95	1.11	1.27	1.38	1.44	1.37	1.26	1.09	0.89
	G	0.85	1.03	1.21	1.39	1.51	1.57	1.49	1.37	1.21	0.97
	H	0.92	1.12	1.31	1.50	1.63	1.70	1.62	1.49	1.29	1.05
	I	0.99	1.20	1.41	1.62	1.76	1.83	1.74	1.60	1.39	1.13
	J	1.06	1.29	1.51	1.73	1.88	1.96	1.87	1.72	1.49	1.21

3.19 Cat-19: modified Levy [21]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\right) = s_1 + \sum_{i=1}^{n^{\text{cont}}-1} (y_i - 1)^2 \left[1 + 0.5 \sin^2(\pi y_{i+1})\right] + s_2 + 0.1 \sum_{i=1}^{n^{\text{cont}}-1} (y_i - 1)(y_{i+1} + 2)^3,$$

with $y_i = 1 + \frac{x_i^{\text{cont}} - 1}{4}$ for $i \in I^{\text{cont}}$, and where

$$s_1 \coloneqq s_1\left(x_1^{\text{cat}}, y_1\right) = \begin{cases} y_1^2 + 0.10, & \text{if } x_1^{\text{cat}} = \text{A}, \\ (y_1 - 0.25)^2 + 0.33, & \text{if } x_1^{\text{cat}} = \text{B}, \\ (y_1 - 0.50)^2 + 0.20, & \text{if } x_1^{\text{cat}} = \text{C}, \\ 0.8\,(y_1 - 0.75)^2 + 0.25, & \text{if } x_1^{\text{cat}} = \text{D}, \\ 1.2\,(y_1 - 1.00)^2 + 0.15, & \text{if } x_1^{\text{cat}} = \text{E}, \\ (y_1 - 1.25)^2 + 0.40, & \text{if } x_1^{\text{cat}} = \text{F}, \\ 0.9\,(y_1 - 1.50)^2 + 0.30, & \text{if } x_1^{\text{cat}} = \text{G}, \\ 1.1\,(y_1 - 0.00)^2 + 0.05, & \text{if } x_1^{\text{cat}} = \text{H}, \end{cases}$$

and

$$s_2 \coloneqq s_2\left(x_2^{\text{cat}}, y_{n^{\text{cont}}}\right) = \begin{cases} (y_{n^{\text{cont}}} - 1)^2, & \text{if } x_2^{\text{cat}} = \text{A}, \\ 0.8\,(y_{n^{\text{cont}}} - 1)^2 + 0.05, & \text{if } x_2^{\text{cat}} = \text{B}, \\ 1.1\,(y_{n^{\text{cont}}} - 1.05)^2 + 0.08, & \text{if } x_2^{\text{cat}} = \text{C}, \\ 0.9\,(y_{n^{\text{cont}}} - 0.95)^2 + 0.10, & \text{if } x_2^{\text{cat}} = \text{D}, \\ -\exp\left(-(y_{n^{\text{cont}}} - 0.5)^2\right) + 1.10, & \text{if } x_2^{\text{cat}} = \text{E}, \\ -0.9\,\exp\left(-(y_{n^{\text{cont}}} - 0.7)^2\right) + 1.05, & \text{if } x_2^{\text{cat}} = \text{F}, \\ -1.1\,\exp\left(-(y_{n^{\text{cont}}} - 0.3)^2\right) + 1.15, & \text{if } x_2^{\text{cat}} = \text{G}, \\ -\exp\left(-(y_{n^{\text{cont}}} - 0.9)^2\right) + 1.00, & \text{if } x_2^{\text{cat}} = \text{H}. \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-10, 10]$ for $i \in \{1, \ldots, n^{\text{cont}}\}$.

3.20 Cat-20: modified Camel [21]

$$\begin{aligned} f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) &= s_1 \times \left(1 + 0.03\left(x_1^{\text{int}} - 5\right) + 0.02\left(x_2^{\text{int}} - 4\right)\right) \\ &\times \left[\left(4 - 2.1(x_1^{\text{cont}})^2 + \tfrac{1}{3}(x_1^{\text{cont}})^4\right)(x_1^{\text{cont}})^2 + x_1^{\text{cont}} x_2^{\text{cont}} + \left(-4 + 4(x_2^{\text{cont}})^2\right)(x_2^{\text{cont}})^2\right] \\ &+ 0.2\left[(x_3^{\text{cont}})^2 + (x_4^{\text{cont}} - 1)^2\right] + 0.05\left(x_3^{\text{cont}} x_1^{\text{cont}} + x_4^{\text{cont}} x_2^{\text{cont}}\right)^2 \end{aligned}$$

$$+ 0.01\Big(x_1^{\text{int}} - 5\Big)\Big(x_2^{\text{int}} - 4\Big)\big(x_3^{\text{cont}} + x_4^{\text{cont}}\big) + 0.1\, s_2.$$

where the images of s_1 are provided by Table 5 and

Table 5: Values of $s_1 \coloneqq s_1(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}			
		A	B	C	D
x_2^{cat}	A	0.80	1.10	0.95	1.20
	B	1.05	0.85	1.25	0.90
	C	0.92	1.18	1.00	0.88
	D	1.15	0.98	0.87	1.30

$$s_2 \coloneqq s_2\big(x_3^{\text{cat}}, x_3^{\text{cont}}\big) = \begin{cases} \big|x_3^{\text{cont}} - 0.5\big| + 0.05, & \text{if } x_3^{\text{cat}} = \text{A}, \\ 1.05\,\big|x_3^{\text{cont}} - 0.5\big| + 0.05, & \text{if } x_3^{\text{cat}} = \text{B}, \\ 0.95\,\big|x_3^{\text{cont}} - 0.5\big| + 0.06, & \text{if } x_3^{\text{cat}} = \text{C}, \\ 0.70 + 0.30\exp\big(-0.5(x_3^{\text{cont}} + 2)\big), & \text{if } x_3^{\text{cat}} = \text{D}, \\ 1.00 + 0.40\exp\big(-0.8(x_3^{\text{cont}} + 1)\big), & \text{if } x_3^{\text{cat}} = \text{E}. \end{cases}$$

- $n^{\text{cat}} = 3$ with $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$ and $x_3^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$;
- $n^{\text{int}} = 2$ with $x_1^{\text{int}}, x_2^{\text{int}} \in \{-10, \ldots, 10\}$;
- $n^{\text{cont}} = 4$ with $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}} \in [-2, 2]$.

3.21 Cat-21: modified Gamma [14]

$$f\big(x^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\big) = x_1^{\text{cont}} \frac{\left(\sqrt{\left(s + x_2^{\text{cont}} + \dfrac{x_3^{\text{cont}} s + x_4^{\text{cont}}}{\big(x_3^{\text{cont}} s + x_4^{\text{cont}}\big)^2 + \varepsilon^2}\right)^2 + \delta^2}\right)^{s+\frac{1}{2}}}{(s+1)^{\,s+\frac{1}{2}}} - 1.$$

where $\varepsilon = 10^{-2}$, $\delta = 10^{-6}$, and where the images of s are given in Table 6

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{“1”}, \text{“2”} \ldots, \text{“61”}\}$
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 4$ and $x_i^{\text{cont}} \in [-1, 1]$ for $i \in \{1, 2, 3, 4\}$.

3.22 Cat-22: modified EVD-61 [14]

$$\begin{aligned} f\big(x^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\big) = {} & (1 + 0.15\, s)\,\big(x_1^{\text{cont}}\big)^2 + (1 + 0.10\,|s|)\,\big(x_5^{\text{cont}}\big)^2 \\ & - (2.0 - 0.25\, s)\,\frac{x_1^{\text{cont}}}{1 + \big(x_2^{\text{cont}} s\big)^2}\left(1 - 0.25\frac{\big|x_3^{\text{cont}} s + x_4^{\text{cont}}\big|}{1 + \big|x_3^{\text{cont}} s + x_4^{\text{cont}}\big|}\right) \\ & - 0.6\,\big(1 + 0.08\, s^2\big)\,\cos\big(0.6 x_6^{\text{cont}} + 1.1 s\big) + 0.02\, s\, x_2^{\text{cont}} x_6^{\text{cont}}. \end{aligned}$$

where

$$s \coloneqq s(x^{\text{cat}}) = \begin{cases} 0.12\,(i-1) + 0.20, & \text{if the number } i \in \mathbb{N} \text{ in } x^{\text{cat}} = \text{“}i\text{”} \text{ is odd}, \\ -0.09\,(i-1) + 0.50, & \text{if the number } i \in \mathbb{N} \text{ in } x^{\text{cat}} = \text{“}i\text{”} \text{ is even}. \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{“1”}, \ldots, \text{“51”}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 6$ and $x_j^{\text{cont}} \in [-10, 10]$ for $j \in \{1, 2, 3, 4, 5, 6\}$.

Table 6: Images of the categorical function $s := s(x^{\text{cat}})$.

x^{cat}	$s(x^{\text{cat}})$
"1"	1.00
"2"	1.26
"3"	1.54
"4"	1.84
"5"	2.16
"6"	2.50
"7"	2.86
"8"	1.12
"9"	1.15
"10"	1.20
"11"	1.25
"12"	1.30
"13"	1.35
"14"	1.40
"15"	3.00
"16"	3.26
"17"	3.54
"18"	3.84
"19"	4.16
"20"	4.50
"21"	4.86
"22"	5.24
"23"	5.64
"24"	6.06
"25"	6.50
"26"	6.96
"27"	7.44
"28"	7.94
"29"	6.76
"30"	7.20
"31"	7.66
"32"	15.00
"33"	14.85
"34"	14.70
"35"	14.55
"36"	12.00
"37"	11.85
"38"	11.70
"39"	11.55
"40"	11.40
"41"	11.25
"42"	11.10
"43"	10.95
"44"	10.80
"45"	10.65
"46"	10.50
"47"	10.35
"48"	10.20
"49"	10.05
"50"	12.15
"51"	11.05
"52"	9.95
"53"	8.85
"54"	7.75
"55"	6.65
"56"	5.55
"57"	4.45
"58"	3.35
"59"	2.25
"60"	1.15
"61"	1.00

3.23 Cat-23: modified Hal-04 [11]

$$f(\boldsymbol{x}^{\text{cat}}, \boldsymbol{x}^{\text{cont}}) = \sum_{i=1}^{5} \left(5x_i^{\text{cont}} + 0.3s \times \left(x_i^{\text{cont}}\right)^4 + (1 - x_i^{\text{cont}})^2 + s \times \sin\left(3\pi x_i^{\text{cont}} + \frac{i\pi}{5}\right)\right) 2^{\frac{i-1}{4}}$$

where

$$s := s(x^{\text{cat}}) = \begin{cases} 0, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{A,D,F}), \\ 0.20, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{A,D,G}), \\ -0.10, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{A,E,F}), \\ 0.25, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{A,E,G}), \\ 0.50, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{B,D,F}), \\ -0.20, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{B,D,G}), \\ -0.50, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{B,E,F}), \\ 0.80, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{B,E,G}), \\ 0.90, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{C,D,F}), \\ -0.50, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{C,D,G}), \\ 1.00, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{C,E,F}), \\ 1.25, & \text{if } \boldsymbol{x}^{\text{cat}} = (\text{C,E,G}). \end{cases}$$

- $n^{\text{cat}} = 3$ and $\boldsymbol{x}^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\} \times \{\text{D}, \text{E}\} \times \{\text{F}, \text{G}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 5$ and $x_1^{\text{cont}} \in [-9, 12]$, $x_2^{\text{cont}} \in [-7, 14]$, $x_3^{\text{cont}} \in [-10, 10]$ $x_4^{\text{cont}} \in [-8, 13]$, and $x_5^{\text{cont}} \in [-6, 15]$.

3.24 Cat-24: modified OET5 [14]

$$f\Big(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{cont}}, x_1^{\text{cont}}, x_2^{\text{cont}}\Big) = \left(\frac{x_2^{\text{int}}\,|x_1^{\text{cont}}|}{1 + |x_2^{\text{int}}|\,|x_1^{\text{cont}}|} - \frac{\sqrt{s}}{1+\sqrt{s}}\right) \times \left(0.03 + \frac{\Big(0.5\,x_1^{\text{cont}} s^2 + x_2^{\text{cont}} s\, x_1^{\text{int}}\Big)^2}{1 + \Big(x_1^{\text{cont}} s^2 + x_2^{\text{cont}} s\,|x_1^{\text{int}}|\Big)^2} + 0.03\big(1 + 0.5\cos\big(2\pi x_3^{\text{cont}} + 0.7\,s\big)\big)\right) \times \Big(0.15\,\cos\big(2\pi x_3^{\text{cont}} + 3\,s\big) + 1\Big).$$

where,

$$s \coloneqq s(x^{\text{cat}}, x_3^{\text{cont}}) = \begin{cases} 0.1 + 0.02\,\dfrac{x_3^{\text{cont}} + 25}{50} + 0.01\,(k-5)\,\dfrac{x_3^{\text{cont}} + 25}{50}, & \text{if the number } k \in \{\text{“1”}, \text{“2”}, \ldots, \text{“10”}\} \textit{ in } x^{\text{cat}} = \text{“}k\text{”}, \\ 0.2 + 0.08\left(\dfrac{x_3^{\text{cont}} + 25}{10} + 0.3\,(k-11)\right)^2, & \text{if the number } k \in \{\text{“11”}, \text{“12”}, \ldots, \text{“21”}\} \textit{ in } x^{\text{cat}} = \text{“}k\text{”}, \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{“1”}, \text{“2”}, \ldots, \text{“21”}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{-5, 4\}$, $x_2^{\text{int}} \in \{-6, 8\}$;
- $n^{\text{cont}} = 3$ and $x_j^{\text{cont}} \in [-25, 25]$ for $j \in \{1, 2, 3\}$.

3.25 Cat-25: modified Wong 3 [14]

$$f\Big(x^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\Big) = g\Big(\boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\Big) + s\Big(x^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\Big),$$

where

$$\begin{aligned} g\Big(\boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\Big) = {} & \Big|x_1^{\text{int}}\Big| + |x_1^{\text{cont}}| + x_1^{\text{int}} x_1^{\text{cont}} - 14 x_1^{\text{int}} - 16 x_1^{\text{cont}} + \Big(x_2^{\text{int}} - 10\Big)^2 + 4\,\big(x_2^{\text{cont}} - 5\big)^2 \\ & + \Big|x_3^{\text{int}} - 3\Big| + 2\,|x_3^{\text{cont}} - 1| + 5\,\Big(x_4^{\text{int}}\Big)^2 + 7\,\big(x_4^{\text{cont}} - 11\big)^2 + 2\,\Big|x_5^{\text{int}} - 10\Big| + |x_5^{\text{cont}} - 7| \\ & + \Big(x_6^{\text{int}} - 9\Big)^2 + 10\,\big(x_6^{\text{cont}} - 1\big)^2 + 5\,\Big|x_7^{\text{int}} - 7\Big| + 4\,|x_7^{\text{cont}} - 14| + 27\,\Big(x_8^{\text{int}} - 1\Big)^2 + \big(x_8^{\text{cont}}\big)^4 \\ & + \Big|x_9^{\text{int}} - 2\Big| + 13\,|x_9^{\text{cont}} - 2| + \Big(x_{10}^{\text{int}} - 3\Big)^2 + \big(x_{10}^{\text{cont}}\big)^2 + 95. \end{aligned}$$

and

$$s\Big(x^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\Big) = 10 \times \begin{cases} 0, & \text{if } x^{\text{cat}} = \text{A}, \\ 3\left(x_1^{\text{int}} - 2\right)^2 + 4\left(x_1^{\text{cont}} - 3\right)^2 + 2\left(x_2^{\text{int}}\right)^2 - 7x_2^{\text{cont}} - 120, & \text{if } x^{\text{cat}} = \text{B}, \\ 5\left(x_1^{\text{int}}\right)^2 + 8x_1^{\text{cont}} + \left(x_2^{\text{int}} - 6\right)^2 - 2x_2^{\text{cont}} - 40, & \text{if } x^{\text{cat}} = \text{C}, \\ 0.5\left(x_1^{\text{int}} - 8\right)^2 + 2\left(x_1^{\text{cont}} - 4\right)^2 + 3\left(x_3^{\text{int}}\right)^2 - x_3^{\text{cont}} - 30, & \text{if } x^{\text{cat}} = \text{D}, \\ \left(x_1^{\text{int}}\right)^2 + 2\left(x_1^{\text{cont}} - 2\right)^2 - 2x_1^{\text{int}}x_1^{\text{cont}} + 14x_3^{\text{int}} - 6x_3^{\text{cont}}, & \text{if } x^{\text{cat}} = \text{E}, \\ 4\left(x_2^{\text{int}}\right)^2 + 5x_2^{\text{cont}} - 3x_4^{\text{int}} + 9x_4^{\text{cont}} - 105, & \text{if } x^{\text{cat}} = \text{F}, \\ 10x_1^{\text{int}} - 8x_1^{\text{cont}} - 17x_4^{\text{int}} + 2x_4^{\text{cont}}, & \text{if } x^{\text{cat}} = \text{G}, \\ -3x_1^{\text{int}} + 6x_1^{\text{cont}} + 12\left(x_5^{\text{int}} - 8\right)^2 - 7x_5^{\text{cont}}, & \text{if } x^{\text{cat}} = \text{H}, \\ -8x_1^{\text{int}} + 2x_1^{\text{cont}} + 5x_5^{\text{int}} - 2x_5^{\text{cont}} - 12, & \text{if } x^{\text{cat}} = \text{I}, \\ x_1^{\text{int}} + x_1^{\text{cont}} + 4x_6^{\text{int}} - 21x_6^{\text{cont}}, & \text{if } x^{\text{cat}} = \text{J}, \\ \left(x_1^{\text{int}}\right)^2 + 5x_6^{\text{int}} - 8x_6^{\text{cont}} - 28, & \text{if } x^{\text{cat}} = \text{K}, \\ 4x_1^{\text{int}} + 9x_1^{\text{cont}} + 5\left(x_7^{\text{int}}\right)^2 - 9x_7^{\text{cont}} - 87, & \text{if } x^{\text{cat}} = \text{L}, \\ 3x_1^{\text{int}} + 4x_1^{\text{cont}} + 3\left(x_7^{\text{int}} - 6\right)^2 - 14x_7^{\text{cont}} - 10, & \text{if } x^{\text{cat}} = \text{M}, \\ 14\left(x_1^{\text{int}}\right)^2 + 35x_8^{\text{int}} - 79x_8^{\text{cont}} - 92, & \text{if } x^{\text{cat}} = \text{N}, \\ 15\left(x_1^{\text{cont}}\right)^2 + 11x_8^{\text{int}} - 61x_8^{\text{cont}} - 54, & \text{if } x^{\text{cat}} = \text{O}, \\ 5\left(x_1^{\text{int}}\right)^2 + 2x_1^{\text{cont}} + 9\left(x_9^{\text{int}}\right)^4 - x_9^{\text{cont}} - 68, & \text{if } x^{\text{cat}} = \text{P}, \\ \left(x_1^{\text{int}}\right)^2 - x_1^{\text{cont}} + 19x_{10}^{\text{int}} - 20x_{10}^{\text{cont}} + 19, & \text{if } x^{\text{cat}} = \text{Q}, \\ 7\left(x_1^{\text{int}}\right)^2 + 5\left(x_1^{\text{cont}}\right)^2 + \left(x_{10}^{\text{int}}\right)^2 - 30x_{10}^{\text{cont}}, & \text{if } x^{\text{cat}} = \text{R}. \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \ldots, \text{R}\}$;
- $n^{\text{int}} = 10$ and $x_i^{\text{int}} \in \{-50, -49, \ldots, 50\}$ for $i \in \{1, \ldots, 10\}$;
- $n^{\text{cont}} = 10$ and $x_j^{\text{cont}} \in [-50, 50]$ for $j \in \{1, \ldots, 10\}$.

3.26 Cat-26: modified Roustant [19]

$$f\Big(x^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\Big) =$$
$$\begin{cases} \left(x_1^{\text{cont}} + 0.01\big(x_1^{\text{cont}} - \frac{1}{2}\big)^2\right)\frac{u}{10}\left(1 + 0.1x_2^{\text{cont}} + 0.05x_3^{\text{cont}}\right) + 0.02\Big(\big(x_4^{\text{cont}}\big)^2 + \big(x_5^{\text{cont}}\big)^2\Big), & \text{if } x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}, \\ 0.9\cos\Big(2\pi\big(x_1^{\text{cont}} + \frac{u-4}{20}\big)\Big)\exp\big(-x_1^{\text{cont}}\big)\left(1 + 0.1x_2^{\text{cont}}\right) + 0.03x_3^{\text{cont}} + 0.02\Big(\big(x_4^{\text{cont}}\big)^2 + x_5^{\text{cont}}\Big), & \text{if } x^{\text{cat}} \in \{\text{E}, \text{F}, \text{G}\}, \\ -0.7\cos\Big(2\pi\big(x_1^{\text{cont}} + \frac{u-7}{20}\big)\Big)\exp\big(-x_1^{\text{cont}}\big)\left(1 + 0.1x_2^{\text{cont}} - 0.05x_3^{\text{cont}}\right) + 0.02x_4^{\text{cont}}x_5^{\text{cont}}, & \text{if } x^{\text{cat}} \in \{\text{H}, \text{I}, \text{J}\}. \end{cases}$$

where

$$u = b\big(x^{\text{cat}}\big) + \delta,$$

with

$$\delta = \text{round}\left(\frac{x_1^{\text{int}} + x_2^{\text{int}}}{10}\right) \in \{-2, -1, 0, 1, 2\},$$

and

$$b(x^{\text{cat}}) = \begin{cases} 0.6, & \text{if } x^{\text{cat}} = \text{A}, \\ 1.1, & \text{if } x^{\text{cat}} = \text{B}, \\ 2.7, & \text{if } x^{\text{cat}} = \text{C}, \\ 3.5, & \text{if } x^{\text{cat}} = \text{D}, \\ 4.3, & \text{if } x^{\text{cat}} = \text{E}, \\ 4.0, & \text{if } x^{\text{cat}} = \text{F}, \\ 3.4, & \text{if } x^{\text{cat}} = \text{G}, \\ 2.3, & \text{if } x^{\text{cat}} = \text{H}, \\ 1.0, & \text{if } x^{\text{cat}} = \text{I}, \\ 0.4, & \text{if } x^{\text{cat}} = \text{J}. \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \ldots, \text{J}\}$;
- $n^{\text{int}} = 3$ and $x_i^{\text{int}} \in \{-10, -9, \ldots, 10\}$ for $i \in \{1, 2, 3\}$;
- $n^{\text{cont}} = 5$ with $x_1^{\text{cont}} \in [0, 1]$ and $x_j^{\text{cont}} \in [-1, 1]$ for $j \in \{2, 3, 4, 5\}$.

3.27 Cat-27: Kowalik-Osborne [15]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \frac{x_1^{\text{cont}}\left(s^2 + x_2^{\text{cont}} s\right)}{s^2 + x_3^{\text{cont}} s + x_4^{\text{cont}}} - s + 0.05\left|x_2^{\text{cont}} - x_3^{\text{cont}} + 0.1 x_1^{\text{int}}\right| + 0.03\left|x_1^{\text{cont}} x_4^{\text{cont}} - 0.05 x_1^{\text{int}}\right| + 0.02\left|x_3^{\text{cont}} + 0.02 x_1^{\text{int}}\right|,$$

where $s = s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$ is given by Table 7.

Table 7: Values of $s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}									
		A	B	C	D	E	F	G	H	I	J
x_2^{cat}	A	0.783	0.391	0.196	0.098	0.049	0.033	0.025	0.020	0.016	0.014
	B	0.779	0.389	0.195	0.097	0.049	0.033	0.024	0.020	0.016	0.014
	C	0.694	0.347	0.174	0.087	0.043	0.029	0.022	0.017	0.014	0.012
	D	0.640	0.320	0.160	0.080	0.040	0.027	0.020	0.016	0.013	0.011
	E	0.338	0.169	0.084	0.042	0.021	0.014	0.011	0.008	0.007	0.006
	F	0.251	0.125	0.063	0.031	0.016	0.011	0.008	0.006	0.005	0.005
	G	0.182	0.091	0.046	0.023	0.011	0.008	0.006	0.005	0.004	0.003
	H	0.137	0.068	0.034	0.017	0.009	0.006	0.004	0.003	0.003	0.002
	I	0.129	0.065	0.032	0.016	0.008	0.005	0.004	0.003	0.003	0.002
	J	0.094	0.047	0.024	0.012	0.006	0.004	0.003	0.002	0.002	0.002

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \ldots, \text{J}\}$;
- $n^{\text{int}} = 1$ and $x_1^{\text{int}} \in \{-50, -49, \ldots, 50\}$;
- $n^{\text{cont}} = 4$ and $x_j^{\text{cont}} \in [-10, 10]$ for $j \in \{1, 2, 3, 4\}$.

3.28 Cat-28: modified Three-humps [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 2s_1^2 - 1.05\, s_1^4 + \frac{s_1^6}{6} + s_1 s_2 + s_2^2 + 0.08\,|s_1| + 0.05\,|s_2 - s_1|,$$

where

$$s_1 \coloneqq s\big(x_1^{\text{cat}}, x_1^{\text{cont}}\big) = \begin{cases} x_1^{\text{cont}} + 0.12\big(x_1^{\text{cont}} - 1\big)^2 + 0.15\,\big|x_1^{\text{cont}}\big|, & \text{if } x_1^{\text{cat}} = \text{A}, \\ x_1^{\text{cont}} + 0.12\big(x_1^{\text{cont}} - 1\big)^2 + 0.10\,\big|x_1^{\text{cont}} - 0.5\big|, & \text{if } x_1^{\text{cat}} = \text{B}, \\ x_1^{\text{cont}} + 0.12\big(x_1^{\text{cont}} - 1\big)^2 + 0.08\,\big|x_1^{\text{cont}} + 0.5\big|, & \text{if } x_1^{\text{cat}} = \text{C}, \\ x_1^{\text{cont}} - 0.10\big(x_1^{\text{cont}} + 1\big)^2 - 0.12\,\big|x_1^{\text{cont}}\big|, & \text{if } x_1^{\text{cat}} = \text{D}, \\ x_1^{\text{cont}} - 0.10\big(x_1^{\text{cont}} + 1\big)^2 - 0.09\,\big|x_1^{\text{cont}} - 0.5\big|, & \text{if } x_1^{\text{cat}} = \text{E}, \\ x_1^{\text{cont}} - 0.10\big(x_1^{\text{cont}} + 1\big)^2 - 0.07\,\big|x_1^{\text{cont}} + 0.5\big|, & \text{if } x_1^{\text{cat}} = \text{F}. \end{cases}$$

and,

$$s_2 \coloneqq s\big(x_2^{\text{cat}}, x_2^{\text{cont}}\big) = \begin{cases} 0.9\,e^{0.35 x_2^{\text{cont}}} + 0.05\,\big|x_2^{\text{cont}}\big|, & \text{if } x_2^{\text{cat}} = \text{A}, \\ 0.6\,e^{0.45 x_2^{\text{cont}}} + 0.04\,\big|x_2^{\text{cont}}\big|, & \text{if } x_2^{\text{cat}} = \text{B}, \\ 0.9\,e^{0.35 x_2^{\text{cont}}} + 0.06\,\big|x_2^{\text{cont}} - 0.4\big|, & \text{if } x_2^{\text{cat}} = \text{C}, \\ 0.6\,e^{0.45 x_2^{\text{cont}}} + 0.05\,\big|x_2^{\text{cont}} - 0.4\big|, & \text{if } x_2^{\text{cat}} = \text{D}, \\ 0.9\,e^{0.35 x_2^{\text{cont}}} + 0.06\,\big|x_2^{\text{cont}} + 0.4\big|, & \text{if } x_2^{\text{cat}} = \text{E}, \\ 0.6\,e^{0.45 x_2^{\text{cont}}} + 0.05\,\big|x_2^{\text{cont}} + 0.4\big|, & \text{if } x_2^{\text{cat}} = \text{F}. \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_i^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}\}$ for $i \in \{1, 2\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 2$ and $x_j^{\text{cont}} \in [-5, 5]$ for $j \in \{1, 2\}$.

3.29 Cat-29: modified McCormick [13]

$$f\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\Big) = \sin(s_1 + s_2) + (s_1 - s_2)^2 - 1.5\,s_1 + 2.5\,s_2 + 1 + 0.06\left|s_1 + \frac{x_1^{\text{int}}}{10}\right| + 0.04\left|s_2 - \frac{x_2^{\text{int}}}{10}\right| + 0.03\left|s_1 - s_2 + \frac{x_3^{\text{int}}}{10}\right|,$$

where

$$s_1 \coloneqq s\big(x_1^{\text{cat}}, x_1^{\text{cont}}, x_3^{\text{cont}}\big) = \begin{cases} x_1^{\text{cont}} + 0.25\,\tanh\big(x_1^{\text{cont}}\big) + 0.15\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{A}, \\ x_1^{\text{cont}} + 0.25\,\tanh\big(x_1^{\text{cont}} - 0.5\big) + 0.12\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{B}, \\ x_1^{\text{cont}} + 0.25\,\tanh\big(x_1^{\text{cont}} + 0.5\big) + 0.10\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{C}, \\ x_1^{\text{cont}} - 0.30\,\tanh\big(x_1^{\text{cont}}\big) + 0.10\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{D}, \\ x_1^{\text{cont}} - 0.30\,\tanh\big(x_1^{\text{cont}} + 0.5\big) + 0.08\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{E}, \\ x_1^{\text{cont}} - 0.30\,\tanh\big(x_1^{\text{cont}} - 0.5\big) + 0.09\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{F}, \\ x_1^{\text{cont}} + 0.18\,\big|x_1^{\text{cont}}\big|^{1.3} - 0.10\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{G}, \\ x_1^{\text{cont}} + 0.18\,\big|x_1^{\text{cont}} + 0.3\big|^{1.3} - 0.12\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{H}, \end{cases}$$

and

$$s_2 \coloneqq s\left(x_2^{\text{cat}}, x_2^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} x_2^{\text{cont}} + 0.35\,\arctan\left(x_2^{\text{cont}}\right) + 0.20\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{A}, \\ x_2^{\text{cont}} + 0.35\,\arctan\left(x_2^{\text{cont}} - 0.6\right) + 0.18\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{B}, \\ x_2^{\text{cont}} + 0.35\,\arctan\left(x_2^{\text{cont}} + 0.6\right) + 0.16\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{C}, \\ x_2^{\text{cont}} + 0.22\,\text{sign}\left(x_2^{\text{cont}}\right)\sqrt{\left|x_2^{\text{cont}}\right|} - 0.12\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{D}, \\ x_2^{\text{cont}} + 0.22\,\text{sign}\left(x_2^{\text{cont}}\right)\sqrt{\left|x_2^{\text{cont}} + 0.4\right|} - 0.10\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{E}, \\ x_2^{\text{cont}} + 0.22\,\text{sign}\left(x_2^{\text{cont}}\right)\sqrt{\left|x_2^{\text{cont}} - 0.4\right|} - 0.11\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{F}, \\ x_2^{\text{cont}} - 0.28\,\log\left(1 + \left(x_2^{\text{cont}}\right)^2\right) + 0.15\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{G}, \\ x_2^{\text{cont}} - 0.28\,\log\left(1 + \left(x_2^{\text{cont}} - 0.4\right)^2\right) + 0.13\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{H}. \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_i^{\text{cat}} \in \{\text{A}, \text{B}, \ldots, \text{H}\}$ for $i \in \{1, 2\}$;
- $n^{\text{int}} = 3$ and $x_i^{\text{int}} \in \{-10, -9, \ldots, 10\}$ for $i \in \{1, 2, 3\}$;
- $n^{\text{cont}} = 4$ with $x_j^{\text{cont}} \in [-4, 4]$ for $j \in \{1, 2\}$, and $x_j^{\text{cont}} \in [-2, 2]$ for $j \in \{3, 4\}$.

3.30 Cat-30: modified Shekel [13]

$$\begin{aligned} f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{cat}}, x_4^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\right) = \\ \sum_{i=1}^{4}\Bigg(s_i + \left(x_1^{\text{cont}} - x_i^{\text{int}}\right)^2 + \left(x_2^{\text{cont}} - x_i^{\text{int}}\right)^2 \\ + 0.08\left(x_5^{\text{cont}} - 0.7x_i^{\text{int}}\right)^2 + 0.06\left(x_6^{\text{cont}} - 0.4x_i^{\text{int}}\right)^2 \\ + 0.05\left|x_1^{\text{cont}} - x_2^{\text{cont}} + 0.4x_5^{\text{cont}} - 0.3x_6^{\text{cont}}\right|\Bigg)^{-1} \\ + 0.12\sum_{i=1}^{4}\left|x_i^{\text{cont}} - x_i^{\text{int}}\right| + 0.04\sum_{i=1}^{4}\sqrt{\left|x_i^{\text{cont}} - x_i^{\text{int}}\right|} \\ + 0.03\left|x_5^{\text{cont}} - x_6^{\text{cont}}\right| + 0.02\left|x_1^{\text{cont}}x_2^{\text{cont}} - x_5^{\text{cont}}x_6^{\text{cont}}\right| \\ + 0.06\sum_{i=1}^{4}\left(1 + 0.3\,s_i\right)\cos\left(\frac{\pi}{5}\left(x_5^{\text{cont}} + x_6^{\text{cont}}\right) + 0.4\,x_i^{\text{int}}\right). \end{aligned}$$

where

$$s_1 \coloneqq s_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} 0.35 + 0.10\log\left(1 + \left(x_1^{\text{cont}} - 3\right)^2 + \left(x_2^{\text{cont}} - 7\right)^2\right), & \text{if } x_1^{\text{cat}} = \text{A}, \\ 0.30 + 0.06\left|x_1^{\text{cont}} - 5\right| + 0.04\left|x_2^{\text{cont}} - 2\right|, & \text{if } x_1^{\text{cat}} = \text{B}, \\ 0.55 + \dfrac{0.80}{1 + \left(x_1^{\text{cont}} - 8\right)^2 + \left(x_2^{\text{cont}} - 1\right)^2}, & \text{if } x_1^{\text{cat}} = \text{C}, \end{cases}$$

and

$$s_2 \coloneqq s_2\left(x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} 0.30 + 0.06\left|x_1^{\text{cont}} - 2\right| + 0.05\left|x_2^{\text{cont}} - 6\right|, & \text{if } x_2^{\text{cat}} = \text{A}, \\ 0.55 + \dfrac{0.85}{1 + \left(x_1^{\text{cont}} - 1\right)^2 + \left(x_2^{\text{cont}} - 9\right)^2}, & \text{if } x_2^{\text{cat}} = \text{B}, \\ 0.33 + 0.10\log\left(1 + \left(x_1^{\text{cont}} - 6\right)^2 + \left(x_2^{\text{cont}} - 4\right)^2\right), & \text{if } x_2^{\text{cat}} = \text{C}, \end{cases}$$

and

$$s_3 \coloneqq s_3\left(x_3^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} 0.55 + \dfrac{0.75}{1 + \left(x_1^{\text{cont}} - 4\right)^2 + \left(x_2^{\text{cont}} - 4\right)^2}, & \text{if } x_3^{\text{cat}} = \text{A}, \\ 0.34 + 0.10\log\left(1 + \left(x_1^{\text{cont}} - 9\right)^2 + \left(x_2^{\text{cont}} - 2\right)^2\right), & \text{if } x_3^{\text{cat}} = \text{B}, \\ 0.28 + 0.07\left|x_1^{\text{cont}} - 7\right| + 0.03\left|x_2^{\text{cont}} - 5\right|, & \text{if } x_3^{\text{cat}} = \text{C}, \end{cases}$$

and

$$s_4 \coloneqq s_4\left(x_4^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} 0.33 + 0.10\log\left(1 + \left(x_1^{\text{cont}} - 2\right)^2 + \left(x_2^{\text{cont}} - 1\right)^2\right), & \text{if } x_4^{\text{cat}} = \text{A}, \\ 0.55 + \dfrac{0.90}{1 + \left(x_1^{\text{cont}} - 7\right)^2 + \left(x_2^{\text{cont}} - 8\right)^2}, & \text{if } x_4^{\text{cat}} = \text{B}, \\ 0.29 + 0.06\left|x_1^{\text{cont}} - 4\right| + 0.05\left|x_2^{\text{cont}} - 7\right|, & \text{if } x_4^{\text{cat}} = \text{C}. \end{cases}$$

- $n^{\text{cat}} = 4$ and $x_i^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$ for $i \in \{1, \ldots, 4\}$;
- $n^{\text{int}} = 4$ and $x_i^{\text{int}} \in \{0, 1, \ldots, 10\}$ for $i \in \{1, \ldots, 4\}$;
- $n^{\text{cont}} = 6$ and $x_j^{\text{cont}} \in [0, 10]$ for $j \in \{1, \ldots, 6\}$.

4 Description of the constrained problems

The following subsections details the constrained problems.

4.1 Cat-cstrs-1: modified Beale [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \left(1.5 - s_1 + x_1^{\text{int}}\left(1 - s_2\right)\right)^2 + \left(2.25 - s_1 + x_2^{\text{int}}\left(1 - s_2\right)^2\right)^2 + \left(2.625 - s_1 + x_3^{\text{cont}}\left(1 - s_2\right)^3\right)^2$$

where

$$s_1 \coloneqq s_1(x_1^{\text{cat}}, x_1^{\text{cont}}) = \begin{cases} \lfloor x_1^{\text{cont}} \rfloor & \text{if } x_1^{\text{cat}} = \text{A} \\ x_1^{\text{cont}} & \text{if } x_1^{\text{cat}} = \text{B} \\ \exp\left(\frac{x_1^{\text{cont}}}{2}\right) & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2 \coloneqq s_2(x_2^{\text{cat}}, x_2^{\text{cont}}) = \begin{cases} \sqrt{|x_2^{\text{cont}}| + 1} & \text{if } x_2^{\text{cat}} = \text{A} \\ \left|x_2^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{B} \\ \left(x_2^{\text{cont}}\right)^2 - 2 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

with constraints

$$\begin{aligned} g_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) &= 3\left(x_1^{\text{cont}} - 2\right)^2 + 4\left(x_2^{\text{cont}} - 3\right)^2 + 2\left(x_3^{\text{cont}}\right) - 100 \leq 0 \\ g_2\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) &= 5\left(x_1^{\text{cont}}\right)^2 + 8x_2^{\text{cont}} + \left(x_3^{\text{cont}} - 6\right)^2 - 30 \leq 0 \\ g_3\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) &= \left(x_1^{\text{cont}}\right)^2 + 2\left(x_2^{\text{cont}} - 2\right)^2 - 2x_1^{\text{cont}}x_2^{\text{cont}} - 6x_3^{\text{cont}} \leq 0 \end{aligned}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-2, -1, 0, 1, 2\}$;
- $n^{\text{cont}} = 3$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}} \in [-4.5, 4.5]$.

4.2 Cat-cstrs-2: modified Branin [17]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} s & \text{if } x_1^{\text{cat}} = \text{A} \text{ and } x_2^{\text{cat}} = \text{A} \\ 0.4s & \text{if } x_1^{\text{cat}} = \text{A} \text{ and } x_2^{\text{cat}} = \text{B} \\ -0.75s & \text{if } x_1^{\text{cat}} = \text{B} \text{ and } x_2^{\text{cat}} = \text{A} \\ -0.5s & \text{if } x_1^{\text{cat}} = \text{B} \text{ and } x_2^{\text{cat}} = \text{B} \end{cases}$$

where

$$s \coloneqq s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) =$$
$$\frac{1}{a}\left[\left(\left(15x_2^{\text{cont}} - \frac{x_1^{\text{int}}}{4\pi^2}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)^2 + \frac{x_1^{\text{int}}}{\pi}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)\right)^2 + \left(x_1^{\text{int}} - \frac{1}{8\pi}\right)\cos\left(15x_1^{\text{cont}} - 5\right) + x_1^{\text{int}}\right) - b\right]$$

with $a = 51.9496$, $b = 54.8104$ and constraint

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} -x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_1^{\text{int}}}{10} & \text{if } x_1^{\text{cat}} = 0 \text{ and } x_2^{\text{cat}} = 0 \\ -1.5x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_2^{\text{int}}}{10} & \text{if } x_1^{\text{cat}} = 0 \text{ and } x_2^{\text{cat}} = 1 \\ -1.5x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_1^{\text{int}}}{5} & \text{if } x_1^{\text{cat}} = 1 \text{ and } x_2^{\text{cat}} = 0 \\ -1.2x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_2^{\text{int}}}{7.5} & \text{if } x_1^{\text{cat}} = 1 \text{ and } x_2^{\text{cat}} = 1 \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{1, 2, 3, 4, 5\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 1]$.

4.3 Cat-cstrs-3: modified Bukin6 [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = 100\sqrt{|s_1 - 0.01s_2|} + 0.01\left|s_2 + x_2^{\text{int}}\right|$$

where

$$s_1 \coloneqq s_1(x_1^{\text{cat}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \sqrt{|x_2^{\text{cont}} + x_3^{\text{cont}}| + 2} & \text{if } x_1^{\text{cat}} = \text{A} \\ \left|x_2^{\text{cont}} + x_3^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{B} \\ \frac{\left(x_2^{\text{cont}} + x_3^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2 \coloneqq s_2(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}) = \begin{cases} \sqrt{|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}| + 1.5} & \text{if } x_2^{\text{cat}} = \text{A} \\ \left|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{x_1^{\text{int}} + \left(x_1^{\text{cont}} + x_4^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

with constraints

$$g_1(x_1^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}) = \begin{cases} 1.5\sin\left(\frac{x_1^{\text{cont}} + x_4^{\text{cont}}}{5}\right) + 0.5x_1^{\text{int}} - 0.1\left(x_1^{\text{cont}}\right)^2 - 2 & \text{if } x_1^{\text{cat}} = \text{A}, \\ 1.5\sin\left(\frac{x_1^{\text{cont}} - x_4^{\text{cont}}}{5}\right) + 0.3x_1^{\text{int}} - 0.1\left(x_4^{\text{cont}}\right)^2 - 2.5 & \text{if } x_1^{\text{cat}} = \text{B}, \\ \exp\left(0.2x_1^{\text{cont}} + 0.1x_4^{\text{cont}}\right) + 0.1x_1^{\text{int}} - 4 & \text{if } x_1^{\text{cat}} = \text{C}. \end{cases}$$

$$g_2(x_2^{\text{cat}}, x_2^{\text{int}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} 1.5\cos\left(\frac{x_2^{\text{cont}} + x_3^{\text{cont}}}{5}\right) + 0.4x_2^{\text{int}} - 0.2\left(x_2^{\text{cont}}\right)^2 - 1.5 & \text{if } x_2^{\text{cat}} = \text{A}, \\ 1.5\cos\left(\frac{x_2^{\text{cont}} - x_3^{\text{cont}}}{5}\right) + 0.6x_2^{\text{int}} - 0.2\left(x_3^{\text{cont}}\right)^2 - 2 & \text{if } x_2^{\text{cat}} = \text{B}, \\ \log\left(1 + \left(x_2^{\text{cont}} + x_3^{\text{cont}}\right)^2\right) + 0.2x_2^{\text{int}} - 3 & \text{if } x_2^{\text{cat}} = \text{C}. \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-5, -4, \ldots, 4, 5\}$;
- $n^{\text{cont}} = 4$ and $\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) \in [-15, 5] \times [-3, 3] \times [-15, 5] \times [-3, 3]$.

4.4 Cat-cstrs-4: modified Dembo-5 [14]

$$f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = -\left(x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}}\right)$$
$$+ \begin{cases} 0 & \text{if } x^{\text{cat}} = \text{A} \\ -10^5 \left(\frac{833.33252 x_4^{\text{cont}}}{x_1^{\text{cont}} x_2^{\text{int}}} + \frac{100}{x_2^{\text{int}}} - \frac{83333.333}{x_1^{\text{cont}} x_2^{\text{int}}} - 1\right) & \text{if } x^{\text{cat}} = \text{B} \\ -10^5 \left(\frac{x_4^{\text{cont}}}{x_3^{\text{int}}} + \frac{1250\left(x_1^{\text{int}} - x_4^{\text{cont}}\right)}{x_2^{\text{cont}} x_3^{\text{int}}} - 1\right) & \text{if } x^{\text{cat}} = \text{C} \\ -10^5 \left(\frac{1250000}{x_3^{\text{cont}} x_4^{\text{int}}} + \frac{x_1^{\text{int}}}{x_4^{\text{int}}} - \frac{2500 x_1^{\text{int}}}{x_3^{\text{cont}} x_4^{\text{int}}} - 1\right) & \text{if } x^{\text{cat}} = \text{D} \end{cases}$$

with constraints

$$g_1\left(x_2^{\text{int}}, x_4^{\text{cont}}\right) = 0.0025\left(x_2^{\text{int}} + x_4^{\text{cont}}\right) - 1$$
$$g_2\left(x_1^{\text{int}}, x_3^{\text{int}}, x_4^{\text{cont}}\right) = 0.0025\left(x_1^{\text{int}}, x_3^{\text{int}}, x_4^{\text{cont}}\right) - 1$$
$$g_3\left(x_1^{\text{int}}, x_4^{\text{int}}\right) = 0.01\left(x_4^{\text{int}} - x_1^{\text{int}}\right) - 1$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}} = 4$ and $x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}} \in \{10, 11, \ldots, 1000\}$;
- $n^{\text{cont}} = 4$ and $x_1^{\text{cont}} \in [100, 10000]$, and $x_2^{\text{cont}} \in [1000, 10000]$, and $x_3^{\text{cont}} \in [1000, 10000]$, and $x_4^{\text{cont}} \in [10, 1000]$

4.5 Cat-cstrs-5: modified EVD52 [14]

$$f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = -1 \times \begin{cases} \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{A} \\ \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}} - 2\right)^2 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{B} \\ x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}} - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{C} \\ x_1^{\text{cont}} + x_2^{\text{cont}} - x_3^{\text{cont}} + 1 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{D} \\ 2\left(x_1^{\text{cont}}\right)^3 + 6\left(x_2^{\text{cont}}\right)^2 + 2\left(5x_3^{\text{cont}} - x_1^{\text{cont}} + 1\right)^2 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{E} \\ \left(x_1^{\text{cont}}\right)^2 - 9x_3^{\text{cont}} + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{F} \end{cases}$$

with constraint

$$g(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \left|x_1^{\text{cont}} + x_2^{\text{cont}} - 5\right|^{1.5} + \left(x_3^{\text{cont}} + 1\right)^2 + \frac{|x^{\text{int}}|}{10} - 30 & \text{if } x^{\text{cat}} = \text{A}, \\ \left(x_1^{\text{cont}} \cdot x_2^{\text{cont}}\right)^2 + \left|x_3^{\text{cont}} - 1\right| + \frac{|x^{\text{int}} - 5|}{10} - 30 & \text{if } x^{\text{cat}} = \text{B}, \\ \left|x_1^{\text{cont}} - x_2^{\text{cont}}\right|^3 + \left(x_3^{\text{cont}}\right)^2 + \frac{|x^{\text{int}} + 5|}{10} - 30 & \text{if } x^{\text{cat}} = \text{C}, \\ \left(x_1^{\text{cont}} + x_2^{\text{cont}}\right)^2 + \left|x_3^{\text{cont}} - 0.5\right|^{1.5} + \frac{|x^{\text{int}} - 10|}{10} - 30 & \text{if } x^{\text{cat}} = \text{D}, \\ \left(x_1^{\text{cont}} - 2x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}} - 1.5\right) + \frac{|x^{\text{int}} + 10|}{10} - 30 & \text{if } x^{\text{cat}} = \text{E}, \\ \left(x_1^{\text{cont}} \cdot x_3^{\text{cont}}\right)^2 + \left|x_2^{\text{cont}} + 1.5\right| + \frac{|x^{\text{int}} - 15|}{10} - 30 & \text{if } x^{\text{cat}} = \text{F}. \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{-25, -24, \ldots, , 24, 25\}$;
- $n^{\text{cont}} = 3$ and $x_i^{\text{cont}} \in [-25, 25]$ for $i \in I^{\text{cont}}$.

4.6 Cat-cstrs-6: modified G-09 [6]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \left(x_1^{\text{cont}} - 10\right)^2 + 5\left(x_2^{\text{cont}} - 12\right)^2 + \left(x_3^{\text{cont}}\right)^4 + 3\left(x_1^{\text{int}} - 11\right)^2$$
$$+ 10\left(x_2^{\text{int}}\right)^6 + 7s_1^2 + s_2^2 - 4s_1 s_2 - 10s_1 - 8s_2$$

where

Table 8: Images of $s_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$.

	$x_1^{\text{cat}} = \text{A}$	$x_1^{\text{cat}} = \text{B}$	$x_1^{\text{cat}} = \text{C}$
$x_2^{\text{cat}} = \text{A}$	-5	2.5	7.5
$x_2^{\text{cat}} = \text{B}$	-5	3.5	8.5
$x_2^{\text{cat}} = \text{C}$	-4	4.5	10

Table 9: Images of $s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$.

	$x_1^{\text{cat}} = \text{A}$	$x_1^{\text{cat}} = \text{B}$	$x_1^{\text{cat}} = \text{C}$
$x_2^{\text{cat}} = \text{A}$	-5	-5	-4
$x_2^{\text{cat}} = \text{B}$	2.5	4.5	3
$x_2^{\text{cat}} = \text{C}$	1	6.5	9

with constraints

$$g_1\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \frac{2\left(x_1^{\text{cont}}\right)^2 + 3\left(x_2^{\text{cont}}\right)^4 + x_3^{\text{cont}} + 4\left(x_1^{\text{int}}\right)^2 + 5x_2^{\text{int}}}{2} - 127$$
$$g_2\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \frac{7x_1^{\text{cont}} + 3x_2^{\text{cont}} + 10\left(x_3^{\text{cont}}\right)^2 + x_1^{\text{int}} - x_2^{\text{int}}}{2} - 282$$
$$g_3\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 23x_1^{\text{cont}} + \left(x_2^{\text{cont}}\right)^2 + s_1 - 8s_2 - 196$$
$$g_4\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = 4\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 - 3x_1^{\text{cont}}x_2^{\text{cont}} + 2\left(x_3^{\text{cont}}\right)^2 + 5s_1 - 11s_2$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-10, -9, \ldots, 10\}$;
- $n^{\text{cont}} = 3$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [-10, 10]$.

4.7 Cat-cstrs-7: Goldstein taken directly from [17]

$$\begin{aligned} f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \\ & 53.3108 + 0.184901x_1^{\text{cont}} \\ & - 5.02914\left(x_1^{\text{cont}}\right)^3 \times 10^{-6} + 7.72522\left(x_1^{\text{cont}}\right)^4 \times 10^{-8} \\ & - 0.0870775x_2^{\text{cont}} - 0.106959s_1 \\ & + 7.98772s_1^3 \times 10^{-6} + 0.00242482s_2 \\ & + 1.32851s_2^3 \times 10^{-6} - 0.00146393x_1^{\text{cont}}x_2^{\text{cont}} \\ & - 0.00301588x_1^{\text{cont}}s_1 - 0.00272291x_1^{\text{cont}}s_2 \\ & + 0.0017004x_2^{\text{cont}}s_1 + 0.0038428x_2^{\text{cont}}s_2 \\ & - 0.000198969s_1s_2 \\ & + 1.86025x_1^{\text{cont}}x_2^{\text{cont}}s_1 \times 10^{-5} \\ & - 1.88719x_1^{\text{cont}}x_2^{\text{cont}}s_2 \times 10^{-6} \\ & + 2.50923x_1^{\text{cont}}s_1s_2 \times 10^{-5} \\ & - 5.62199x_2^{\text{cont}}s_1s_2 \times 10^{-5} \end{aligned}$$

where

$$s_1 \coloneqq s_1\left(x_1^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_1^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_1^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_1^{\text{cat}} = \text{C}, \end{cases} \quad \text{and} \quad s_2 \coloneqq s_2\left(x_2^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_2^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_2^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_2^{\text{cat}} = \text{C}. \end{cases}$$

with constraints

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = s_1(x_1^{\text{cat}})\sin\left(\frac{x_1^{\text{cont}}}{100}\right)^3 + s_2(x_2^{\text{cat}})\sin\left(\frac{x_2^{\text{cont}}}{10}\right)^3$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 100]$.

4.8 Cat-cstrs-8: modified Himmelblau [13]

$$f\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \left(\left(x_1^{\text{cont}}\right)^2 + x_2^{\text{cont}} - 6 - x_1^{\text{int}}\right)^2 + \left(x_1^{\text{cont}} + \left(x_2^{\text{cont}}\right)^2 - 2 - x_2^{\text{int}}\right)^2 + 10$$

with constraints

$$g_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = -s_1 x_1^{\text{cont}} - x_2^{\text{cont}} + 0.5$$
$$g_2\left(x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 3x_1^{\text{cont}} - s_2 x_2^{\text{cont}} + 1$$

where

$$s_1 \coloneqq s_1\left(x_1^{\text{cat}}\right) = \begin{cases} 0.75 & \text{if } x_1^{\text{cat}} = \text{A} \\ 1.5 & \text{if } x_1^{\text{cat}} = \text{B} \\ 2.25 & \text{if } x_1^{\text{cat}} = \text{C} \\ 3 & \text{if } x_1^{\text{cat}} = \text{D} \\ 3.75 & \text{if } x_1^{\text{cat}} = \text{E} \end{cases} \quad \text{and} \quad s_2 \coloneqq s_2\left(x_2^{\text{cat}}\right) = \begin{cases} -1.25 & \text{if } x_2^{\text{cat}} = \text{A} \\ -0.5 & \text{if } x_2^{\text{cat}} = \text{B} \\ 0.25 & \text{if } x_2^{\text{cat}} = \text{C} \\ 1 & \text{if } x_2^{\text{cat}} = \text{D} \\ 1.75 & \text{if } x_2^{\text{cat}} = \text{E} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{0, 1, \ldots, 5\}$
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [-5, 5]$.

4.9 Cat-cstrs-9: modified HS144 [14]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}\right) =$$
$$- 5.04x_1^{\text{cont}} - 0.035x_2^{\text{cont}} - 10x_3^{\text{cont}} - 3.36x_5^{\text{cont}} + 0.063x_2^{\text{int}}x_4^{\text{cont}} + s_1 + s_2$$

where

$$s_1 \coloneqq s_1\left(x_1^{\text{cat}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) =$$
$$\begin{cases} -500\left(1.12x_1^{\text{cont}} + 0.13167x_3^{\text{int}}x_1^{\text{cont}} - 0.00667\left(x_3^{\text{int}}\right)^2 x_1^{\text{cont}} - \dfrac{1}{0.99}x_4^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A} \\ \ \ 500\left(1.12x_1^{\text{cont}} + 0.13167x_3^{\text{int}}x_1^{\text{cont}} - 0.00667\left(x_3^{\text{int}}\right)^2 x_1^{\text{cont}} - \dfrac{1}{0.99}x_4^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B} \end{cases}$$

and

$$s_2 \coloneqq s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}\right) =$$
$$\begin{cases} -500\left(1.098x_3^{\text{int}} - 0.038\left(x_3^{\text{int}}\right)^2 + 0.325x_1^{\text{int}} - \dfrac{1}{0.99}x_2^{\text{int}} + 57.425\right) & \text{if } x_2^{\text{cat}} = \text{A} \\ \ \ 500\left(1.098x_3^{\text{int}} - 0.038\left(x_3^{\text{int}}\right)^2 + 0.325x_1^{\text{int}} - \dfrac{1}{0.99}x_2^{\text{int}} + 57.425\right) & \text{if } x_2^{\text{cat}} = \text{B} \end{cases}$$

with constraints

$$g_1\left(x_2^{\text{cont}}, x_5^{\text{cont}}\right) = 0.02x_2^{\text{cont}} + 0.1x_5^{\text{cont}} - 100$$
$$g_2\left(x_2^{\text{cont}}, x_5^{\text{cont}}\right) = -0.1x_2^{\text{cont}} - 0.5x_5^{\text{cont}} + 100$$
$$g_3\left(x_1^{\text{cont}}, x_4^{\text{cont}}\right) = -0.5x_4^{\text{cont}} + 0.1x_1^{\text{cont}} + 500$$
$$g_4\left(x_1^{\text{cont}}, x_4^{\text{cont}}\right) = 0.2x_4^{\text{cont}} - 0.2x_1^{\text{cont}} - 500$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$;
- $n^{\text{int}} = 3$ and $x_1^{\text{int}} \in \{83, 84, \ldots, 93\}$, and $x_2^{\text{int}} \in \{90, 91, \ldots, 95\}$, and $x_3^{\text{int}} \in \{3, 4, \ldots, 12\}$.
- $n^{\text{cont}} = 5$ and $x_1^{\text{cont}}, x_5^{\text{cont}} \in [10^{-5}, 2000]$. and $x_2^{\text{cont}} \in [10^{-5}, 16000]$, and $x_3^{\text{cont}} \in [10^{-5}, 120]$ and $x_4^{\text{cont}} \in [10^{-5}, 5000]$.

4.10 Cat-cstrs-10: modified Pentagon [14]

$$f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} \sqrt{\left(x_1^{\text{cont}} - x_1^{\text{int}}\right)^2 + \left(x_2^{\text{cont}} - x_2^{\text{int}}\right)^2} & \text{if } x^{\text{cat}} = \text{A} \\ \sqrt{\left(x_1^{\text{int}} - x_3^{\text{cont}}\right)^2 + \left(x_2^{\text{int}} - x_4^{\text{cont}}\right)^2} & \text{if } x^{\text{cat}} = \text{B} \\ \sqrt{\left(x_3^{\text{cont}} - x_1^{\text{cont}}\right)^2 + \left(x_4^{\text{cont}} - x_2^{\text{cont}}\right)^2} & \text{if } x^{\text{cat}} = \text{C} \end{cases}$$

with constraints

$$\begin{aligned} g_1\left(x_1^{\text{int}}, x_2^{\text{int}}\right) &= x_1^{\text{int}} \cos\left(\frac{2\pi}{5}\right) + x_2^{\text{int}} \sin\left(\frac{2\pi}{5}\right) - 1 \\ g_2\left(x_1^{\text{int}}, x_2^{\text{int}}\right) &= x_1^{\text{int}} \cos\left(\frac{8\pi}{5}\right) + x_2^{\text{int}} \sin\left(\frac{8\pi}{5}\right) - 1 \\ g_3\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= x_1^{\text{cont}} \cos\left(\frac{2\pi}{5}\right) + x_2^{\text{cont}} \sin\left(\frac{2\pi}{5}\right) - 1 \\ g_4\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= x_1^{\text{cont}} \cos\left(\frac{8\pi}{5}\right) + x_2^{\text{cont}} \sin\left(\frac{8\pi}{5}\right) - 1 \\ g_5\left(x_3^{\text{cont}}, x_4^{\text{cont}}\right) &= x_3^{\text{cont}} \cos\left(\frac{2\pi}{5}\right) + x_4^{\text{cont}} \sin\left(\frac{2\pi}{5}\right) - 1 \\ g_6\left(x_3^{\text{cont}}, x_4^{\text{cont}}\right) &= x_3^{\text{cont}} \cos\left(\frac{8\pi}{5}\right) + x_4^{\text{cont}} \sin\left(\frac{8\pi}{5}\right) - 1 \end{aligned}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-3, -2, \ldots, 3\}$
- $n^{\text{cont}} = 4$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}} \in [-\pi, \pi]$.

4.11 Cat-cstrs-11: modified Pressure-vessel [6]

$$f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 0.6224\, x_1^{\text{int}} x_1^{\text{cont}}\, s + 1.7781\, x_2^{\text{int}}\, s^2 + 3.1661 \left(x_1^{\text{int}}\right)^2 x_1^{\text{cont}} + 19.84 \left(x_1^{\text{int}}\right)^2 s.$$

where

$$s := s\left(x_1^{\text{cat}}, x_2^{\text{cont}}\right) = \begin{cases} 20 + 0.8 x_2^{\text{cont}} + 15 \sin\left(0.02 x_2^{\text{cont}}\right) & \text{if } x^{\text{cat}} = \text{A} \\ 50 + 0.85 x_2^{\text{cont}} + 18 \sin\left(0.025 x_2^{\text{cont}} + 0.3\right) & \text{if } x^{\text{cat}} = \text{B} \\ 10 + 0.75 x_2^{\text{cont}} + 12 \sin\left(0.018 x_2^{\text{cont}} - 0.4\right) & \text{if } x^{\text{cat}} = \text{C} \\ 40 + 0.82 x_2^{\text{cont}} + 20 \sin\left(0.022 x_2^{\text{cont}} + 0.5\right) & \text{if } x^{\text{cat}} = \text{D} \\ 200 - 0.8 x_2^{\text{cont}} - 15 \sin\left(0.02 x_2^{\text{cont}}\right) & \text{if } x^{\text{cat}} = \text{E} \\ 160 - 0.85 x_2^{\text{cont}} - 18 \sin\left(0.025 x_2^{\text{cont}} + 0.3\right) & \text{if } x^{\text{cat}} = \text{F} \\ 220 - 0.75 x_2^{\text{cont}} - 12 \sin\left(0.018 x_2^{\text{cont}} - 0.4\right) & \text{if } x^{\text{cat}} = \text{G} \\ 180 - 0.82 x_2^{\text{cont}} - 20 \sin\left(0.022 x_2^{\text{cont}} + 0.5\right) & \text{if } x^{\text{cat}} = \text{H} \end{cases}$$

with constraints

$$\begin{aligned} g_1\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{cont}}\right) &= -x_1^{\text{int}} + 0.0193\, s \\ g_2\left(x^{\text{cat}}, x_2^{\text{int}}, x_2^{\text{cont}}\right) &= -x_2^{\text{int}} + 0.00954\, s \\ g_3\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= -\pi x_1^{\text{cont}}\, s^2 - \frac{4\pi}{3}\, s^3 + 1296000 \end{aligned}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} = 0.0625 a_1$, $x_2^{\text{int}} = 0.0625 a_2$ with $a_1, a_2 \in \{1, 2, \ldots, 99\}$
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [10, 200]$.

Table 10: Images of $s := s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$.

	$x_1^{\text{cat}} = \text{A}$	$x_1^{\text{cat}} = \text{B}$	$x_1^{\text{cat}} = \text{C}$	$x_1^{\text{cat}} = \text{D}$	$x_1^{\text{cat}} = \text{E}$
$x_2^{\text{cat}} = \text{A}$	0.2	0.52	0.83	1.13	1.45
$x_2^{\text{cat}} = \text{B}$	0.27	0.58	0.87	1.19	1.49
$x_2^{\text{cat}} = \text{C}$	0.33	0.63	0.91	1.24	1.54
$x_2^{\text{cat}} = \text{D}$	0.38	0.68	0.96	1.3	1.62
$x_2^{\text{cat}} = \text{E}$	0.42	0.73	1.01	1.35	1.66

4.12 Cat-cstrs-12: modified Reinforced-concrete-beam [6]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 29.4s + 0.6x^{\text{int}}\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right)$$

where the images of s are provided by Table 10, with (modified) constraints

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 5\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 4x^{\text{int}} + 2s - 2.5$$
$$g_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 2.5s^2 + 25x^{\text{int}} - s\,x^{\text{int}}\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 2.5$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{28, 29, \ldots, 40\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [5, 10]$.

4.13 Cat-cstrs-13: modified Rosenbrock [13]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \left|x_1^{\text{int}}\right| + s + \begin{cases} \sum_{i=1}^{n^{\text{cont}}-1} 100\left(x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right)^2 + x_2^{\text{int}}(x_i^{\text{cont}} - 1)^2 & \text{if } x_1^{\text{cat}} = \text{A} \\ \sum_{i=1}^{n^{\text{cont}}-1} 100\left|x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right| + 5x_2^{\text{int}}\left|x_i^{\text{cont}} - 1\right| & \text{if } x_1^{\text{cat}} = \text{B} \end{cases}$$

where

$$s := s(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}) = \begin{cases} \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} 1.1 \max\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{A} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} -0.9 \min\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} \left|x_i^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

with constraint

$$g_1\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = -\sqrt{(x_1^{\text{cont}})^2 + (x_2^{\text{cont}})^2 + (x_3^{\text{cont}})^2 + (x_4^{\text{cont}})^2} + \left(\frac{x_1^{\text{int}}}{2}\right)^2 + \begin{cases} 4.25^2 & \text{if } x_2^{\text{cat}} = \text{A} \\ 5.5^2 & \text{if } x_2^{\text{cat}} = \text{B} \\ 8^2 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}} \in \{\text{A}, \text{B}\}$, $x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{-2, -1, 0, 1, 2\}$, $x_2^{\text{int}} \in \{-5, -4, \ldots, 4, 5\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-10, 10]$ for $i \in I^{\text{cont}}$.

4.14 Cat-cstrs-14: modified Styblinski-Tang [13]

$$f\left(x^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \frac{1}{2}\sum_{i=1}^{n^{\text{cont}}}\left(\left(x_i^{\text{cont}}\right)^4 - 16\left(x_i^{\text{cont}}\right)^2 + 5x_i^{\text{int}} + 8s\right)$$

where $n^{\text{cont}} = n^{\text{int}}$ and

$$s := s\left(x^{\text{cat}}, x_i^{\text{cont}}\right) = \begin{cases} \left|x_i^{\text{cont}} - 1\right| & \text{if } x^{\text{cat}} = \text{A} \\ \frac{\left(\left(x_i^{\text{cont}}\right)^2 + 1\right)}{2} & \text{if } x^{\text{cat}} = \text{B} \\ \exp\left(\left|x_i^{\text{cont}} + 1\right|\right) - 1 & \text{if } x^{\text{cat}} = \text{C} \\ \frac{\left(x_i^{\text{cont}}\right)^2}{\left(1 + \left|x_i^{\text{cont}}\right|\right)} & \text{if } x^{\text{cat}} = \text{D} \\ 1 - \exp\left(-\left(x_i^{\text{cont}}\right)^2\right) & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

with constraints

$$g_1\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} \exp\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 10 & \text{if } x^{\text{cat}} = \text{A} \\ \exp\left(x_1^{\text{cont}} + 2x_2^{\text{cont}}\right) - 18 & \text{if } x^{\text{cat}} = \text{B} \\ \exp\left(2x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 18 & \text{if } x^{\text{cat}} = \text{C} \\ \exp\left(x_1^{\text{cont}} - x_2^{\text{cont}}\right) + \log\left(1 + \left|x_2^{\text{cont}}\right|\right) - 12 & \text{if } x^{\text{cat}} = \text{D} \\ \exp\left(x_1^{\text{cont}} - 0.5x_2^{\text{cont}}\right) + \log\left(1 + \left|x_2^{\text{cont}}\right|\right) - 11 & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

$$g_2\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} \left(x_1^{\text{cont}} + 2\right)^3 + \left(x_2^{\text{cont}} - 1\right)^2 + 0.1\left(x_1^{\text{int}} - 1\right)^2 - 50 & \text{if } x^{\text{cat}} = \text{A} \\ \left(x_1^{\text{cont}} - 1\right)^3 + \left(x_2^{\text{cont}} + 2\right)^2 + 0.1\left(x_1^{\text{int}} + 2\right)^2 - 40 & \text{if } x^{\text{cat}} = \text{B} \\ \left(x_1^{\text{cont}}\right)^3 + \left(x_2^{\text{cont}} - 2\right)^2 + 0.1\left(x_1^{\text{int}} - 3\right)^2 - 45 & \text{if } x^{\text{cat}} = \text{C} \\ \sin\left(\frac{x_1^{\text{cont}} + x_2^{\text{cont}}}{10}\right) + \left(x_1^{\text{cont}}\right)^2 + 0.2\left|x_1^{\text{int}} - 1\right| - 3 & \text{if } x^{\text{cat}} = \text{D} \\ \sin\left(\frac{x_1^{\text{cont}} + x_2^{\text{cont}}}{12.5}\right) + \left(x_1^{\text{cont}}\right)^2 + 0.2\left|x_1^{\text{int}} - 2\right| - 4 & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$
- $n^{\text{int}} \in \mathbb{N}$ and $x_i^{\text{int}} \in \{-5, 10\}$ for $i \in I^{\text{int}}$.
- $n^{\text{cont}} \in \mathbb{N}$ and $x_j^{\text{cont}} \in [-5, 10]$ for $j \in I^{\text{cont}}$.

4.15 Cat-cstrs-15: modified Toy [16]

$$f\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = 5 +$$
$$\begin{cases} \cos\left(3.6\pi\left(x_1^{\text{cont}} - 2\right) + x_2^{\text{cont}}\right) + x_3^{\text{cont}} - 1 + \left(x_4^{\text{cont}}\right)^2 & \text{if } x^{\text{cat}} = \text{A}, \\ 2\cos\left(1.1\pi\exp\left(x_1^{\text{cont}}\right)\right) - \frac{x_2^{\text{cont}}}{2} + \left(x_3^{\text{cont}}\right)^2 + 2\log\left(1 + \left(x_4^{\text{cont}}\right)^2\right) & \text{if } x^{\text{cat}} = \text{B}, \\ \cos\left(2\pi x_1^{\text{cont}}\right) + \frac{x_2^{\text{cont}}}{2} + x_3^{\text{cont}} x_4^{\text{cont}} & \text{if } x^{\text{cat}} = \text{C}, \\ x_1\cos\left(3.4\pi\left(x_1^{\text{cont}} - 1\right)\right) - x_2^{\text{cont}} - 1 + x_3^{\text{cont}} + \left(x_4^{\text{cont}}\right)^3 & \text{if } x^{\text{cat}} = \text{D}, \\ -\frac{\left(x_1^{\text{cont}}\right)^2}{2} + \log\left(1 + \left(x_2^{\text{cont}}\right)^2\right) + \left(x_3^{\text{cont}}\right)^2 + x_4^{\text{cont}} & \text{if } x^{\text{cat}} = \text{E}, \\ 2\cos\left(\frac{\pi}{4}\exp\left(-\left(x_1^{\text{cont}}\right)^4\right)\right)^2 - \frac{x_2^{\text{cont}}}{2} + x_3^{\text{cont}} x_4^{\text{cont}} + 1 & \text{if } x^{\text{cat}} = \text{F}, \\ x_1\cos\left(3.4x_1^{\text{cont}}\right) - \frac{x_2^{\text{cont}}}{2} + x_3^{\text{cont}} + \left(x_4^{\text{cont}}\right)^3 + 1 & \text{if } x^{\text{cat}} = \text{G}, \\ x_1^{\text{cont}}\left(-\cos\left(\frac{7}{2\pi}\right)\frac{x_2^{\text{cont}}}{2}\right) + x_3^{\text{cont}} + x_4^{\text{cont}} + 2 & \text{if } x^{\text{cat}} = \text{H}, \\ -\frac{\left(x_1^{\text{cont}}\right)^3}{2} + \left(x_2^{\text{cont}}\right)^2 + x_3^{\text{cont}} x_4^{\text{cont}} + 1 & \text{if } x^{\text{cat}} = \text{I}, \\ -\cos\left(5\pi x_1^{\text{cont}}\right)^2\sqrt{x_1^{\text{cont}}} - \frac{-\log\left(x_2^{\text{cont}} + x_3^{\text{cont}} + 0.5\right)}{2} + \left(x_4^{\text{cont}}\right)^3 - 1.3 & \text{if } x^{\text{cat}} = \text{J}. \end{cases}$$

with constraints

$$g_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = \sqrt{\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + \left(x_4^{\text{cont}}\right)^2} - 0.25^2$$
$$g_2\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = -\sqrt{\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + \left(x_4^{\text{cont}}\right)^2} + 0.1^2$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 4$ and $x_i^{\text{cont}} \in [0, 1]$ for $i \in I^{\text{cont}}$.

4.16 Cat-cstrs-16: modified Wong-2 [14]

$$f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\right) =$$
$$-s + \begin{cases} 0 & \text{if } x^{\text{cat}} = \text{A} \\ 10\left(3\left(x_1^{\text{cont}} - 2\right)^2 + 4\left(x_1^{\text{int}} - 3\right)^2 + 2\left(x_2^{\text{cont}}\right)^2 - 7\left(x_2^{\text{int}}\right)^2 - 120\right) & \text{if } x^{\text{cat}} = \text{B} \\ 10\left(5\left(x_1^{\text{cont}}\right)^2 + 8x_1^{\text{int}} + 6\left(x_2^{\text{cont}} - 6\right)^2 - 2x_2^{\text{int}} - 40\right) & \text{if } x^{\text{cat}} = \text{C} \\ 10\left(0.5\left(x_1^{\text{cont}} - 8\right)^2 + 2\left(x_1^{\text{int}} - 4\right)^2 + 3\left(x_3^{\text{cont}}\right)^2 - x_3^{\text{int}} - 30\right) & \text{if } x^{\text{cat}} = \text{D} \\ 10\left(\left(x_1^{\text{cont}}\right)^2 + 2\left(x_1^{\text{int}} - 2\right)^2 - 2x_1^{\text{cont}} x_1^{\text{int}} + 14x_3^{\text{cont}} - 6x_3^{\text{int}}\right) & \text{if } x^{\text{cat}} = \text{E} \\ 10\left(-3x_1^{\text{cont}} + 6x_1^{\text{int}} + 12\left(x_5^{\text{cont}} - 8\right)^2 - 7x_6^{\text{cont}}\right) & \text{if } x^{\text{cat}} = \text{F} \end{cases}$$

where

$$s := s(x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}) =$$
$$\left(x_1^{\text{cont}}\right)^2 + \left(x_1^{\text{int}}\right)^2 + x_1^{\text{cont}} x_1^{\text{int}} - 14x_1^{\text{cont}} - 16x_1^{\text{int}} + \left(x_2^{\text{cont}} - 10\right)^2 + 4\left(x_2^{\text{int}} - 5\right)^2 + \left(x_3^{\text{cont}} - 3\right)^2$$
$$+ 2\left(x_3^{\text{int}} - 1\right)^2 + 5\left(x_4^{\text{cont}}\right) + 7\left(x_4^{\text{int}} - 11\right)^2 + 2\left(x_5^{\text{cont}} - 10\right)^2 + \left(x_6^{\text{cont}} - 7\right)^2 + 45$$

with constraints

$$g_1\left(x_1^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} 4x_1^{\text{cont}} + 5x_1^{\text{int}} - 3x_4^{\text{cont}} + 9x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \text{A} \\ 3x_1^{\text{cont}} + 6x_1^{\text{int}} - 3x_4^{\text{cont}} + 9x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \text{B} \\ 4x_1^{\text{cont}} + 4x_1^{\text{int}} - 2x_4^{\text{cont}} + 9x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \text{C} \\ 4x_1^{\text{cont}} + 5x_1^{\text{int}} - 4x_4^{\text{cont}} + 10x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \text{D} \\ 5x_1^{\text{cont}} + 5x_1^{\text{int}} - 3x_4^{\text{cont}} + 8x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \text{E} \\ 3x_1^{\text{cont}} + 4x_1^{\text{int}} - 1x_4^{\text{cont}} + 8x_4^{\text{int}} - 95 \le 0 & \text{if } x_1^{\text{cat}} = \text{F} \end{cases}$$

$$g_2\left(x_1^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} 10x_1^{\text{cont}} - 8x_1^{\text{int}} - 17x_4^{\text{cont}} + 2x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{A} \\ 8x_1^{\text{cont}} - 6x_1^{\text{int}} - 17x_4^{\text{cont}} + 2x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{B} \\ 10x_1^{\text{cont}} - 10x_1^{\text{int}} - 15x_4^{\text{cont}} + 2x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{C} \\ 10x_1^{\text{cont}} - 8x_1^{\text{int}} - 19x_4^{\text{cont}} + 4x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{D} \\ 12x_1^{\text{cont}} - 8x_1^{\text{int}} - 19x_4^{\text{cont}} + 1x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{E} \\ 9x_1^{\text{cont}} - 9x_1^{\text{int}} - 18x_4^{\text{cont}} + 1x_4^{\text{int}} + 10 \le 0 & \text{if } x_1^{\text{cat}} = \text{F} \end{cases}$$

$$g_3\left(x_1^{\text{int}}, x_1^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\right) = \begin{cases} -8x_1^{\text{cont}} + 2x_1^{\text{int}} + 5x_5^{\text{cont}} - 2x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{A} \\ -4x_1^{\text{cont}} + 4x_1^{\text{int}} + 5x_5^{\text{cont}} - 2x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{B} \\ -8x_1^{\text{cont}} + 1x_1^{\text{int}} + 10x_5^{\text{cont}} - 2x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{C} \\ -8x_1^{\text{cont}} + 2x_1^{\text{int}} + 2.5x_5^{\text{cont}} - 4x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{D} \\ -16x_1^{\text{cont}} + 2x_1^{\text{int}} + 5x_5^{\text{cont}} - 1x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \text{E} \\ -4x_1^{\text{cont}} + 1x_1^{\text{int}} + 10x_5^{\text{cont}} - 4x_6^{\text{int}} + 10 \le 0 & \text{if } x_1^{\text{cat}} = \text{F} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}\}$;
- $n^{\text{int}} = 4$ and $x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}} \in \{0, 1, \ldots, 10\}$
- $n^{\text{cont}} = 6$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}} \in [0, 10]$.

4.17 Cat-cstrs-17: modified Speed Reducer

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\right) = 0.7854\, x_1^{\text{cont}} \left(x_2^{\text{cont}}\right)^2 \left(3.3333\, s^2 + 14.9334\, s - 43.0934\right)$$
$$- 1.508\, x_1^{\text{cont}} \left(\left(x_5^{\text{cont}}\right)^2 + \left(x_6^{\text{cont}}\right)^2\right) + 7.477 \left(\left(x_5^{\text{cont}}\right)^3 + \left(x_6^{\text{cont}}\right)^3\right) + 0.7854 \left(x_3^{\text{cont}} \left(x_5^{\text{cont}}\right)^2 + x_4^{\text{cont}} \left(x_6^{\text{cont}}\right)^2\right),$$

where $s \coloneqq s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$ and images defined by Table 11

Table 11: Values of $s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}							
		A	B	C	D	E	F	G	H
x_2^{cat}	A	23	19	27	21	18	24	20	26
	B	20	25	18	24	22	19	27	21
	C	26	22	24	19	27	21	18	23
	D	19	27	21	23	20	26	22	18
	E	24	18	23	20	26	22	19	27
	F	21	23	20	26	22	18	24	19
	G	17	24	22	19	27	21	23	20
	H	22	19	26	22	18	23	20	25

with constraints

$$g_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\right) = 27 - x_1^{\text{cont}} \left(x_2^{\text{cont}}\right)^2 s \leq 0,$$
$$g_2\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\right) = 397.5 - x_1^{\text{cont}} \left(x_2^{\text{cont}}\right)^2 s^2 \leq 0,$$
$$g_3\left(x_2^{\text{cont}}, x_3^{\text{cont}}, x_5^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\right) = 1.93 \left(x_3^{\text{cont}}\right)^3 - x_2^{\text{cont}} s \left(x_5^{\text{cont}}\right)^4 \leq 0,$$
$$g_4\left(x_2^{\text{cont}}, x_4^{\text{cont}}, x_6^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\right) = 1.93 \left(x_4^{\text{cont}}\right)^3 - x_2^{\text{cont}} s \left(x_6^{\text{cont}}\right)^4 \leq 0,$$
$$g_5\left(x_2^{\text{cont}}, x_3^{\text{cont}}, x_5^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\right) = \sqrt{(745.0\, x_3^{\text{cont}})^2 + 16.9 \times 10^6 \left(x_2^{\text{cont}}\right)^2 s^2} - 110\, x_2^{\text{cont}} s \left(x_5^{\text{cont}}\right)^3 \leq 0,$$
$$g_6\left(x_2^{\text{cont}}, x_4^{\text{cont}}, x_6^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\right) = \sqrt{(745.0\, x_4^{\text{cont}})^2 + 157.5 \times 10^6 \left(x_2^{\text{cont}}\right)^2 s^2} - 85\, x_2^{\text{cont}} s \left(x_6^{\text{cont}}\right)^3 \leq 0,$$
$$g_7\left(x_2^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\right) = x_2^{\text{cont}} s - 40 \leq 0,$$
$$g_8\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 5 x_2^{\text{cont}} - x_1^{\text{cont}} \leq 0,$$
$$g_9\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) = x_1^{\text{cont}} - 12 x_2^{\text{cont}} \leq 0,$$
$$g_{10}\left(x_3^{\text{cont}}, x_5^{\text{cont}}\right) = 1.9 + 1.5 x_5^{\text{cont}} - x_3^{\text{cont}} \leq 0,$$
$$g_{11}\left(x_4^{\text{cont}}, x_6^{\text{cont}}\right) = 1.9 + 1.1 x_6^{\text{cont}} - x_4^{\text{cont}} \leq 0.$$

- $n^{\text{cat}} = 2$ and $x_i^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}\}$ for $i \in \{1, 2\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 6$ and $x_1^{\text{cont}} \in [2.6, 3.6]$, $x_2^{\text{cont}} \in [0.7, 0.8]$, $x_3^{\text{cont}} \in [7.3, 20.0]$, $x_4^{\text{cont}} \in [0.5, 8.3]$, $x_5^{\text{cont}} \in [2.6, 3.9]$ and $x_6^{\text{cont}} \in [5.0, 5.5]$.

4.18 Cat-cstrs-18: modified Spring [6]

$$f\left(x_1^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = (s+2)\, x_1^{\text{cont}} \left(x_2^{\text{cont}}\right)^2 + 0.05\, x_1^{\text{int}} \left(x_1^{\text{cont}}\right)^2 + 0.03 \left(x_2^{\text{int}}\right)^2 x_2^{\text{cont}} + 0.01\, x_1^{\text{int}} x_2^{\text{int}}\, s,$$

where

$$s \coloneqq s\left(x_1^{\text{cat}}\right) = \begin{cases} 10 + 0.3\, k + 0.7 \sin\left(\dfrac{\pi k}{6}\right), & \text{if the number } k \in \mathbb{N} \text{ } in \text{ } x_1^{\text{cat}} = \text{“}k\text{”} \text{ is even,} \\ 25 \exp(-0.05\, k) + 1.5, & \text{if the number } k \in \mathbb{N} \text{ } in \text{ } x_1^{\text{cat}} = \text{“}k\text{”} \text{ is odd.} \end{cases}$$

with constraints

$$g_1\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, s\Big) = 71785\left(x_2^{\text{cont}}\right)^4 - \left(x_1^{\text{cont}}\right)^3 s + 500\, x_1^{\text{int}} x_2^{\text{int}} \leq 0,$$
$$g_2\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\Big) = 5108\left(x_2^{\text{cont}}\right)^2\Big(4\left(x_1^{\text{cont}}\right)^2 - x_1^{\text{cont}} x_2^{\text{cont}}\Big) + 12566\Big(x_1^{\text{cont}}\left(x_2^{\text{cont}}\right)^3 - \left(x_1^{\text{cont}}\right)^4\Big)$$
$$- 64187128\left(x_2^{\text{cont}}\right)^5\left(x_1^{\text{cont}} - x_2^{\text{cont}}\right) + 2000\Big(\Big(x_1^{\text{int}}\Big)^2 - x_2^{\text{int}}\Big) \leq 0,$$
$$g_3\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, s\Big) = \left(x_1^{\text{cont}}\right)^2 s - 140.45\, x_2^{\text{cont}} + 10\, x_1^{\text{int}} \leq 0,$$
$$g_4\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = x_1^{\text{cont}} + x_2^{\text{cont}} - 1.5 \leq 0,$$
$$g_5\Big(x_1^{\text{int}}, x_2^{\text{int}}\Big) = x_1^{\text{int}} + 2x_2^{\text{int}} - 15 \leq 0.$$

- $n^{\text{cat}} = 1$ and $x_1^{\text{cat}} \in \{\text{“1”}, \text{“2”}, \ldots, \text{“50”}\}$;
- $n^{\text{int}} = 2$ and $x_i^{\text{int}} \in \{-2, -1, \ldots, 2\}$ for $i \in \{1, 2\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}} \in [0.25, 1.3]$ and $x_2^{\text{cont}} \in [0.05, 2.0]$.

4.19 Cat-cstrs-19: mixed G07 [6]

$$f\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{cat}}, x_4^{\text{cat}}, x_5^{\text{cat}}, x_6^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\Big) = s_1^2 + s_2^2 + s_1 s_2 - 14 s_1 - 16 s_2 + (s_3 - 10)^2$$
$$+ 4\,(s_4 - 5)^2 + (s_5 - 3)^2 + 2\,(s_6 - 1)^2 + 5\left(x_1^{\text{cont}}\right)^2 + 7\left(x_2^{\text{cont}} - 11\right)^2$$
$$+ 2\left(x_1^{\text{int}} - 10\right)^2 + \left(x_2^{\text{int}} - 7\right)^2 + 45,$$

where, for $i \in \{1, \ldots, 6\}$,

$$s_i \coloneqq s_i\big(x_i^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \begin{cases} a_i + b_i \log\Big(1 + \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2\Big), & \text{if } x_i^{\text{cat}} = \text{A}, \\ c_i + d_i \exp\Big(0.02\left(x_1^{\text{cont}} - x_2^{\text{cont}}\right)\Big), & \text{if } x_i^{\text{cat}} = \text{B}. \end{cases}$$

Table 12: Coefficients for $s_i\big(x_i^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big)$, $i \in \{1, \ldots, 6\}$.

i	a_i	b_i	c_i	d_i
1	2.00	1.10	1.40	1.80
2	3.10	0.95	2.60	1.55
3	9.80	0.85	9.10	1.25
4	5.00	0.60	4.60	1.05
5	3.20	0.90	2.05	1.08
6	1.10	0.55	0.90	0.85

with constraints

$$g_1\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = -105 + 4s_1 + 5s_2 - 3x_1^{\text{cont}} + 9x_2^{\text{cont}} \leq 0,$$
$$g_2\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = 10s_1 - 8s_2 - 17x_1^{\text{cont}} + 2x_2^{\text{cont}} \leq 0,$$
$$g_3\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = -8s_1 + 2s_2 + 5x_1^{\text{cont}} - 2\left(x_2^{\text{cont}}\right)^2 - 12 \leq 0,$$
$$g_4\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = 3\,(s_1 - 2)^2 + 4\,(s_2 - 3)^2 + 2s_3^2 - 7s_4 - 220 \leq 0,$$
$$g_5\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = 5s_1^2 + 8s_2 + (s_3 - 6)^2 - 2s_4 - 95 \leq 0,$$
$$g_6\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = s_1^2 + 2\,(s_2 - 2)^2 - 2s_1 s_2 + 14s_5 - 6s_6 - 20 \leq 0,$$
$$g_7\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = 0.5\,(s_1 - 8)^2 + 2\,(s_2 - 4)^2 + 3s_5^2 - s_6 - 40 \leq 0,$$
$$g_8\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) = -3s_1 + 6s_2 + 3\left(x_1^{\text{cont}} - 8\right)^2 - 7\left(x_2^{\text{cont}}\right)^2 \leq 0.$$

- $n^{\text{cat}} = 6$ and $x_i^{\text{cat}} \in \{\text{A}, \text{B}\}$ for $i \in \{1, \ldots, 6\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-10, \ldots, 10\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}} \in [-10, 10]$ and $x_2^{\text{cont}} \in [-10, 10]$.

4.20 Cat-cstrs-20: modified Car-side-impact [6]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = \\ 1.98 + 4.90\, x_1^{\text{cont}} + 6.67\, x_2^{\text{cont}} + 6.98\, x_3^{\text{cont}} + 4.01\, x_4^{\text{cont}} + 1.78\, x_5^{\text{cont}} + 2.73\, x_2^{\text{int}} + 0.60\, s \\ + 0.20\, |s - 1.20| + 0.12\, \left|x_4^{\text{cont}} - 1.00\right| + 0.06\, \left|x_1^{\text{int}} + 2\right| + 0.25\, \left(x_1^{\text{cont}} - 1.0\right)^2 \left|x_2^{\text{cont}} - 0.9\right|,$$

where the images are given by Table 13.

Table 13: Values of $s := s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}									
		A	B	C	D	E	F	G	H	I	J
x_2^{cat}	A	0.45	0.65	0.78	0.87	0.93	0.92	0.89	0.81	0.63	0.44
	B	0.46	0.66	0.81	0.90	0.96	0.97	0.89	0.80	0.63	0.43
	C	0.46	0.69	0.83	0.96	1.02	1.01	0.92	0.85	0.69	0.49
	D	0.52	0.75	0.90	1.02	1.10	1.07	1.03	0.93	0.75	0.54
	E	0.63	0.84	0.99	1.12	1.19	1.15	1.13	1.01	0.83	0.66
	F	0.75	0.93	1.12	1.23	1.28	1.29	1.23	1.12	0.97	0.72
	G	0.87	1.07	1.25	1.34	1.40	1.41	1.37	1.26	1.06	0.86
	H	1.04	1.21	1.39	1.50	1.57	1.56	1.49	1.39	1.23	1.05
	I	1.20	1.41	1.57	1.67	1.72	1.70	1.67	1.56	1.43	1.22
	J	1.42	1.62	1.76	1.87	1.91	1.92	1.84	1.77	1.59	1.39

with constraints

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_6^{\text{cont}}\right) = 1.16 - 0.3717\, x_2^{\text{cont}} x_4^{\text{cont}} - 0.00931\, x_2^{\text{cont}} x_6^{\text{cont}} - 0.484\, x_3^{\text{cont}}\, s \\ + 0.01343\, x_1^{\text{int}} x_6^{\text{cont}} - 1 \leq 0.$$

$$g_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}, , x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = 0.261 - 0.0159\, x_1^{\text{cont}} x_2^{\text{cont}} - 0.188\, x_1^{\text{cont}} s \\ - 0.019\, x_2^{\text{cont}} x_2^{\text{int}} + 0.0144\, x_3^{\text{cont}} x_5^{\text{cont}} + 0.0008757\, x_5^{\text{cont}} x_6^{\text{cont}} + 0.08045\, x_1^{\text{int}} s \\ + 0.00139\, s\, x_7^{\text{cont}} + 0.000001575\, x_6^{\text{cont}} x_7^{\text{cont}} - 0.32 \leq 0.$$

$$g_3\left(x_1^{\text{cat}}, x_2^{\text{cat}}, , x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = 0.214 + 0.00817\, x_5^{\text{cont}} - 0.131\, x_1^{\text{cont}} s \\ - 0.0704\, x_1^{\text{cont}} s + 0.03099\, x_2^{\text{cont}} x_1^{\text{int}} - 0.018\, x_2^{\text{cont}} x_2^{\text{int}} + 0.0208\, x_3^{\text{cont}} s + 0.121\, x_3^{\text{cont}} s \\ - 0.00364\, x_5^{\text{cont}} x_1^{\text{int}} + 0.0007715\, x_5^{\text{cont}} x_6^{\text{cont}} - 0.0005354\, x_1^{\text{int}} x_6^{\text{cont}} \\ + 0.00121\, s\, x_7^{\text{cont}} - 0.32 \leq 0.$$

$$g_4\left(x_1^{\text{cat}}, x_2^{\text{cat}}, , x_2^{\text{int}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_6^{\text{cont}}\right) = 0.74 - 0.061\, x_2^{\text{cont}} - 0.163\, x_3^{\text{cont}} s + 0.001232\, x_3^{\text{cont}} x_6^{\text{cont}} - 0.166\, x_2^{\text{int}} s \\ + 0.02\, \left(x_2^{\text{int}}\right)^2 - 0.32 \leq 0.$$

$$g_5\left(x_1^{\text{cat}}, x_2^{\text{cat}}, , x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\right) = 28.98 + 3.818\, x_3^{\text{cont}} - 4.2\, x_1^{\text{cont}} x_2^{\text{cont}} + 0.0207\, x_5^{\text{cont}} x_6^{\text{cont}} \\ + 6.63\, x_1^{\text{int}} s - 7.7\, x_2^{\text{int}} s + 0.32\, s\, x_6^{\text{cont}} - 32 \leq 0.$$

$$g_6\left(x_1^{\text{cat}}, x_2^{\text{cat}}, , x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\right) = 33.86 + 2.95\, x_3^{\text{cont}} + 0.1792\, x_6^{\text{cont}} - 5.057\, x_1^{\text{cont}} x_2^{\text{cont}} - 11.02\, x_2^{\text{cont}} s - 0.0215\, x_5^{\text{cont}} x_6^{\text{cont}} - 9.98\, x_2^{\text{int}} s + 22.0\, s^2 - 32 \leq 0.$$

$$g_7\left(x_1^{\text{cat}}, x_2^{\text{cat}}, , x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_6^{\text{cont}}\right) = 46.36 - 9.9\, x_2^{\text{cont}} - 12.9\, x_1^{\text{cont}} s + 0.1107\, x_3^{\text{cont}} x_6^{\text{cont}} - 32 \leq 0.$$

$$g_8\left(x_1^{\text{int}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = 0.62 - 0.5\, x_4^{\text{cont}} - 0.19\, x_2^{\text{cont}} x_3^{\text{cont}} - 0.0122\, x_4^{\text{cont}} x_6^{\text{cont}} + 0.009325\, x_1^{\text{int}} x_6^{\text{cont}} + 0.000191 \left(x_7^{\text{cont}}\right)^2 - 4 \leq 0.$$

$$g_9\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_6^{\text{cont}}\right) = 10.58 - 0.674\, x_1^{\text{cont}} x_2^{\text{cont}} - 1.95\, x_2^{\text{cont}} s + 0.02054\, x_3^{\text{cont}} x_6^{\text{cont}} - 0.0198\, x_4^{\text{cont}} x_6^{\text{cont}} + 0.028\, x_1^{\text{int}} x_6^{\text{cont}} - 9.9 \leq 0.$$

$$g_{10}\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = 16.45 - 0.489\, x_3^{\text{cont}} x_2^{\text{int}} - 0.843\, x_5^{\text{cont}} x_1^{\text{int}} + 0.432\, s\, x_6^{\text{cont}} - 0.0556\, s\, x_7^{\text{cont}} - 0.000786 \left(x_7^{\text{cont}}\right)^2 - 15.7 \leq 0.$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-10, -9, \ldots, 25\}$;
- $n^{\text{cont}} = 7$ and $x_1^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}} \in [0.5, 1.5]$, $x_2^{\text{cont}} \in [0.45, 1.35]$ and $x_5^{\text{cont}} \in [0.875, 2.625]$.

4.21 Cat-cstrs-21: modified Dembo 7 [14]

$$f\left(x^{\text{cat}}, x_1^{\text{cont}}, \ldots, x_{16}^{\text{cont}}\right) = s + a \times \left(x_{12}^{\text{cont}} + x_{13}^{\text{cont}} + x_{14}^{\text{cont}} + x_{15}^{\text{cont}} + x_{16}^{\text{cont}}\right) + b \times \left(x_1^{\text{cont}} x_{12}^{\text{cont}} + x_2^{\text{cont}} x_{13}^{\text{cont}} + x_3^{\text{cont}} x_{14}^{\text{cont}} + x_4^{\text{cont}} x_{15}^{\text{cont}} + x_5^{\text{cont}} x_{16}^{\text{cont}}\right),$$

where with $a = 1.262626$, $b = -1.231060$, $c = 0.034750$, $d = 0.009750$, and

$$s := s\left(x^{\text{cat}}, x_1^{\text{cont}}, \ldots, x_{16}^{\text{cont}}\right) =$$

$$10^3 \begin{cases} c\, x_1^{\text{cont}} \left(x_6^{\text{cont}}\right)^{-1} + 100d\, x_1^{\text{cont}} - d\, x_1^{\text{cont}} \left(x_6^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{A}, \\ c\, x_2^{\text{cont}} \left(x_7^{\text{cont}}\right)^{-1} + 100d\, x_2^{\text{cont}} - d\, x_2^{\text{cont}} \left(x_7^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{B}, \\ c\, x_3^{\text{cont}} \left(x_8^{\text{cont}}\right)^{-1} + 100d\, x_3^{\text{cont}} - d\, x_3^{\text{cont}} \left(x_8^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{C}, \\ c\, x_4^{\text{cont}} \left(x_9^{\text{cont}}\right)^{-1} + 100d\, x_4^{\text{cont}} - d\, x_4^{\text{cont}} \left(x_9^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{D}, \\ c\, x_5^{\text{cont}} \left(x_{10}^{\text{cont}}\right)^{-1} + 100d\, x_5^{\text{cont}} - d\, x_5^{\text{cont}} \left(x_{10}^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{E}, \\ c\, x_6^{\text{cont}} \left(x_7^{\text{cont}}\right)^{-1} + x_1^{\text{cont}} \left(x_5^{\text{cont}}\right)^{-1} x_{11}^{\text{cont}} x_{12}^{\text{cont}} - x_6^{\text{cont}} \left(x_5^{\text{cont}}\right)^{-1} x_1^{\text{cont}} x_2^{\text{cont}} - 1, & \text{if } x^{\text{cat}} = \text{F}, \\ x_7^{\text{cont}} \left(x_8^{\text{cont}}\right)^{-1} + 0.002\left(x_7^{\text{cont}} - x_2^{\text{cont}}\right) x_1^{\text{cont}} x_8^{\text{cont}} x_{12}^{\text{cont}} - 0.002\left(x_7^{\text{cont}} - x_2^{\text{cont}}\right) x_5^{\text{cont}} - 1, & \text{if } x^{\text{cat}} = \text{G}, \\ x_8^{\text{cont}} + 0.002\left(x_8^{\text{cont}} - x_2^{\text{cont}}\right) x_5^{\text{cont}} x_8^{\text{cont}} + 0.002\left(x_3^{\text{cont}} - x_9^{\text{cont}}\right) x_{14}^{\text{cont}} + x_9^{\text{cont}} - 1, & \text{if } x^{\text{cat}} = \text{H}, \\ \left(x_3^{\text{cont}}\right)^{-1} x_9^{\text{cont}} + \left(x_4^{\text{cont}} - x_8^{\text{cont}}\right) \left(x_3^{\text{cont}}\right)^{-1} \left(x_{14}^{\text{cont}}\right)^{-1} x_{15}^{\text{cont}} + 500\left(x_{10}^{\text{cont}} - x_9^{\text{cont}}\right) \left(x_3^{\text{cont}}\right)^{-1} \left(x_{14}^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{I}, \\ \left(\left(x_4^{\text{cont}}\right)^{-1} x_6^{\text{cont}} - 1\right) \left(x_{15}^{\text{cont}}\right)^{-1} x_{16}^{\text{cont}} + \left(x_4^{\text{cont}}\right)^{-1} x_{10}^{\text{cont}} + 500\left(1 - \left(x_4^{\text{cont}}\right)^{-1} x_{10}^{\text{cont}}\right) \left(x_{15}^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{J}, \\ 0.9 \left(x_4^{\text{cont}}\right)^{-1} + 0.002\left(1 - \left(x_4^{\text{cont}}\right)^{-1} x_5^{\text{cont}}\right) x_{16}^{\text{cont}} - 1, & \text{if } x^{\text{cat}} = \text{K}, \\ x_{11}^{\text{cont}} \left(x_{12}^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{L}, \\ x_4^{\text{cont}} \left(x_5^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{M}, \\ x_3^{\text{cont}} \left(x_4^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{N}, \\ x_2^{\text{cont}} \left(x_3^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{O}, \\ x_1^{\text{cont}} \left(x_2^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{P}, \\ x_9^{\text{cont}} \left(x_{10}^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{Q}, \\ x_8^{\text{cont}} \left(x_9^{\text{cont}}\right)^{-1} - 1, & \text{if } x^{\text{cat}} = \text{R}. \end{cases}$$

with constraints

$$
\begin{aligned}
g_1\big(x_{11}^{\text{cont}}, x_{12}^{\text{cont}}\big) &= 0.002\big(x_{11}^{\text{cont}} - x_{12}^{\text{cont}}\big) - 1 \leq 0,\\
g_2\big(x_4^{\text{cont}}, x_5^{\text{cont}}\big) &= x_4^{\text{cont}}\big(x_5^{\text{cont}}\big)^{-1} - 1.05 \leq 0,\\
g_3\big(x_3^{\text{cont}}, x_4^{\text{cont}}\big) &= x_3^{\text{cont}}\big(x_4^{\text{cont}}\big)^{-1} - 1.05 \leq 0,\\
g_4\big(x_8^{\text{cont}}, x_9^{\text{cont}}\big) &= x_8^{\text{cont}}\big(x_9^{\text{cont}}\big)^{-1} - 1.05 \leq 0,\\
g_5\big(x_6^{\text{cont}}, x_7^{\text{cont}}\big) &= x_6^{\text{cont}}\big(x_7^{\text{cont}}\big)^{-1} - 1.10 \leq 0,\\
g_6\big(x_{13}^{\text{cont}}, x_{14}^{\text{cont}}\big) &= x_{13}^{\text{cont}} - 0.8\, x_{14}^{\text{cont}} \leq 0,\\
g_7\big(x_2^{\text{cont}}, x_{16}^{\text{cont}}\big) &= x_2^{\text{cont}} + x_{16}^{\text{cont}} - 8 \leq 0.
\end{aligned}
$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}, \text{K}, \text{L}, \text{M}, \text{N}, \text{O}, \text{P}, \text{Q}, \text{R}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 16$ and $x_i^{\text{cont}} \in [0.1, 10]$ for $i \in \{1, \ldots, 16\}$.

4.22 Cat-cstrs-22: modified MAD [14]

$$
f\big(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \begin{cases}
\big(x_1^{\text{cont}}\big)^2 + \big(x_2^{\text{cont}}\big)^2 + x_1^{\text{cont}} x_2^{\text{cont}} - 1, & \text{if } x^{\text{cat}} = \text{A},\\
\sin\big(x_1^{\text{cont}}\big) + 0.05\,\big|x_2^{\text{cont}}\big|, & \text{if } x^{\text{cat}} = \text{B},\\
-\cos\big(x_2^{\text{cont}}\big) + 0.03\,\big|x_1^{\text{cont}}\big|, & \text{if } x^{\text{cat}} = \text{C},\\
0.95\Big(\big(x_1^{\text{cont}}\big)^2 + \big(x_2^{\text{cont}}\big)^2 + x_1^{\text{cont}} x_2^{\text{cont}} - 1\Big) + 0.10\big(x_1^{\text{cont}} - x_2^{\text{cont}}\big), & \text{if } x^{\text{cat}} = \text{D},\\
\sin\big(x_1^{\text{cont}}\big) + 0.08\big(x_1^{\text{cont}} - 1\big)^2, & \text{if } x^{\text{cat}} = \text{E},\\
-\cos\big(x_2^{\text{cont}}\big) + 0.06\big(x_2^{\text{cont}} - 1\big)^2, & \text{if } x^{\text{cat}} = \text{F},\\
-\exp\big(x_1^{\text{cont}} - x_2^{\text{cont}}\big), & \text{if } x^{\text{cat}} = \text{G},\\
\sinh\big(x_1^{\text{cont}} - 1\big) - 1 + 0.03\,\big|x_2^{\text{cont}} - 1\big|, & \text{if } x^{\text{cat}} = \text{H},\\
-\log\big(x_2^{\text{cont}}\big) - 1 + 0.04\,\big|x_1^{\text{cont}}\big|, & \text{if } x^{\text{cat}} = \text{I},\\
-0.92\,\exp\big(x_1^{\text{cont}} - x_2^{\text{cont}}\big) - 0.05\big(x_1^{\text{cont}} - x_2^{\text{cont}}\big), & \text{if } x^{\text{cat}} = \text{J},\\
1.05\big(\sinh\big(x_1^{\text{cont}} - 1\big) - 1\big) + 0.02\big(x_1^{\text{cont}} - 1\big)^2, & \text{if } x^{\text{cat}} = \text{K},\\
-\log\big(x_2^{\text{cont}}\big) - 1 + 0.06\big(x_2^{\text{cont}} - 0.5\big)^2, & \text{if } x^{\text{cat}} = \text{L}.
\end{cases}
$$

with constraints

$$
g_1\big(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \begin{cases}
0.5 - \Big(\big(x_1^{\text{cont}}\big)^2 + \big(x_2^{\text{cont}}\big)^2\Big) \leq 0, & \text{if } x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\},\\
3x_1^{\text{cont}} + x_2^{\text{cont}} + 2.5 \leq 0, & \text{if } x^{\text{cat}} \in \{\text{D}, \text{E}, \text{F}\},\\
x_2^{\text{cont}} - 0.05 x_1^{\text{cont}} - 0.5 \leq 0, & \text{if } x^{\text{cat}} \in \{\text{G}, \text{H}, \text{I}\},\\
0.9 x_1^{\text{cont}} - x_2^{\text{cont}} + 1 \leq 0, & \text{if } x^{\text{cat}} \in \{\text{J}, \text{K}, \text{L}\}.
\end{cases}
$$

$$
\begin{aligned}
g_2\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) &= \big(x_1^{\text{cont}} - 1\big)^2 + \big(x_2^{\text{cont}} - 1\big)^2 - 0.5 \leq 0,\\
g_3\big(x_1^{\text{cont}}, x_2^{\text{cont}}\big) &= x_1^{\text{cont}} + x_2^{\text{cont}} - 1 \leq 0.
\end{aligned}
$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}, \text{K}, \text{L}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}} \in [-3, 3]$, $x_2^{\text{cont}} \in [0.1, 3]$.

4.23 Cat-cstrs-23: modified Wong 3

$$
f\Big(x^{\text{cat}}, x_1^{\text{int}}, \ldots, x_{10}^{\text{int}}, x_1^{\text{cont}}, \ldots, x_{10}^{\text{cont}}\Big) =
$$

$$
\begin{aligned}
&s + \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + x_1^{\text{cont}} x_2^{\text{cont}} - 14x_1^{\text{cont}} - 16x_2^{\text{cont}} + \left(x_3^{\text{cont}} - 10\right)^2 + 4\left(x_4^{\text{cont}} - 5\right)^2 \\
&+ \left(x_5^{\text{cont}} - 3\right)^2 + 2\left(x_6^{\text{cont}} - 1\right)^2 + 5\left(x_7^{\text{cont}}\right)^2 + 7\left(x_8^{\text{cont}} - 11\right)^2 + 2\left(x_9^{\text{cont}} - 10\right)^2 + \left(x_{10}^{\text{cont}} - 7\right)^2 \\
&+ \left(x_1^{\text{int}} - 9\right)^2 + 10\left(x_2^{\text{int}} - 1\right)^2 + 5\left(x_3^{\text{int}} - 7\right)^2 + 4\left(x_4^{\text{int}} - 14\right)^2 + 27\left(x_5^{\text{int}} - 1\right)^2 + \left(x_6^{\text{int}}\right)^2 \\
&+ \left(x_7^{\text{int}} - 2\right)^2 + 13\left(x_8^{\text{int}} - 2\right)^2 + \left(x_9^{\text{int}} - 3\right)^2 + \left(x_{10}^{\text{int}}\right)^2 + 95 \\
&+ 0.30\sin\left(x_1^{\text{cont}}\right) + 0.25\cos\left(0.5\, x_4^{\text{cont}}\right) + 0.20\sin\left(0.3\, x_9^{\text{cont}}\right) \\
&+ 0.08\left|x_5^{\text{cont}} - 3\right|\left|x_6^{\text{cont}} - 1\right| + 0.05\left|x_3^{\text{int}} - 7\right|\left|x_4^{\text{int}} - 14\right| + 0.03\left(x_7^{\text{cont}} - 0.5x_8^{\text{cont}}\right)\left(x_1^{\text{int}} - 9\right) \\
&+ 0.02\left(x_3^{\text{cont}} - 10\right)\left(x_2^{\text{int}} - 1\right),
\end{aligned}
$$

and

$$
s\left(x^{\text{cat}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_5^{\text{int}}, x_6^{\text{int}}, x_7^{\text{int}}, x_8^{\text{int}}, x_9^{\text{int}}, x_{10}^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_8^{\text{cont}}, x_9^{\text{cont}}, x_{10}^{\text{cont}}\right) =
$$

$$
10 \times \begin{cases}
3\left(x_1^{\text{cont}} - 2\right)^2 + 4\left(x_2^{\text{cont}} - 3\right)^2 + 2\left(x_3^{\text{cont}}\right)^2 - 7x_4^{\text{cont}} - 120, & \text{if } x^{\text{cat}} = \text{A}, \\
5\left(x_1^{\text{cont}}\right)^2 + 8x_2^{\text{cont}} + \left(x_3^{\text{cont}} - 6\right)^2 - 2x_4^{\text{cont}} - 40, & \text{if } x^{\text{cat}} = \text{B}, \\
0.5\left(x_1^{\text{cont}} - 8\right)^2 + 2\left(x_2^{\text{cont}} - 4\right)^2 + 3\left(x_5^{\text{cont}}\right)^2 - x_6^{\text{cont}} - 30, & \text{if } x^{\text{cat}} = \text{C}, \\
\left(x_1^{\text{cont}}\right)^2 + 2\left(x_2^{\text{cont}} - 2\right)^2 - 2x_1^{\text{cont}} x_2^{\text{cont}} + 14x_5^{\text{cont}} - 6x_6^{\text{cont}}, & \text{if } x^{\text{cat}} = \text{D}, \\
-3x_1^{\text{cont}} + 6x_2^{\text{cont}} + 12\left(x_8^{\text{cont}} - 8\right)^2 - 7x_{10}^{\text{cont}}, & \text{if } x^{\text{cat}} = \text{E}, \\
\left(x_1^{\text{cont}}\right)^2 + 5x_1^{\text{cont}} - 8x_2^{\text{cont}} - 28, & \text{if } x^{\text{cat}} = \text{F}, \\
4x_1^{\text{cont}} + 9x_2^{\text{cont}} + 5\left(x_3^{\text{int}}\right)^2 - 9x_4^{\text{int}} - 87, & \text{if } x^{\text{cat}} = \text{G}, \\
3x_1^{\text{cont}} + 4x_2^{\text{cont}} + 3\left(x_3^{\text{int}} - 6\right)^2 - 14x_4^{\text{int}} - 10, & \text{if } x^{\text{cat}} = \text{H}, \\
14\left(x_2^{\text{int}}\right)^2 + 35x_5^{\text{int}} - 79x_6^{\text{int}} - 92, & \text{if } x^{\text{cat}} = \text{I}, \\
15\left(x_5^{\text{int}}\right)^2 + 11x_5^{\text{int}} - 61x_6^{\text{int}} - 54, & \text{if } x^{\text{cat}} = \text{J}, \\
5\left(x_1^{\text{cont}}\right)^2 + 2x_2^{\text{cont}} + 9\left(x_7^{\text{int}}\right)^4 - x_8^{\text{int}} - 68, & \text{if } x^{\text{cat}} = \text{K}, \\
\left(x_1^{\text{cont}}\right)^2 - x_9^{\text{cont}} + 19x_9^{\text{int}} - 20x_{10}^{\text{int}} + 19, & \text{if } x^{\text{cat}} = \text{L}, \\
12\left(x_2^{\text{cont}}\right)^2 + \left(x_9^{\text{cont}}\right)^2 - 30x_{10}^{\text{int}}, & \text{if } x^{\text{cat}} = \text{M}.
\end{cases}
$$

with constraints

$$
g_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_7^{\text{cont}}, x_8^{\text{cont}}, x_1^{\text{int}}\right) = 4x_1^{\text{cont}} + 5x_2^{\text{cont}} - 3x_7^{\text{cont}} + 9x_8^{\text{cont}} - 105 + 0.15\left(x_1^{\text{cont}}\right)^2 + 0.05\left|x_1^{\text{int}}\right| \le 0,
$$

$$
g_2\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_7^{\text{cont}}, x_8^{\text{cont}}, x_2^{\text{int}}\right) = 10x_1^{\text{cont}} - 8x_2^{\text{cont}} - 17x_7^{\text{cont}} + 2x_8^{\text{cont}} + 0.10\left(x_2^{\text{cont}} - 1\right)^2 + 0.03\left(x_2^{\text{int}} - 1\right)^2 \le 0,
$$

$$
g_3\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_9^{\text{cont}}, x_{10}^{\text{cont}}, x_3^{\text{int}}\right) = -8x_1^{\text{cont}} + 2x_2^{\text{cont}} + 5x_9^{\text{cont}} - 2x_{10}^{\text{cont}} - 12 + 0.06\left(x_9^{\text{cont}} - 8\right)^2 + 0.02\left|x_3^{\text{int}} - 6\right| \le 0,
$$

$$
g_4\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, x_4^{\text{int}}\right) = x_1^{\text{cont}} + x_2^{\text{cont}} + 4x_1^{\text{int}} - 21x_2^{\text{int}} + 0.08\left(x_3^{\text{cont}} - 10\right)^2 + 0.05\left|x_4^{\text{int}} - 14\right| \le 0.
$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \ldots, \text{M}\}$;
- $n^{\text{int}} = 10$ and $x_i^{\text{int}} \in \{-20, \ldots, 20\}$ for $i \in \{1, \ldots, 10\}$;
- $n^{\text{cont}} = 10$ and $x_j^{\text{cont}} \in [-20, 20]$ for $j \in \{1, \ldots, 10\}$.

4.24 Cat-cstrs-24: Welded beam

Let $L = 14$ and define

$$
\begin{aligned}
h\left(x_1^{\text{cont}}, x_1^{\text{int}}\right) &:= 0.18 + 0.22\, x_1^{\text{cont}} + 0.006\left(x_1^{\text{int}} + 6\right), \\
\ell\left(x_2^{\text{cont}}, x_2^{\text{int}}\right) &:= 2.5 + 4.8\, x_2^{\text{cont}} + 0.08\left(x_2^{\text{int}} + 6\right), \\
t\left(x_3^{\text{cont}}, x_1^{\text{int}}\right) &:= 5.0 + 4.5\, x_3^{\text{cont}} + 0.10\, x_1^{\text{int}}, \\
b\left(x_4^{\text{cont}}\right) &:= 0.20 + 0.30\, x_4^{\text{cont}}, \\
P\left(x_2^{\text{cont}}, x_2^{\text{int}}\right) &:= 2600 + 1600\, x_2^{\text{cont}} + 60\, x_2^{\text{int}}.
\end{aligned}
$$

then,

$$f\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = \Big(1.35 - 0.25\,s\Big) \times \Big(1.10471\,h\Big(x_1^{\text{cont}}, x_1^{\text{int}}\Big)^2\,\ell\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big) + 0.04811\,t\Big(x_3^{\text{cont}}, x_1^{\text{int}}\Big)\,b\big(x_4^{\text{cont}}\big)\,\big(L + \ell\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big)\big)\Big) + 10\,s\left|\frac{x_1^{\text{int}} - x_2^{\text{int}}}{24}\right| + 0.08\,|s - 1.35|\,.$$

where

Table 14: Values of $s \coloneqq s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}									
		A	B	C	D	E	F	G	H	I	J
x_2^{cat}	A	0.40	0.44	0.50	0.60	0.74	0.86	0.92	0.84	0.70	0.54
	B	0.62	0.70	0.80	0.94	1.12	1.28	1.36	1.22	1.02	0.78
	C	0.82	0.94	1.08	1.26	1.50	1.70	1.78	1.58	1.30	0.98
	D	0.98	1.12	1.30	1.54	1.84	2.05	2.08	1.84	1.50	1.12
	E	1.08	1.26	1.48	1.76	2.12	2.28	2.20	1.94	1.58	1.18
	F	1.02	1.18	1.38	1.62	1.96	2.14	2.16	1.90	1.54	1.14
	G	0.90	1.02	1.18	1.38	1.66	1.82	1.90	1.74	1.42	1.06
	H	0.74	0.84	0.96	1.12	1.34	1.46	1.56	1.50	1.24	0.94
	I	0.56	0.62	0.70	0.82	0.98	1.06	1.14	1.12	1.00	0.78
	J	0.36	0.40	0.46	0.54	0.66	0.72	0.78	0.80	0.72	0.56

with constraints

$$g_1\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = \tau\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, s\Big) - 13600\,s \le 0,$$
$$g_2\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = \sigma\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\Big) - 30000\,s \le 0,$$
$$g_3\Big(x_1^{\text{cont}}, x_1^{\text{int}}, x_4^{\text{cont}}\Big) = h\Big(x_1^{\text{cont}}, x_1^{\text{int}}\Big) - b\big(x_4^{\text{cont}}\big) \le 0,$$
$$g_4\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = \delta\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, s\Big) - 0.25 \le 0,$$
$$g_5\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = P\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big) - P_c\Big(x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_2^{\text{int}}, s\Big) \le 0,$$

where

$$h\Big(x_1^{\text{cont}}, x_1^{\text{int}}\Big) = 0.18 + 0.22\,x_1^{\text{cont}} + 0.006\,(x_1^{\text{int}} + 6),$$
$$\ell\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big) = 2.5 + 4.8\,x_2^{\text{cont}} + 0.08\,(x_2^{\text{int}} + 6),$$
$$t\Big(x_3^{\text{cont}}, x_1^{\text{int}}\Big) = 5.0 + 4.5\,x_3^{\text{cont}} + 0.10\,x_1^{\text{int}},$$
$$b\big(x_4^{\text{cont}}\big) = 0.20 + 0.30\,x_4^{\text{cont}},$$
$$P\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big) = 2600 + 1600\,x_2^{\text{cont}} + 60\,x_2^{\text{int}},$$

$$\tau\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\Big) = \sqrt{(\tau')^2 + 2\,\tau'\tau''\,\frac{\ell(x_2^{\text{cont}}, x_2^{\text{int}})}{2R(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}})} + (\tau'')^2},$$

$$\tau' = \frac{P\big(x_2^{\text{cont}}, x_2^{\text{int}}\big)}{\sqrt{2}\,h(x_1^{\text{cont}}, x_1^{\text{int}})\;\ell(x_2^{\text{cont}}, x_2^{\text{int}})},$$

$$M\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big) = P\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big)\left(L + \frac{\ell\big(x_2^{\text{cont}}, x_2^{\text{int}}\big)}{2}\right),$$

$$R\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\Big) = \sqrt{\frac{\ell(x_2^{\text{cont}}, x_2^{\text{int}})^2}{4} + \left(\frac{h(x_1^{\text{cont}}, x_1^{\text{int}}) + t(x_3^{\text{cont}}, x_1^{\text{int}})}{2}\right)^2},$$

$$J\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\Big) = 2\big(\sqrt{2}\, h\Big(x_1^{\text{cont}}, x_1^{\text{int}}\Big)\ \ell\Big(x_2^{\text{cont}}, x_2^{\text{int}}\Big)\big)\left(\frac{\ell\big(x_2^{\text{cont}}, x_2^{\text{int}}\big)^2}{12} + \frac{\big(h\big(x_1^{\text{cont}}, x_1^{\text{int}}\big) + t\big(x_3^{\text{cont}}, x_1^{\text{int}}\big)\big)^2}{4}\right),$$

$$\tau'' = \frac{M\big(x_2^{\text{cont}}, x_2^{\text{int}}\big)\ R\big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\big)}{J(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}})},$$

$$\sigma\Big(x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\Big) = \frac{6\, P\big(x_2^{\text{cont}}, x_2^{\text{int}}\big)\ L}{b(x_4^{\text{cont}})\ t(x_3^{\text{cont}}, x_1^{\text{int}})^2},$$

$$\delta\Big(x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, s\Big) = \frac{4\, P\big(x_2^{\text{cont}}, x_2^{\text{int}}\big)\ L^3}{E(s)\, b(x_4^{\text{cont}})\ t(x_3^{\text{cont}}, x_1^{\text{int}})^3},$$

$$P_c\Big(x_3^{\text{cont}}, x_4^{\text{cont}}, x_1^{\text{int}}, s\Big) = \frac{4.013\, E(s)\sqrt{\dfrac{t\big(x_3^{\text{cont}}, x_1^{\text{int}}\big)^2\ b\big(x_4^{\text{cont}}\big)^6}{36}}}{L^2}\left(1 - \frac{t\big(x_3^{\text{cont}}, x_1^{\text{int}}\big)}{2L}\sqrt{\frac{E(s)}{4G(s)}}\right),$$

$$E(s) = 30 \times 10^6\,(0.55 + 0.35\, s),$$
$$G(s) = 12 \times 10^6\,(0.60 + 0.30\, s).$$

- $n^{\text{cat}} = 2$ and $x_i^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$ for $i \in \{1, 2\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-6, \ldots, 18\}$;
- $n^{\text{cont}} = 4$ and $x_1^{\text{cont}} \in [0.7, 1.5]$, $x_2^{\text{cont}} \in [0.5, 1.4]$, $x_3^{\text{cont}} \in [0.6, 1.6]$ and $x_4^{\text{cont}} \in [0.7, 1.5]$.

4.25 Cat-cstrs-25: modified Three-Bar Truss [18]

$$f\big(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = 100\left(2\sqrt{2}\, x_1^{\text{cont}} + x_2^{\text{cont}}\right)\left(1 + 10^{-3}\, s\right),$$

where

$$s \coloneqq s\big(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = 50 \times \begin{cases} \big|x_1^{\text{cont}} - 0.70\big| + 0.30\,\big|x_2^{\text{cont}} - 0.35\big| - 0.25, & \text{if } x^{\text{cat}} = \text{A}, \\ \big(x_1^{\text{cont}} - 0.80\big)^2 + 0.60\,\big|x_2^{\text{cont}} - 0.40\big| - 0.18, & \text{if } x^{\text{cat}} = \text{B}, \\ \big|x_1^{\text{cont}} + x_2^{\text{cont}} - 1.15\big| + 0.25\,\big(x_2^{\text{cont}} - 0.45\big)^2 - 0.22, & \text{if } x^{\text{cat}} = \text{C}, \\ \max\big(0,\, x_2^{\text{cont}} - 0.55\big)^2 + 0.40\,\big|x_1^{\text{cont}} - 0.75\big| - 0.20, & \text{if } x^{\text{cat}} = \text{D}, \\ \big|x_1^{\text{cont}} x_2^{\text{cont}} - 0.30\big| + 0.15\,\big|x_1^{\text{cont}} - x_2^{\text{cont}}\big| - 0.15, & \text{if } x^{\text{cat}} = \text{E}, \\ \big|x_1^{\text{cont}} - 0.65\big|^{-1}\,\big(x_2^{\text{cont}} - 0.25\big)^2 - 0.10, & \text{if } x^{\text{cat}} = \text{F}, \\ \big|x_2^{\text{cont}} - 0.35\big|^{-1}\,\big(x_1^{\text{cont}} - 0.85\big)^2 - 0.10, & \text{if } x^{\text{cat}} = \text{G}, \\ \big|x_1^{\text{cont}} - 0.90\big| + \big|x_2^{\text{cont}} - 0.20\big| - 0.30, & \text{if } x^{\text{cat}} = \text{H}, \\ \max\big(0,\, 1.05 - x_1^{\text{cont}} - x_2^{\text{cont}}\big)^2 - 0.08, & \text{if } x^{\text{cat}} = \text{I}, \\ 0.50\,\big|x_1^{\text{cont}} - 0.78\big| + 0.50\,\big|x_2^{\text{cont}} - 0.42\big| - 0.19, & \text{if } x^{\text{cat}} = \text{J}. \end{cases}$$

with constraints

$$g_1\big(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \frac{2\big(\sqrt{2}\,x_1^{\text{cont}} + x_2^{\text{cont}}\big)}{\sqrt{2\,(x_1^{\text{cont}})^2 + 2x_1^{\text{cont}}x_2^{\text{cont}}}} - 2 + 0.002\,s - 0.04 \le 0,$$

$$g_2\big(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \frac{2\,x_2^{\text{cont}}}{\sqrt{2\,(x_1^{\text{cont}})^2 + 2x_1^{\text{cont}}x_2^{\text{cont}}}} - 2 + 0.0015\,\big|s\big| \le 0,$$

$$g_3\big(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \frac{2}{x_1^{\text{cont}} + \sqrt{2}\,x_2^{\text{cont}}} - 2 + 0.001 s^2 \le 0.$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}} \in [0.05, 1.00]$ and $x_2^{\text{cont}} \in [0.05, 1.00]$.

4.26 Cat-cstrs-26: modified constrained Three hump [18]

$$f\big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = 2s_1^2 - 1.05\,s_1^4 + \frac{s_1^6}{6} + s_1 s_2 + s_2^2 + 0.08\,|s_1| + 0.05\,|s_2 - s_1|\,,$$

where

$$s_1 \coloneqq s\big(x_1^{\text{cat}}, x_1^{\text{cont}}\big) = \begin{cases} x_1^{\text{cont}} + 0.12\big(x_1^{\text{cont}} - 1\big)^2 + 0.15\,\big|x_1^{\text{cont}}\big|\,, & \text{if } x_1^{\text{cat}} = \text{A}, \\ x_1^{\text{cont}} + 0.12\big(x_1^{\text{cont}} - 1\big)^2 + 0.10\,\big|x_1^{\text{cont}} - 0.5\big|\,, & \text{if } x_1^{\text{cat}} = \text{B}, \\ x_1^{\text{cont}} + 0.12\big(x_1^{\text{cont}} - 1\big)^2 + 0.08\,\big|x_1^{\text{cont}} + 0.5\big|\,, & \text{if } x_1^{\text{cat}} = \text{C}, \\ x_1^{\text{cont}} - 0.10\big(x_1^{\text{cont}} + 1\big)^2 - 0.12\,\big|x_1^{\text{cont}}\big|\,, & \text{if } x_1^{\text{cat}} = \text{D}, \\ x_1^{\text{cont}} - 0.10\big(x_1^{\text{cont}} + 1\big)^2 - 0.09\,\big|x_1^{\text{cont}} - 0.5\big|\,, & \text{if } x_1^{\text{cat}} = \text{E}, \\ x_1^{\text{cont}} - 0.10\big(x_1^{\text{cont}} + 1\big)^2 - 0.07\,\big|x_1^{\text{cont}} + 0.5\big|\,, & \text{if } x_1^{\text{cat}} = \text{F}, \end{cases}$$

and,

$$s_2 \coloneqq s\big(x_2^{\text{cat}}, x_2^{\text{cont}}\big) = \begin{cases} 0.9\,e^{0.35x_2^{\text{cont}}} + 0.05\,\big|x_2^{\text{cont}}\big|\,, & \text{if } x_2^{\text{cat}} = \text{A}, \\ 0.6\,e^{0.45x_2^{\text{cont}}} + 0.04\,\big|x_2^{\text{cont}}\big|\,, & \text{if } x_2^{\text{cat}} = \text{B}, \\ 0.9\,e^{0.35x_2^{\text{cont}}} + 0.06\,\big|x_2^{\text{cont}} - 0.4\big|\,, & \text{if } x_2^{\text{cat}} = \text{C}, \\ 0.6\,e^{0.45x_2^{\text{cont}}} + 0.05\,\big|x_2^{\text{cont}} - 0.4\big|\,, & \text{if } x_2^{\text{cat}} = \text{D}, \\ 0.9\,e^{0.35x_2^{\text{cont}}} + 0.06\,\big|x_2^{\text{cont}} + 0.4\big|\,, & \text{if } x_2^{\text{cat}} = \text{E}, \\ 0.6\,e^{0.45x_2^{\text{cont}}} + 0.05\,\big|x_2^{\text{cont}} + 0.4\big|\,, & \text{if } x_2^{\text{cat}} = \text{F}, \end{cases}$$

with constraints

$$g_1\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_1^{\text{cat}}, x_2^{\text{cat}}\Big) = \frac{\big(x_1^{\text{cont}} - 1\big)^2}{0.10} + \frac{\big(x_2^{\text{cont}} + 0.6 s_2 - 0.54\big)^2}{0.06} + \frac{\big(x_1^{\text{int}} + 2 - 2|s_1|\big)^2}{9} - 1 \le 0,$$

$$g_2\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cat}}, x_2^{\text{cat}}\Big) = \frac{s_2 - s_1}{0.10} + \frac{|x_1^{\text{int}}|}{2} + \frac{|x_2^{\text{int}}|}{2} - 1 \le 0,$$

$$g_3\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cat}}, x_2^{\text{cat}}\Big) = \frac{(s_1 - 1.15)^2}{0.05} + \frac{(s_2 - 0.90)^2}{0.05} + \frac{|x_1^{\text{int}} + x_2^{\text{int}}|}{6} - 1 \le 0,$$

$$g_4\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_2^{\text{int}}, x_1^{\text{cat}}, x_2^{\text{cat}}\Big) = \left(\frac{1}{1 + \exp(-1.2(x_1^{\text{cont}} - x_2^{\text{cont}}))} - 0.73\right)^2 + \left(\frac{|s_1|}{3.0}\right)^2 + \left(\frac{|x_2^{\text{int}}|}{10}\right)^2 - 0.12 \le 0,$$

$$g_5\big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{cat}}, x_2^{\text{cat}}\big) \;=\; \left|x_1^{\text{cont}} - \frac{x_2^{\text{cont}}}{10} - 1\right| \;+\; 0.45\,|s_2 - 0.90| \;+\; 0.10\,|s_1 - 1.15| \;-\; 0.12 \;\leq\; 0,$$

$$g_6\Big(x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cat}}, x_2^{\text{cat}}\Big) = \frac{\big(x_2^{\text{cont}} - 0.40 s_1\big)^2}{0.20} + \frac{\big(x_1^{\text{cont}} + 0.30 s_2 - 1.27\big)^2}{0.20} + 0.015|x_1^{\text{int}}| + 0.015|x_2^{\text{int}}| - 1 \leq 0.$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-2, -1, 0, 1, 2\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [-5, 5]$.

4.27 Cat-cstrs-27: modified constrained McCormick

$$f\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = \sin(s_1 + s_2) + (s_1 - s_2)^2 - 1.5\,s_1 + 2.5\,s_2 + 1 + 0.06\left|s_1 + \frac{x_1^{\text{int}}}{10}\right| + 0.04\left|s_2 - \frac{x_2^{\text{int}}}{10}\right| + 0.03\left|s_1 - s_2 + \frac{x_3^{\text{int}}}{10}\right|,$$

where $s_1 \coloneqq s\big(x_1^{\text{cat}}, x_1^{\text{cont}}, x_3^{\text{cont}}\big)$ and $s_2 \coloneqq s\big(x_2^{\text{cat}}, x_2^{\text{cont}}, x_4^{\text{cont}}\big)$ are defined by

$$s_1 = s\big(x_1^{\text{cat}}, x_1^{\text{cont}}, x_3^{\text{cont}}\big) = \begin{cases} x_1^{\text{cont}} + 0.25\tanh\big(x_1^{\text{cont}}\big) + 0.15\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“1”}, \\ x_1^{\text{cont}} + 0.25\tanh\big(x_1^{\text{cont}} - 0.5\big) + 0.12\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“2”}, \\ x_1^{\text{cont}} + 0.25\tanh\big(x_1^{\text{cont}} + 0.5\big) + 0.10\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“3”}, \\ x_1^{\text{cont}} - 0.30\tanh\big(x_1^{\text{cont}}\big) + 0.10\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“4”}, \\ x_1^{\text{cont}} - 0.30\tanh\big(x_1^{\text{cont}} + 0.5\big) + 0.08\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“5”}, \\ x_1^{\text{cont}} - 0.30\tanh\big(x_1^{\text{cont}} - 0.5\big) + 0.09\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“6”}, \\ x_1^{\text{cont}} + 0.18\,\big|x_1^{\text{cont}}\big|^{1.3} - 0.10\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“7”}, \\ x_1^{\text{cont}} + 0.18\,\big|x_1^{\text{cont}} + 0.3\big|^{1.3} - 0.12\,x_3^{\text{cont}}, & \text{if } x_1^{\text{cat}} = \text{“8”}, \end{cases}$$

and

$$s_2 = s\big(x_2^{\text{cat}}, x_2^{\text{cont}}, x_4^{\text{cont}}\big) = \begin{cases} x_2^{\text{cont}} + 0.35\arctan\big(x_2^{\text{cont}}\big) + 0.20\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“1”}, \\ x_2^{\text{cont}} + 0.35\arctan\big(x_2^{\text{cont}} - 0.6\big) + 0.18\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“2”}, \\ x_2^{\text{cont}} + 0.35\arctan\big(x_2^{\text{cont}} + 0.6\big) + 0.16\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“3”}, \\ x_2^{\text{cont}} + 0.22\,\text{sign}\big(x_2^{\text{cont}}\big)\,\sqrt{|x_2^{\text{cont}}|} - 0.12\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“4”}, \\ x_2^{\text{cont}} + 0.22\,\text{sign}\big(x_2^{\text{cont}}\big)\,\sqrt{|x_2^{\text{cont}} + 0.4|} - 0.10\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“5”}, \\ x_2^{\text{cont}} + 0.22\,\text{sign}\big(x_2^{\text{cont}}\big)\,\sqrt{|x_2^{\text{cont}} - 0.4|} - 0.11\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“6”}, \\ x_2^{\text{cont}} - 0.28\log\Big(1 + \big(x_2^{\text{cont}}\big)^2\Big) + 0.15\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“7”}, \\ x_2^{\text{cont}} - 0.28\log\Big(1 + \big(x_2^{\text{cont}} - 0.4\big)^2\Big) + 0.13\,x_4^{\text{cont}}, & \text{if } x_2^{\text{cat}} = \text{“8”}, \end{cases}$$

with constraints

$$g_1\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = \frac{(s_1 - 0.80)^2}{0.55} + \frac{(s_2 - 0.55)^2}{0.60} + \frac{\big(x_3^{\text{cont}} - 0.40\big)^2}{0.90} + \frac{\big(x_1^{\text{int}} - 4\big)^2}{36} - 1 \leq 0,$$

$$g_2\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\Big) = |\sin(s_1 + s_2 - 0.35)| + \frac{\big|x_2^{\text{int}} + 2\big|}{20} + 0.30\max(0, |s_2 - s_1| - 0.35) - 0.80 \leq 0,$$

$$g_3\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = \left(\frac{1}{1+\exp(-1.1\,(x_2^{\text{cont}} - x_1^{\text{cont}} - 0.4))} - 0.60\right)^2 + \left(\frac{x_4^{\text{cont}} + 0.6}{1.8}\right)^2 + \frac{\left|x_3^{\text{int}} - 1\right|}{25} + \frac{0.10}{1+|s_1|} + \frac{0.08}{1+|s_2|} - 0.22 \leq 0.$$

- $n^{\text{cat}} = 2$ and $x_i^{\text{cat}} \in \{\text{“1”}, \text{“2”}, \ldots, \text{“8”}\}$ for $i \in \{1, 2\}$;
- $n^{\text{int}} = 3$ and $x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}} \in \{-10, \ldots, 10\}$;
- $n^{\text{cont}} = 4$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [-4, 4]$ and $x_3^{\text{cont}}, x_4^{\text{cont}} \in [-2, 2]$.

4.28 Cat-cstrs-28: G06

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \left(x_1^{\text{cont}} - 25\right)^2 + \left(x_2^{\text{cont}} - 40\right)^2 + 6\,(s-18)^2 + 0.20\left(x_1^{\text{int}}\right)^2 + 0.15\left(x_2^{\text{int}}\right)^2 + 0.60\left|x_1^{\text{int}} - x_2^{\text{int}}\right| + 0.04\left|x_2^{\text{cont}} - 2s\right| + 12,$$

where the images are given by Table 15.

Table 15: Categorical mapping values $s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}									
		A	B	C	D	E	F	G	H	I	J
x_2^{cat}	A	11.8	12.3	12.9	13.6	14.1	14.4	14.6	14.7	14.8	14.9
	B	11.2	12.1	13.0	13.9	14.6	15.0	15.3	15.5	15.6	15.7
	C	10.8	11.8	13.2	14.6	15.5	16.0	16.4	16.7	16.9	17.0
	D	10.5	11.6	13.1	15.3	16.8	17.6	18.2	18.6	18.9	19.1
	E	10.3	11.3	12.8	15.0	17.9	19.0	19.8	20.4	20.9	21.2
	F	10.2	11.1	12.4	14.5	17.2	20.1	21.1	21.9	22.6	23.1
	G	10.1	10.9	12.0	13.8	16.3	19.3	22.0	23.0	23.8	24.4
	H	10.0	10.8	11.7	13.2	15.3	18.2	21.0	23.5	24.6	25.4
	I	10.0	10.7	11.4	12.6	14.4	16.9	19.6	22.2	24.2	25.7
	J	10.0	10.6	11.2	12.1	13.5	15.5	17.7	20.1	22.4	25.0

with constraints

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\right) = \left(x_1^{\text{cont}} - 35\right)^2 + \left(x_2^{\text{cont}} - 55\right)^2 - 900 + 20(s-18) + 6x_1^{\text{int}} \leq 0,$$

$$g_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\right) = \left(x_1^{\text{cont}} - 20\right)\left(x_2^{\text{cont}} - 30\right) - 700 + 12\,|s-16| + 2\left|x_2^{\text{int}}\right| \leq 0,$$

$$g_3\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\right) = \frac{x_2^{\text{cont}}}{100} + \frac{\left|x_1^{\text{int}}\right|}{12} + 0.05(s-10) - 1 \leq 0,$$

$$g_4\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\right) = \left|x_2^{\text{cont}} - (2s+5)\right| - \left(18 - 0.6\left|x_2^{\text{int}}\right|\right) \leq 0,$$

$$g_5\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_1^{\text{int}}, x_2^{\text{int}}\right) = \left(x_1^{\text{int}} + x_2^{\text{int}}\right)^2 - 60 + 4(s-18) \leq 0.$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-10, \ldots, 10\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}} \in [13, 100]$ and $x_2^{\text{cont}} \in [0, 100]$.

4.29 Cat-cstrs-29 constrained modified Shekel

$$f\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{cat}}, x_4^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\Big) =$$
$$\sum_{i=1}^{4}\Bigg(s_i + \Big(x_1^{\text{cont}} - x_i^{\text{int}}\Big)^2 + \Big(x_2^{\text{cont}} - x_i^{\text{int}}\Big)^2 + 0.08\,\Big(x_5^{\text{cont}} - 0.7x_i^{\text{int}}\Big)^2 + 0.06\,\Big(x_6^{\text{cont}} - 0.4x_i^{\text{int}}\Big)^2$$
$$+ 0.05\,\big|x_1^{\text{cont}} - x_2^{\text{cont}} + 0.4x_5^{\text{cont}} - 0.3x_6^{\text{cont}}\big|\Bigg)^{-1}$$
$$+ 0.12\sum_{i=1}^{4}\Big|x_i^{\text{cont}} - x_i^{\text{int}}\Big| + 0.04\sum_{i=1}^{4}\sqrt{|x_i^{\text{cont}} - x_i^{\text{int}}|} + 0.03\,\big|x_5^{\text{cont}} - x_6^{\text{cont}}\big| + 0.02\,\big|x_1^{\text{cont}}x_2^{\text{cont}} - x_5^{\text{cont}}x_6^{\text{cont}}\big|$$
$$+ 0.06\sum_{i=1}^{4}\Big(1 + 0.3\,s_i\Big)\cos\Big(\frac{\pi}{5}\,\big(x_5^{\text{cont}} + x_6^{\text{cont}}\big) + 0.4\,x_i^{\text{int}}\Big).$$

where

$$s_1 \coloneqq s_1\big(x_1^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \begin{cases} 0.35 + 0.10\log\Big(1 + \big(x_1^{\text{cont}} - 3\big)^2 + \big(x_2^{\text{cont}} - 7\big)^2\Big), & \text{if } x_1^{\text{cat}} = \text{A},\\ 0.30 + 0.06\,\big|x_1^{\text{cont}} - 5\big| + 0.04\,\big|x_2^{\text{cont}} - 2\big|, & \text{if } x_1^{\text{cat}} = \text{B},\\ 0.55 + \dfrac{0.80}{1 + \big(x_1^{\text{cont}} - 8\big)^2 + \big(x_2^{\text{cont}} - 1\big)^2}, & \text{if } x_1^{\text{cat}} = \text{C}, \end{cases}$$

and

$$s_2 \coloneqq s_2\big(x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \begin{cases} 0.30 + 0.06\,\big|x_1^{\text{cont}} - 2\big| + 0.05\,\big|x_2^{\text{cont}} - 6\big|, & \text{if } x_2^{\text{cat}} = \text{A},\\ 0.55 + \dfrac{0.85}{1 + \big(x_1^{\text{cont}} - 1\big)^2 + \big(x_2^{\text{cont}} - 9\big)^2}, & \text{if } x_2^{\text{cat}} = \text{B},\\ 0.33 + 0.10\log\Big(1 + \big(x_1^{\text{cont}} - 6\big)^2 + \big(x_2^{\text{cont}} - 4\big)^2\Big), & \text{if } x_2^{\text{cat}} = \text{C}, \end{cases}$$

and

$$s_3 \coloneqq s_3\big(x_3^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \begin{cases} 0.55 + \dfrac{0.75}{1 + \big(x_1^{\text{cont}} - 4\big)^2 + \big(x_2^{\text{cont}} - 4\big)^2}, & \text{if } x_3^{\text{cat}} = \text{A},\\ 0.34 + 0.10\log\Big(1 + \big(x_1^{\text{cont}} - 9\big)^2 + \big(x_2^{\text{cont}} - 2\big)^2\Big), & \text{if } x_3^{\text{cat}} = \text{B},\\ 0.28 + 0.07\,\big|x_1^{\text{cont}} - 7\big| + 0.03\,\big|x_2^{\text{cont}} - 5\big|, & \text{if } x_3^{\text{cat}} = \text{C}, \end{cases}$$

and

$$s_4 \coloneqq s_4\big(x_4^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\big) = \begin{cases} 0.33 + 0.10\log\Big(1 + \big(x_1^{\text{cont}} - 2\big)^2 + \big(x_2^{\text{cont}} - 1\big)^2\Big), & \text{if } x_4^{\text{cat}} = \text{A},\\ 0.55 + \dfrac{0.90}{1 + \big(x_1^{\text{cont}} - 7\big)^2 + \big(x_2^{\text{cont}} - 8\big)^2}, & \text{if } x_4^{\text{cat}} = \text{B},\\ 0.29 + 0.06\,\big|x_1^{\text{cont}} - 4\big| + 0.05\,\big|x_2^{\text{cont}} - 7\big|, & \text{if } x_4^{\text{cat}} = \text{C}. \end{cases}$$

with constraints

$$g_1\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{cat}}, x_4^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\Big) =$$
$$\big(s_1 + s_2\big)\big(s_3 + s_4\big) + \frac{x_1^{\text{int}} + x_2^{\text{int}} + x_3^{\text{int}} + x_4^{\text{int}}}{12} - 2.60 \le 0.$$

$$g_2\Big(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\Big) = \big(x_3^{\text{cont}} - x_4^{\text{cont}}\big)^2 + \big(x_5^{\text{cont}} - x_6^{\text{cont}}\big)^2 + 0.20\,\Big|x_1^{\text{cont}} - x_1^{\text{int}}\Big|$$
$$+ 0.20\,\Big|x_2^{\text{cont}} - x_2^{\text{int}}\Big| - 3.20 \le 0.$$

$$g_3\Big(x_1^{\text{cat}}, x_2^{\text{cat}}, x_4^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\Big) = \big(x_1^{\text{cont}} - 4\big)\,\big(x_2^{\text{cont}} - 6\big) + 0.30\,\Big|x_1^{\text{int}} - x_2^{\text{int}}\Big| + 0.20\,|s_2 - s_4| - 2.50 \le 0.$$

- $n^{\text{cat}} = 4$ and $x_i^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$ for $i \in \{1, \ldots, 4\}$;
- $n^{\text{int}} = 4$ and $x_i^{\text{int}} \in \{0, 1, \ldots, 10\}$ for $i \in \{1, \ldots, 4\}$;
- $n^{\text{cont}} = 6$ and $x_j^{\text{cont}} \in [0, 10]$ for $j \in \{1, \ldots, 6\}$.

4.30 Cat-cstrs-30: modified constrained Ishigami [21]

$$
\begin{aligned}
f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}\right) = \\
s^2 \times \left[\sin(\pi x_1^{\text{cont}} + 7\sin^2(\pi x_2^{\text{cont}}) + 0.1(\pi x_3^{\text{cont}})^4 \sin(\pi x_1^{\text{cont}})\right] \\
+ s \times \left[\left(\left(x_4^{\text{cont}}\right)^2 + \left(x_5^{\text{cont}}\right)^2 - 1/2\right) + \frac{x^{\text{int}}}{20}\sin\left(4\left(x_4^{\text{cont}} + x_5^{\text{cont}}\right)\right)\right] \\
+ 0.3\left[\left(\left(x_6^{\text{cont}}\right)^2 + \left(x_7^{\text{cont}}\right)^2 - 1\right)^2 + \frac{x^{\text{int}}}{20}\cos\left(4\left(x_6^{\text{cont}} - x_7^{\text{cont}}\right)\right)\right]
\end{aligned}
$$

where the images of s are provided by Table 16

Table 16: Images for $s := s(x_1^{\text{cat}}, x_2^{\text{cat}})$, where columns correspond to x_1^{cat} and rows correspond to x_2^{cat}.

		x_1^{cat}						
		A	B	C	D	E	F	G
x_2^{cat}	A	0.40	0.66	0.91	1.06	0.89	0.61	0.37
	B	0.56	0.81	1.07	1.26	1.09	0.78	0.52
	C	0.71	0.97	1.22	1.43	1.24	0.94	0.69
	D	0.91	1.17	1.43	1.67	1.47	1.14	0.85
	E	1.12	1.37	1.63	1.92	1.70	1.31	1.03
	F	1.36	1.61	1.94	2.23	1.96	1.57	1.26
	G	1.61	1.93	2.24	2.53	2.22	1.82	1.51

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{1, 2, \ldots, 20\}$;
- $n^{\text{cont}} = 7$ and $x_i^{\text{cont}} \in [-1, 1]$ for $i \in \{1, 2, \ldots, 7\}$.

with constraints

$$
\begin{aligned}
g_1(x_1^{\text{cat}}, x_2^{\text{cat}}, x_4^{\text{cont}}, x_5^{\text{cont}}) &= \left(\left(x_4^{\text{cont}}\right)^2 + \left(x_5^{\text{cont}}\right)^2 - \left(0.35 + 0.25\,s\right)\right)^2 - 0.0064 \leq 0, \\
g_2(x_1^{\text{cat}}, x_2^{\text{cat}}, x_6^{\text{cont}}, x_7^{\text{cont}}) &= \left(\left(x_6^{\text{cont}}\right)^2 + \left(x_7^{\text{cont}}\right)^2 - \left(0.55 + 0.15\,s\right)\right)^2 - 0.0100 \leq 0,
\end{aligned}
$$

$$
\begin{aligned}
g_3(x_1^{\text{cat}}, x_2^{\text{cat}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \\
\left(\sin(\pi x_1^{\text{cont}}) + 0.5\sin(\pi x_2^{\text{cont}}) + 0.25 x_3^{\text{cont}} + \frac{x^{\text{int}} - 10}{20} - 0.15\,s(x_1^{\text{cat}}, x_2^{\text{cat}})\right)^2 - 0.0064 \leq 0.
\end{aligned}
$$

References

[1] M.A. Abramson, C. Audet, J.W. Chrissis, and J.G. Walston. Mesh Adaptive Direct Search Algorithms for Mixed Variable Optimization. Optimization Letters, 3(1):35–47, 2009.

[2] C. Audet, A. Batailly, and S. Kojtych. Escaping Unknown Discontinuous Regions in Blackbox Optimization. SIAM Journal on Optimization, 32(3):1843–1870, 2022.

[3] C. Audet and J.E. Dennis, Jr. Pattern Search Algorithms for Mixed Variable Programming. SIAM Journal on Optimization, 11(3):573–594, 2001.

[4] C. Audet, K.J. Dzahini, M. Kokkolaras, and S. Le Digabel. Stochastic mesh adaptive direct search for blackbox optimization using probabilistic estimates. Computational Optimization and Applications, 79(1):1–34, 2021.

[5] C. Audet, E. Hallé-Hannan, and S. Le Digabel. A General Mathematical Framework for Constrained Mixed-variable Blackbox Optimization Problems with Meta and Categorical Variables. Operations Research Forum, 4(12), 2023.

[6] A.-S. Crélot, C. Beauthier, D. Orban, C. Sainvitu, and A. Sartenaer. Combining Surrogate Strategies with MADS for Mixed-Variable Derivative-Free Optimization. Technical Report G–2017–70, Les cahiers du GERAD, 2017.

[7] J. Cuesta-Ramirez, R. Le Riche, O. Roustant, G. Perrin, C. Durantin, and A. Gliere. A comparison of mixed-variables Bayesian optimization approaches. Advanced Modeling and Simulation in Engineering Sciences, 9(1):6, 2022.

[8] E.C. Garrido-Merchán and D. Hernández-Lobato. Dealing with categorical and integer-valued variables in Bayesian Optimization with Gaussian processes. Neurocomputing, 380:20–35, 2020.

[9] N.I.M. Gould, D. Orban, and Ph.L. Toint. CUTEst: a Constrained and Unconstrained Testing Environment with safe threads for mathematical optimization. Computational Optimization and Applications, 60(3):545–557, 2015. Code available at https://ccpforge.cse.rl.ac.uk/gf/project/cutest/wiki.

[10] E. Hallé-Hannan, C. Audet, Y. Diouane, S. Le Digabel, and P. Saves. A graph-structured distance for mixed-variable domains with meta variables. Technical Report G–2024–33, Les cahiers du GERAD, 2024.

[11] M. Halstrup. Black-Box Optimization of Mixed Discrete-Continuous Optimization Problems. PhD thesis, Technical University of Denmark, 2016.

[12] T. Hastie, R. Tibshirani, and J. Friedman. The Elements of Statistical Learning. Springer Series in Statistics. Springer New York Inc., New York, NY, USA, 2001.

[13] M. Jamil and X.-S. Yang. A literature survey of benchmark functions for global optimisation problems. International Journal of Mathematical Modelling and Numerical Optimisation, 4(2):150–194, 2013.

[14] L. Lukšan and J. Vlček. Test Problems for Nonsmooth Unconstrained and Linearly Constrained Optimization. Technical Report V-798, ICS AS CR, 2000.

[15] J.J. Moré, B.S. Garbow, and Kenneth E. Hillstrom. Testing unconstrained optimization software. ACM Transactions on Mathematical Software, 7(1):17–41, 1981.

[16] M. Munoz Zuniga and D. Sinoquet. Global optimization for mixed categorical-continuous variables based on Gaussian process models with a randomized categorical space exploration step. INFOR: Information Systems and Operational Research, 58(2):310–341, 2020.

[17] J. Pelamatti, L. Brevault, M. Balesdent, E.-G. Talbi, and Y. Guerin. Efficient global optimization of constrained mixed variable problems. Journal of Global Optimization, 73(3):583–613, 2019.

[18] T. Ray and K.M. Liew. A swarm metaphor for multiobjective design optimization. Engineering Optimization, 34(2):141–153, 2002.

[19] O. Roustant, E. Padonou, Y. Deville, A. Clément, G. Perrin, J. Giorla, and H. Wynn. Group kernels for Gaussian process metamodels with categorical inputs. Uncertainty Quantification, 8(2):775–806, 2020.

[20] P. Saves, Y. Diouane, N. Bartoli, T. Lefebvre, and J. Morlier. A mixed-categorical correlation kernel for Gaussian process. Neurocomputing, 550:126472, 2023.

[21] S. Surjanovic and D. Bingham. Virtual Library of Simulation Experiments: Test Functions and Datasets. Technical report, Simon Fraser University, 2025.

[22] E.-G. Talbi. Metaheuristics for variable-size mixed optimization problems: A unified taxonomy and survey. Swarm and Evolutionary Computation, 89:101642, 2024.

[23] F. Wang, H. Zhang, and A. Zhou. A particle swarm optimization algorithm for mixed-variable optimization problems. Swarm and Evolutionary Computation, 60:100808, 2021.