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V. N. Motta, M. F. Anjos, M. Gendreau

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Improved generalized Benders decomposition for stochastic unit commitment models with demand response

Vinicius N. Motta [a, c]

Miguel F. Anjos [b, c]

Michel Gendreau [a, d]

[a] *Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal (Qc), Canada, H3T 1J4*

[b] *School of Mathematics, University of Edinburgh, Scotland, United Kingdom, EH9 3FD*

[c] *GERAD, Montréal (Qc), Canada, H3T 1J4*

[d] *CIRRELT, Montréal (Qc), Canada, H3T 1J4*

vinicius.neves-motta@polymtl.ca
anjos@stanfordalumni.org
michel.gendreau@polymtl.ca

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Abstract : The increasing penetration of renewable electricity generation as well as the implementation of demand response programs has led to new challenges in the operation of the power grid. The output of renewable sources fluctuates, and this adds uncertainties to the problem. The distributed nature of the demand response resources is an additional operational challenge that is normally addressed by the creation of aggregators that manage these resources. The impacts of the power transmission system must also be taken into account. We propose a short-term unit commitment model to allocate demand response resources considering the variability of renewable sources and the needs of the grid. We formulate this as a mixed nonlinear integer optimization problem that is challenging to solve, which motivates, first, to relax the problem by applying a semidefinite relaxation to it, and, second, the use of Generalized Benders Decomposition (GBD) to tackle it. It is well known that the GBD algorithm can suffer from slow convergence to an optimal solution, therefore we use a Benders-based Branch-and-Cut with various enhancement methods to improve its performance. In order to choose which enhancement methods should be used, we analyze their impact on the performance of the GBD algorithm using the IEEE RTS-96 network. We conclude that while all of the enhancement methods considered improve the convergence rate and solution time for our model, the Pareto-Optimal cuts are the most significant improvement, both in terms of convergence rate and computational time.

Keywords : Demand response, stochastic optimization, unit commitment, optimal power flow, Benders decomposition

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1 Introduction

The operation of electric power grids is becoming more challenging with the growing penetration of renewable electricity sources such as wind and solar generation, as well as the implementation of demand response (DR) programs. The fundamental challenges are that renewable generation fluctuates by its very nature, and that DR resources that can provide flexible support are normally small in scale and distributed geographically, which makes their management more complex.

The operation of DR resources requires specific instructions for each individual residential or commercial source of DR. This challenge has led to the creation of entities called aggregators. An aggregator is tasked with the management of a set of DR resources so as to supply their flexibility to (presumed) Independent System Operator (ISO). They effectively act as intermediaries between the DR service providers and the ISO [7]. However, designing aggregators adequately is also a challenging task, and there are several works that propose aggregator models, such as [6, 9, 10]. There is also the need to coordinate the operation of the power grid in the presence of aggregators.

Furthermore, because renewable energy generation is variable, there will always be a degree of uncertainty when predicting it. The models used for finding the optimal commitment and dispatch of generating units as well as the optimal use of DR resources need to consider the uncertain aspects of renewable generation. This leads to a unit commitment (UC) problem that is an optimization problem under uncertainty that integrates the operation of the power grid with the dispatch of DR resources (through the aggregators).

Usually, the UC problem considers which generating units are available, which ones should be committed, and how much energy they should produce to meet the demand. It is also possible to consider transmission system security when making the aforementioned decisions, which leads to the security-constrained unit commitment (SCUC) problem. Both problems are modelled as either mixed-integer linear or mixed-integer nonlinear optimization problems under uncertainty.

For modelling this type of problem, there are two main approaches, stochastic optimization and robust optimization. There are several works using the former, such as in [30, 34, 19, 21, 23, 12, 33, 35, 40, 2, 38, 41, 36, 17, 32, 26, 14, 16, 29, 1]. There are also different alternatives in regards to modelling the transmission system, such as not considering it or considering either a DC Optimal Power Flow (DCOPF) or an Alternating Current Optimal Power Flow (ACOPF) model.

There are some models that do not consider the transmission system, such as [40, 12, 14]. In [40], a fuzzy chance-constrained stochastic UC model is proposed to minimize the generation and start-up costs. It considers the wind power and DR capacity as uncertainties.

In [12], the authors propose a two-stage UC model that aims to minimize generation, DR use, solar energy generation curtailment and battery use costs under solar energy generation uncertainty. The decisions taken before the uncertainty realization are the UC and determining the incentive charge for DR. In [14], they propose a bi-level stochastic SCUC model which aims to minimize generator and DR use costs under wind power uncertainty. The upper-level of the problem determines the operation of the power grid. The lower level problem determines the Incentive-based DR successful bidders.

However, the fact that no transmission system model is taken into consideration may give us solutions that are too optimistic. Therefore, in several projects it was decided to consider a DCOPF model, as seen in [19, 2, 38, 41, 36, 17, 32, 29, 21, 33, 26].

In [2], they propose a dynamic multistage stochastic UC (SUC) model in which the demand is uncertain with the aim to minimize generators operating costs. Similarly, [19] developed a model that considers wind and solar energy uncertainties instead.

In [17], the authors propose a two-stage SUC model aiming to minimize generation and DR operating costs taking into consideration wind generation as an uncertain parameter. Similarly, [29] aims to minimize generation operating costs. The decision taken before the uncertainty realization is the commitment of slow generation units. Finally, we also have models that aim at guaranteeing a secure operation of the transmission system, such as [33], which proposes a chance-constrained stochastic SCUC model.

In [41], a two-stage chance-constrained UC model is proposed with the aim of minimizing generators operating costs wind power and demand uncertainties. The UC and initial generators dispatch are decided before the uncertainty realization. In [38], the authors propose a model that also considers the demand response and wind spillage costs. In the first stage, they consider, additionally, the required up/down spinning reserve. Finally, [26] propose a stochastic SCUC model to minimize generator operating costs that uses compressed air energy storage to manage the uncertainty.

In [21], a two-stage SCUC model is proposed. This model considers the wind generation and the equipment outages as uncertainties and it aims to minimize the generation and reserve operating costs. In [32], they propose a two-stage SUC model considering transmission line and generators outages and demand uncertainties with the objective of minimizing generation and DR use costs. The DR reserve capacity and UC are decided before the uncertainty realization. In [36], it is proposed a two-stage SUC model in which the wind generation, demand and the outage of generating units and transmission lines are considered uncertain. The objective is to minimize generation, DR, reserves and wind spillage operating costs.

Nonetheless, a DCOPF model does not consider certain aspects of the transmission system, such as transmission losses. This may lead to optimistic solutions and thus some works consider a more detailed representation of the transmission system using an ACOPF model, such as in [35, 16, 1]

In [16], they propose a two-stage stochastic SCUC model that aims to minimize generators operating costs and takes into consideration wind power uncertainty. In [1], the authors propose a two-stage stochastic SCUC model with the objective of minimizing generation, DR use and load loss costs under wind power uncertainty. In the first stage, the UC problem is first solved without considering a transmission system, afterwards its solution feasibility is verified for both DCOPF and ACOPF transmission systems. Finally, in the second stage the wind power scenarios are considered and it is solved using the first-stage solution as a starting solution. In [35], the authors build up on the previous work and consider additionally DR uncertainties with the aim of minimizing generators and DR operating costs and maximizing the operator's revenues. However, it should be noted that they use a linearization of the ACOPF model to solve the second-stage problem, thus not solving it exactly.

Finally, there are also works on robust optimization models for the UC problem under uncertainty, as seen in [37, 39, 42], with some of them considering the SCUC problem.

[37] propose a multi-objective hybrid stochastic and robust UC model with the objective of both minimizing generation and emission costs, and risks under demand, transmission lines and generation outages uncertainties. In [39], the authors propose a two-stage distributionally robust optimization UC model that aims to minimize generators operating costs under wind power uncertainty. In [42], a robust UC model is proposed with the aim of minimizing generators and DR use costs taking into consideration wind power and solar power uncertainties.

Table 1 summarizes the different contributions made by each of the papers mentioned in this literature review. We can observe that most of the works do not consider a detailed representation of the transmission system as well as the impact of DR.

As it can be seen, none of the aforementioned works consider the DR as a decision to be taken before any uncertain parameter is realized. Also, most of them do not consider a detailed representation of the transmission system but instead ignore it or use a DCOPF representation. Models that consider a full representation of the transmission system with an ACOPF model will be nonconvex nonlinear optimization problems that are generally extremely hard to solve, with limited or no guarantee of finding a global optimal solution. There is a need for a model that provides a solution accounting for these aspects and that can be solved in a reasonable amount of time.

Table 1: List of contributions of reviewed papers.

Reference	DCOPF	ACOPF	Stochastic Optimization	Robust Optimization	Uncertainty	Demand Response
[19]	X		X		WP	
[40]			X		WP, DR	1st,2nd and 3rd stages
[2]	X		X		Demand	
[38]	X		X		WP, Demand	
[41]	X		X		WP, Demand	
[36]	X		X		WP, Outages, Demand	2nd stage
[17]	X		X		WP	2nd stage
[32]	X		X		Outages, Demand	2nd stage
[29]	X		X		WP	
[12]			X		WP	1st and 2nd stages
[21]	X		X		WP, Outages	
[33]	X		X		WP	
[35]		X	X		WP, PV, DR	2nd stage
[26]	X		X		WP, Demand	2nd stage
[30]	X		X		Outages	1st and 2nd stages
[14]			X		WP	2nd stage
[16]		X	X		WP	
[1]		X	X		WP	
[37]			X	X	Outages, Demand	
[39]	X			X	WP	
[42]	X			X	WP, PV, DR	3rd stage

Furthermore, there is also a need to consider the limitations of the methods to solve nonlinear stochastic optimization problems, which is not explored by the works presented in our literature review. We are particularly interested in the Generalized Benders Decomposition (GBD) algorithm, which allows us to solve convex nonlinear stochastic optimization problems by decomposing them into smaller problems and finding the solution in an iterative fashion. However, it is known that the GBD can possibly have convergence and performance issues. Because the model proposed in this paper is a day-ahead operation planning problem, it needs to be possible to solve it in a 24-hour time frame. This has prompted us to explore methods for accelerating the GBD.

In this paper, we develop a stochastic UC model with the objective of maximizing the profit from supplying energy to meet unexpected demands. Specifically, the unexpected demand is a demand for energy that may appear after determining the operation schedule for the grid, which means that we are interested in using the resources available to supply this unexpected demand. An example of unexpected demand would be when the system operator of one power grid requests energy from a different power grid. In this problem, we consider the renewable generation and the unexpected demand availability as sources of uncertainty, and both the DR use and the UC will be considered as the decisions taken before the realization of the uncertainties. Therefore, our idea is to develop a short-term horizon model, in which we allocate the DR resources and available generation optimally under uncertainty considering all of the aspects of the transmission system topology with the objective of maximizing the generators' profits.

Specifically, it is modeled as a short-term scheduling problem with DR resources, which traditionally uses an AC power flow transmission model to describe the grid's transmission. However, considering an AC power flow leads to a nonconvex optimization model, which is computationally costly to solve, since it is a NP-hard problem ([22]). Therefore, we consider a convex relaxation of the AC transmission model to turn this problem into a tractable one and this allows us to use the GBD algorithm. In other words, we solve large-scale problems in a reasonable amount of time, opening the path to use this model for large-scale networks.

The main contributions of this paper are the following:

- We apply a relaxation of the standard stochastic UC-ACOPF problem, which is a nonlinear stochastic optimization problem, that allows us to have a tractable model. Thus we are able to solve the UC-ACOPF problem taking into consideration wind and solar energy generation, as well as the demand availability uncertainties in a reasonable amount of computational time.
- We use various enhancement methods to improve the performance of GBD for solving our problem. To the best of our knowledge, there have been no such investigations of the use of GBD acceleration methods for the solution of UC-ACOPF models. In this paper, we show the possible time gains from using such methods.

This paper is organized in the following way. In Section 2, we present our proposed model for solving the stochastic UC-ACOPF problem. In Section 3, we present the solution methodology, the GBD and its acceleration methods. Finally, in Section 4, we present the computational experiments in which we analyze the performance of the different acceleration methods. Section 5 summarizes the findings of our work.

2 Model

We consider a stochastic UC model that maximizes the revenue of exporting demand while minimizing the cost of supplying it with extra generation or with DR resources. In our model, we consider that the wind and solar energy generation are uncertain parameters as well as the export demand. Our formulation of the deterministic model is as we now describe.

The objective function (1) considers the cost of the extra generation and DR that may be necessary to meet the external demand, as well as the revenue generated by supplying this external demand.

- Objective function:

$$\begin{aligned}&\min \sum_{t=1}^{|T|}\left[-\sum_{e\in\Phi} r_{et}\Lambda_e^t + \sum_{d\in\Psi} c_{dt}\Gamma_d^t + \right.\\ &\left.\sum_{m=1}^{N}\left(\sum_{j\in U_m} y_{jm}^t c_{jm} + a_j^U\left((T_{jm}^t)^2 - \left(\underline{T_{jm}^t}\right)^2\right) + b_j^U(T_{jm}^t - \underline{T_{jm}^t})\right)\right]\end{aligned} \tag{1}$$

where N is the set of buses, U_m is the set of thermal plants connected to bus m, and T is the set of time periods. We denote by Φ the set of buses that have external demand offers and by Ψ the set of buses that have DR resources. Furthermore, T_{jm}^t is the active power generation, $\underline{T_{jm}^t}$ being its lower bound, Γ_m^t is the amount of DR provided at bus m at time period t, and Λ_m^t is the external demand at time t. There are also the coefficients of the generation cost function, a_j^U, b_j^U, c_{jm}, where the latter is the start-up cost for plant j at bus m, and the variable y_{jm} is a binary variable equal to 1 if plant j at bus m starts up at time period t. Moreover, c_{dt} is the cost of using DR, while r_{et} is the revenue for supplying external demand.

- Active power balance constraint:

$$\begin{aligned}&\sum_{j\in U_m} T_{jm}^t + \sum_{\{m,n\}\in\Omega} I_{mnt}^{pf} + \sum_{\{n,m\}\in\Omega} I_{mnt}^{pe} - G'_m {V_m^t}^2\\ &+\Gamma_m^t - \Delta D_m^t = \Lambda_m^t + D_m^t - W_m^t - FV_m^t \quad \forall m\in N, \forall t\in T\end{aligned} \tag{2}$$

- Reactive power balance constraint:

$$\begin{aligned}&\sum_{j\in U_m} QT^t_{jm} + \sum_{\{m,n\}\in\Omega} I^{qf}_{mnt} + \sum_{\{n,m\}\in\Omega} I^{qe}_{mnt} + B'_m {V^t_m}^2 \\ &+ \Delta Q^t_m = Q^t_m - QW^t_m - QFV^t_m \qquad \forall m\in N, \forall t\in T\end{aligned} \tag{3}$$

where QT^t_{jm} is the reactive power generation; the active and reactive power injections at the "to" point of the branch are, respectively, $I^{pe}_{mnt}, I^{qe}_{mnt}$, and at the "from" point of the branch are $I^{pf}_{mnt}, I^{qf}_{mnt}$, respectively. ΔQ^t_m is the reactive power demand adjustment, and ΔD^t_m is the demand shift caused by the use of DR. V^t_m is the voltage magnitude at bus m at time t, B'_m, G'_m are the shunt susceptance and the shunt conductance, respectively, and D^t_m, Q^t_m are the active and reactive power demands. Finally, we have W^t_m, QW^t_m for the active and reactive wind energy generation, FV^t_m, QFV^t_m for the active and reactive photovoltaic generation. We also have the set Ω of transmission lines.

In (2), we observe the addition of Λ^t_m on the demand side of the constraint. Unlike D^t_m, Λ^t_m is a variable, meaning that the generator can decide how much of the external demand to supply. This decision is directly connected to how profitable it is to supply this extra demand. Otherwise, (2) is a standard power balance constraint, guaranteeing that the generation summed to the eventual energy transmitted or received through transmission lines is equal to the demand.

We note that in (2), the DR is composed of both demand reduction and demand shift actions. To compensate the demand shift that occurs because of Γ^t_m, we have the variable ΔD^t_m to guarantee that any demand that is shifted at time period t will be supplied in another time period.

The reactive power balance constraint (3) has the additional term ΔQ^t_m that adjusts reactive power demand according to the decision to supply a certain amount of the external demand.

- Transmission constraints in rectangular ACOPF form:

$$\begin{aligned}&I^{pf}_{mnt} + \jmath I^{qf}_{mnt} = -\frac{V^t_m}{\Upsilon_{mn}}\left[\left(\jmath\frac{B_{mn}}{2} + Y_{mn}\right)\frac{V^t_m}{\Upsilon_{mn}} - Y_{mn}V^t_n\right] \\ &\qquad \forall\{m,n\}\in\Omega, \forall t\in T\end{aligned} \tag{4}$$

$$\begin{aligned}&I^{pe}_{mnt} + \jmath I^{qe}_{mnt} = -\frac{V^t_n}{\Upsilon_{mn}}\left[\left(\jmath\frac{B_{mn}}{2} + Y_{mn}\right)V^t_n - Y_{mn}\frac{V^t_m}{\Upsilon_{mn}}\right] \\ &\qquad \forall\{m,n\}\in\Omega, \forall t\in T\end{aligned} \tag{5}$$

$$\underline{V^t_m} \le V^t_m \le \overline{V^t_m} \quad \forall m\in N, \forall t\in T \tag{6}$$

$$(I^p_{mnt})^2 + (I^q_{mnt})^2 \le \overline{S^t_{mn}}^2 \quad \forall\{m,n\}\in\Omega, \forall t\in T \tag{7}$$

where $\jmath = \sqrt{-1}$, and B_{mn}, Y_{mn} and Υ_{mn} are the transmission line susceptance, admittance and turns ratio respectively for line mn. Moreover, $\overline{S^t_{mn}}^2$ is the maximum transmission capacity for line mn, and $\overline{V^t_m}$ and $\underline{V^t_m}$ are the maximum and minimum voltage at bus m.

- Thermal plants bounds:

$$\underline{T^t_{jm}}x^t_{jm} \le T^t_{jm} \le \overline{T^t_{jm}}x^t_{jm} \quad \forall m\in N, \forall j\in U_m, \forall t\in T \tag{8a}$$

$$\underline{QT^t_{jm}}x^t_{jm} \le QT^t_{jm} \le \overline{QT^t_{jm}}x^t_{jm} \quad \forall m\in N, \forall j\in U_m, \forall t\in T \tag{8b}$$

- Demand Constraints:

$$0 \le \Gamma^t_d \le \overline{\Gamma^t_d} \quad \forall d\in\Psi, \forall t\in T \tag{9a}$$

$$-\overline{\Delta Q^t_m} \le \Delta Q^t_m \le \overline{\Delta Q^t_m} \quad \forall m\in N, \forall t\in T \tag{9b}$$

$$0 \le \Lambda^t_e \le \overline{\Lambda^t_e} \quad \forall e\in\Phi, \forall t\in T \tag{9c}$$

where $\underline{T_{jm}^t}$ is the minimum active power generation, $\underline{QT_{jm}^t}, \overline{QT_{jm}^t}$ are the maximum and minimum reactive power generation, $\overline{\Gamma_d^t}$ is the upper bound for DR resource allocation, $\overline{\Delta Q_m^t}$ is the maximum reactive power demand adjustment, and $\overline{\Lambda_e^t}$ is the unexpected demand offer. Finally, x_{jm}^t is the generating unit on/off state variable.

The bounds on generation, transmission, demand response and extra demand are enforced in (8)–(12). It should be noted, however, that the generation bounds are dependent on whether the unit has been committed or not.

- Start-up and Shutdown Constraints:

$$x_{jm}^{t-1} - x_{jm}^t + y_{jm}^t - z_{jm}^t = 0 \quad \forall m \in N, \forall j \in U_m, \forall t \in T \tag{10a}$$

$$x_{jm}^t, y_{jm}^t, z_{jm}^t \in \{0,1\} \quad \forall m \in N, \forall j \in U_m, \forall t \in T \tag{10b}$$

- Ramping Constraints:

$$T_{jm}^t - T_{jm}^{t-1} \leq R_{jm}^U x_{jm}^{t-1} + S_{jm}^U y_{jm}^t \quad \forall m \in N, \forall j \in U_m, \forall t \in T \tag{11a}$$

$$T_{jm}^{t-1} - T_{jm}^t \leq R_{jm}^D x_{jm}^t + S_{jm}^D z_{jm}^t \quad \forall m \in N, \forall j \in U_m, \forall t \in T \tag{11b}$$

- Demand Shift Constraints:

$$\sum_{t=24(\omega-1)+1}^{24\omega} (f_\Gamma \Gamma_d^t - \Delta D_d^t) = 0 \quad \forall d \in \Psi, \forall \omega \tag{12}$$

where y_{jm}^t and z_{jm}^t are the start-up and shutdown variables, and S_{jm}^U, S_{jm}^D are the start-up and shutdown rates. Moreover, R_{jm}^U and R_{jm}^D are the maximum ramp-up and ramp-down rate of unit m. f_Γ is the proportion of the demand reduced because of the demand shift. Finally, ω represents a week day.

In (10)–(11), start-up, shutdown, as well as the generation ramping constraints are defined. Finally, the demand shift constraint (12) guarantees that the total demand of the system remains unchanged regardless of the amount of DR used. It also should be observed that in this work we consider only incentive-based DR, where the ISO pays the user to shift or reduce demand.

As it can be seen, the proposed model is a nonconvex mixed-integer optimization problem. The fact that the model is nonconvex makes finding an optimal solution very time consuming and there is no way to guarantee that it will find the global optimum. In order to overcome these issues, we consider the use of semidefinite relaxations. Specifically, we choose to use the Tight-and-Cheap relaxation (TCR) [3] that allows us to find a solution close to the optimal one in a reasonable amount of time. In [3], it has been shown that the optimization gap between the solution given by TCR and the original formulation of the scheduling problem is relatively small.

2.1 TCR relaxation

First, we reformulate the problem by defining $VM^t = V^t(V^t)^H$. With that we can reformulate the transmission constraints and eliminate their non-convexity. The resulting constraints can be seen below:

$$\begin{aligned} &I_{mnt}^{pf} + \jmath I_{mnt}^{qf} = -\frac{1}{|Tn_{mn}|^2}\left(-\jmath\frac{B_{mn}}{2} + Y_{mn}^*\right) VM_{mm}^t \\ &-\frac{Y_{mn}^*}{Tn_{mn}} VM_{mn}^t \qquad \forall \{m,n\}, \in \Omega, \forall t \in T \end{aligned} \tag{13}$$

$$\begin{aligned} &I_{mnt}^{pe} + \jmath I_{mnt}^{qe} = -\frac{Y_{mn}^*}{Tn_{mn}^*} VM_{nm}^t + \\ &\left(-\jmath\frac{B_{mn}}{2} + Y_{mn}\right) VM_{nn}^t \qquad \forall \{m,n\}, \in \Omega, \forall t \in T \end{aligned} \tag{14}$$

$$\underline{V_m^t}^2 \leq VM_{mm}^t \leq \overline{V_m^t}^2 \quad \forall m \in N, \forall t \in T \tag{15}$$

$$M^t = V^t(V^t)^H \tag{16}$$

However, the problem is still nonconvex (and nonlinear) so we replace the constraint $VM^t = V^t(V^t)^H$ by the following constraints:

$$\begin{bmatrix} VM_{11}^t & VM_{1m}^t & VM_{1n}^t \\ (VM_{1m}^t)^* & VM_{mm}^t & VM_{mn}^t \\ (VM_{1n}^t)^* & (VM_{mn}^t)^* & VM_{nn}^t \end{bmatrix} \succeq 0 \quad \forall \{m,n\}, \in \Omega, \forall t \in T \tag{17}$$

thus defining a convex relaxation of the original problem. With a convex problem, it is possible to guarantee the convergence of the solution algorithm towards a global optimal solution.

2.2 Stochastic model

In order to solve this problem, we consider it as a two-stage stochastic optimization problem.

In the first stage, decisions are made about UC and the use of DR resources. The first-stage optimization problem can be seen below:

- Objective function:

$$\max \sum_{t=1}^{T} \sum_{m=1}^{N} -c_{mt}^D \Gamma_m^t - (\sum_{j \in Th_m} y_{jm}^t c_{jm}) \tag{18}$$

- Demand Shift Constraints:

$$\sum_{t=24(\omega-1)+1}^{24\omega} (f_\Gamma \Gamma_d^t - \Delta D_d^t) = 0 \quad \forall d \in \Psi, \forall \omega \tag{19}$$

- Start-up and Shutdown Constraints:

$$x_{jm}^{t-1} - x_{jm}^t + y_{jm}^t - z_{jm}^t = 0 \quad \forall m \in N, \forall j \in Th_m, \forall t \in T \tag{20a}$$

$$x_{jm}^t, y_{jm}^t, z_{jm}^t \in \{0,1\} \quad \forall m \in N, \forall j \in Th_m, \forall t \in T \tag{20b}$$

As it can be seen, the dispatch as well as the offer of external demand are not considered on the first-stage.

In the second stage, the final dispatch and the external demand to be supplied are determined, and an ACOPF transmission system model is used to determine the dispatch. Finally, in the second stage we consider the realizations of wind and solar energy generation as well as the realization of the external demand offered:

$$\max \sum_{t=1}^{T} \left(\sum_{e \in \Phi} r_{et} \Lambda_e^t - \sum_{m=1}^{N} (\sum_{j \in Th_m} a_j^{Th} ((T_{jms}^t)^2) + b_j^{Th} (T_{jms}^t))) \right) \tag{21}$$

subject to (2)–(3), (8a)–(11), (13)–(17) where $s \in S$, and S is the set of scenarios considered in the problem.

In this problem, the UC and DR variables are considered as parameters and the values of the parameters are obtained from the solution of the first-stage. In the case of DR resources, that happens because we do not consider the uncertainty of the DR and we also consider that all of the DR resources purchased through the aggregators have to be guaranteed in the second stage. As a consequence, all of the constraints related to DR resources and unit commitment are not considered.

3 Generalized Benders Decomposition

In order to solve this mixed-integer nonlinear stochastic optimization problem, we use the Generalized Benders Decomposition (GBD), which is a generalization of the Benders Decomposition method for convex nonlinear problems ([13]), that can be also applied to stochastic problems. In general, it allows us to solve a problem in the form of:

$$\begin{aligned} \max \quad & dx + cy \\ s.t. \quad & Ax + Cy - b \geq 0 \\ & x \in X, y \in Y \end{aligned} \tag{22}$$

In this problem, y is a vector of complicating variables, meaning that if we had a fixed value for y, the problem would be much easier to solve. Dividing this problem in two stages, we define the master problem:

$$\begin{aligned} \max \quad & y_0 \\ s.t. \quad & y_0 \leq L^*(y, u^j) \\ & L_*(y, \lambda^j) \geq 0 \\ & y \in Y \end{aligned} \tag{23}$$

where

$$L^*(y, u) = \underset{x \in X}{max} \{dx + cy + u^t\,(Ax + Cy - b)\} \tag{24}$$

$$L_*(y, \lambda) = \underset{x \in X}{max} \{\lambda^t\,(Ax + Cy - b)\} \tag{25}$$

$y_0 \leq L^*(y, u^j)$ is the optimality cut, $L_*(y, \lambda^j) \geq 0$ is the feasibility cut, and j is the number of iterations of the GBD algorithm.

Afterwards, we can define the subproblem as:

$$\begin{aligned} \max \quad & dx + cy_{j-1} \\ s.t. \quad & Ax + Cy_{j-1} - b \geq 0 \\ & x \in X \end{aligned} \tag{26}$$

In order to find the optimal solution the following algorithm was devised:

1. Solve the problem (26) with an user supplied initial solution $\hat{y}$, retrieve the optimal multiplier vector u_0 and create $L^*(y, u_0)$. Store the objective function of (26) as the lower bound (LBD).
2. Solve the master problem, (23), adding either $y_0 \leq L^*(y, u_j - 1)$ or $L_*(y, \lambda^j - 1) \geq 0$ as a constraint to it, and find the optimal solution $\hat{y}^0$. If $\hat{y}^0 - LBD \leq \epsilon$ for a suitably small choice of ϵ, stop the algorithm, the optimal solution has been found. Otherwise, proceed to the next step.
3. Solve the revised subproblem using the master problem solution.
   (a) If the objective function has a finite value and is greater than LBD, update LBD. If $\hat{y}^0 - LBD \leq \epsilon$, the optimal solution has been found, stop the algorithm. Otherwise, retrieve the optimal multiplier vector u_j and create $L^*(y, u_j)$. Go to step 2.
   (b) If the problem is infeasible, determine a λ^j, calculate $L_*(y, \lambda^j)$ and go to step 2.

Unfortunately, the GBD algorithm has performance issues due to the master problem being an integer problem. When solving its linear relaxation, there is also the same issue ([24]). In order to be able to attain an acceptable performance for our algorithm, it will be necessary to use different enhancement techniques, which are aimed at either choosing good cuts, providing a stronger formulation for our problem or avoiding solving the integer master problem several times. These techniques are presented in the coming subsections.

It should be noted that the impact of the aforementioned methods can only be measured experimentally.

3.1 DCOPF inequalities

In our model, the first-stage problem's constraints provide little information to find the optimal solution. As a consequence, it may take an enormous amount of time for the GBD algorithm to converge and find the optimal solution. The addition of new constraints that give us more information about our problem and, consequently, improve its lower bound, becomes necessary.

We can improve our first-stage problem by adding the constraints for determining the scheduling with a DCOPF model. With this addition, there will be more information for determining the unit commitments in the first-stage. The following constraints are added to our model:

- Power balance constraint:

$$\sum_{j \in Th_m} T^t_{jm} - \sum_{\{n,m\} \in \Omega} I^p_{nmt} + \Gamma^t_m - \Delta D^t_m = D^t_m \quad \forall m \in N, \forall t \in T \tag{27}$$

- Transmission constraints:

$$I^p_{mnt} = B_{mn}(\theta^t_n - \theta^t_m) \quad \forall \{m,n\}, \in \Omega, \forall t \in T \tag{28a}$$

$$-\overline{P^t_{mn}} \leq (I^p_{mnt}) \leq \overline{P^t_{mn}} \quad \forall \{m,n\}, \in \Omega, \forall t \in T \tag{28b}$$

- Thermal plants bounds:

$$\underline{T^t_{jm}} x^t_{jm} \leq T^t_{jm} \leq \overline{T^t_{jm}} x^t_{jm} \quad \forall m \in N, \forall j \in Th_m, \forall t \in T \tag{29}$$

- Demand Constraints:

$$0 \leq \Gamma^t_d \leq \overline{\Gamma^t_d} \quad \forall e \in \Phi, \forall t \in T \tag{30}$$

When adding these constraints, we consider the scheduling costs, as well, and we change the objective function accordingly, as can be seen below:

$$min \sum_{t=1}^{T} \left(\sum_{d \in \Psi} c_{dt} \Gamma^t_d + \sum_{m=1}^{N} (\sum_{j \in Th_m} y^t_{jm} c_{jm} + a^{Th}_j \left(T^t_{jm}\right)^2 + b^{Th}_j T^t_{jm}) \right) \tag{31}$$

However, we now have a mixed-integer nonlinear optimization problem, which is computationally costly. If we can replace the current objective function with a linear one, the problem becomes considerably easier to solve because it becomes a mixed-integer optimization problem (MILP). Thus, we linearize the objective function by representing it as a piecewise linear function as seen in [11, 31, 20, 18, 8].

In our first-stage problem we have, for the generation cost, the following function:

$$f(T_i, x_i) = aT_i^2 + bT_i + cx_i \tag{32}$$

We choose $k+1$ points in the interval $[\underline{T_i}, \overline{T_i}]$, $T^0_i, T^1_i, \ldots, T^k_i$, and propose a piecewise linear function in which its linear functions are interpolations of the chosen points two by two, giving us an upper approximation. We also create k new variables named δ^i_{ls} that represent each segment of the piecewise linear function. With that, we can create the constraints and modify the objective function:

- Constraints:

$$T_i = \sum_{ls=1}^{k} \delta^i_{ls} + \underline{T_i} x_i \quad \forall i \tag{33}$$

$$0 \leq \delta^i_{ls} \leq T^{ls}_i - T^{ls-1}_i \quad \forall i, \forall ls \tag{34}$$

- We define the linear cost for each segment as:

$$F^i_{ls} = \frac{f(T^{ls}_i) - f(T^{ls-1}_i)}{T^{ls}_i - T^{ls-1}_i} \tag{35}$$

- Objective function:

$$\sum_{i=1}^{N}(f(\underline{T_i})x_i + \sum_{ls=1}^{k} F^i_{ls}\delta^i_{ls} \tag{36}$$

In this way, the quadratic objective function is approximated as a linear one and our first stage problem becomes a MILP, which we can solve in a reasonable amount of time.

3.2 Benders Based Branch-and-Cut

Solving a stochastic MILP can be very computationally costly, principally when there are many integer variables in the problem. Its linear programming (LP) relaxation, however, is significantly faster to solve, and, according to [25], all of the optimality and feasibility cuts generated when solving the LP version of our problem are valid for our original problem.

Based on that, the Benders Based Branch-and-Cut algorithm was proposed, in which we first solve the stochastic optimization problem at the root node to optimality. Afterwards, we build the branch-and-cut tree and add new cuts to the pool whenever we reach a node that gives us an integer solution. According to [25], all cuts generated by solving the sub-problems, no matter what is the given first-stage problem, are global cuts. If the node becomes infeasible or does not give us an integer solution after the addition of new cuts, it is pruned from the tree. The branch-and-cut tree is explored until we find an optimal solution ([27]). We use this algorithm to improve the performance of GBD.

3.3 Mixed-Integer Rounding cuts

When using the Benders Based Branch-and-Cut approach, we initially solve the root node problem, which is a LP relaxation. However, the cuts that are generated by this relaxed problem do not take into consideration that some of our variables are integer, which can lead to a weak LP relaxation. Because of that, we want to generate constraints that will add integrality restrictions to our problem, which we call Mixed-Integer Rounding (MIR) cuts [4].

In each iteration of the GBD, we generate a Benders cut, $y_0 \geq a - \sum_{i=1}^{N} b_i y_i$. Let $y_0 \geq a^{it} - \sum_{i}^{N} b^{it}_i y^{it}_i$ denote the last generated Benders cut. Define $f_0 = \beta(a - a^{it}) - \lfloor \beta(a - a^{it}) \rfloor$ and $f_i = \beta(b_i - b^{it}_i) - \lfloor \beta(b_i - b^{it}_i) \rfloor$, where β is a parameter that respects the condition $0 < \beta \leq 1$. With that we can build the following cut:

$$y_0 \geq d_0 - \sum_{i=1}^{N} d_i y_i,$$

where

$$d_0 = a^{it} + \frac{f_0 \lceil \beta(a - a^{it}) \rceil}{\beta}$$
$$d_i = \frac{\min\{f_0 \lceil \beta(b_i - b^{it}_i) \rceil, f_i + f_0 \lfloor \beta(b_i - b^{it}_i) \rfloor\}}{\beta} + b^{it}_i.$$

Because this cut is generated by combining the last generated cut with another existing cut, we have to choose which of the potential cuts is the best suited for the problem. Therefore, we verify which of the generated cuts is the most violated by the solution given in the last iteration and we choose it as the MIR cut. Our criterion for making this choice is the scaled violation defined as:

$$\frac{\max\{d_0 - \sum_{i=1}^{N} d_i y^{it}_i - y_0, 0\}}{\| (1, d) \|_2}$$

where $d = (d_0, ..., d_N)$. Finally, we choose the cut that give us the maximal scaled violation.

It should be noted that in [4], the proposed family of cuts is shown to be valid inequalities, and, as such, they will always contribute to the convergence of GBD.

3.4 Pareto Optimal cuts

In [24], the authors propose a method for finding the best possible cut at each iteration of the GBD algorithm. We state that a cut $f(x,y) + u_1^t g(x,y)$ dominates another cut $f(x,y) + u_2^t g(x,y)$ when $f(x,y) + u_1^t g(x,y) \geq f(x,y) + u_2^t g(x,y)$, and we define it as a nondominated cut or Pareto optimal (PO) cut when no other cut dominates it. When a cut dominates another cut, we say that it is a stronger cut and that it contributes more to the convergence of the GBD algorithm.

Let us define a core point y_0 as a point such that it is in the relative interior of the convex hull of the master problem solution space. In order to find such a cut, we need to find a core point of the master problem, which will be used to solve a modified version of the dual of the sub-problem. First, we define the sub-problem:

$$\begin{aligned} \min \quad & dx \\ s.t. \quad & Ax = b - Cy \\ & x \in X \end{aligned} \tag{37}$$

Here, we define $\hat{y}$ as the optimal solution of the master problem and $v(\hat{y})$ as the value of the objective function of the subproblem when considering $\hat{y}$. Then, we can solve the following optimization problem to find the PO cut:

$$\begin{aligned} s.t. \quad & uA \leq d \\ & u(b - C\hat{y}) = v(\hat{y}) \\ & x \in X \end{aligned} \tag{38}$$

However, finding y^0 and solving the proposed optimization problem is often difficult. Because of that, [28] proposes an enhancement to the method proposed in [24]. Instead of solving the problem initially proposed, we solve the problem with the constraint $u(b - C\hat{y}) = v(\hat{y})$ removed. In [28], the author proves that this new problem also generates a PO cut. Furthermore, to avoid the cost of finding a core point each time we want to find a PO cut, we find an initial core point y^0 and use the equation $\bar{y} = \frac{y_0 + \hat{y}}{2}$ to update the core point in each iteration ([28]).

4 Computational results

4.1 Test network

In order to analyze the performance of the enhancement techniques that we have chosen to improve the convergence speed of GBD, we apply our proposed method to the IEEE RTS-96 test network.

The IEEE RTS-96 is a 73 bus-system that can be divided into 3 zones with the same number of buses, except for the last zone, which has one more. We consider a one-week time horizon with 168 hourly time periods. We took the data for this case study from [15] but made small changes to the generators' installed capacity, bus demands, load profile, and operating costs. Furthermore, the load profile data for the period was taken from [5] taking into consideration the number of buses in our case study.

We also made some changes to the demand and load profile of some buses. Specifically, in our study, buses 317, 318 and 321 have demand of 160 MW, 403 MW and 220 MW respectively. Concerning the load profile, a value of 0.06 was subtracted from all buses for hours 13 and 14.

Regarding the generators, we increased the installed capacity by 21% for all buses except those shown in Table 2. In this table, we have the generation data for the plants that have had their installed capacities modified. We also note that we added wind or solar energy generation at some of the buses; the generation capacity and type of plant added on each of these buses is reported in Table 3.

Table 2: Generators installed capacity.

Generator	Capacity(MW)
121	464
123-1	179.8
123-2	179.8
123-3	406
218	580
221	580
223-1	643.8
318	139.2
321	255.2

Table 3: Generation capacity for wind and solar plants.

Node	Capacity Installed (MW)	Energy Source
103	150	Solar
105	50	Wind
108	150	Solar
206	100	Wind
209	150	Wind
211	250	Solar
219	300	Wind
221	50	Solar
223	600	Wind
316	120	Wind

Besides that, DR can be activated in all buses with active demand greater than 0 in zones 1 and 2, and in the buses 314, 318 and 321, being limited to a maximum of 10% of the demand with a cost of $25.55 per MWh. Finally, buses 106, 112, 119, 120, 319, and 320 will offer the possibility of supplying extra demand up to a maximum of 18% of the demand. Node 317 offers this possibility as well, but up to a maximum of 200 MW. All of them offer a revenue of $85.55 per MWh.

4.2 Algorithms performance comparison

The proposed model was implemented using Julia 1.2.0 and we have used CPLEX 12.10.0.0 to solve our problems, adding the optimality and feasibility cuts in the Benders based Branch-and-cut tree through the use of callbacks. We used a PC with an Intel Core i7-9750H CPU 2.60 GHz and 16 GB of RAM memory to solve all of the problem's instances.

For our analysis, we solved our model using this test network considering 5 and 20 scenarios, so that we can see how our performance is impacted by the size of the problem. That will also give us the opportunity to analyze the impact of the different enhancements methods in the Benders based Branch-and-cut performance in regards to the size of the problem.

We set the maximum number of iterations on the root node to 500 and we imposed a time execution limit of 24 hours. Thus, if the algorithm is able to find an integer solution with an integrality gap of less than 1.00% in less than 24 hours, its execution is stopped. Finally, we add MIR cuts only at the root node. The MIR cuts are added after every three iterations, since our tests have shown that adding them more frequently does not help in finding an integer solution faster and it also slows down the convergence of the algorithm.

The computational results from our experiments are summarized in Tables 4 and 5. They were obtained by solving several instances of the problem for the two cases of 5 and 20 scenarios. In each of these tables, we have the information about the best, worst and average time of execution to find the optimal solution. In addition, we also have the number of iterations at the root node, Benders decomposition gap and integrality gap, which are the columns It, S-Gap and I-Gap, respectively, for the instance with the smallest execution time.

Table 4: Results for the 5 scenarios instances, with a limit of 500 iterations in the root node.

PO	MIR	DC	Avg. time (s)	Min. time (s)	Max time (s)	S-Gap	I-Gap	It
✓	✓	✓	16307	5714	41648	0.97%	0.96%	32
✗	✓	✓	54177	36999	86400	0.99%	1.00%	57
✓	✗	✓	42177	16393	77408	0.97%	0.99%	30
✓	✓	✗	42993	12993	76092	0.93%	0.94%	91

Table 5: Results for the 20 scenarios instances, with a limit of 500 iterations in the root node.

PO	MIR	DC	Avg. time (s)	Min. time (s)	Max time (s)	S-Gap	I-Gap	It
✓	✓	✓	39835	6353	86400	0.70%	0.84%	28
✗	✓	✓	74863	28719	86400	0.82%	2.13%	68
✓	✗	✓	61208	27476	86400	0.86%	1.00%	30
✓	✓	✗	59506	15419	86400	0.58%	0.96%	59

Analyzing the results, we can conclude that all of the proposed acceleration methods for the Benders decomposition contribute positively to improve its performance. Considering the average time of execution for the instances with 5 and 20 scenarios, we can see that the PO cuts is the method that has the most impact on GBD's performance. Also, we can see that it has a significant impact on the convergence of the root node problem. We observe the same thing when analyzing the impact of the DCOPF constraints: there was a bigger impact in the solution time of the root node problem, which translates in less iterations, and little change on the time spent exploring the B&C tree. Finally, we note that the MIR has significant impact in the time spent exploring the B&C tree, as well as a negligible impact on the time to find a solution for the root node.

Besides that, although all of the acceleration methods have a significant impact on the execution time, the addition of the DCOPF constraints has the biggest impact. Also, when solving the various instances, we were able to conclude that the improvement in the execution time by using MIR cuts is highly dependent on the value of β. It is necessary to test various values of β to possibly find one such that it improves the performance of the GBD algorithm. However, it should be noted that even if the chosen β does not improve performance, the quality of the solution found is not affected, it will merely take more time to find the optimal solution.

When analyzing the impact of the number of scenarios on the solution time, we can see that when we use all of the available acceleration methods, the impact of considering more scenarios on the performance is minimized. When any of the acceleration methods is not considered, there is a significantly larger impact on the performance, which is also dependent on which method is not used. Also, we can observe that both the PO cuts and the DCOPF constraints have a more significant impact on the time of solution of the root node problem, due to their impact on the number of iterations. Finally, we also see that the PO cuts also have a larger impact in the performance of the Benders based branch-and-cut algorithm.

Finally, computational experiments with a version of the code without PO, MIR and DCOPF on a subset of instances show that these improvements, in fact, reduce solution times for both 5-scenarios and 20-scenarios instances. More specifically, it reduces the solution time, on average, by 74.15%, from $24,841$s to $6,421$s, for the instances with 5 scenarios and by 74.84%, from $60,324$s to $15,175$s, for the instances with 20 scenarios. Other experiments that involve using only a subset of the proposed enhancement methods did not yield results that were as clear: sometimes CPU times are improved, sometimes they deteriorate.

5 Conclusion

In this paper, we proposed a model that maximizes the profit of supplying external demand using an ACOPF model with DR under uncertainty. We have also proposed the use of several different methods to improve the performance of the GBD algorithm in order to solve our problem in a reasonable amount of time. We were able to observe that the proposed acceleration methods were successfully able to improve GBD's performance significantly. That allowed us to solve instance with more scenarios and it also enables us to solve instances with larger power grids.

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