Les Cahiers du GERAD ISSN: 0711–2440

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G–2024–28

April 2024

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2024
– Bibliothèque et Archives Canada, 2024

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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Joint optimization of electric bus scheduling and fast charging infrastructure location planning

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April 2024
Les Cahiers du GERAD
G–2024–28

Abstract : Transit authorities are rapidly replacing conventional buses with electric ones because of the increasing concerns about air quality, greenhouse gas emissions, and energy demand. Many mathematical optimization models have been developed for scheduling conventional buses. However, such models would not fit electric buses (EBs) due to their limited travelling range and long charging time. Moreover, such operational differences have prompted new research into the literature on the charging station location problem for EBs. This study combines EB scheduling with fast-charging infrastructure location planning with the objective of minimizing the total costs of scheduling, including deadhead trips, electricity and ownership costs of EBs, as well as the cost of establishing fast-chargers. We propose a Mixed-Integer Linear Programming (MILP) formulation for an arc-based model and an Integer Linear Programming (ILP) formulation for a path-based model and solve them with Cplex solver and branch-and-price algorithm, respectively. The two solution approaches have been tested for various instances with different numbers of trips and potential charging locations. The computational experiments show that the branch-and-price algorithm is more computationally efficient in terms of execution time compared to the arc-based model solved with Cplex. Finally, a sensitivity analysis was conducted to identify the most cost-effective EB type, considering the real characteristics of different EB types. Moreover, we assessed how changes in battery capacity and the maximum travel range of EBs impact the optimal solution.

Keywords : Transportation, electric bus scheduling, charging station location, charging scheduling, branch-and-price

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Acknowledgements: This research was undertaken, in part, thanks to funding from the Canada Excellence Research Chairs Program with grant number CERC2018-00005.

Declaration : Declaration of competing interest: none

1 Introduction

Energy sources that release greenhouse gases are a major contributor to global warming. Therefore, replacing such energy sources with clean and renewable ones has become vital in recent decades. Among the various ways to mitigate the impact of greenhouse gases, electrifying vehicles has gained attention as a particularly effective measure due to increasing concerns about the environment, air quality, and energy consumption. Thus, there has been a significant increase in the utilization of electric vehicles and electric buses. According to the recent research by Bloomberg New Energy Finance Electric, EBs are projected to replace more than 47% of the world's city bus fleet by 2025 (An, 2020). The European electric bus market is projected to increase by 18.6% between 2022 and 2027, indicating a shift towards electric buses from conventional buses.

Transit authorities are increasingly focused on enhancing the efficiency of their public bus transit systems and one of the most important factors affecting that is operational processes (Kang and Meng, 2017). Therefore, optimizing bus operation planning has become essential to reduce operational costs and decrease passengers waiting time. Bus scheduling is one of the most vital processes in bus operations and has a considerable impact on operational costs. Bus/vehicle scheduling (VS) is the process of assigning buses to the trips of a given timetable and aiming to use the minimum number of buses while minimizing operating costs. EBs have battery capacity limitations and their scheduling and charging processes differ from those of conventional buses (Yao et al., 2020). The operating range of EBs is less than conventional buses and their charging time is longer than refueling diesel buses. Li (2014) demonstrates how the introduction of electric buses affects the bus scheduling procedure.

The limited travelling range of EBs has prompted new research into the literature, which is the problem of locating charging stations for electric buses. This problem identifies the ideal places for installing fast-charging infrastructures on the bus transportation network, as well as the optimal number of such stations to minimize the total establishment costs. Public transportation agencies have adopted fast-charging (FC) technology using high-voltage electricity to address the challenges of long charging times and limited driving range of electric buses. FC technology allows EBs to recharge in just a few minutes. Bus transit systems that use fast-charging technologies are gaining popularity. Also, charging stations at bus terminals are less costly than depot charging and better suited to bus electrification throughout the life cycle (Lajunen, 2018). However, using such technology makes EBs scheduling more complex (He et al., 2020). For example, the location of fast chargers affects the travelling time of charging trips of electric buses; hence, EB scheduling should be customized to consider this charging trip time. Also, the time of charging due to the limited availability and capacity of charging stations should be coordinated. Thus, as highlighted by Bie et al. (2021), a reasonable charging strategy is necessary to improve bus scheduling, and it is crucial to study the fast-charging infrastructure location planning problem for electric buses.

Transit authorities opting for fast-charging solutions will employ pantograph technology capable of delivering up to 350 kW of power. Although initially more costly, pantograph systems could ultimately decrease the total cost of ownership (TCO) and operational expenses for EBs as they allow continuous opportunity charging along routes. This reduces the need for large battery capacities and energy consumption. Moreover, pantograph charging enables an automatic connection from the bus roof, which can be integrated into existing bus networks by installing fast chargers at particular stops. This integration supports continuous bus operation with lower maintenance costs and higher availability, allowing buses to run around the clock (Al-Saadi et al., 2022). This mechanism is a well-established method for linking electric buses with their power sources. A special inverted pantograph automatically lowers when a bus arrives at the charging station. After completing the safety checks, the system provides the bus with a swift and powerful recharge.

Transitioning from traditional buses to electric buses requires further research on public transit operation planning steps. These steps are network route design, timetable development, vehicle scheduling, and crew scheduling. First, the network route design outlines the bus routes and their bus stops based on the regions' geographical layout and travel demand. Next, the timetable development sets bus departure frequencies in order to cover the travel demand of passengers. In the vehicle scheduling step, buses will be efficiently assigned to bus routes based on the timetables. Finally, the last step determines the schedule for bus drivers. The limited range and longer charging time of EBs necessitate a reassessment of these steps. The majority of studies discuss public transportation operation planning in a sequential manner, where the output of one planning phase is used as input for the next stage. This approach is less effective compared to a fully integrated strategy. The fully integrated method analyzes the problem as a whole, considering all phases of public transportation planning simultaneously. For instance, making minor adjustments to the bus timetable might lead to better vehicle scheduling, and locating fast-charging infrastructure based on bus schedules could lower the operational costs of bus scheduling. Thus, in this research, we aim to integrate charging station location planning and bus scheduling steps of electric bus transit operation planning to improve the overall efficiency of the transit system.

In the network design step, the travel demand of each region is analyzed to design the bus routes and their bus stop locations based on the geographical features and travel demand of that region. It also includes determining the length of bus routes ensuring the routes are of a length that EBs can complete their cycles without battery depletion. Finding the best spots to locate charging stations or continuous charging infrastructures is another aspect of the network design step for EBs. Also, this step includes choosing the most suitable charging technology based on the EBs' specifications and bus network features. The vehicle or bus scheduling step involves assigning the buses to each route based on the vehicle's range, charging requirements, given timetable, and route's length. The aim is to cover all the given trips in timetables with minimum operational costs. In this step, planners seek to minimize the total number of required EBs, total deadhead trips, and charging costs while covering all the given trips. Also, The charging scheduling of EBs will be analyzed in this step.

In this research, we aim to integrate the network design step and vehicle scheduling step of public bus transit operation planning, which, in our case, are fast-charging location planning and electric bus scheduling (FCLP-EBS), respectively. We seek to minimize the total operational costs, including deadhead trips, electricity and ownership costs of EBs while covering all the given trips. Also, we find the best locations and the optimum number of fast chargers to be placed on the bus network to meet the charging demand for EBs with minimum cost. By doing so, we improve the overall efficiency and performance of operating EBs and fleet management.

We present two mathematical optimization models for the problem: an arc-based model (ABM) and a path-based model (PBM). The former is solved using the Cplex solver, while the latter employs the branch-and-price algorithm.

The contributions of this study are as follows:

- Proposing the integrated problem of fast-charging infrastructure location planning and electric bus scheduling, while addressing the charging scheduling of electric buses, taking into account the capacity of fast-charging stations.
- Developing an exact branch-and-price (B&P) algorithm to solve the FCLP-EBS problem.
- Performing sensitivity analysis on different EB types based on original equipment manufacturers' real data and analyzing the added cost due to the increase in the battery capacity.

The remainder of this article is structured as follows. A literature review on charging station location planning for EBs and electric bus scheduling problems is presented in Section 2. In Section 3, we provide a detailed description of the fast-charging location planning and electric bus scheduling problem. Section 4 describes the proposed models, including arc-based and path-based models. This section also provides an overview of the branch-and-price algorithm used to solve the path-based model. Then, Section 5 reports computational results. Finally, conclusions are drawn in Section 6.

2 Literature review

The following sections will provide a brief overview of the related works and theoretical background of charging station location planning and EB scheduling. Section 2.1 describes the different charging station location models found in the literature. In Section 2.2, we review the literature on electric bus scheduling problems and different methods, including exact and heuristic solution approaches, adopted to solve such problems. Finally, the integration of charging station location and EB scheduling problems are briefly discussed in Section 2.3.

2.1 Charging station location

Fast-charging for electric buses is becoming more common. Due to this growth, extensive research is required to determine the best locations for installing charging station infrastructure along bus routes. Kunith et al. (2014) conducted one of the initial studies on the planning of fast-charging stations. The study aimed to identify the optimal locations for fast-chargers throughout the network, taking into account operational restrictions and serving the required daily energy demand of EBs. For the same problem, Wu et al. (2021) added the power distribution network and bus operation network. Their proposed approach aimed to reduce the overall installation costs of fast-chargers, as well as their operation and maintenance costs, travel costs to the chargers, and power loss costs. The affinity propagation approach was used to determine the number of fast-charging infrastructures needed, and binary particle swarm optimization was utilized to determine the best places for chargers and their maximum capacity.

Enhanced heuristic descent gradient is another optimization technique to address fast-charging infrastructure location planning. This technique was employed by Othman et al. (2020) to identify the best spots for fast-charger locations based on the given timetable. Csonka (2021) considered partial charging for EBs, i.e. how much to charge during each charging event. The author determined the best places to locate charging infrastructures to meet the total energy demand of EBs by solving the problem of both static and dynamic charging technologies. Conductive charging stations and overhead wire charging lanes are referred to as static charging and dynamic charging, respectively.

A mixed-integer linear programming approach was suggested by Kunith et al. (2017) to jointly optimize the battery capacity for each line and location planning of the fast-charging infrastructure for electric buses. The optimal places and best number of chargers were found using a set covering problem. He et al. (2019) solved this problem by considering an added energy storage system (ESS) to store the energy during off-peak hours and sending it to fast-charging infrastructure during peak hours. Comparing the findings of this study to those of Kunith et al. (2017) study yielded a 9.2% reduction in the overall system costs. Zhou et al. (2023) aimed to determine the optimal number and types of EB chargers required at bus terminals and depots. The objective was to fulfill the daily charging needs of electric buses efficiently while also minimizing the overall cost. To address this, the authors developed a two-stage stochastic programming model that handles uncertainties related to travel time and battery degradation.

In another study, Olmos et al. (2019) looked into the problem of locating opportunity charging infrastructure for hybrid and fully electric buses to cover the required charging activities that fulfill the energy demand of EBs. Determining the size of energy storage systems and the power rates of opportunity charging infrastructures were the other two objective functions in their research. Berthold et al. (2017) studied the charging station location problem along with battery aging and partial charging. The goal was to decrease overall costs, including the price of installing charging stations and buying new buses. Liu et al. (2018) also considered uncertain energy consumption for battery-electric buses for this problem. To determine the lowest overall implementation cost, the authors proposed a mixed-integer linear programming model based on a robust optimization methodology. The optimal capacity for EBs (Kunith et al., 2017), constructing transit route networks (Zhang et al., 2021), and finding out the charging schedule for each fast-charging facility (He et al., 2020) are other extensions of this problem.

In earlier literature, most studies treat the energy demand for EBs as a constant. Furthermore, these studies assume that the bus schedule is predetermined. Consequently, the primary focus was identifying the best locations for charging stations and their optimum number, ensuring EBs can complete their trips based on fixed schedules. However, our research introduces flexibility by treating the bus schedule as a variable that could be changed according to the charging station locations and the number of such facilities. Thus, this research seeks to find the optimum solution for the fast charging station location planning while optimizing the scheduling of electric buses. This integrated approach promises more effective solutions than the traditional sequential methods where the EB schedules are set in advance.

The combination of the opportunity charging location problem and the charging scheduling problem of EBs was investigated in a recent work by Hu et al. (2022). They took into account electricity prices based on time of usage and included passenger waiting times associated with the charging procedure during trips as a penalty cost. The authors sought to decrease the cost of procuring opportunity chargers and batteries for electric buses, as well as the overall cost of charging and the amount of extra waiting time for passengers. A robust optimization strategy was developed to deal with trip time uncertainty and passenger travel demand. The main difference between Hu et al. (2022) and our study is that we consider the EB scheduling and number of required buses to cover all the trips, while they do not consider these features. A summary of relevant studies with more detailed information is provided in Table 1. This table allows readers to compare different approaches and methodologies used in FCLP problems, highlighting the diversity and innovation in solving these problems with different objective functions.

Table 1: Studies on electric buses fast-charging location planning

Paper	Objective(s)	Fast-charger Location	Model	Algorithm	Case Study
Kunith et al. (2014)	Min construction cost	Depot	MILP	Standard solver	-
Kunith et al. (2017)	Min the total cost and number of chargers	Depot	MILP	Standard solver	Berlin, Germany
Liu et al. (2018)	Min the cost of installing FC and batteries	Bus stop	MILP	AARC	Utah, USA
He et al. (2019)	Minimizing the total cost of installing FC, ESS, and EB batteries	Bus stop	MILP	Standard solver	Utah, USA
Lin et al. (2019)	Min the total operating, establishing, and grid power loss costs	Depot	MISOCP	Spatial-temporal approach	Shenzhen, China
Liu and Ceder (2020)	Min the required number of EBs and FC infrastructure	Terminal	DF and IP	Adjusted max-flow	Singapore
Othman et al. (2020)	Min the operational costs and energy consumption	Specific locations	EHDG	Voronoi diagram	Toronto, Canada
Zhang et al. (2021)	Min the total costs, passengers' travel time, and operator cost	Terminal	MINLP & MILP	Modified GA	Swiss
Wu et al. (2021)	Min maintenance, FC station construction, travel to charging stations costs and power loss of FC	Terminal	BPSO	Mathematical program	Yangjiang, China
Li et al. (2022)	Min the deadhead trips and charging services	Terminal	MILP	Standard solver	Chengdu, China

2.2 Electric bus scheduling

Vehicle scheduling (VS) is the process of assigning buses to the routes of a given timetable and the timetable is the input for the vehicle scheduling problem. VS aims to cover the given timetable efficiently while satisfying all operational constraints by finding the necessary number of buses and reducing the deadhead trip costs. The problem of electric bus scheduling across multi-depots falls within the category of NP-hard problems Carpaneto et al. (1989). A well-known approach to model vehicle scheduling problems was introduced by Bertossi et al. (1987) and is known as the time-space network framework. In this framework, nodes show the trips and their starting and ending times, while arcs show the routes taken by vehicles to get from one node to another.

Rinaldi et al. (2018) investigated the electric bus scheduling problem with service factor and charging factor limitations in order to find the sequence of electric and hybrid buses departing from a multi-line bus terminal. They proposed a mixed-integer linear program (MILP) to reduce total operating costs. Tang et al. (2019) addressed the electric bus scheduling problem by developing static and dynamic scheduling models to minimize total operating costs by implementing a buffer-distance approach and rescheduling buses based on current traffic conditions. The authors used a branch-and-price method to tackle the problem. This is one of the few publications that dealt with the EB scheduling problem using an exact solution technique. Alwesabi et al. (2020) investigated the EB scheduling problem by reducing the total cost by determining the optimal number of electric buses while considering the number of charging stations and battery size constraints.

2.2.1 Heuristic solution approaches

One of the earliest investigations on scheduling electric buses with a constrained travel range and a constrained recharge period was conducted by Wang and Shen (2007). The aims were to reduce the number of buses and the time spent on deadhead trips. The authors used the ant colony optimization (ACO) technique to tackle the problem. The charging, scheduling, and operation of electric buses and conventional buses for four bus routes were discussed by Paul and Yamada (2014). They intended to increase the overall travel distance of the EBs while minimizing CO_2 emissions and fuel costs. By using a real use-case in Japan, the authors used a k-greedy algorithm to solve the suggested problem and validate it. Scheduling problems for electric buses could also be resolved using simulation models. Sung et al. (2022) aims to reduce the cost of charging stations, batteries, and buses and power usage. To accomplish this, a simulation model and heuristic algorithm were created. The optimal number and type of buses and charging stations are the primary findings of this study. Various metaheuristic solution techniques were combined to establish the ideal bus departure timings. Ke et al. (2022) combined genetic algorithm (GA), Particle Swarm Optimization (PSO), and Simulated Annealing (SA) methods to reduce both energy costs and overall greenhouse gas emissions. For a heterogeneous bus fleet (conventional and electric buses), joint optimization of EB scheduling and crew scheduling was investigated in Wang et al. (2022a).

Another method for solving the vehicle scheduling problem is the adaptive genetic algorithm (AGA). This approach was applied by Li et al. (2020) to address the integration of stationary charger deployment and electric bus scheduling for a real-world case study in Anting Town, Shanghai. They investigated partial charging and time-varying energy pricing and tried to reduce the overall construction and maintenance costs of charging infrastructure and bus scheduling operational costs. In order to determine the optimal electric buses departure intervals, Guo et al. (2022) looked at uncertainties in the number of arriving passengers, their wait time, and the energy consumption of electric buses. Based on an uncertain bi-level programming model (UBPM), the problem was solved. The higher level tries to reduce the cost of transportation for the passengers, while the lower level seeks to reduce how much energy is used by electric buses. A real-world case study was solved using GA. The objective function also includes reducing in-service costs and electricity usage. Yao et al. (2020) conducted research on a multi-vehicle form of electric bus scheduling in 2020. The authors sought to lower operational costs associated with deadhead trips while also decreasing the required investment for purchasing EBs and charging infrastructures. A heuristic strategy was used to solve the NP-hard problem in a reasonable amount of time.

In a multiple bus line transit system with a partial charging policy, Liu and Ceder (2020) proposed a bi-objective integer programming method to reduce the number of electric buses and fast-charging infrastructures. The authors proposed two approaches to address this problem: a lexicographic approach and a modified max-flow technique. They examined their model in a real-world case study in Singapore. Zhou et al. (2020) investigated the combined optimization of electric bus scheduling and charging scheduling. This research developed a multi-objective bi-level programming model for reducing carbon emissions and operating costs, such as passenger trip costs, deadhead trip costs and power consumption costs. To tackle the electric bus scheduling and charging scheduling problem, an iterative neighborhood search and a greedy dynamic search technique were used.

2.2.2 Exact solution approaches

Alwesabi et al. (2020) investigated the electric bus scheduling problem by reducing total cost, determining the optimal number of electric buses, by considering the number of charging stations and battery size constraints. Li (2014) looked into the scheduling of EBs that used fast-charging or battery swapping technologies. He also investigated this problem using a restricted trip range form for buses powered by various sources of energy. For these problems, mixed-integer programming methods have been presented. Because the battery swapping time is almost equivalent to the fast-charging time, the author asserts that the suggested methodology will be used for both charging modes. The column generation technique was employed to solve this scheduling problem. Rinaldi et al. (2018) investigated the scheduling problem of electric buses with service factor and charging-factor constraints in order to determine the order of EBs and hybrid buses departing from a multi-line bus terminal. A mixed-integer linear program to reduce total operating costs was proposed to solve the problem.

Two methods for battery-electric bus scheduling with a limited-travel-range limitation were presented by van Kooten Niekerk et al. (2017). The first one is a simplified model that assumes buses can be charged linearly, ignoring the time-of-use (TOU), and ignoring the effects of depth-of-discharge (DOD). The second one modified those assumptions to make the model more realistic. The proposed model for this problem was solved for small and medium-sized problems in an acceptable amount of time using integer linear programming. The authors offered two more strategies based on the column generation method for finding near-optimal solutions to large-scale problems. According to Wang et al. (2017), the travel range constraint of electric buses will be addressed and removed using the optimal recharging approach. Thus, the study aimed to determine the best strategy for charging EBs. Finally, the authors applied their approach to a real-world case study in California. Avishan et al. (2023) studied the electric bus scheduling under energy consumption and travel time uncertainty. The objectives were to find the best number of required buses, determine the optimum schedule and find the best strategy for charging EBs. The authors developed a MILP model and solved it through a robust optimization approach.

Table 2 provides an overview of the objective functions of numerous bus scheduling studies and the different main features of the studied problems. In this table, each row represents a research work. The second column represents the objectives that the studies seek to reach, which are the number of buses used to cover all the trips, deadhead trip costs, and electricity cost of operating EBs, respectively. The next two columns indicate whether the problems were considered on either single or multi-depot and single or multi-line bus networks. The "Veh. type" column specifies whether the considered fleet is homogeneous or heterogeneous. Finally, the last two columns present the models and the solution approaches adopted to solve EB scheduling problems. This table reveals that scheduling electric buses involves multiple factors, highlighting the complexity of the problem. It shows that research in this area must consider diverse operational, economic, and infrastructure considerations to create effective urban bus systems. It indicates that most of the studies on EB scheduling used heuristic or metaheuristic solution approaches to deal with the problem. Additionally, it is evident that MILP models are the most commonly used for modelling various types of EB scheduling problems.

Table 2: Studies on electric buses fast-charging location planning. DH: Deadhead trips, EC: Electricity Cost, S/M-D: Single/Multi Depot, S/M-L: Single/Multi Line, Veh. type: Vehicle Type

Paper	Objective			S/M-D	S/M-L	Veh. type	Model	Method
	No. buses	DH	EC					
Li et al. (2019)	✓		✓	Multi	Multi	Het[1]	ILP	TSE[4] network
Zhang et al. (2020)	✓			Single	Single	Het	MIP	GA
Yao et al. (2020)	✓	✓	✓	Multi	Multi	Het	MILP	GA
Teng et al. (2020)	✓		✓	Single	Single	Het	MIP	Multi-objective PSO
Zhou et al. (2020)		✓	✓	Single	Multi	Het	Bi-level programming	Iterative Neighborhood Search
Liu and Ceder (2020)	✓			Multi	Multi	Hom[2]	DF[3] and IP	Adjusted max-flow
Li et al. (2020)	✓		✓	Single	Multi	Hom	Nonconvex model	AGA
Bie et al. (2021)	✓		✓	Single	Single	Hom	ILP	B&P
Sung et al. (2022)	✓		✓	Multi	Multi	Hom	Simulation	Heuristic
Wang et al. (2022a)	✓			Single	Multi	Hom	MILP	MILP
Jiang and Zhang (2022)	✓	✓	✓	Multi	Multi	Hom	MILP	B&P
Zhang et al. (2022)	✓		✓	Multi	Multi	Het	MILP	ALNS
Guo et al. (2022)			✓	Single	Single	Hom	UBPM	GA
Wu et al. (2022)		✓		Single	Multi	Hom	MILP	B&P
Gkiotsalitis et al. (2023)	✓			Multi	Multi	Hom	MILP	B&C[5]
This study	✓	✓	✓	Multi	Multi	Hom	MILP	B&P

[1] Heterogeneous
[2] Homogeneous
[3] Time-Space-Energy
[4] Deficit Function
[5] Branch-and-Cut

2.3 Integrated fast-charging station location and bus scheduling

The integration of fast-charging infrastructure site planning with electric bus scheduling has not received much attention in the literature. The study by Stumpe et al. (2021) was the most comparable. The authors investigated the simultaneous optimization of opportunity charging location planning and electric bus scheduling. They suggested an innovative mixed-integer linear formulation and used variable neighborhood search to solve it. Wang et al. (2022b) addressed joint optimization of battery electric bus scheduling, pantograph charger location planning, and battery capacity. The goal was to minimize the overall cost of the fleet, and they developed a mixed-integer linear programming model to address the problem. Tzamakos et al. (2023) investigated fast-wireless-charging infrastructure location planning, taking into account the impact of delays caused by buses lining up to charge at charger locations. They proposed a MILP approach to reduce the total costs of installing wireless charging.

Olsen and Kliewer (2022) studied the combination of depot charging planning with EB scheduling. The goal was to reduce the total cost, which included the operational costs, costs of installing depot chargers, and vehicle costs. To address this problem, a metaheuristic solution method based on variable neighborhood search (VNS) was proposed. They proved that optimizing these two problems simultaneously outperforms the sequential planning approach. Li et al. (2019) investigated the multidepot and multi-vehicle form of this problem for refueling charging stations. The authors sought to reduce the number of buses and refueling stations necessary, as well as energy consumption, maintenance costs, and external emission costs. To handle small-scale problems, the authors presented an arc-based model, and to tackle large-scale problems, they suggested a time-space bus flow network. Alwesabi et al. (2021) provided a mixed-integer linear programming formulation for determining the battery capacity, optimal fleet size, and dynamic wireless charging locations all at the same time.

The study by Liu and Ceder (2020) addressed the combination of electric bus scheduling with the optimization of charging infrastructure. However, they focused on depot charging and did not address fast-charging infrastructures. Thus, with the rapid increase in fully electric buses, an investigation on the integration of bus scheduling and fast-charging infrastructure location problems is necessary.

Although electric bus scheduling, charging scheduling, and charging station location problem have been the subject of various studies, there is still more work to be done to bridge the gap between theory and practice. If we explore the literature, we can find that no work has been done on the influence of electric bus scheduling on the location of fast-charging infrastructure and vice versa. There are currently few papers that deal with the location problem of fast-charging infrastructures. Based on these papers, the charging station location problem is treated as an individual optimization problem.

3 Problem description

This paper addresses the joint optimization of fast charging station location planning and electric bus scheduling. The aim is to schedule a homogenous fleet of EBs that start their trip from multiple depots and return to the same depots after completing their daily trips with minimum total operational costs and simultaneously find the best locations for fast-chargers to be placed on predetermined spots on the bus network to satisfy the energy demand of EBs. Since EBs have a limited travel range, they need to be charged at charging stations frequently throughout the day to complete their daily trips. The objective is to minimize the required number of EBs needed to fulfill the given trips and their total operational costs by reducing their deadhead trips and electricity costs while minimizing the total installing cost of fast chargers. Figure 1 represents a schematic view of the problem.

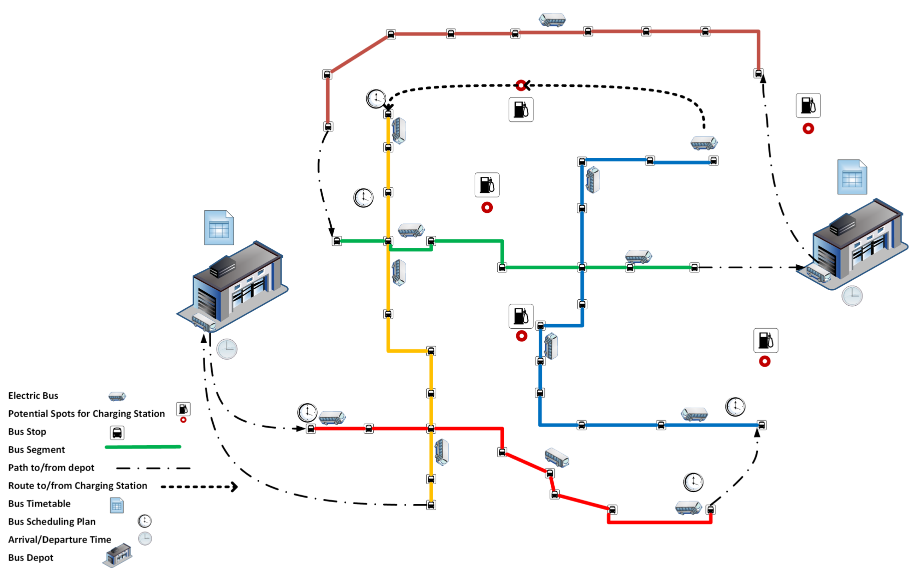

Figure 1: A schematic view of the problem

Henceforth, the term *trip* will refer to the scheduled timetable for each bus route. Take, for instance, a bus route that begins its first journey at 06:00 AM with a frequency of 30 minutes until 08:00 AM, with each trip lasting 45 minutes. For this bus route, there would be a series of six trips, starting at 06:00, 06:30, 07:00, 07:30, and 08:00, and correspondingly ending at 06:45, 07:15, 07:45, 08:15, and 08:45. For the FCLP-EBS problem, we are given a set of trips denoted as S and a set of potential locations for locating fast-charging stations as B. Each trip in S has a starting time a_i and an ending time e_i. It's important to note that each bus can only be assigned to one trip at a time. An EB can start a new trip if it finishes its previous one with enough time to travel to the starting point of the next trip as a deadhead trip and still has enough energy. Deadhead trips refer to the transit between trips, in which buses travel without any passengers. They occur mainly to relocate buses for their next trip. Deadhead trips can be from the depots to bus routes and vice versa or between charging stations and bus routes and vice versa.

Let d_{ji} denote the deadhead trip distance from j to i, where j is either a depot, a charging station, or the end location of a trip, and i is either a depot, a charging station, or the beginning location of a trip, plus the trip distance of trip i, which is denoted by $\tilde{d}_i$ if $i \in S$. Similarly, let τ_{ji} denote the travel time from j to i plus the travel time of trip i, which is denoted by Δ_i if $i \in S$. Let d_{max} indicate the maximum distance that a fully charged battery can travel. The unit cost of a vehicle's travelled distance is represented by c_d, while c_w stands for the cost of the vehicle's waiting time. Meanwhile, the cost of charging at a fast-charging station at time t is indicated by C_t, and C_v denotes the fixed annual cost of each EB.

Electric buses starting from a depot fully charged can operate along a series of feasible connecting trips. They can cover these trips as long as they have enough energy to return to either the same depot or a charging station. At each charging station, at most W, buses can be charged simultaneously and we suppose a fixed charging time of U for each charging event. For each $b \in B$, let V_b be the establishment cost of a fast-charger at location b.

The aim of this paper is to find the set of feasible connecting trips in order to cover all the given trips with the minimum total costs. The cost function in the objective function includes three main components. The first component is the total annual purchase cost of the required number of EBs. The second component is the annual cost of establishing fast chargers along the network. Finally, the third component is the operational costs of EBs, which are divided into two components. The annual travelling costs and electricity costs due to charging the EBs. The first one is the sum of the travelling costs of EBs from depots to the trips and vice versa, from trips to trips, and from trips to the charging stations and vice versa. The electricity cost is the cost of charging electric buses based on the given electricity tariff.

4 Methodology

This section explains the fast-charging location planning and electric bus scheduling (FCLP-EBS) problem in more detail and describes the methodology used to solve the problem. Section 4.1 presents an arc-based mathematical formulation and Section 4.2 presents a path-based model for FCLP-EBS problem. Although the arc-based formulation can be solved using standard integer programming techniques, the path-based formulation cannot due to its high number of variables. We propose a branch-and-price algorithm to solve the path-based formulation in Section 4.3.

4.1 Arc-Based Model (ABM)

Consider $G = (N, A)$ as the bus network, where N is the trip nodes and A is the feasible connecting arcs. The set of *origins* and *destinations* nodes linked with the depots are labelled as O and D, respectively. The set of nodes N includes the EB trips, the depots, and the time-expanded nodes of charging stations.

We model the charging station capacity, which is denoted by W as time-expanded station nodes, by discretizing the range of service start times into a set of discretized nodes. One minute is specified as the sample time step. The time period set T consists of $\{1, 2, \ldots, |T|\}$, for the planning horizon for charging electric buses at charging stations and T' be the set of time periods, $\{1, 2, \ldots, |T| - U\}$, from the first period to the maximum time period reduced by the fixed charging time of electric buses. The set of time-expanded nodes of a potential spot $b \in B$ for locating a charging station is denoted by T_b.

The set of arcs A is divided into five categories. There are depot-to-trip arcs between O and $j \in S$. There are trip-to-trip arcs between $j \in S$ and $i \in S$ if $a_j + \Delta_j + \tau_{ji} \leq e_i$. There are trip-to-charger arcs from $j \in S$ to time-expanded node t for potential spot $b \in B$ if $a_j + \Delta_j + \tau_{jt} \leq |T| - U$, and charger-to-trip arcs from time-expanded node t for potential spot $b \in B$ to $j \in S$ if $t + \tau_{bj} \leq e_j$. Finally, there are trip-to-depot arcs between $j \in S$ and D. A' is the set of all possible arcs between trips and charging stations and vice versa. The set of candidate locations for locating fast-chargers denoting by B. The network used in the arc-based formulation is presented in Figure 2. The cost of arc ji is defined as: (1) $c_{ji} = c_d d_{ji} + C_v$ if $j = O$; (2) $c_{ji} = c_d d_{ji} + c_w w_{ji} + C_t$ if $i \in B$; and (3) $c_{ji} = c_d d_{ji} + c_w w_{ji}$ for other arcs.

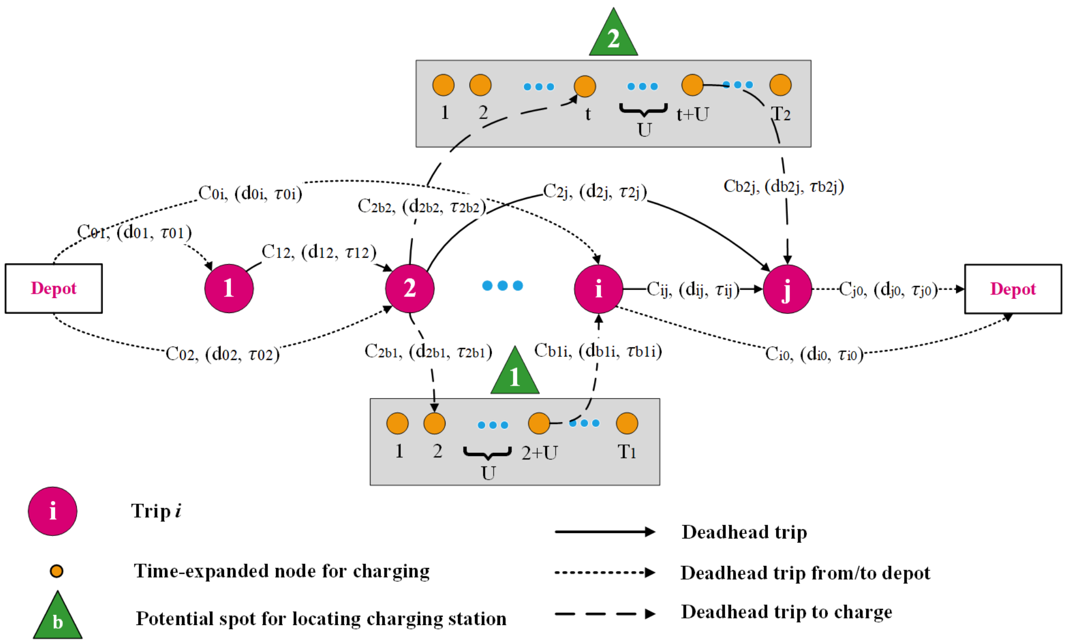

Figure 2: Arc-based network

We now present an arc-based formulation of the integrated EB scheduling and fast charging station location planning problem. The decision variables for the problem are as follows. Let z_b be a binary decision variable with $z_b = 1$ if a fast-charger is located at location $b \in B$, and $z_b = 0$ otherwise. Let x^o_{ji} be a binary decision variable, with $x^o_{ji} = 1$ if an electric bus is assigned to node i immediately after trip j starting at depot $o \in O$, and $x^o_{ji} = 0$ otherwise. Lastly, g^o_i is a continuous decision variable, representing the accumulative distance travelled from depot o to the ending point of trip $i \in S$ since the latest battery renewal.

The arc-based model can be formulated as a mixed-integer programming model as follows:

$$\min \left(\sum_{o \in O} \sum_{(j,i) \in A \setminus A'} c^o_{ji} x^o_{ji} + \sum_{o \in O} \sum_{(j,t) \in A'} c^o_{jt} x^o_{jt} + \sum_{b \in B} V_b z_b \right) \tag{1}$$

s.t.

$$\sum_{o \in O} \sum_{\substack{j \\ (j,i) \in A}} x^o_{ji} = 1 \qquad \forall i \in S \tag{2}$$

$$\sum_{\substack{i \\ (j,i) \in A}} x^o_{ji} - \sum_{\substack{i \\ (i,j) \in A}} x^o_{ij} = 0 \qquad \forall j \in S, \forall o \in O \tag{3}$$

$$\sum_{\substack{j \\ (j,t \in T_b) \in A'}} x^o_{jt} = \sum_{\substack{j \\ (j,t \in T_b) \in A'}} x^o_{tj} \qquad \forall b \in B, \forall o \in O \tag{4}$$

$$g_i^o = \sum_{\substack{j \\ (j,i)\in A\setminus A'}} (g_j^o + d_{ji})x_{ji}^o \qquad \forall i \in S, \forall o \in O \tag{5}$$

$$g_t^o \leq (1 - x_{jt}^o)d_{max} \qquad \forall (j,t) \in A', \forall o \in O \tag{6}$$

$$g_i^o \leq d_{max} \qquad \forall i \in S \cup T, \forall o \in O \tag{7}$$

$$\sum_{o\in O} \sum_{\substack{i \\ (i,t\in T_b)\in A'}} x_{it}^o + \sum_{o\in O} \sum_{\substack{i \\ (i,t\in T_b)\in A'}} \sum_{s=1}^{U} x_{it}^{o,t+s} \leq W \qquad \forall b \in B, \forall t \in T' \tag{8}$$

$$\sum_{\substack{i \\ (i,t\in T_b)\in A'}} x_{it}^o \leq z_b \qquad \forall b \in B, \forall o \in O \tag{9}$$

$$x_{ji}^o \in \{0,1\} \qquad \forall (j,i) \in A, \forall o \in O \tag{10}$$

$$z_b \in \{0,1\} \qquad \forall b \in B \tag{11}$$

$$g_j^o \geq 0 \qquad \forall j \in S, \forall o \in O \tag{12}$$

The objective function (1) includes three terms. The first term calculates the total travelling and waiting costs between trips plus the annual purchase cost of each electric bus, the second term determines the charging cost and travelling cost between trips and charging stations, and the last term indicates the total establishing cost of fast-chargers. Constraints (2) and (3) represent the covering and flow conservation of trips. Constraint (4) ensures that the inbound and outbound trips to/from a charging station should be equal. Constraint (5) determines the accumulative distance travelled since the latest battery renewal. This particular constraint introduces nonlinearity to the mathematical model. A linear version of our model can be found in the appendix. Constraint (6) ensures that when an EB recharges at a station, its total travelled distance resets to zero. Constraint (7) forces that the total travelled distance of each EB cannot exceed its maximum travel range. Constraint (8) ensures that the station capacity constraint must be satisfied. Constraints (9) ensure that EBs can be charged only at the locations where a charger has been installed.

4.2 Path-Based Model (PBM)

In this section, we convert the arc-based model to a path-based model in which the number of variables is huge and the number of constraints is reasonable. This makes the optimization model capable to exploit the branch-and-price algorithm.

We denote P as the set of all possible paths from O to D. Such paths should satisfy the maximum distance constraint prior to battery renewal. Each path $p \in P$ has an operational cost of c_p. Let $\varpi_{ji}^p = 1$ if arc $(j,i) \in A$ is in path p, otherwise $\varpi_{ji}^p = 0$; thus, $\sum_{(j,i)\in A} \varpi_{ji}^p c_{ji} = c_p$. Let $\delta_i^p = 1$ if trip $i \in S$ is covered by path p, and otherwise $\delta_i^p = 0$. Let $\sigma_{tb}^p = 1$ if path p is recharging at time t at station b, otherwise $\sigma_{tb}^p = 0$. Let binary variable $y_p = 1$, if path p is selected in the solution, otherwise $y_p = 0$. The path-based formulation of FCLP-EBS problem is as follows:

$$\min \left(\sum_{p\in P} c_p y_p + \sum_{b\in B} V_b z_b \right) \tag{13}$$

s.t.

$$\sum_{p\in P} \delta_i^p y_p = 1 \qquad \forall i \in S \tag{14}$$

$$\sum_{p\in P} y_p \delta_t^p \leq z_b \qquad \forall b \in B, \forall t \in T_b \tag{15}$$

$$\sum_{p\in P} \sigma_{tb}^p y_p \leq W \qquad \forall b \in B, \forall t \in T' \tag{16}$$

$$y_p \in \{0,1\} \qquad \forall p \in P \tag{17}$$

$$z_b \in \{0,1\} \qquad \forall b \in B \tag{18}$$

The objective function (13) aims to minimize the total operational costs of EBs and the total establishment costs of fast-chargers. The first term calculates the cost of selected paths, and the second term calculates the cost of installing the required number of fast-chargers. Constraint (14) ensures that each trip is served once. Constraint (15) states that to charge an EB at a charging station, a charger should have been installed at that point. Constraint (16) ensures that the charging station capacity limitation must always be met.

4.3 Branch-and-price

Branch-and-price is a powerful optimization method used to solve combinatorial optimization problems, which involve a large number of variables and constraints. This method combines two well-known techniques: branch-and-bound and column generation.

Column generation is an optimization method used to solve large-scale linear programming problems. This technique involves iteratively generating columns, also known as variables, by solving the *Pricing Problem* and adding them to the *Restricted Master Problem* (RMP) until an optimal solution is reached. The RMP is the linear relaxation of the master problem, restricted to only some of its columns. Column generation is particularly useful for problems with a large number of variables, where generating all possible columns upfront is computationally infeasible.

To generate these new columns, a pricing problem, which is typically formulated as a shortest path problem or a knapsack problem, is formulated and solved to find the best columns to add to the master problem and the optimization process continues. The master problem formulation includes all previously generated columns and the new ones, and it is solved again using the optimization algorithm. This process of generating new columns and adding them to the problem formulation continues until there are no other columns that can be added to master problem and improve the solution. In this case, the optimum solution is reached. Figure 3 depicts the column generation algorithm.

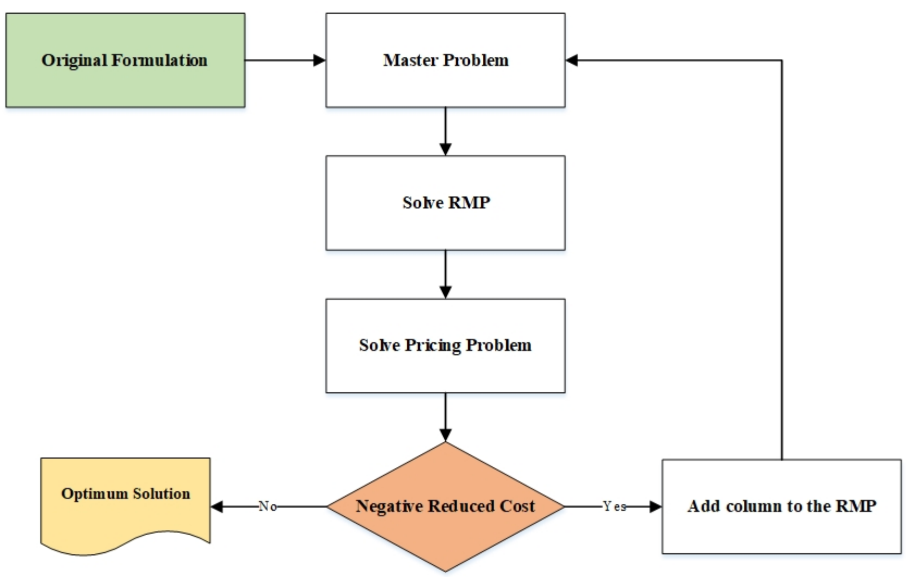

Figure 3: Column generation flowchart

The linear relaxation of the path-based model mentioned in the previous section is the RMP in the CG method. The restricted master problem (RMP) of the master problem in which P is replaced by

$\bar{P}$, which is a subset of paths or columns. By iteratively solving the RMP and a pricing subproblem, the proposed path-based model will be solved through the column generation algorithm. It is not guaranteed that the optimal solution obtained is integral even if the linear relaxation of the path-based model is solved to optimality via column generation. Thus, we develop a branch-and-price algorithm to integrate the column generation algorithm into a branch-and-bound search framework.

The idea behind branch-and-price is to start with a small subset of the problem variables and solve it using branch-and-bound, dividing the problem into smaller subproblems and recursively solving them. A depth-first search strategy is used to explore the branch-and-bound tree. At each node in the search tree, the linear relaxation is solved using CG to obtain a lower bound on the objective function. If the RMP at a given node is infeasible or produces an integer solution, the node is pruned. Otherwise, the branch-and-bound algorithm selects a variable with a fractional value and creates two subproblems by branching on that variable. The algorithm ends when all nodes have been explored.

To apply the branch-and-price technique, we model the pricing subproblem as a constrained shortest-path problem on an acyclic network. This subproblem network is similar but not identical to the network in the path-based formulation and is described as follows. Consider $G^s = (N^s, A^s)$, which N^s stands for the node sets and A^s for the arc sets. There is one *origin* node for each depot $o \in O$ where EBs initiate their journeys and one *destination* node for the same depot but with a different notation of $d \in D$ where EBs finish their daily trips. The set of nodes contains *origin* nodes, O, where EBs initiate their journeys and *destination* nodes, D, where EBs finish their daily trips; One *trip* node for each trip in S; and for each fast-charging location, we create two sets of node: there is one *time-expanded* node in T per each fast charger signifying available charging intervals at one-minute samples, and one *dummy* trip node per trip in S which is a replica of the actual trips to retain the network's acyclic nature. The set of all dummy nodes is denoted by S' and the set of dummy nodes associated with each fast charger $b \in B$ is denoted by S'_b. The set of arcs contains four different sets: beginning and ending arcs, deadhead trips, dummy arcs, and waiting time arcs for fast chargers. The *beginning* trip arcs connect the depots, O, to the trip nodes, S, to start a path for an EB and the *ending* trip arcs connect each trip node to the destination depots, D to finish the path for that EB. Deadhead arcs are divided into two groups: First, the arcs linking *trip nodes to each other*. Second, the arcs linking *trip nodes to the time-expanded nodes* of fast chargers. Let t^e_{ib} be the earliest time an EB can be at charging station $b \in B$ after completing trip $i \in S$. This deadhead arc links trip node i to the time node of station b that corresponds to t^e_{ib}. Dummy arcs are divided into two groups: First, the arcs linking *time-expanded nodes to the dummy trips* associated with each charging station time-expanded nodes. These dummy arcs link the time-expanded nodes, $t \in T_b$, of each fast charger, $b \in B$, to all dummy trip nodes, $i \in S'_b$, of the corresponding fast charger. Second, the dummy arcs linking *dummy trips to the trip nodes*. The final set illustrates the waiting times at each charger, sequentially connecting time-expanded nodes to indicate one-minute waiting periods at charging stations.

In this network, as shown in Figure 4, the charging stations are depicted by a block containing a group of two sets of node types: time-expanded nodes and dummy trips. This is created to model the charging scheduling of EBs at these stations based on the capacity of the charging stations. The time-expanded nodes indicate the arrival time of EBs at each charging station. Also, the waiting time of EBs at charging stations could be calculated using time-expanded nodes in the given network structure. The purpose of including dummy trips in the network is to eliminate cycles and make it possible to solve efficiently in the Gencol library. When trips involve travelling to charging stations and returning, it produces a cycle in the network, which we avoid by adding dummy trips.

Our problem has two limited resources, which are *time* and *travel distance* since the last trip node or time-expanded node. Traversing arcs in the network consumes resources. Every constrained resource at each node must lie within a specified range, known as the resource window. Let R represent the set of resources. For each node $i \in N^s$ and each resource $r \in R$, the resource window is given by $[l^r_i, u^r_i]$. As an illustration, the time resource window at a trip node $i \in S$ is set to $[e_i, e_i]$. For all nodes (except the origin and destination nodes), the distance resource window is $[0, D]$. While, for

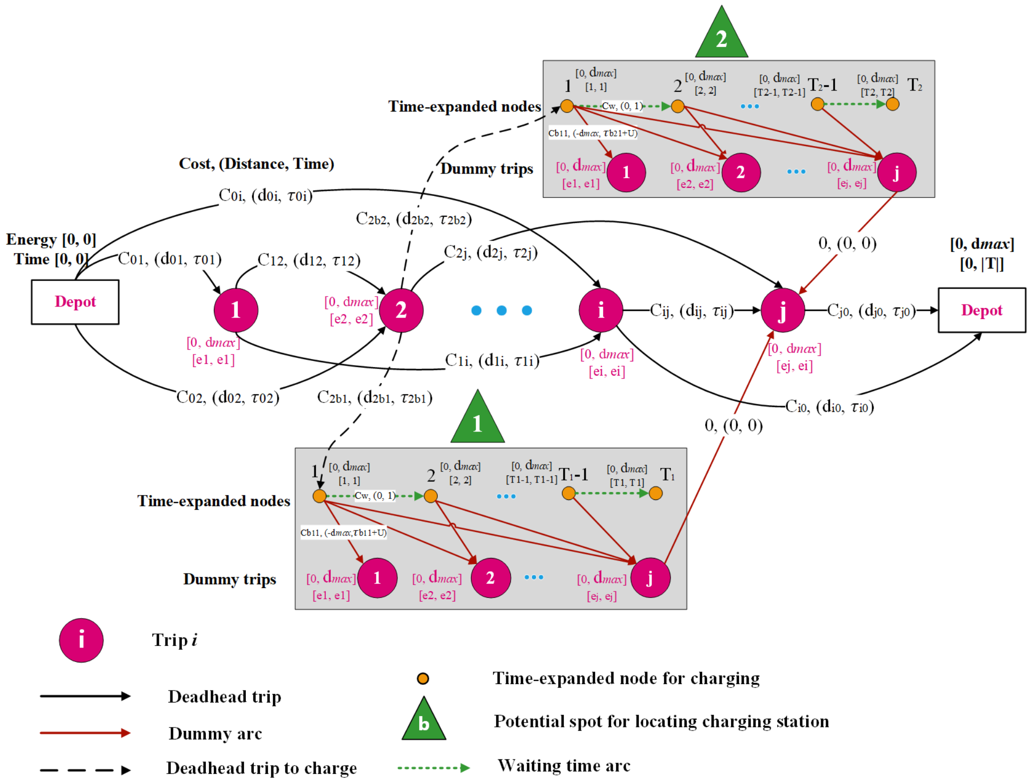

Figure 4: Acyclic network of the problem to deploy branch-and-price method

the origin node, the range is fixed at $[0, 0]$, implying that all EBs begin their journey fully charged. In our network, time resource windows are categorized based on node types. Trip nodes and dummy trips have a time window matching their ending time, $[e_i, e_i]$, meaning each trip must be completed by its designated end time, if not earlier. Note that the lower bound for such nodes is soft, meaning the nodes can be visited even when the arrival time falls below the lower bound. In that case, the value of the resource gets updated to the lower bound. On the other hand, nodes associated with fast charging stations, termed as time-expanded nodes, indicate when an EB can arrive for charging. For instance, a time-expanded node for fast charging at a specific time t has a time window of $[t, t]$. It is noteworthy to mention that the time window for the destination nodes is $[0, |T|]$.

Each arc has three components. The first component is the reduced cost of travelling through that arc. This cost, $\bar{c}_{ji}$, depending on the origin and destination nodes, includes the travelling, waiting time, and electricity costs of EBs and dual variables. For example, the reduced cost of dummy arcs from the time-expanded nodes to the dummy trips is the reduced cost of travelling from that charging station to the dummy trip plus the electricity costs of charging the EB. However, the reduced cost of the second type of dummy arcs, which connect the dummy trips to the trips, is zero. Secondly, we consider the time resource and, finally, the travel distance resource used by each arc. For instance, travelling from a depot to a specific trip consumes a certain amount of time (usage from the time resource) and a specific distance (usage from the travel distance resource). The only exception to positive resource usage is when travelling from charging stations to dummy trips. In such cases, the travel range usage is set as $-d_{max}$, ensuring that the EBs be recharged completely before continuing their journey. It is important to highlight that all the resource windows for nodes and resource usages for arcs should be integers that we could use the Gencol library to apply the branch-and-price algorithm.

The dual variables associated with the restricted master problem are outlined as follows. λ_i^1 are the vector of dual variables associated with constraints (14), λ_b^2 are the vector of dual variables associated with constraints (15) and λ^3_{bt} are the vector of dual variables associated with constraints (16). Thus, the reduced cost of path p is as follows:

$$\bar{c}_p \equiv c_p - \sum_{i \in S} \delta^r_i \lambda^1_i + \sum_b \sum_t \sigma_t b^r \lambda^2_b + \sum_b \sum_t \sigma_t b^p \lambda_b t^3 \tag{19}$$

The reduced cost of path p is decomposed to the reduced cost of the arcs composing that path. The reduced cost of arc $\bar{c}_{ji}$, then equals: $c_{ji} - \lambda^1_i$ if $i \in S$; $c_{ji} + \sum_t b^r \lambda^2_i + \sum_t b^p \lambda_i t^3$ if $i \in B$; and c_{ji} if $i \in D$. The pricing subproblem, in our case, is the problem of finding a route with the minimum reduced cost from the origin depot $o \in O$ to the destination depot $d \in D$. The pricing subproblem can be classified as a type of constrained shortest-path problem, and this particular problem has been shown to be NP-hard by Garey Michael and Johnson David (1979) in 1979. Thus, it can be concluded that the suggested pricing subproblem is also NP-hard.

5 Results and analysis

We present the results for different sizes of the problem and show the performance of the proposed method on the developed model in Section 5.1. Then, in Section 5.2, a brief sensitivity analysis on the critical parameters will be done to check the validity of the model and observe the impacts of changing such parameters' values on the different terms of the objective function. The arc-based model was solved using the C++ programming language and IBM ILOG CPLEX Optimization Studio. The experiments were conducted on the Intel Core ™ i7-8750 CPU 2.20 GHz processor with 16GB of RAM using a single thread. To solve the path-based model, we utilized the specialized Gencol library version 4.5, which is specifically designed for the branch-and-price algorithm in a Linux computer with an Intel i7-8700 CPU clocked at 3.20 GHz without parallel computing and compared with the results of Cplex in terms of execution time.

5.1 Instance description and parameter settings

For our computational experiments, we used the larger instances of the benchmark dataset introduced by Carpaneto et al. (1989) in 1989 in order to investigate the computational complexity of our approach. Most of the parameters to generate different instances with various scales are collected from Li (2014) and Gkiotsalitis et al. (2023).

The starting and ending points of the trips, the coordinates of the depots, and the coordinates of potential spots for locating fast-chargers are located in a 10 km to 10 km square on the Euclidean plane, with an even distribution. The travel time between any two points is proportional to the Euclidean distance between them, and assuming that, on average, a vehicle can travel 60 km in an hour. Based on Bloomberg New Energy Finance (2018), the capital cost of pantograph chargers and electric buses are $230,000 and $530,000, respectively. The lifetime of both EBs and pantograph chargers is 15 years. Thus, the annualized purchase cost of an EB and annualized installation cost of fast chargers will be around $35,333 and $15,333, respectively. The actual cost may vary depending on the location where the charger is installed, and it will be multiplied by a factor within the range of [1, 1.25]. According to Li (2014), the cost of waiting time is 0.1$/min , and the unit cost of EBs travelled distance is 6.61935 cent/km. Also, the maximum travel range of EBs is assumed to be 70 km. This information and other required parameters are given in Table 3. It is important to note that the operational costs shown in Table 3 will be multiplied by the number of days in a year. This will convert them into annual costs, allowing for a direct comparison with the yearly expenses of acquiring a new bus and installing a fast-charging station.

We solved instances of different sizes for two depots. For each instance, we list the number of trips, potential spots to locate fast-chargers and the number of time-expanded nodes as the available time to charge with pantograph chargers. The number of trips, potential spots for establishing charging

Table 3: Parameters settings for generating random instances

Parameter	Description	Value	Reference
C_v	Annual cost of using a new bus	\$ 35,333	Bloomberg New Energy Finance (2018)
V_b	Annual cost of establishing fast-chargers	\$ 15,333	Bloomberg New Energy Finance (2018)
c_w	Cost of waiting time	0.1 \$/min	Li (2014)
U	Charging time	10 min	Li (2014)
c_{ji}	Unit travel cost	0.06 \$/km	Li (2014)
d_{max}	Maximum travel range of EBs	70 km	
T	Number of time periods	$\max_{i\in S}\{e_i\}$	

stations, and time periods, are represented by S, B, and T, respectively. For example, S10_B2_T250 indicates that there are ten trips, two potential places to install fast-chargers, and 250 number of time-expanded nodes. 21 instances have been generated with this representation. Instances are categorized based on the number of trips : those with fewer than 30 trips are labelled as small, instances with 30 or more trips are categorized as medium-scale, and, finally, instances that involve more than 50 trips are classified as large-scale problems.

The results of arc-based model solving with Cplex and the results of path-based model solving using the branch-and-price algorithm are presented in Table 4. The operational costs (OC) column indicates the total travelling cost, waiting time cost and cost of charging. The next column shows the total annual installation cost (IC) of fast chargers, and the next two columns present the objective value and execution time, respectively. The objective value includes the total operational costs, electricity costs, and the annual cost of purchasing EBs and installing fast-chargers.

Table 4: Comparison of the results of Cplex and branch-and-price method

Intance	No. chargers	No. EB	OC (\$)	IC (\$)	Obj (\$)	CPU (s)	
						Cplex	Branch-and-Price
S10-B3-T250	1	4	16549.1	15564	173445.1	1.5	0.1
S10-B5-T250	1	4	15556.3	15680	172568.3	2.3	0.2
S10-B8-T250	1	4	15687.7	15680	172699.7	4.3	0.7
S20-B3-T300	1	7	34572.8	15564	297467.8	84.8	1.7
S20-B5-T300	1	6	35441.5	15708	263147.5	32.5	1.2
S20-B8-T300	1	6	35353.9	15708	263059.9	65.4	4.1
S30-B3-T404	2	8	51866.5	34374	368904.5	117.5	2.8
S30-B5-T404	2	8	52764.4	31272	366700.4	205.4	16.4
S30-B8-T404	1	8	54268.2	16811	353743.2	255.9	76.5
S40-B3-T404	3	10	65440.8	53242	472012.9	91.8	9.3
S40-B5-T404	3	10	65448.2	50198	468976.1	193.7	31.7
S40-B8-T404	3	10	64867.8	50543	468740.8	393.9	152.8
S50-B3-T444	1	14	85873.5	15564	596099.5	604.3	53.3
S50-B5-T444	1	14	82730.9	15564	592956.9	615.2	124.2
S50-B8-T444	2	12	80409.5	31272	535677.5	1256.8	553.0
S60-B3-T444	2	15	95432.9	34374	659801.9	###	394.9
S60-B5-T444	2	15	95630.0	31272	656897.0	###	781.8
S60-B8-T444	2	15	91888.7	32836	654719.7	###	5043.5
S70-B3-T444	3	16	106937.7	46781	719046.7	###	564.2
S70-B5-T444	4	16	106839.1	65820	737987.1	###	12194.4
S70-B8-T444	3	16	106061.7	49999	721388.7	###	37105.6

Comparing the results of Cplex and branch-and-price methods shows that the proposed branch-and-price algorithm for the path-based model improves the computational performance compared to the arce-based model solved with Cplex. In all cases, the computational time of branch-and-price is less than the execution time of the Cplex solver. Altering the parameters of the instances significantly impacts the difficulty of solving of those instances. For example, in the S40-B5-T404 case, adding three more locations for fast chargers, and changing it to S40-B8-T404, results in a doubling of CPU processing time, indicating a rise in complexity. In a more extreme scenario, adding ten extra trips to the S50-B3-T444 instance makes it so challenging that the Cplex solver cannot find the optimal solution within a day. Note that the optimality gap has been set to zero for all the instances.

5.2 Sensitivity analysis

In this section, we assess the model's performance and accuracy by introducing various changes to the problem parameters and analyzing the corresponding results. First, we check the results for different electric bus types. Then, we focus on the sensitivity analysis of the maximum travel range of EBs.

5.2.1 Bus selection

The differences in electric bus types are based on their purchasing cost, maximum travel range, and charging time. Note that the lifetime of all EBs is assumed to be 15 years. Table 5 illustrates these characteristics for four real-world electric bus types according to Proterra Electric Vehicle Technology Manufacturer (2016) and Gallo et al. (2014). We conduct individual model runs for each electric bus type and present the corresponding results. This analysis can assist transit authorities in determining the most suitable electric bus type for their fleet, considering factors such as cost-effectiveness and scale of the problem.

Table 5: Different features of five types of well-known EBs

Electric bus	Purchase cost of EB ($)	Maximum travel range (km)	Charging Time (min)
Proterra-40 Ft. Catalyst	799000	49	10
Proterra-40 Ft. Catalyst+	1000000	62	13
New Flyer-40 ft	1300000	72	6
Hengtong-40ft	1220000	50	10

The aim is to find the EB type with the lowest total costs among the given electric bus types. In this comparison, we focus on two specific instances: S30-B5-T404 and S60-B5-T444. By conducting these comparisons, we can identify the most cost-effective electric bus type for the given instance. The results of such analysis are represented in Tables 6 and 7.

Table 6: Comparison of results for different EB types for 30 trips

Electric bus	No, chargers	No. EB	EB purchase cost ($)	OC ($)	IC ($)	Obj ($)
Proterra-40 Ft. Catalyst	1	9	479394	54782.6	16307	548483.6
Proterra-40 Ft. Catalyst+	2	9	599994	51950.8	32174	684118.8
New Flyer-40 ft	1	8	693328	45939.6	16307	755574.6
Hengtong-40ft	1	9	731997	52428.6	16307	800732.6

Table 7: Comparison of results for different EB types for 60 trips

Electric bus	No. chargers	No. EB	EB purchase cost ($)	OC ($)	IC ($)	Obj ($)
Proterra-40 Ft. Catalyst	4	16	852256	89782.7	66584	1008622.7
Proterra-40 Ft. Catalyst+	2	17	1133322	99102.2	33155	1265579.2
New Flyer-40 ft	1	14	1213324	95097.1	16307	1324728.1
Hengtong-40ft	4	16	1301328	89768.1	66584	1457680.1

The general understanding from this analysis is that purchasing expensive buses with longer ranges is not cost-effective, mainly because the cost of EBs heavily influences the objective function. According to Table 6, more charging stations are needed as the charging time for the Catalyst+ bus increases. Also, the longer charging time of Catalyst+ prevents EBs from starting their subsequent trips on time. For that reason, the model has decided to install an additional charging station as a more cost-effective solution than purchasing new EBs. Additionally, in this case, the total operational costs will decrease. This decrease is due to fewer deadhead trips to charging stations because of the longer travel range of

Catalyst+ bus and reduced waiting time costs, which outweigh the higher electricity costs from longer charging times. In contrast, the New Flyer bus, with its short charging time and longer maximum travel range compared to the other EB types given in Table 5, shows a greater reduction in charging costs compared to its increase in EB waiting times and deadhead trips for recharging due to fewer established fast chargers. It is noteworthy to highlight that because of the longer travel range of the New Flyer bus, the number of deadhead trips to the charging stations would be less than EBs with a shorter travel range. However, the travel distance for such EBs to reach the fewer available charging stations is greater compared to the scenario where more charging stations are installed.

For the S60-B5-T44 instance and according to Table 7, increasing the charging time for electric buses from 10 to 13 minutes requires adding an extra EB to the system, raising total costs. However, there is a reduced need for charging stations for the Catalyst+ bus, which has a longer travel range than the Catalyst. This decrease is also due to the higher number of EBs for Catalyst+ buses. The New Flyer bus, despite needing the least number of EBs, comes with high initial purchase costs. However, its advantage lies in its longer range and shorter charging time, which means a transit system using these buses would only require a single charging station.

This analysis, however, has a limitation as it only accounts for fixed charging times. If variable charging times were considered, the Catalyst+ bus would outperform the Catalyst in all aspects except for its higher purchase cost. It is crucial to note that this experiment was conducted for specific instances, and the results may vary if the scale of the problem changes. Therefore, it is essential first to determine the scale of the bus network, consider the number of given trips, and identify potential locations for fast chargers. Once these factors are established, transit authorities can then assess which electric bus type best suits their fleet. By utilizing the proposed model and conducting a thorough analysis, transit authorities gain valuable insights to make well-informed decisions before purchasing or expanding their electric bus fleet. This approach empowers them to select the most suitable electric bus type that aligns with their specific requirements, resulting in a more efficient and cost-effective public transportation system.

5.2.2 Range sensitivity analysis

In this sensitivity analysis, we aim to explore the relationship between battery capacity and the objective function value by running the model on S30-B5-T404 and S60-B5-T444 instances with the same parameters described in Table 3. According to the research by Bi et al. (2018), electric buses typically require an average of 1.24 kWh for every kilometer travelled and based on Chen et al. (2018), the unit manufacturing cost of an EB battery is 570 $/kWh of capacity. To assess the costs on an annual basis, we calculate the yearly added expense of batteries to the purchase cost of EBs. This is achieved by dividing the total cost of batteries by the electric bus's lifespan, which is 15 years. This process allows us to understand the annual impact of battery costs on the overall cost of an EB. Based on this information, we can calculate how the cost increases as the maximum travel range of an electric bus is extended. This calculation will take into account the energy requirement per kilometer and the manufacturing cost of the EB battery. By identifying sensitive points in battery capacity, this analysis provides crucial insights for informed decision-making regarding electric bus specifications and the optimal battery size to achieve cost-effective and efficient operations.

Table 8 shows that EBs with a 45 km maximum travel range have the highest objective value compared to those with longer ranges. Increasing the range by 5 km reduces the need for fast chargers from 2 to 1, significantly lowering the objective value. However, extending the range to 60 km does not notably affect the objective value. Also, the total cost decreases as the maximum range of EBs increases from 45 to 60 km.

A notable drop in the objective value occurs at a 65 km range due to the reduction of EBs from 9 to 8. When the range is expanded to 75 km, the requirement for fast chargers decreases again, from 2 to 1, substantially lowering the objective value. Between 75 km and 150 km, there are no significant changes in the objective value. This is because the increase in range is insufficient to reduce the number of EBs or fast chargers. The most cost-effective point is at 75 km, with a total cost of 352,419.1. Beyond this point, total costs rise due to the higher cost of making more capable batteries. Finally, no fast chargers are needed for EBs with 200 and 300 km ranges. These EBs can complete all trips and return to the depots to recharge for the next day.

Table 8: Sensitivity analysis on the maximum travel range of EBs for 30 trips

d_{max} (km)	Battery capacity (kWh)	Added cost ($)	No. chargers	EB	OC ($)	IC ($)	Obj ($)	Total cost ($)
45	55.8	0	2	9	47782.1	34067	399846.1	399846.1
50	62.0	2850	1	9	52428.6	16307	386732.6	386922.6
55	68.2	5700	1	9	51873.8	16307	386177.8	386557.8
60	74.4	8550	1	9	51483.2	16307	385787.2	386357.2
65	80.6	11400	2	8	49559.7	32174	364397.7	365157.7
75	93.0	17100	1	8	52308.1	16307	351279.1	352419.1
100	124.0	31350	1	8	51844.6	16307	350815.6	352905.6
120	148.8	42750	1	8	51844.6	16307	350815.6	353665.6
150	186.0	59850	1	8	51800.8	16307	350771.8	354761.8
200	248.0	141360	0	8	63517.3	0	346181.3	355605.3
300	372.0	212040	0	8	63291.0	0	345955.0	360091.0

Table 9 indicates that with a maximum travel range of 45 km, EBs cannot feasibly cover all the given trips. Therefore, EBs with a range exceeding 45 km are necessary. When the range is increased to 50 km, the situation becomes feasible with a requirement of 16 EBs and four fast chargers distributed across the bus network to meet the EBs' energy demands. Increasing the maximum range to 55 km is expected to significantly lower the objective value and the total cost, as it reduces the need for fast chargers from 4 to 3.

Table 9: Sensitivity analysis on the maximum travel range of EBs for 60 trips

d_{max} (km)	Battery capacity (kWh)	Added cost ($)	No. chargers	EB	OC ($)	IC ($)	Obj ($)	Total cost ($)
45	55.8	0	-	-	-	-	Infeasible	Infeasible
50	62.0	2850	4	16	89768.1	66584	721680.1	721870.1
55	68.2	5700	3	16	91348.5	49736	706412.5	706792.5
60	74.4	8550	3	15	91954.4	49736	671685.4	672255.4
65	80.6	11400	3	15	91698.9	49736	671429.9	672189.9
75	93.0	17100	2	15	93031.2	33155	656181.2	657321.2
100	124.0	31350	1	14	105514.2	16307	616483.2	618573.2
120	148.8	42750	1	14	104831.6	15867	615360.6	618210.6
150	186.0	59850	1	14	104773.2	15867	615302.2	619292.2
200	248.0	141360	1	14	104773.2	15867	615302.2	624726.2
300	372.0	212040	1	14	104751.3	15867	615280.3	629416.3

A significant decrease in the objective value is observed when the range reaches 60 km. At this point, the fleet requires 15 EBs, and the number of chargers remains the same. Another notable reduction in objective value and total cost occurs when the range extends from 65 km to 75 km, mainly due to a decrease in the total number of fast chargers. Increasing the range from 75 km to 100 km results in removing one EB from the fleet, further decreasing the objective value. From a 45 km range up to 120 km, total costs gradually decline. However, total costs begin to rise beyond 120 km due to the added cost of manufacturing higher-capacity batteries. In this scenario, the optimal range for covering 60 trips is 120 km, with a total cost of 618,210.6. Beyond this range of up to 300 km, there are no significant changes in the objective value, though total costs continue to increase.

In the sensitivity analysis for EBs' maximum travel range, we observe a significant impact on the objective value. As the maximum travel range increases, the objective value decreases, indicating that higher battery capacities lead to more marginal savings. Tables 8 and 9 show that the total operating costs of EBs vary. This is because of changes in costs related to transportation and waiting times. These changes are affected by how far the EBs can travel on a single charge and how many charging stations are available. Interestingly, if the number of charging stations stays the same, but the buses can travel further, the total costs will increase. Additionally, if the buses can cover greater distances, fewer buses are required to manage the planned trips. This leads to a decrease in overall costs. As a result, this analysis represents a trade-off between the increase in the maximum travel range and the cost associated with increasing the capacity of EBs' batteries.

6 Conclusion

Electric bus scheduling is a challenging problem due to the limited travel range of EBs and their time-consuming charging process. Additionally, location planning of charging stations and scheduling the charging events further complicates decision-making. To address these complexities, this study introduced an arc-based model through a mixed-integer linear formulation for the electric bus scheduling problem. Also, we suggested a path-based model through an integer-linear programming formulation. To solve the arc-based model and path-based model, we used Cplex solver and developed a branch-and-price algorithm to deal with larger instances. We evaluated the performance of these algorithms using instances randomly generated in a practical setting. The proposed algorithm is compared with the MILP solver of Cplex and extensive computational studies are conducted to assess their effectiveness.

We carried out multiple sensitivity analyses to validate the accuracy of the proposed model and to examine how adjustments to the crucial parameters impact total costs. These sensitivity analyses offer valuable insights and recommendations for transit authorities, helping them select the most suitable EB type with specific parameters. Additionally, our examination of the maximum travel range of electric buses can assist transit authorities in determining the optimal battery size for EBs. By doing so, the operational costs can be effectively reduced in the long-term planning horizon.

The proposed study has the potential for several extensions. One key area for improvement is to incorporate uncertainty into the model for EBs travel time and energy consumption. Travel time for EBs is influenced by various factors such as passenger volume, driving behavior, traffic congestion, road conditions, weather, and more. By considering stochastic travel time, the model can become more robust and better suited to real-world situations. Similarly, energy consumption for electric buses also varies depending on factors like passenger load, battery life, and road conditions. Future research could consider these factors to improve the accuracy of the model.

Another area of interest to explore is the inclusion of different vehicle types in the proposed model. Currently, the model assumes that all the EBs in the fleet are homogeneous, while many cities have public bus transportation systems with heterogeneous fleets. Additionally, the model can be adapted to account for different charging technologies with varying establishment costs, charging power, and charging times. It may even be possible to enable the model to consider partial charging for electric buses, making it more adaptable to real-world scenarios.

To enhance the public transportation operation planning for electric buses, several additional elements could be added to the proposed model in future studies. For example, the second stage of public transportation planning, which is timetable development could be included as a decision variable, like the first and third stages, which are network design and bus scheduling, respectively. This means that departure times and trip frequencies could be optimized rather than being fixed parameters. Furthermore, since pantograph chargers have high electricity loads, it is important to consider the impacts of locating such facilities in neighborhoods. Future works could investigate the power loss incurred in connecting such infrastructure to the grid and their effects on energy sources.

Appendix

A Mathematical formulation

The arc-based formulation in Section 4.1 is nonlinear due to the multiplication of binary variable, x_{ji} to the continuous variable g_j^o. We convert such formulations to linear ones by introducing a new variable denoted by h_{ij}^o, which is equal to $x_{ji}g_j^o$. Thus, the linear formulation of the first model will be as follows.

$$\min \left(\sum_{o\in O} \sum_{(j,i)\in A\setminus A'} c_{ji}^o x_{ji}^o + \sum_{o\in O} \sum_{(j,t)\in A'} c_{jt}^o x_{jt}^o + \sum_{b\in B} V_b z_b \right) \tag{20}$$

s.t.

$$\sum_{o\in O} \sum_{\substack{j \\ (j,i)\in A}} x_{ji}^o = 1 \qquad \forall i \in S \tag{21}$$

$$\sum_{\substack{i \\ (j,i)\in A}} x_{ji}^o - \sum_{\substack{i \\ (i,j)\in A}} x_{ij}^o = 0 \qquad \forall j \in S, \forall o \in O \tag{22}$$

$$\sum_{\substack{j \\ (j,t\in T_b)\in A'}} x_{jt}^o = \sum_{\substack{j \\ (j,t\in T_b)\in A'}} x_{tj}^o \qquad \forall b \in B, \forall o \in O \tag{23}$$

$$g_i^o = \sum_{j:(j,i)\in A} (h_{ji}^o + d_{ji} x_{ji}^o) \qquad \forall i \in S, \forall o \in O \tag{24}$$

$$h_{ji}^o \leq d_{max} x_{ji}^o \quad (j,i) \in A, \qquad \forall o \in O \tag{25}$$

$$h_{ji}^o \leq g_j^o \quad \forall j \in S, \qquad \forall o \in O \tag{26}$$

$$h_{ji}^o \geq g_j^o - (1 - x_{ji}^o) d_{max} \quad (j,i) \in A, \qquad \forall o \in O \tag{27}$$

$$g_t^o \leq (1 - x_{jt}^o) d_{max} \quad (j,t) \in A', \qquad \forall o \in O \tag{28}$$

$$g_i^o \leq d_{max} \qquad \forall i \in S, \forall o \in O \tag{29}$$

$$\sum_{o\in O} \sum_{\substack{i \\ (i,t\in T_b)\in A'}} x_{it}^o + \sum_{o\in O} \sum_{\substack{i \\ (i,t\in T_b)\in A'}} \sum_{s=1}^{U} x_{it}^{o,t+s} \leq W \qquad \forall b \in B, \forall t \in T', \forall o \in O \tag{30}$$

$$\sum_{\substack{i \\ (i,t\in T_b)\in A'}} x_{it}^o \leq z_b \qquad \forall b \in B, \forall o \in O \tag{31}$$

$$x_{ji}^o \in \{0,1\} \qquad \forall (j,i) \in A, \forall o \in O \tag{32}$$

$$z_b \in \{0,1\} \qquad \forall b \in B \tag{33}$$

$$g_j^o \geq 0 \qquad \forall j \in S, \forall o \in O \tag{34}$$

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