Les Cahiers du GERAD ISSN: 0711–2440

A novel approach to nonlinear short-term hydropower optimization using a combination of heuristic and meta-heuristic algorithm

M. Jafari Aminabadi, S. Séguin, I. Fofana

G–2023–64

December 2023

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2023
– Bibliothèque et Archives Canada, 2023

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2023
– Library and Archives Canada, 2023

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

A novel approach to nonlinear short-term hydropower optimization using a combination of heuristic and meta-heuristic algorithm

Mohammad Jafari Aminabadi [a, c]

Sara Séguin [b, c]

Issouf Fofana [a, d]

[a] *Department of applied sciences, Université du Québec à Chicoutimi, Saguenay (Qc), Canada, G7H 2B1*

[b] *Department of computer science and mathematics, Université du Québec à Chicoutimi, Saguenay (Qc), Canada, G7H 2B1*

[c] *GERAD, Montréal (Qc), Canada, H3T 1J4*

[d] *Canada Research Chair tier 1, in Aging of Oil-Filled Equipment on High Voltage Lines (ViAHT)*

mjaminabad@etu.uqac.ca
sara.seguin@uqac.ca
issouf.fofana@uqac.ca

December 2023
Les Cahiers du GERAD
G–2023–64

Abstract : In this paper, a mixed integer nonlinear model for the short-term hydropower optimization problem considering operational constraints such as demand and startup costs, is presented. The complexity of the problem is reduced by using maximum energy output rather than working with individual turbines or turbine combinations. In order to solve the model, three methods are proposed: method A, a binary genetic algorithm; method B, an iterative heuristic method; and method C, using the iterative heuristic method in the genetic algorithm. Based on computational results in a case study, method B converges to a solution very quickly and with few iterations, whereas methods A and C perform more efficiently. A comparison between methods A and C indicates that method C not only reduces the computational burden for convergence but also yields better results. The proposed methods are evaluated by comparing them with optimal solutions for the mixed integer nonlinear model. Results indicate that the proposed methods are highly effective in achieving favorable results.

Keywords: Short-term hydropower optimization, nonlinear programming, genetic algorithm, heuristic algorithm, meta-heuristic algorithm

---

Acknowledgements: This research was funded by a Discovery grant provided by the Natural Sciences and Engineering Research Council of Canada. We extend our sincere gratitude to E.K.Aasgård and SINTEF for access to SHOP.

Notation	
$t \in \{1, 2, 3, ..., T\}$	Index of planning periods.
$c \in \{1, 2, 3, ..., C\}$	Index of hydropower plants.
$j \in \{1, 2, 3, ..., J_t^c\}$	Index of the number of active turbines in plant c at period t.
$r \in \{1, 2, 3, ..., r^c\}$	Index of hydropower plants upstream of plant c.
q_t^c	Water discharge at plant c and period t (m^3/s).
v_t^c	Reservoir volume of plant c at period t. (Mm^3/h)
ϵ^c	Penalty factor for the start-up of the turbines for plant c.
λ_t	The volume of demand in period $t \in T$.
Δ_t^c	Number of turbines turned on at period t and plant c.
α	Non-coverage of the demand penalty factor.
β	Oversupply reward factor.
L_t	Supply shortage at period t (MW).
U_t	Oversupply of energy in period t (MW).
δ_t	Inflow in period $t \in T$ (Mm^3).
ζ	Conversion factor from (m^3/s) to (Mm^3/h).
g_t^c	Water spillage at plant c and period t (m^3/s).
$z_{j,t}^c$	$\begin{cases} 1 & \text{if surface j is chosen at period t for plant c.} \\ 0 & \text{otherwise.} \end{cases}$
$\chi_{j,t}^c$	Power production function for surface j at period t and plant c (MW).
w_t	Duration of period t (h).
q_{min}^c	Minimum water discharge at plant c (m^3/s).
q_{max}^c	Maximum water discharge at plant c (m^3/s).
v_{min}^c	Minimal volume of plant c reservoir (Mm^3).
v_{max}^c	Maximum volume of plant c reservoir (Mm^3).

1 Introduction

Hydropower is one of the most significant renewable energy sources for producing electrical energy in the world and plays a decisive role in meeting global energy requirements [1]. Electricity producers aim to maximize revenue or minimize costs. However, the efficient management of hydropower systems presents intricate challenges due to the complexity of the system. Therefore, hydropower optimization processes are categorized into long-term, mid-term, and short-term problems [2, 3]. Long-term scheduling is typically based on stochastic models with uncertain variables for maximizing future production potential [4]. In general, mid-term models have a one-year horizon and are used to manage the reservoir trajectories [4, 5]. Short-term hydropower models have planning times between one day and one week, considering operational constraints in order to determine the optimal daily production strategy. This planning mainly involves daily physical operations and is usually solved as a deterministic problem [3], although stochastic models have proven to be useful when there is variability in the inflows [6].

In view of the multi-dimensional relationships between variables such as water discharge, reservoir volume, and turbine efficiency, short-term hydropower production planning is naturally a nonlinear problem. The status (on/off) of turbines is determined by binary variables, so integer programming must be used. Since it is difficult to work with nonlinear models with binary variables, either the production functions are linearized or the head effect is neglected [7]. The nonlinearity and non-convexity of this problem, as well as the large dimensions of the problem, including a large number of decision variables, integer variables, and operational limitations, have made handling this system very challenging. Therefore, classical algorithms are not always sufficient to solve this problem, and other powerful methods are required [2, 8]. Various exact methods have been developed to optimize the hydropower problem, such as Linear Programming (LP) [9], Mixed Integer linear Programming (MILP) [5, 10], Mixed Integer Nonlinear Programming (MINLP) [11], Lagrangian Relaxation (LR) [12] and Dynamic Programming (DP) [13]. These algorithms can provide good results depending on the problem conditions, but they each have their own limitations. The optimal global solution can be obtained by linearizing the objective function and constraints in the LP model [14]. MILP has a high computational cost, especially if there are many turbines involved. Lagrange multipliers are difficult to find in LR, although it is a fast method. DP can handle the nonlinearity of the problem, but it suffers from the "curse of dimensions" as the dimensions of the problem increase [15]. Generally, the nonlinear effect is neglected or linearized due to the difficulty of working with nonlinear models, especially with integer variables [7]. A two-phase model is presented in [11], which provides another way to deal with integer variables and nonlinear aspects of the hydropower problem. The power output, water discharge, reservoir volume, and the number of active turbines are determined using a MINLP in the first phase, the loading problem. In the second phase, the start-up costs are penalized based on the unit commitment problem. In order to use this model, the unimodularity conditions in [16] must be satisfied. Meta-heuristic algorithms can improve performance in complex problems and in large hydro systems [17]. Thus, Ant Colony (AC) [18], Particle Swarm Optimization (PSO) [19], Simulated Annealing (SA) [20], and Artificial Bee Colony (ABC) [21] algorithms are used to optimize the hydropower problem. Additionally, hybrid algorithms such as [22, 23, 24] have been used to improve the performance and efficiency of meta-heuristic methods. Compared to classical methods, meta-heuristics and hybrid algorithms have higher flexibility in dealing with the complexity and limitations of the problem and can reach high-quality results at the right time. In a hybrid algorithm, the advantages of each algorithm are combined to improve the search space for the problem and speed up the convergence process.

The Genetic Algorithm (GA) is one of the most widely used algorithms based on population. It is inspired by natural selection mechanisms and has fast convergence and the ability to create a variety of solutions and search for the optimal result [25, 26]. Genetic algorithms are used for complex and nonlinear optimization of hydropower reservoir systems and multi-reservoirs in [27, 28, 29] and unit commitment problems [30]. The implementation of such a genetic algorithm is usually straightforward, and it can be easily hybridized with other optimization methods [25]. In [31], a hybrid Chaos optimization algorithm is used to improve GA performance and increase convergence speed. Using a hybrid algorithm of genetic algorithms and cellular automation to optimize reservoir operation problems, [32] demonstrates that the proposed algorithm is superior to genetic algorithms in achieving better results. Furthermore, modified genetic algorithms that increase the efficiency and speed of convergence of the algorithm have been developed for the hydropower optimization problem, including [33, 34, 35].

This paper presents a mixed integer nonlinear mathematical formulation of the short-term hydropower problem. Instead of linearization and discretization and to consider the nonlinearity effect of the problem and reduce the states of decision variables, the maximum energy output surface of the water discharge and volume of the reservoir for each turbine combination are used. In this model, demand, operational constraints and start-up costs are considered and the existence of integer variables increases the problem complexity. The exact solution to this problem presents many challenges, and is often impossible in most cases; therefore, we propose three solution methods. All presented methods are based on the principle of reducing the complexity of a problem by handling binary variables. Therefore, in method A, the binary genetic algorithm is employed to solve the model since GA has a high degree of efficiency despite its ease of implementation and can be easily hybridized with other algorithms. In method B, an iterative heuristic method is employed to solve the problem and determine the number of active turbines. The iterative heuristic method is applied to the GA in method C.

The paper is organized as follows. Section 2 presents a mixed integer nonlinear hydropower model that considers start-up costs and demand constraints and aims to maximize revenue. Section 3.1 presents the genetic algorithm, in Section 3.2, we introduce the iterative heuristic method, and in Section 3.3, we apply the iterative heuristic method to the MINLP. The results and method evaluation are discussed in Section 4, and the conclusion is presented in Section 5.

2 The short-term hydro-power problem

The purpose of the short-term hydropower optimization problem is to maximize revenue or energy generation within a time frame ranging from one day to one week. Various factors are discussed in this section regarding the short-term hydropower problem. The parameters in the power production function for a single turbine are gravitational acceleration g in m/s^2, the efficiency of the turbine η, water discharge q in m^3/s, the net water head h in m, which depend on the total water discharge Q (sum of water discharge and spillage) and volume of the reservoir v in (m^3). Additionally, ρ_d is the density of water (kg/m^3) [36]. Power output (W), in a single turbine is given as

$$p(q,h) = g * \eta(q) * q * h(Q,v) * \rho_d, \tag{1}$$

The net water head is calculated by a function as shown in Equation (2).

$$h(Q,v) = fb(v) - tl(Q) - pl(Q,q), \tag{2}$$

where fb is the forebay elevation of the reservoir(m), tl is the tailrace elevation of the reservoir (m), and pl is the penstock losses of the unit (m). Turbine efficiency specifically affects power production, and since each turbine has its own efficiency, the turbines produce different energy in the same conditions in terms of water head and water discharge. The binary variables are used to determine the active turbine in the problem's formulation, and the turbines can be active simultaneously, so there are different combinations for the number of active turbines. For example, all combinations for 4 turbines are shown in Table 1.

Table 1: All combination of four turbines

1 active turbines	2 active turbines	3 active turbines	4 active turbines
1-2 3-4	12-23-34-13-32 24-14-31-21	123-124 134-234	1234

Many turbine combinations and integer variables increase the complexity of the problem. As shown in [11], instead of working with the turbine, the maximum energy output surface can be used for each number of active turbines. On this surface, the maximum power production is obtained by considering the reservoir volume and the water discharge for each number of active turbines, so the nonlinear factors are also considered in the model. Instead of 18 combinations for 4 turbines, 4 maximum energy surfaces are used. Besides determining the number of active turbines, other important concepts can be considered, including start-up, and coverage of demand. A large number of startups increase maintenance costs and reduce the turbine life cycle, so the startup costs can be considered as another variable. The mathematical formulation of the short-term hydropower optimization problem is presented in this section. The purpose of the MINLP is to maximize revenue. The MINLP is given by:

$$\max \sum_{c\in C}\sum_{t\in T}\sum_{j\in J} \rho_t \times \chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} \times \gamma_t + \sum_{t\in T}(\beta \times \rho_t \times U_t - \alpha \times \rho_t \times L_t) - \sum_{c\in C}\sum_{t\in T} \epsilon^c \times \Delta^c_t \tag{3}$$

subject to:

$$v^c_{t+1} = v^c_t - \zeta \times w_t \times (q^c_t + g^c_t) + \zeta \times \delta_t + \sum_{r\in R} \zeta \times w_t \times (q^r_t + g^r_t), \qquad \forall t \in T, c \in C, \tag{4}$$

$$\sum_{j\in J} z^c_{j,t} = 1, \qquad \forall t \in T, c \in C, \tag{5}$$

$$\sum_{c\in C}\sum_{t\in T}\sum_{j\in J} \chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} \times \gamma_t - \lambda_t = U_t - L_t, \qquad \forall t \in T, \tag{6}$$

$$\Delta^c_t = j^c_t \times z^c_{j,t} - j^c_{t-1} \times z^c_{j,t-1}, \qquad \forall j \in J, c \in C, t \in T \setminus \{1\}, \tag{7}$$

$$v_1^c = v_{Initial}^c, \qquad \forall c \in C, \tag{8}$$
$$v_T^c \geq v_{final}^c, \qquad \forall c \in C, \tag{9}$$
$$q_{min}^c \leq q_t^c \leq q_{max}^c, \qquad \forall t \in T, c \in C, \tag{10}$$
$$v_{min}^c \leq v_t^c \leq v_{max}^c, \qquad \forall t \in T, c \in C, \tag{11}$$
$$v_t^c \geq 0, q_t^c \geq 0, \qquad \forall t \in T, c \in C, \tag{12}$$
$$z_{j,t}^c \in B, \qquad \forall t \in T, j \in J, c \in C. \tag{13}$$

The objective function in Equation (3) includes four parts. The first is power production at each selected number of active turbines and at each hour, which is multiplied by prices. Since the energy produced must cover the committed demand, in the second part of the objective function, the excess supply, U_t, is rewarded and the non-supply of demand, L_t, is penalized at each hour. β is the reward factor for oversupply, and α is the penalty factor for an undersupply of demand. Instead of working with all combinations of turbines, the maximum output surface is used, which reduces complexity and speeds up the solving process. In this method, it is not possible to determine exactly which turbine is working, but the minimum start-up costs can be considered. Suppose that at hour t, the number of active turbines is 3 and at hour $t+1$ the number of active turbines is 4, so we know that at least one turbine has been turned on. Therefore, by considering the average start-up costs, ϵ^c, and the number of activated turbines at hour t, Δ_t^c the minimum start-up is penalized in the third part. The reservoir balance constraints are in Equation (4), and described by Equation (5) limit the model to choose only one active turbine combination per hour t. Shown in Equation (6) is the imbalance between demand volume and energy production. Equation (7) shows the number of turbines turned on per hour which is achieved by switching between maximum energy output surfaces. Equation (8) specifies initial volumes. The objective function penalizes the lack of energy production if it is less than the demand. Bounds on the variables are given in Equation (10)–(11). Finally, Equation (12) imposes nonnegativity and Equation (13) defines binary variables.

3 Methodolgy

As mentioned to solve the presented model, a solution should be provided that considers demand constraints and start-up costs by keeping the formulation with combinations of turbines. Therefore, three methods including the binary genetic algorithm, the iterative heuristic method and using the iterative heuristic method in the genetic algorithm are introduced to solve this problem in this section.

3.1 Binary genetic algorithm (Method A)

Genetics and natural selection are the inspiration for the genetic algorithm [37]. Genetic algorithms are well-known algorithms that have been used in various optimization problems. Various operators are used to create a population, so the algorithm searches the problem space efficiently, and it can also be combined with other algorithms [25]. In this method, randomized operators such as selection, crossover, and mutation are used, and it is generally divided into two groups: binary GA and real GA [29].

Due to the difficulty in solving the MINLP, the binary GA is used to determine the number of active turbines at each hour of the planning horizon. By using the maximum output surface, there is no need to check all the combinations of the turbines, reducing the search space and increasing the speed of the algorithm. As shown in Figure 1, the number of active turbines is determined randomly by considering the condition that only one of the maximum energy surfaces can be selected at each hour of the planning horizon. Afterwards, the number of active turbines, the integer variable, is fixed, and the nonlinear short-term hydropower problem is solved. Thus, all the equations presented in section **??** are taken into account, but instead of the integer variable $z_{j,t}^c$, the parameter $z_{Fixed,t}^c$ is fixed, and the nonlinear model is solved. In this method, since integer variables are fixed, a nonlinear model can solve the short-term hydropower problem continuously and quickly, and there is no need to estimate or assign fitness values to other variables. After evaluating the initial population, the parents' chromosomes are randomly selected, and the crossover operation is performed on the number of active turbines. For crossover, hybrid operators such as one-point, two-point, and uniform are used, and then the results are evaluated. In the next step, the mutation is done using a suitable strategy, single-point and multi-point mutation. As in the previous steps, the nonlinear model is continuously solved by fixing the number of turbines, and its results are evaluated. Based on the best result from the previous step, the population is updated, and this process continues until convergence conditions are met. It is considered the termination criterion of an algorithm if the objective function of NLP does not change after a number of successive iterations. In this method, instead of working with individual turbines, a combination of turbines is employed, which reduces the number of binary states and the number of iterations required to reach a solution. It is possible to obtain the objective function value and other variables by solving the nonlinear problem in each iteration by using the combination of genetic algorithms in the exact solution method.

The MINLP short-term hydropower problem can be simplified by using the maximum energy output surface instead of working with all possible combinations of turbines, by fixing the integer variable, and by using GA to determine the number of active turbines. In the genetic algorithm, a systematic structure and iterations are employed to determine the number of active turbines per hour.

3.2 Iterative heuristic method (Method B)

The genetic algorithm works randomly to determine the number of active turbines, and its results improve with some iterations. The purpose of this method is to determine the number of active turbines using a rule rather than random processes and to solve the hydropower problem more quickly and with fewer iterations than method A. As shown in Figure 2, the process starts with an initial estimate of the number of turbines, and like in Sections 3.1, the integer variable is fixed, and the nonlinear continuous model is solved. The number of turbines is updated according to the result and the heuristic method, which is explained in the following text.

The output of the model, the reservoir volume, water discharge, and power generated at each hour, can be obtained after solving the nonlinear problem as shown in box 4 of Figure 2. The next step is to determine whether the change in the active turbines every hour increases the objective function's value. Therefore, the output obtained from the initial guess is used as an input to the maximum energy output surface equations. The number of active turbines is altered if there is a number of active turbines that provide greater energy production with the same input in the surface equation. This means that there are a number of active turbines that will produce more energy with the same input, including water discharge and reservoir volume. Suppose there are four turbines in the hydro plant, so there are four maximum output surface equations as shown in Figure 3, drawn in two dimensions for simplicity. Also, three active turbines are considered as the initial guess at time t and the NLP model is solved with fixed integer values.

After solving the model with the fixed variable, its results are available every hour. This means the amount of water discharge at hour t, q_t^{*c}, the reservoir volume at hour t, v_t^{*c}, and energy production at hour t are known, which is obtained from the following equation:

$$\chi^c_{3,t}(q_t^{*c}, v_t^{*c})\, z^c_{3,t}, \tag{14}$$

The energy output values of one turbine $\chi^c_{1,t}(q_t^{*c}, v_t^{*c})$, two turbines $\chi^c_{2,t}(q_t^{*c}, v_t^{*c})$, and four turbines $\chi^c_{4,t}(q_t^{*c}, v_t^{*c})$ can be determined by putting the inputs into maximum energy output surface equations. The number of active turbines at the time t changes from three to another, which has the highest increase in energy production with the same input. At the time t, the number of active turbines does not change if no situation increases energy production. Therefore, in Figure 3, the number of active turbines changes from three to one when the amount of production with three active turbines is in

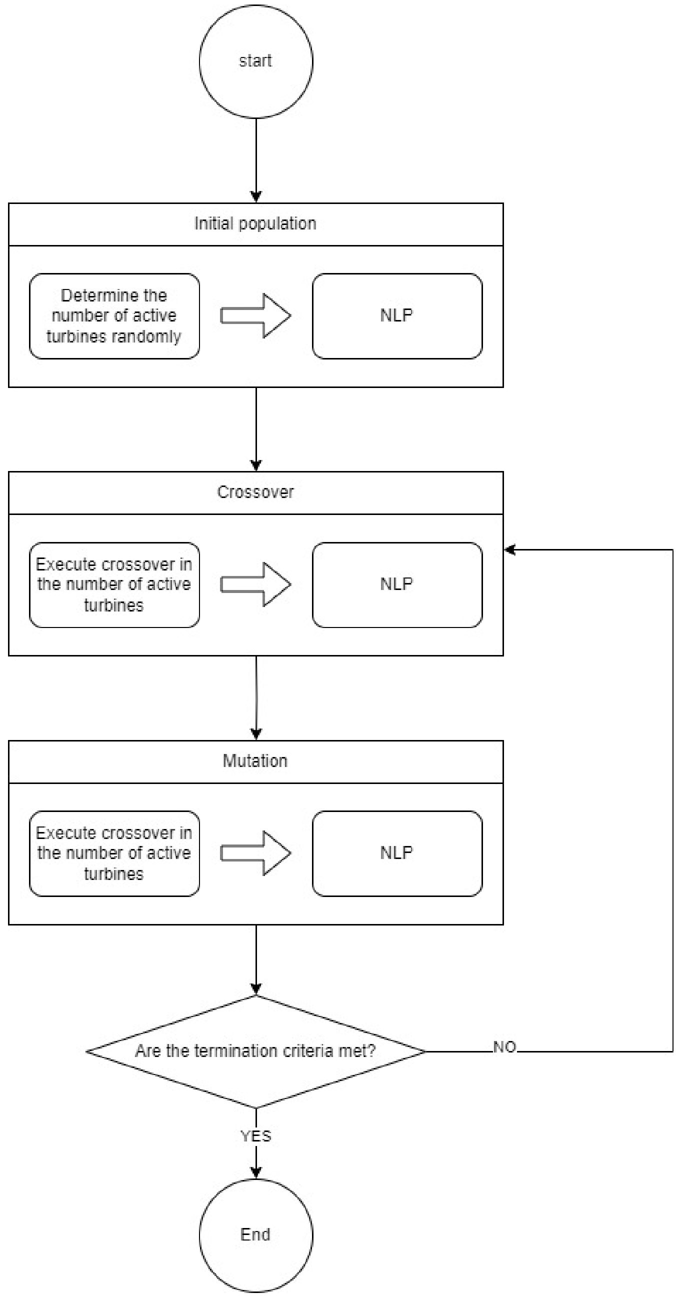

Figure 1: Flowchart of the binary genetic algorithm (method A).

highlighted area A, and it changes to 2 when it is in highlighted area B, and finally, it changes to 4 when the amount of production with three turbines is in highlighted area C.

$$\begin{cases} \text{if} \quad \chi^c_{1,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{2,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{3,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{4,t}(q^{*c}_t, v^{*c}_t) \quad \Rightarrow \text{Area A} \\ z^{*,c}_t = z^c_{1,t} \\ \text{if} \quad \chi^c_{2,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{1,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{3,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{4,t}(q^{*c}_t, v^{*c}_t) \quad \Rightarrow \text{Area B} \\ z^{*,c}_t = z^c_{2,t} \\ \text{if} \quad \chi^c_{4,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{3,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{2,t}(q^{*c}_t, v^{*c}_t) \geq \chi^c_{1,t}(q^{*c}_t, v^{*c}_t) \quad \Rightarrow \text{Area C} \\ z^{*,c}_t = z^c_{4,t} \end{cases} \tag{15}$$

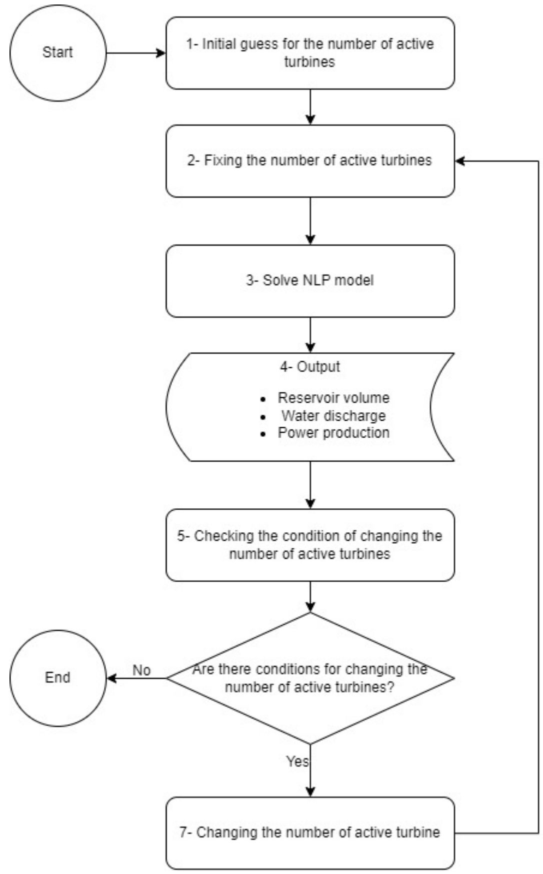

Figure 2: Flowchart of a heuristic method (method B).

Consequently, instead of randomly changing the number of turbines per hour from the planning horizon or checking other situations, this method is used to change the number of active turbines for each hour, and then the NLP model is solved with the applied changes, and this process continues until there is no change in the number of active turbines.

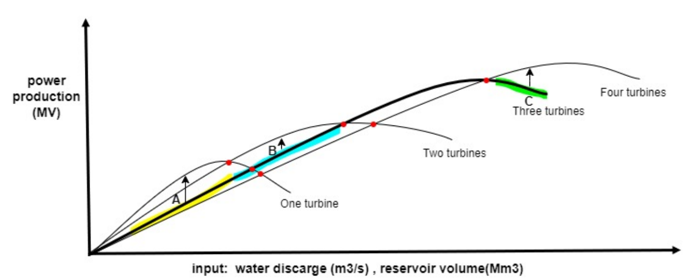

Figure 3: Maximum energy output surface for four turbines.

Despite having advantages such as the appropriate efficiency in finding the number of turbines per hour, this method may have problems and cannot work well at breaking points. This method can also be affected by the initial guess of the number of turbines. Therefore, a method that covers the search space of the problem well and reaches the right solution in a shorter number of iterations is required.

3.3 Using the iterative heuristic method in the GA (Method C)

As mentioned, using the maximum output surface instead of all turbine combinations can expedite solution-finding. The GA, despite its appropriate speed, which uses a random process at all steps, may not be effective for large, complex problems. The heuristic iterative method uses a fast algorithm instead of random methods to determine the number of active turbines. However, the efficiency of this method depends on the initial guess of the number of turbines and it also does not work well at breakpoints. Therefore, a method is introduced that benefits from the advantages of each method presented in the previous sections. The GA searches the problem area well, and the heuristic algorithm can approach the optimal solution at a suitable speed, so, as shown in Figure 4, the heuristic algorithm can be used inside the GA. To avoid limiting the search space in the genetic algorithm, only the alpha percentage of the initial population, derived from the heuristic algorithm results, is utilized. This approach is thus applied to the creation of the initial population in the GA. As discussed in Section 3.1, the entire process, including crossover and mutation, continues until a termination criteria is met.

4 Numerical results

Data extracted from Short-term Hydro Optimization Program (SHOP) runs are used to test the entire methods presented in the previous section. SHOP [38] is an optimization model for planning hydropower systems provided as software by SINTEF Energy Research. Two power plants are connected in series in this case study. The first power plant has two turbines with a maximum reservoir volume of 41.66 Mm^3 and a maximum energy production capacity of 240 MWh. The second power plant consisting of four turbines, has a maximum energy output of 345 MWh, and has a maximum reservoir volume of 104.16 Mm^3.

All methods are implemented in Julia [39], and the optimization software to solve NLP is Ipopt [40]. In order to solve the models, a laptop computer equipped with an Intel Core i5 processor and 8 GB of RAM is used.

4.1 Results

According to the data that includes reservoir volume, water discharge, and energy production, the maximum energy output surfaces are obtained for both powerhouses. Instead of considering all turbine combinations, two output surfaces are used for the first powerhouse when one and two turbines are active, and four output surfaces for the second powerhouse when one, two, three, and four turbines are active. To obtain the nonlinear equations for the maximum energy output surface for each number of active turbines, a polynomial approximation is fitted to the data. The planning horizon is 24 hours, and prices and inflows are deterministic. The methods were tested on 54 instances with different input parameters when reservoirs were almost empty, half full, and full. In order to evaluate the methods, the problem with different demand constraints was investigated and the demand graph for different hours can be seen in Figure 5.

The results of instances are presented in Table 2, which includes the revenue, the number of iterations and computation time for each method. Each method was repeated five times for each instance, and the average value of the objective function, the average number of iterations, and the average execution time are reported.

Due to the use of maximum output surfaces rather than all turbine combinations, the number of problem states and the complexity of the problem are reduced, and the presented methods converge to the result after a suitable number of iterations. As mentioned earlier, Method A searches the problem space entirely randomly, while Method B determines the number of active turbines using rules after an initial guess. Method C uses both methods A and B to reach the result. Table 2 shows method C has a better value of the objective function in most instances than methods A and B. In cases 13,

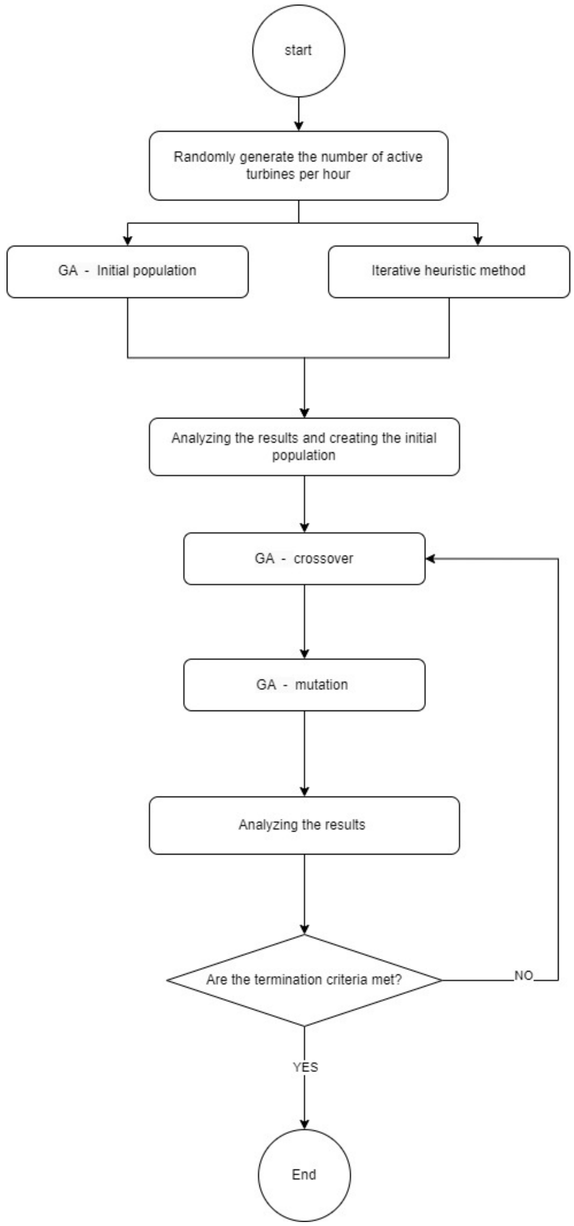

Figure 4: Using the iterative heuristic method in the GA.

16 and 38, method A has better results than method C, although their difference is also quite small. The objective function value in method C is on average 0.09% greater than method A and 0.63% greater than method B. According to a comparison of the solution times, method B has the shortest execution time and converges to the result in the shortest number of iterations, while method C has a shorter execution time and fewer iterations than method A. According to the results, the average calculation time for methods A, B, and C is 138, 2 and 74 seconds, respectively. and the average number of iterations is 78, 5, and 42, respectively. The average number of iterations in method A is 78, method B is 5, and method C is 42.

The objective function for three methods are compared using Equation (16), in which the result of each method is divided by the maximum value obtained in each method. For example, the maximum value in instance 1 in Table 2 is 447,910 (EUR), which is obtained from method C. Therefore, the results in instance one are divided by the maximum value. Figure 6 shows the percentage of similarity

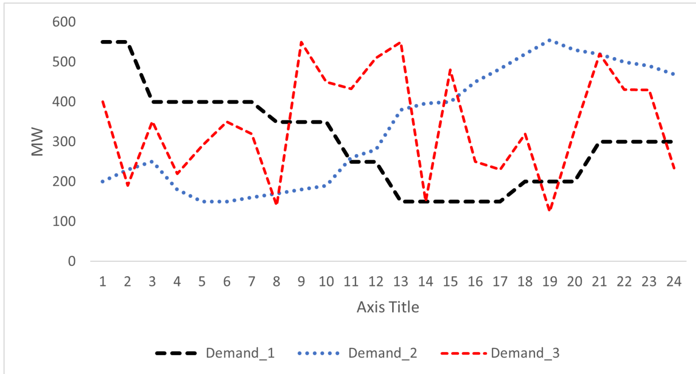

Figure 5: Demand constraint for different hours.

of each method with the best result. Accordingly, any method with the best result will have the same numerator and denominator in equation Equation (16), and the percentage of similarity will be 100 %. As illustrated in the Figure 6, in most instances, method C obtained the best results. Despite method B has weaker results than the other two methods, and it has a maximum difference of 1.60% in instance 53.

$$\text{Similarity with the best result}(\%) = \left(\frac{\text{The objective function of the method A, B and C (€)}}{\text{Maximum objective function (€)}}\right) \times 100 \tag{16}$$

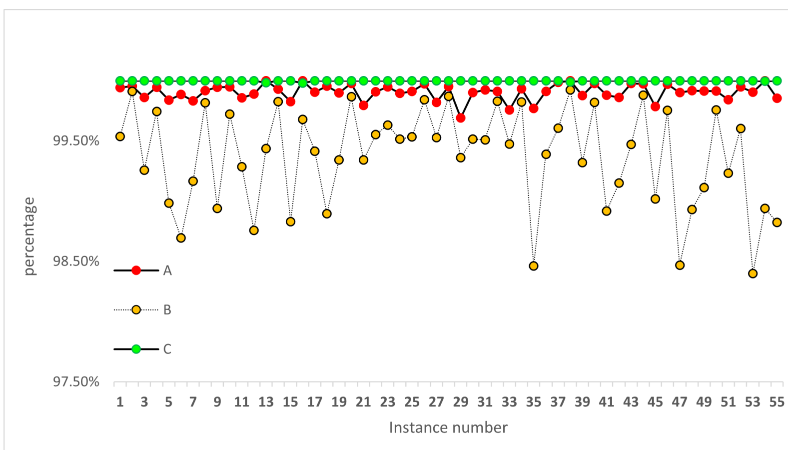

Figure 6: Comparison of the value of the objective function in three methods.

In order to compare the number of iterations of methods B and C the instances 5, 11, 31, and 47 were randomly selected from Table 2 and results are shown in Figure 7. In Figure 7, the horizontal axis represents the number of iterations and the vertical axis represents the value of the objective function. The result of method C converges with a smaller number of iterations compared to method B.

The results show that method C has a shorter execution time than method B and that the objective function value is higher in most cases. By using a heuristic approach in genetic algorithms, method C utilizes the advantages of methods A and B, and brings good chromosomes into the problem solving process. As a result, method C can improve their performance and allow them to achieve better results in a shorter period of time.

Table 2: Comparison of the results obtained using all three methods.

Demand	Inst.	Inflows	Reservoir 1	Reservoir 2	Method A			Method B			Method C		
					Obj(€)	Iter.	Time(s)	Obj(€)	Iter.	Time(s)	Obj(€)	Iter.	Time(s)
1	1	High	Half full	Half full	601,545	108	117.6	593,965	5	1.7	602,310	44	86.7
	2		Almost full	Almost full	806,073	82	122.7	805,695	6	1.6	806,435	29	66.3
	3		Almost empty	Almost full	525,588	71	165.8	515,057	6	1.9	526,265	44	90.0
	4		Almost full	Almost empty	552,653	70	113.7	551,018	6	1.7	553,446	28	43.4
	5		Half full	Almost full	433,352	79	201.5	427,139	6	1.9	435,158	37	85.7
	6		Almost full	Half full	539,930	75	136.8	527,273	5	1.5	540,730	45	80.8
	7	Medium	Half full	Half full	575,440	76	141.8	569,284	5	1.5	576,630	33	58.2
	8		Almost full	Almost full	780,077	79	137.6	779,181	6	1.8	780,781	25	43.9
	9		Almost empty	Almost full	496,614	68	167.8	489,572	5	1.6	497,049	42	98.4
	10		Almost full	Almost empty	530,739	58	100.4	529,697	6	1.8	531,466	30	53.6
	11		Half full	Almost full	401,425	69	174.3	394,923	6	1.9	402,568	37	92.4
	12		Almost full	Half full	513,449	68	119.6	502,238	7	2.1	513,813	68	121.8
	13	Low	Half full	Half full	548,741	79	137.5	542,439	5	1.5	549,146	30	54.5
	14		Almost full	Almost full	753,176	90	139.5	752,783	6	1.7	753,573	29	45.2
	15		Almost empty	Almost full	466,260	68	164.1	454,840	5	1.5	467,652	43	99.3
	16		Almost full	Almost empty	508,131	68	120.5	505,692	5	1.5	508,229	40	68.9
	17		Half full	Almost full	367,048	67	172.2	361,844	6	1.3	368,559	37	93.3
	18		Almost full	Half full	485,402	81	161.6	473,349	4	1.8	485,882	46	87.8
2	19	High	Half full	Half full	550,390	78	221.7	549,045	7	2.2	551,923	37	96.0
	20		Almost full	Almost full	748,408	73	103.0	747,262	5	1.5	748,388	25	35.8
	21		Almost empty	Almost full	476,642	60	183.6	470,783	5	1.8	477,856	35	95.7
	22		Almost full	Almost empty	496,276	75	200.0	495,382	4	1.3	496,772	28	75.5
	23		Half full	Almost full	383,410	77	155.1	374,026	6	2.2	384,141	39	105.4
	24		Almost full	Half full	491,362	71	175.4	484,971	6	2.0	492,818	44	107.9
	25	Medium	Half full	Half full	525,778	79	212.5	521,690	6	2.1	526,979	31	84.0
	26		Almost full	Almost full	722,661	70	101.1	722,426	6	1.8	723,357	33	51.2
	27		Almost empty	Almost full	448,815	68	196.2	441,392	6	2.2	449,899	43	95.2
	28		Almost full	Almost empty	474,881	70	198.3	474,422	5	1.7	475,182	24	55.9
	29		Half full	Almost full	347,837	83	207.3	340,270	5	1.8	349,494	35	84.0
	30		Almost full	Half full	465,842	90	207.6	455,564	6	2.1	465,369	42	96.0
	31	Low	Half full	Half full	499,988	79	225.7	497,338	4	1.5	500,852	31	81.2
	32		Almost full	Almost full	696,455	78	116.4	695,588	6	1.8	697,015	28	40.4
	33		Almost empty	Almost full	419,410	74	200.8	410,483	5	1.9	419,624	45	120.6
	34		Almost full	Almost empty	451,774	78	195.8	452,342	5	1.8	453,321	35	80.4
	35		Half full	Almost full	311,959	92	238.1	303,914	6	2.2	311,941	39	99.6
	36		Almost full	Half full	438,173	86	301.8	428,579	5	1.8	438,620	42	108.2
3	37	High	Half full	Half full	553,271	76	198.3	547,866	6	2.0	553,143	35	96.8
	38		Almost full	Almost full	754,713	68	92.0	754,554	5	1.7	754,927	28	42.2
	39		Almost empty	Almost full	474,909	72	203.7	464,428	5	2.0	474,965	59	169.9
	40		Almost full	Almost empty	500,866	83	211.3	500,479	6	2.4	501,532	38	104.2
	41		Half full	Almost full	372,600	76	182.6	367,112	6	2.3	372,773	51	134.5
	42		Almost full	Half full	490,679	80	194.8	476,443	6	2.3	491,222	56	137.4
	43	Medium	Half full	Half full	526,321	77	198.6	521,765	6	2.4	526,630	39	94.1
	44		Almost full	Almost full	729,370	79	96.8	728,377	5	1.8	729,764	29	37.8
	45		Almost empty	Almost full	443,992	73	198.1	431,699	5	2.0	444,251	48	128.7
	46		Almost full	Almost empty	478,745	71	180.5	477,731	5	2.0	478,866	28	70.7
	47		Half full	Almost full	335,362	82	207.1	329,323	5	2.1	335,421	44	93.7
	48		Almost full	Half full	462,627	68	186.9	446,114	6	2.3	463,026	67	172.6
	49	Low	Half full	Half full	498,660	72	201.7	493,868	6	2.5	499,336	55	154.4
	50		Almost full	Almost full	702,566	74	100.1	701,969	6	2.1	703,347	26	39.6
	51		Almost empty	Almost full	411,372	75	194.8	404,818	5	2.1	411,764	58	176.0
	52		Almost full	Almost empty	455,593	74	178.3	453,572	6	2.6	455,827	40	105.6
	53		Half full	Almost full	296,378	67	161.2	291,358	6	2.5	297,179	45	103.2
	54		Almost full	Half full	432,999	81	225.0	417,161	6	2.5	432,802	49	125.8

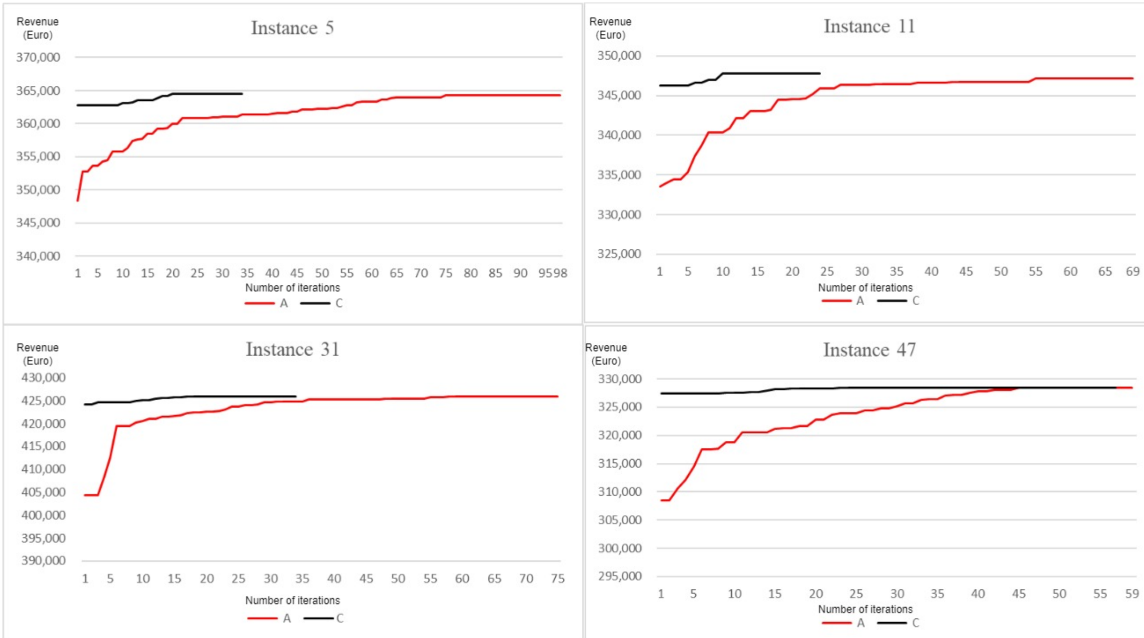

Figure 7: Comparison of the changes in the objective function value in each iteration, instances 5, 11, 31 and 47.

4.2 Validation of methods

The efficiency of the methods presented in the previous sections can be evaluated if an optimal solution can be obtained. As mentioned, the MINLP are difficult to solve, so it is imperative to reduce their complexity to compare their results. The MINLP short-term hydropower problem can be solved if the totally unimodular condition is satisfied, as illustrated in [11]. Therefore, to compare the methods' results with the optimal solution, some model conditions, including start-up costs, and demand constraints, are ignored. Thus, the loading problem formulation for evaluating the methods is as follows:

$$\max \sum_{c\in C}\sum_{t\in T}\sum_{j\in J} \rho_t \times \chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} \tag{17}$$

subject to:

$$v^c_{t+1} = v^c_t - \zeta \times w_t \times (q^c_t + g^c_t) + \zeta \times \delta_t + \sum_{r\in R} \zeta \times w_t \times (q^r_t + g^r_t), \qquad \forall t \in T, c \in C, \tag{18}$$

$$\sum_{j\in J} z^c_{j,t} = 1, \qquad \forall t \in T, c \in C, \tag{19}$$

$$q^c_{min} \le q^c_t \le q^c_{max}, \qquad \forall t \in T, c \in C, \tag{20}$$

$$v^c_{min} \le v^c_t \le v^c_{max}, \qquad \forall t \in T, c \in C, \tag{21}$$

$$v^c_1 = v^c_{Initial}, \qquad \forall c \in C, \tag{22}$$

$$v^c_T = v^c_{final}, \qquad \forall c \in C, \tag{23}$$

$$v^c_t \ge 0, q^c_t \ge 0, \qquad \forall t \in T, c \in C, \tag{24}$$

$$z^c_{j,t} \in B, \qquad \forall t \in T, j \in J, c \in C. \tag{25}$$

Five instances from Table 2 were randomly selected for comparison, and Equation (26) was used to determine the percentage of similarity with the optimal value.

$$\text{Similarity}(\%) = \left(\frac{\text{Avarage of objective function of presented method (EUR)}}{\text{Objective function of Loading problem (EUR)}}\right) \times 100 \tag{26}$$

It was repeated five times in each method in order to obtain the average value of the objective function for each instance. The percentage of similarity for each instance is shown in Table 3.

Table 3: Comparison of all three methods with the optimal solution.

Instance	Inflows	Reservoir 1	Reservoir 2	Method A (%)	Method B (%)	Method C (%)
10	Medium	Almost full	Almost empty	99.99%	99.95%	99.99%
21	High	Almost empty	Almost full	99.97%	99.49%	99.98%
29	Medium	Half full	Almost full	99.98%	99.40%	99.99%
42	High	Almost full	Half full	99.97%	99.25%	99.98%
49	Low	Half full	Half full	99.99%	99.72%	99.98%

The results show that Method A, Genetic Algorithms, and Method C, which utilizes a heuristic algorithm within Genetic Algorithms, are capable of approaching the optimal solution very well, and the difference between them and the optimal solution is relatively small. Although it was demonstrated that method B has reasonable accuracy to reach the result and can reach the solution in a short period of time in different conditions of the input parameters, the initial guess can affect the results and this method does not perform well at breaking points, as mentioned previously.

5 Conclusion

In this paper, a mixed integer nonlinear model for short-term hydropower problems with demand constraints and startup costs is presented. The complexity of the problem is reduced significantly by fixing the binary variable, the number of active turbines, and using the maximum energy output surface. Thus, rather than estimating other variables, the exact solver was used to solve the nonlinear problem, and three methods were presented. In method A, a binary genetic algorithm was used to solve the non-linear problem. In method B, an iterative heuristic approach was employed to determine the number of active turbines and solve the problem. An iterative heuristic approach was applied to the genetic algorithm in Method C. Based on the results, method C utilizes the advantages of both methods, searches the problem space well, and converges to the result with fewer iterations than method A. The average result from method C is 0.09% better than method A and 0.63% better than method B. For future studies, uncertainties such as prices and inflows can be taken into account in the model and solution methods. Metaheuristic algorithms such as PSO, AC, SA, etc, can also be used to solve this problem and the results can be compared.

References

[1] Hira Singh Sachdev, Ashok Kumar Akella, and Niranjan Kumar. Analysis and evaluation of small hydropower plants: A bibliographical survey. Renewable and Sustainable Energy Reviews, 51:1013–1022, 2015.

[2] I. Parvez, J. Shen, M. Khan, and C. Cheng. Modeling and solution techniques used for hydro generation scheduling. Water, 11(7):1392, 2019.

[3] Nils Flatabø, Arne Haugstad, Birger Mo, and Olav B Fosso. Short-term and medium-term generation scheduling in the norwegian hydro system under a competitive power market structure. In EPSOM'98 (International Conference on Electrical Power System Operation and Management), Switzerland, 1998.

[4] OB Fosso and MM Belsnes. Short-term hydro scheduling in a liberalized power system. In 2004 International Conference on Power System Technology, 2004. PowerCon 2004., volume 2, pages 1321–1326. IEEE, 2004.

[5] Maissa Daadaa, Sara Séguin, Kenjy Demeester, and Miguel F Anjos. An optimization model to maximize energy generation in short-term hydropower unit commitment using efficiency points. International Journal of Electrical Power & Energy Systems, 125:106419, 2021.

[6] Sara Séguin, Charles Audet, and Pascal Côté. Scenario-tree modeling for stochastic short-term hydropower operations planning. Journal of Water Resources Planning and Management, 143(12):04017073, 2017.

[7] E. K. Aasgård, S.-E. Fleten, M. Kaut, K. Midthun, and G. A. Perez-Valdes. Hydropower bidding in a multi-market setting. Energy Systems, 10(3):543–565, 2019.

[8] Mohammad Reza Sharifi, Saeid Akbarifard, Mohamad Reza Madadi, Kourosh Qaderi, and Hossein Akbarifard. Optimization of hydropower energy generation by 14 robust evolutionary algorithms. Scientific Reports, 12(1):7739, 2022.

[9] Ziad K Shawwash, Thomas K Siu, and SO Russel. The bc hydro short term hydro scheduling optimization model. In Proceedings of the 21st International Conference on Power Industry Computer Applications. Connecting Utilities. PICA 99. To the Millennium and Beyond (Cat. No. 99CH36351), pages 183–189. IEEE, 1999.

[10] Alberto Borghetti, Claudia D'Ambrosio, Andrea Lodi, and Silvano Martello. An milp approach for short-term hydro scheduling and unit commitment with head-dependent reservoir. IEEE Transactions on power systems, 23(3):1115–1124, 2008.

[11] S. Séguin, P. Côté, and C. Audet. Self-scheduling short-term unit commitment and loading problem. IEEE Transactions on Power Systems, 31(1):133–142, 2016.

[12] Erlon C Finardi, Edson L da Silva, and Claudia Sagastizábal. Solving the unit commitment problem of hydropower plants via lagrangian relaxation and sequential quadratic programming. Computational & applied mathematics, 24:317–342, 2005.

[13] Alicia Arce, Takaaki Ohishi, and Sérgio Soares. Optimal dispatch of generating units of the itaipu hydroelectric plant. IEEE Transactions on power systems, 17(1):154–158, 2002.

[14] Zhong-kai Feng, Wen-jing Niu, Wen-chuan Wang, Jian-zhong Zhou, and Chun-tian Cheng. A mixed integer linear programming model for unit commitment of thermal plants with peak shaving operation aspect in regional power grid lack of flexible hydropower energy. Energy, 175:618–629, 2019.

[15] Peng Lu, Jianzhong Zhou, Chao Wang, Qi Qiao, and Li Mo. Short-term hydro generation scheduling of xiluodu and xiangjiaba cascade hydropower stations using improved binary-real coded bee colony optimization algorithm. Energy Conversion and Management, 91:19–31, 2015.

[16] Laurence A Wolsey. Integer programming. John Wiley & Sons, 2020.

[17] Nkechi Neboh, Josiah Adeyemo, Abimbola Enitan, and Oludayo Olugbara. A review on applications of evolutionary algorithms to reservoir operation for hydropower production. International Journal of Geological and Environmental Engineering, 9(9):1153–1159, 2015.

[18] K Vaisakh and LR Srinivas. Evolving ant colony optimization based unit commitment. Applied soft computing, 11(2):2863–2870, 2011.

[19] Baohong Lu, Kunpeng Li, Hanwen Zhang, Wei Wang, and Huanghe Gu. Study on the optimal hydropower generation of zhelin reservoir. Journal of Hydro-Environment Research, 7(4):270–278, 2013.

[20] Ramesh SV Teegavarapu and Slobodan P Simonovic. Optimal operation of reservoir systems using simulated annealing. Water Resources Management, 16:401–428, 2002.

[21] Xiang Liao, Jianzhong Zhou, Shuo Ouyang, Rui Zhang, and Yongchuan Zhang. An adaptive chaotic artificial bee colony algorithm for short-term hydrothermal generation scheduling. International journal of electrical power & energy systems, 53:34–42, 2013.

[22] Ieda G Hidalgo, Regiane S de Barros, Jéssica Fernandes, João Paulo F Estrócio, and Paulo B Correia. Metaheuristic approaches for hydropower system scheduling. Journal of Applied Mathematics, 2015.

[23] Peng Lu, Jianzhong Zhou, Huifeng Zhang, Rui Zhang, and Chao Wang. Chaotic differential bee colony optimization algorithm for dynamic economic dispatch problem with valve-point effects. International Journal of Electrical Power & Energy Systems, 62:130–143, 2014.

[24] Li Mo, Peng Lu, Chao Wang, and Jianzhong Zhou. Short-term hydro generation scheduling of three gorges–gezhouba cascaded hydropower plants using hybrid macs-ade approach. Energy Conversion and Management, 76:260–273, 2013.

[25] Sourabh Katoch, Sumit Singh Chauhan, and Vijay Kumar. A review on genetic algorithm: past, present, and future. Multimedia Tools and Applications, 80:8091–8126, 2021.

[26] Manlin Wang, Yu Zhang, Yan Lu, Xinyu Wan, Bin Xu, and Lei Yu. Comparison of multi-objective genetic algorithms for optimization of cascade reservoir systems. Journal of Water and Climate Change, 13(11):4069–4086, 2022.

[27] Robin Wardlaw and Mohd Sharif. Evaluation of genetic algorithms for optimal reservoir system operation. Journal of water resources planning and management, 125(1):25–33, 1999.

[28] Onur Hınçal, A Burcu Altan-Sakarya, and A Metin Ger. Optimization of multireservoir systems by genetic algorithm. Water resources management, 25:1465–1487, 2011.

[29] Aida Tayebiyan, Thamer Ahmed Mohammed Ali, Abdul Halim Ghazali, and MA Malek. Optimization of exclusive release policies for hydropower reservoir operation by using genetic algorithm. Water Resources Management, 30:1203–1216, 2016.

[30] V Senthil Kumar and MR Mohan. Solution to security constrained unit commitment problem using genetic algorithm. International journal of electrical power & energy systems, 32(2):117–125, 2010.

[31] Chun-Tian Cheng, Wen-Chuan Wang, Dong-Mei Xu, and Kwok Wing Chau. Optimizing hydropower reservoir operation using hybrid genetic algorithm and chaos. Water Resources Management, 22:895–909, 2008.

[32] M Azizipour and MH Afshar. Adaptive hybrid genetic algorithm and cellular automata method for reliability-based reservoir operation. Journal of Water Resources Planning and Management, 143(8):04017046, 2017.

[33] Kan Yang, Jiao Zheng, Min Yang, Ran Zhou, and Guoshuai Liu. Adaptive genetic algorithm for daily optimal operation of cascade reservoirs and its improvement strategy. Water resources management, 27:4209–4235, 2013.

[34] Yousif H Al-Aqeeli, TS Lee, and S Abd Aziz. Enhanced genetic algorithm optimization model for a single reservoir operation based on hydropower generation: case study of mosul reservoir, northern iraq. SpringerPlus, 5:1–21, 2016.

[35] Yanlai Zhou, Shenglian Guo, Fi-John Chang, and Chong-Yu Xu. Boosting hydropower output of mega cascade reservoirs using an evolutionary algorithm with successive approximation. Applied energy, 228:1726–1739, 2018.

[36] E. Finardi and E. Silva. Unit commitment of single hydroelectric plant. Electric Power Systems Research, 75, 08 2005.

[37] Ling Shang, Yizi Shang, Lianxing Hu, and Jianlin Li. Performance of genetic algorithms with different selection operators for solving short-term optimized reservoir scheduling problem. Soft Computing, 24(9):6771–6785, 2020.

[38] M.M. Belsnes, O. Wolfgang, T. Follestad, and E.K. Aasgård. Applying successive linear programming for stochastic short-term hydropower optimization. Electric Power Systems Research, 130:167–180, 2016.

[39] Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B Shah. Julia: A fresh approach to numerical computing. SIAM review, 59(1):65–98, 2017.

[40] A. Wächter and L. T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Mathematical programming, 106(1):25–57, 2006.