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A heuristic approach for the integrated production-transportation problem with process flexibility

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Abstract : We study an integrated multi-product production and distribution problem considering a network of multiple plants and customers, who are geographically dispersed, with direct shipment from the plants to the costumers. In addition to the decisions on production and distribution, a decision needs to be taken on the level of process flexibility in the network, i.e., which products can be produced in which plants. There is a clear trade-off between these decisions. On one hand, a network with total flexibility where each plant can produce all types of products allows for lower transportation costs, but requires large investments in flexibility and frequent setups. On the other hand, a network with a limited amount of flexibility where each plant produce only few products, will increase the transportation costs, but requires a lower investment in flexibility. We model this problem as an extension of the capacitated lot-sizing problem. We limit the investment in flexibility by a budget constraint and minimize the operational costs. Varying this budget allows us to analyze different levels of flexibility. In this paper, we propose mathematical models and a hybrid solution method that combines a mixed integer programming-based approach and a kernel search heuristic. Our computational results using data sets from the literature show that the proposed hybrid method produces on average better solutions with significantly lower computational times when compared with the results produced by a state-of-the-art optimization software. Additional computational results are presented by varying key parameters and analyzing their impact on the value of flexibility. These computational experiments indicate that some of the main managerial insights which were derived in the literature for the case without transportation costs are no longer valid when we consider transportation costs.

Keywords: Heuristics, lot sizing, transportation costs, multiple plants, process flexibility, production, distribution

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1 Introduction

In this research, the focus is on problems that appear in the context of industrial production and distribution planning. These problems involve the production of several products in multiple plants, and the distribution of these products to customers via direct shipments. These are complex tasks and need to be performed routinely. In general, production and distribution planning deals with decisions about the necessary production activities to transform the raw materials into finished products, and the transportation of these products to customers.

The planning of the production activities relates to the decisions about the quantity of products which must be produced. At the core, there is the lot sizing problem (Pochet and Wolsey (2006)) which consists of determining the quantity of products to be produced in each plant and period of a planning horizon. Production, setup, inventory, and backlog costs are considered. In addition, since we suppose that in the network the multiple plants and customers are geographically dispersed, transportation costs must also be taken into account. When the transportation between the plants and the customers is done using direct shipments, the problem is called the two-level production-transportation problem (Melo and Wolsey (2012)). According to Melo and Wolsey (2012), this is a general problem that covers several other problems arising in the literature and in practice, such as problems where the bottlenecks are on operations of mixing and packing (Molina et al. (2016)), the one-warehouse multi-retailer problem (Cunha and Melo (2016)) and other extensions.

In addition to the decisions on production and distribution, we look at this problem in the context of a network of existing plants that can be flexible to make one or more different products. Nowadays, with the advancement of information technologies, aiming to be more competitive, companies' strategies give more importance to the benefits of flexibility. In line with this, researchers have recognized that flexibility concepts are important for building sustainable supply chains since they enable firms to be reactive, even in large-scale production, without sacrificing cost efficiency. The seminal paper of Jordan and Graves (1995) analyzed the value of manufacturing process flexibility in a stochastic model with a single period. Since then, several authors have extended and analyzed the concept of limited flexibility configurations considering different stochastic environments. The main idea of the process flexibility studied by Jordan and Graves (1995) is that a limited amount of flexibility, applied in the right way, can provide benefits close to the level offered by full flexibility. This is true even in a deterministic multi-period production planning environment, as studied by Fiorotto et al. (2018). This new work extends the latter paper by considering the transportation decisions to the customers, thereby capturing a more complex and more realistic trade-off.

This paper has the following contributions. First, we propose a new optimization problem that considers a network of customers and plants with specific transportation costs between each plant and customer. The decision on which product to make in which plant also has to take into account the trade-off with the transportation cost and hence the geographical dispersion of the demand. Second, we propose two mathematical models for the analyzed problem, being one based on a classical formulation and the other a reformulation as a facility location problem. After analyzing the quality of the lower bounds, a third mathematical model is proposed which combines the first two formulations. Third, a hybrid solution method that combines a mixed integer programming (MIP)-based approach and a kernel search heuristic is proposed to solve the problem. The idea is to use a mixed integer programming-based approach to find an initial solution, and an intensification phase based on a kernel search heuristic tries to improve the initial solution. Computational results are presented comparing the proposed method with the use of a state-of-the-art commercial optimization package. Fourth, we present additional computational results aiming to analyze how different parameters impact the value of flexibility.

The paper is organized as follows. In Section 2, we present a literature review on related papers. In Section 3, we introduce the mathematical formulations. In Section 4, a hybrid solution approach is proposed, which combines a MIP-based approach and a kernel search heuristic. In Section 5, we present the computational results and, finally, in Section 6 the conclusions and future research.

2 Literature review

In the literature, there has been a broad effort to extend decision models and methods for the lot sizing problems in order to include more relevant industrial features. The lot sizing decision is crucial for companies since the production, inventory and setup costs represent a significant portion of the total product cost. Increased competition has forced companies to obtain a competitive position by paying attention to their complete supply chain. In that perspective, particular attention must be given to the integration of lot sizing decisions with other operational decisions such as the transportation decision and with higher-level decisions such as the level of process flexibility. As we did not find any paper that simultaneously considers all the integrated aspects that are taking into account in the present paper (lot sizing on multiple plants, transportation with direct shipment and limited process flexibility), we we will discuss each of the important issues separately. First we will discuss the lot sizing problem with multiple plants and transportation between plants, then we give an overview of the problem with several plants and geographically dispersed customers, and finally we discuss the lot sizing problem with limited process flexibility.

There are some studies on the lot sizing problem with several plants and transportation costs among the plants, but using only aggregate demands without geographically dispersed costumers and considering total flexibility (each plant can produce all types of products). In de Matta and Miller (2004) the lot sizing decisions are integrated with the transportation of the items between the plants so that some plants produce intermediate products and others the final products. Sambasivan and Yahya (2005) propose a Lagrangian-based heuristic for a similar problem by relaxing the capacity constraints. Their research was motivated by a practical application in steel rolled production. Guimarães et al. (2012) present a formulation for a problem with multiple plants in a beverage industry. The authors studied the planning operations that define the scheduling and size of the production, in which the objective is to satisfy the demand by minimizing production, overtime and transfer costs. Carvalho and Nascimento (2016) address this problem considering that all plants produce the same set of items (each one of the plants with a single machine) and that the demands must be satisfied without backlog. Considering the problem with multiple plants and setup carryover, Carvalho and Nascimento (2018) apply a meta-heuristic approach to find feasible solutions. The authors pointed out that the set of feasible solutions becomes significantly bigger considering the possibility of setup carryover.

Regarding the problem with several plants (with total flexibility) and geographically dispersed costumers, Park (2005) develops a local improvement heuristic which obtains better results when compared to a two-stage hierarchical approach. By analyzing the input parameters, the author identified conditions in which the integrated approach provides substantial benefits compared to the sequential approach. Ekşioğlu et al. (2007) consider that the inventory at the costumers' level is forbidden and plants have limited production capacity. The authors propose a Lagrangian-based heuristic and present computational tests on randomly generated data. Melo and Wolsey (2012) extend the paper of Park (2005) and propose different formulations for this problem, as well as a formulation-based heuristic procedure. Computational results are presented and show the quality of the proposed heuristic. It is worth mentioning that, the problem with geographically dispersed customers and one plant is also know as the One-Warehouse Multi-Retailer problem (Solyalı and Süral (2012) and Cunha and Melo (2016)). Gruson et al. (2019) compare several formulations for a three-level lot sizing and replenishment problem with a distribution structure. They consider a single type of item that is produced in a single plant which replenishes several warehouses and then retailers, using direct shipments. The papers including routing decisions are out of the scope of this research. We refer to Adulyasak et al. (2015) for a review on production-routing problems.

Jordan and Graves (1995) is the first study to analyze the benefits of using a limited amount of resource flexibility. Analyzing a manufacturing system with stochastic demand, they show that a chained configuration of products and plants performs almost as well as the full flexibility configuration in terms of average sales and capacity utilization. This means that it is not necessary that all the plants need to be able to produce all types of products in order to capture most of the benefits of flexibility. After the studies presented by Jordan and Graves (1995), several other works were proposed to analyze the value of resource flexibility in a context of stochastic demand (Koste and Malhotra (1999), Graves and Tomlin (2003), Bertrand (2003), Muriel et al. (2006), Mak and Shen (2009), Gurumurthi and Benjaafar (2004), Andradóttir et al. (2013)).

Some limited research has been done on lot sizing problems with limited flexibility. This has been mostly for the case of production with parallel machines. In most cases, the assumption is made that all machines can produce all types of products (Toledo and Armentano (2006), Mateus et al. (2010), Fiorotto and de Araujo (2014), Fiorotto et al. (2015), Vincent et al. (2020)) which corresponds to full flexibility. In some other cases, different machines can produce a different set of products, but the flexibility configuration is an input and not a decision. In their application in the tire industry, Jans and Degraeve (2004) discuss a problem where not all types of tires can be produced on all types of heaters. Xiao et al. (2015) study the capacitated lot sizing problem with parallel resources in the semiconductor industry where not all resources are eligible to produce all items. A proposed hybrid heuristic based on Lagrangian relaxation and simulated annealing method outperformed the numerical results observed in the literature. The solutions of the model connect the eligible resources to the items by satisfying a particular set of constraints that reduce the flexibility configurations. Wu et al. (2018a) propose different mathematical formulations for the lot sizing problem with nonidentical parallel machines, which are capable of producing a predefined subset of items, and analyze the per-item and per-period decomposition for these formulations. Besides the limited flexibility given as input, the authors also consider a constraint that restricts the number of machines that can produce a same type of item per period. In Wu et al. (2018b), the authors extend the problem studied in Wu et al. (2018a) by adding carbon emission constraints. Moreover, in each period, the number of machines that can be setup for a type of item is restricted to only one. The authors propose a progressive selection heuristic which was able to obtain superior results when compared with the state-of-the-art approaches found in the literature.

Recently, Fiorotto et al. (2018) addressed process flexibility and the chaining principle in lot sizing problems by analyzing the value of the resource flexibility in balanced systems (the numbers of items and resources are equals). The comparison of different limited flexibility configurations led to the conclusion that the benefits of the best long chain and the full flexibility configurations are practically the same. Also, when the flexibility level of the resource is a decision variable of the model, it is possible to obtain a new configuration with a smaller number of links than the best chain configuration which gets almost the same benefits as the complete flexibility configuration. Finally, they also pointed out that the importance of flexibility value increases when the data are heterogeneous. More specifically, the authors analyse separately the backlog cost and demand heterogeneity, as well as, the case with setup times. Inspired by a semiconductor manufacturing system, where machines must be qualified to process a product, Perraudat et al. (2022) propose mixed-integer mathematical models for a tactical qualification management problem. The authors propose both deterministic and robust formulations for the problem. Computational results, using industrial data, are presented and provide some very interesting managerial insights, such as: the price of uncertainty is acceptable, often less than a few additional qualifications for each machine; it is possible to achieve the same level of robustness as the case where all new qualifications are performed by only performing a restricted number of relevant qualifications; using the nominal set of qualifications can lead to significant capacity constraint violations.

3 Problem formulations

We model a production planning problem with multiple items, plants and clients and transportation costs from plants to clients. The planning horizon is finite and subdivided into several periods. The plants have a predetermined production capacity and a limited amount of flexibility. In order to be able to produce a certain type of product, the plant needs to make a specific investment. The level of flexibility in each plant is a decision variable and a flexibility constraint is modeled by imposing a budget limit on the total amount invested in flexibility over all plants. The use of backlogging is allowed. In addition, to produce a given item in a specific period, a setup must be performed. The goal of the problem is to find a production plan that satisfies all constraints by minimizing the production, setup, inventory, backlog, and transportation costs. Figure 1 illustrates the integrated production-transportation problem considering 5 items, 3 production plants and 6 customers. Observe that for this example, there is a limited amount of flexibility in which production plant one can produce items one and four, production plant two can produce items three and four and production plant three can produce items two and five. Moreover, it is also important to note that all production plants can deliver their products to all customers with respective transportation costs.

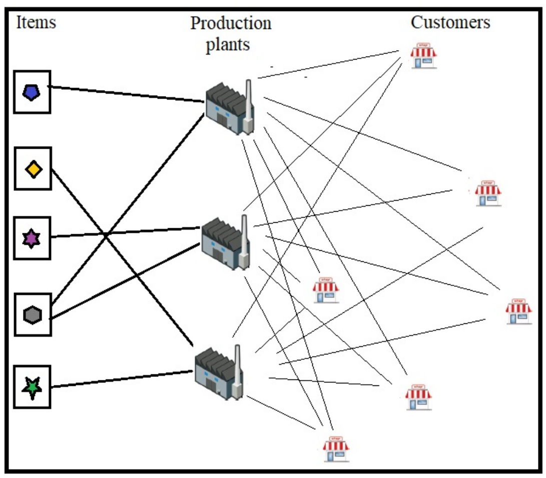

Figure 1: Graphical representation of the integrated production-transportation problem.

The mixed integer programming models are presented next. Before presenting the models, consider the following sets, parameters and decision variables for the problem based on the classical formulation for the multi-plant lot sizing problem proposed by Sambasivan (1994).

Sets	
$\mathcal{I}$	set of items, $\mathcal{I} = \{1, \ldots, n\}$ (index i);
$\mathcal{F}$	set of plants, $\mathcal{F} = \{1, \ldots, f\}$ (index j);
$\mathcal{K}$	set of clients, $\mathcal{K} = \{1, \ldots, c\}$ (index k);
$\mathcal{P}$	set of periods, $\mathcal{P} = \{1, \ldots, p\}$ (indexes t and u);
Parameters	
d_{itk}	demand of item i in period t for client k;
sd_{itu}	sum of the demands of item i, from period t to period u;
st_{ij}	setup time for item i at plant j;
vt_{ij}	unit processing time of item i at plant j;
sc_{ij}	setup cost of item i at plant j;
hc_{ij}	unit inventory cost of item i at plant j;
bc_{itk}	unit backlog cost of item i for client k in period t;
vc_{ij}	unit production cost of item i at plant j;
tc_{ijk}	unit transportation cost of item i from plant j to client k;
fc_{ij}	flexibility investment cost for item i at plant j;
Cap_{jt}	production capacity (in units of time) of plant j in period t;
$Fmax$	maximum budget to invest in flexibility;

Decision variables	
x_{ijt}	amount of item i to be produced at plant j in period t;
s_{ijt}	amount of item i in stock at plant j at the end of period t;
tr_{ijkt}	amount of item i to be transported from plant j to client k in period t;
b_{itk}	amount of backlog for item i and client k in period t;
y_{ijt}	binary variable that assumes value 1, if plant j is setup to produce item i in period t and 0, otherwise;
z_{ij}	binary variable that assumes value 1, if plant j is configured to produce item i and 0, otherwise.

The capacitated lot sizing problem with multiple plants, process flexibility and transport costs (MPCLSP-PFT) is modeled as follows:

$$(\mathbf{M1}) \quad \min \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\left[sc_{ij}y_{ijt} + vc_{ij}x_{ijt} + hc_{ij}s_{ijt} + \sum_{k\in\mathcal{K}}(tc_{ijk}tr_{ijkt})\right] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{itk}b_{itk}) \quad (1)$$

subject to:

$$s_{ijt} = x_{ijt} + s_{ij,t-1} - \sum_{k\in\mathcal{K}} tr_{ijkt} \qquad \forall(i,j,t) \quad (2)$$

$$d_{itk} = \sum_{j\in\mathcal{F}} tr_{ijkt} + b_{itk} - b_{i,t-1,k} \qquad \forall(i,t,k) \quad (3)$$

$$x_{ijt} \leq \min\{sd_{i1p}, (Cap_{jt} - st_{ij})/vt_{ij}\}y_{ijt} \qquad \forall(i,j,t) \quad (4)$$

$$\sum_{i\in\mathcal{I}}(st_{ij}y_{ijt} + vt_{ij}x_{ijt}) \leq Cap_{jt} \qquad \forall(j,t) \quad (5)$$

$$y_{ijt} \leq z_{ij} \qquad \forall(i,j,t) \quad (6)$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}} fc_{ij}z_{ij} \leq Fmax \quad (7)$$

$$y_{ijt} \in \{0,1\},\ z_{ij} \in \{0,1\},\ x_{ijt} \geq 0,\ tr_{ijkt} \geq 0 \qquad \forall(i,j,t,k) \quad (8)$$

$$s_{ijt} \geq 0,\ s_{ij0} = 0,\ s_{ijp} = 0,\ b_{itk} \geq 0,\ b_{i0k} = 0 \qquad \forall(i,j,t,k) \quad (9)$$

Objective function (1) minimizes the sum of setup, production, inventory, backlog and transportation costs. Constraints (2) model the flow balance at the plants, whereas (3) model the flow balance at the customers. Note that at the customers, demand cannot be fulfilled early: so no inventory is allowed, but if the demand cannot be delivered on time, backlogging is allowed. Constraints (4) guarantee that if an item is produced at a specific plant, a set up is done. Constraints (5) limit the available capacity for production and setup in each plant and period. For each plant j, item i and period t, constraints (6) ensure that a setup cannot be performed if plant j is not configured to produce item i. Constraint (7) limits the budget invested in flexibility. Finally, the domains of the decision variables are defined by constraints (8) and (9).

Problem reformulation

Consider the following parameters and decision variables for the MPCLSP-PFT reformulated as a facility location problem as proposed by Krarup and Bilde (1977) for the basic lot sizing problem.

Parameters	
$Cost_{ijtuk}$	unit cost for production, inventory and transportation of item i produced at plant j in period t to satisfy the demand of client k in period u. The cost is calculated as follows: $Cost_{ijtuk} = vc_{ij} + tc_{ijk} + (u-t)hc_{ij}$, for all $(i,j,t,u \geq t,k)$;
$Costb_{ijtuk}$	unit cost for production and transport of item i produced as backlog at plant j in period t to fulfill the demand of client k in period u; The cost is set by making $Costb_{ijtuk} = vc_{ij} + tc_{ijk} + \sum_{l=u}^{t-1} bc_{ilk}$, for all $(i,j,t,u < t,k)$;

Decision variables

x'_{ijtuk}	amount of item i to be produced at plant j in period t to satisfy the demand of client k in period u;
b'_{ipuk}	amount of item i backlogged at the end of the planning horizon (i.e., period p), necessary to fulfill the unmet demand of client k in period u;

The MPCLSP-PFT reformulated as a facility location problem can be modeled as follows:

$$
(\mathbf{M2}) \quad \min \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}(sc_{ij}y_{ijt}) + \sum_{i\in\mathcal{I}}\sum_{u\in\mathcal{P}}\sum_{k\in\mathcal{K}}\left[(\sum_{t=u}^{p} bc_{itk})b'_{ipuk}\right] + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}\left[\left(\sum_{u=t}^{p}(Cost_{ijtuk}x'_{ijtuk})\right) + \left(\sum_{u=1}^{t-1}(Costb_{ijtuk}x'_{ijtuk})\right)\right] \tag{10}
$$

subject to:

$$\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}} x'_{ijtuk} + b'_{ipuk} = d_{iuk} \qquad \forall(i,u,k) \tag{11}$$

$$x'_{ijtuk} \le \min\{d_{iuk}, (Cap_{jt} - st_{ij})/vt_{ij}\}y_{ijt} \qquad \forall(i,j,t,u,k) \tag{12}$$

$$\sum_{i\in\mathcal{I}}\left(st_{ij}y_{ijt} + \sum_{u\in\mathcal{P}}\sum_{k\in\mathcal{K}} vt_{ij}x'_{ijtuk}\right) \le Cap_{jt} \qquad \forall(j,t) \tag{13}$$

$$y_{ijt} \le z_{ij} \qquad \forall(i,j,t) \tag{14}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}} fc_{ij}z_{ij} \le Fmax \tag{15}$$

$$y_{ijt} \in \{0,1\},\ z_{ij} \in \{0,1\},\ x'_{ijtuk} \ge 0,\ b'_{ipuk} \ge 0 \qquad \forall(i,j,t,u,k) \tag{16}$$

Objective function (10) minimizes the sum of setup, production, inventory, backlog and transportation costs. Constraints (11) model the flow balance at the plants and at the customers. Note that at the customers, demand cannot be fulfilled early: so no inventory is allowed, but if the demand cannot be delivered on time, backlogging is allowed. In constraint (11) we use an additional variable b'_{ipuk} since it is possible that some part of the demand will not be satisfied by any production in the case there is backlog at the end of the horizon. Constraints (12) guarantee that if an item is produced at a specific plant, a set up is done. Constraints (13) limit the available capacity for production and setup in each plant and period. For each plant j, item i and period t, constraints (14) ensure that a setup cannot be performed if plant j is not configured to produce item i. Constraints (15) limit the budget invested in flexibility. Finally, the domains of the decision variables are defined by constraints (16).

As it will be explained in more detail in Section 4.2, the proposed solution method considers the solution of the linear relaxed problem to assist the solution procedure. Having this in mind, we analyzed the lower bounds obtained with both the formulations $M1$ and $M2$ so we can have the best results in the solution method. Based on the analyses presented in A, we notice that for the problems with tight capacity $M1$ provides better bounds while $M2$ obtained better bounds for the problems with normal capacity.

Based on the results presented in A, we consider a third formulation ($M3$) to the MPCLSP-PFT that always gives a lower bound that is equal or better than those found with the formulations $M1$ and $M2$. The formulation $M3$ consists in minimizing the objective function value $\zeta \in \mathbb{R}$, subject to constraints (2)–(9), (11)–(13) and (16)–(19).

$$\zeta \geq \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\left[sc_{ij}y_{ijt} + vc_{ij}x_{ijt} + hc_i s_{ijt} + \sum_{k\in\mathcal{K}}(tc_{ijk}tr_{ijkt})\right] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{itk}b_{itk}) \tag{17}$$

$$\zeta \geq \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}(sc_{ij}y_{ijt}) + \sum_{i\in\mathcal{I}}\sum_{u\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{ipk}b'_{ipuk}) + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}\left[\left(\sum_{u=t}^{p}(Cost_{ijtuk}x'_{ijtuk})\right) + \left(\sum_{u=1}^{t-1}(Costb_{ijtuk}x'_{ijtuk})\right)\right] \tag{18}$$

$$\sum_{u\in\mathcal{P}}\sum_{k\in\mathcal{K}} x'_{ijtuk} = x_{ijt} \qquad \forall(i,j,t) \tag{19}$$

Constraints (17) and (18) limit the value ζ of the objective function to be equal or greater than the values of the objective functions of $M1$ and $M2$, respectively. For each tuple of item, plant and period, constraints (19) guarantee the correspondence between the two types of production decision variables.

4 Solution method

To search for good feasible solutions to the MPCLSP-PFT, we propose a solution method denoted by KS, which consists in the hybridization of a MIP-based approach and a kernel search (KS) heuristic. The strategy is an adaption of solution methods proposed in (Carvalho and Nascimento, 2021). While an MIP-based approach provides an initial solution, an intensification phase based on a kernel search heuristic (Angelelli et al., 2007) tries to improve the initial solution.

The following sections describe in detail the structure of the applied MIP-based and kernel search strategies.

4.1 MIP-based approach

The proposed approach takes advantage of the outstanding performance of commercial MIP solvers to solve some classes of combinatorial problems, in particular, to solve the addressed problem when the decision variables associated to the flexibility are fixed.

To find an initial solution, we use a variation of the model (1)–(9) by adding constraints (20). The main idea behind this model, here denoted by DModel, is to fix the decision variables z_{ij} associated to the process flexibility such that each item is produced by only one plant. To this end, consider a set $\mathcal{S}$ composed of preselected pairs of items and plants (i, j) satisfying the aforementioned condition.

$$z_{ij} = 1 \qquad \forall(i,j)\in\mathcal{S} \tag{20}$$

Based on the set $\mathcal{S}$, constraint (20) imposes a solution to the flexibility related variables by fixing to the value 1 the decision variables z_{ij}, $\forall\ (i,j)\in\mathcal{S}$.

To decide the pairs (i, j) of items and plants to be in the set $\mathcal{S}$ we used the cost of sending the demands of an item i at plant j to the clients. Let sdk_{ik} be the total demand of an item i ordered by a client k during the whole planning horizon. The cost of transportation PT_{ij} of item i from plant j to all the clients is calculated as presented in Equation (21).

$$\mathsf{PT}_{ij} = \sum_{k\in\mathcal{K}} tc_{ijk}sdk_{ik} \tag{21}$$

For each item i starting from 1 and ending at n the pair (i, j) is chosen, as shown in Algorithm 1, and it is inserted in the set $\mathcal{S}$.

Algorithm 1: Decision on the set $\mathcal{S}$.

```
Data: PT_ij
Result: S
S ← ∅;
assignedPlants ← ∅;
for i = 1 to n do
    min ← ∞;
    bestJ ← 0;
    for j = 1 to f do
        if (PT_ij < min) & (j ∉ assignedPlants) then
            min ← PT_ij;
            bestJ ← j;
    Add (i, bestJ) to S;
    Add j to assignedPlants;
    if |assignedPlants| = |I| then
        assignedPlants ← ∅;
Return S.
```

After solving DModel, we apply the KS heuristic to the current solution as described in the next section.

4.2 KS heuristic

In this section we present an improvement strategy based on a math-heuristic known as Kernel Search (KS). The KS heuristic has presented good solutions to some combinatorial optimization problems including to lot sizing problems (Guastaroba et al., 2017; Carvalho and Nascimento, 2018).

Introduced first in (Angelelli et al., 2007) to solve a portfolio problem, the KS is inspired on ideas of the core-based algorithms proposed by Balas and Zemel (1980); Pisinger (1994); Martello et al. (2000) to address knapsack problems. Generally speaking, the KS is a strategy composed of two phases where several reduced problems are solved by re-optimizing a dynamic set of promising variables, denoted by *kernel*, and fixing the remaining variables at 0.

In the first phase of KS, called the Initialization phase, the solution for the linear relaxation of the original problem is commonly used to decide which variables are most likely to be in the optimal solution of the investigated problem and to build the structure of a KS, which is composed of a kernel and a sequence of buckets. Therefore, initially, all variables with a linear relaxation value greater than 0 are addressed to the kernel and the remaining variables are sorted according to their associated reduced costs in a sequence of nb evenly-sized buckets. Then, the first reduced subproblem is solved by considering the re-optimization of the variables in the kernel and by fixing the variables in the buckets at 0. The re-optimization here is modelled as a MIP.

During the second phase, known as the Extension phase, the method keeps investigating the solution space of the problem by iteratively re-optimizing, besides the variables in the kernel, those from one of the sequence of nb buckets defined on the Initialization phase. Therefore, at each iteration a reduced problem restricted to the kernel and the next bucket from an ordered sequence of buckets, and fixing all the remaining variables at 0, is solved. Moreover, to speed-up the solution process and to find new promising variables, each reduced problem is also subject to two additional constraints. Whilst one constraint imposes an upper bound on the objective function, a second constraint guarantees that at least one variable of the bucket being investigated is in the solution of the current subproblem. If a feasible solution is found, then the kernel is enlarged by adding to it all variables of the bucket with value greater than 0 in the solution of the current iteration. The method ends after investigating all the buckets. For more details on the KS heuristic we refer the reader to (Angelelli et al., 2007, 2010; Guastaroba et al., 2017).

Based on the aforementioned standard KS, next, we describe in detail both the Initialization and the Extension phases of a slightly adapted KS heuristic applied here to solve the MPCLSP-PFT.

Initialization phase

The linear relaxation solution (including the reduced cost) associated to the relaxed variables of the original problem are commonly used to guide the construction of the kernel and the sequence of buckets. A difference in the adopted KS is the use of a feasible solution found by the MIP-based approach, described in Section 4, to build the kernel and the sequence of buckets, besides the use of the linear solution. The Initialization phase can be described as follows.

1. *Step 1 - Solve the linear relaxation of the MPCLSP-PFT*
   Let $(x, y^{LR}, z^{LR}, b, w, tr)$ be the decision variables of the linear relaxation of the MPCLSP-PFT. First, the KS method solves the problem $(P2)$.

$$(\mathbf{P2}) \quad \min \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\left[sc_{ij}y^{LR}_{ijt} + vc_{ij}x_{ijt} + hc_i s_{ijt} + \sum_{k\in\mathcal{K}}(tc_{ijk}tr_{ijkt})\right] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{itk}b_{itk}) \tag{22}$$

   subject to:

$$\text{Constraints } (2) - (7), \tag{23}$$

$$z^{LR}_{ij} = 1 \qquad \forall (i,j)\in\mathcal{S} \tag{24}$$

$$y^{LR}_{ijt}\in[0,1],\ z^{LR}_{ij}\in[0,1],\ x_{ijt}\geq 0,\ b_{itk}\geq 0,\ s_{ijt}\geq 0,\ tr_{ijkt}\geq 0 \qquad \forall (i,j,t,k) \tag{25}$$

   Where the optimal solution to problem $(P2)$ is represented by tuple $(x^{(P2)}, y^{(P2)}, z^{(P2)}, b^{(P2)}, w^{(P2)}, tr^{(P2)})$. Note that in (24) we fix some flexibility decisions as defined for the MIP-based approach. The aim is to combine the information of both solutions to have a promising set of decision variables to create the kernel.

2. *Step 2 - Build the kernel*
   The kernel, represented by Π, is the set responsible for keeping the promising variables. In this approach we consider the variables y_{ijt}, for all (t), and z_{ij} as promising if they satisfy at least one of the following conditions:

   (a) their associated pair of indices $(i,j)\in\mathcal{S}$ or z^{P2}_{ij} value is equal to 1;

   (b) at least one reduced cost associated to the linear variables y^{P2}_{ijt}, $t\in\mathcal{P}$, is greater than the average of the positive reduced costs associated to the set y.

   The complementary set $\overline{\Pi}$ receives all the variables y_{ijt} and z_{ij}, that are not in the kernel.
   It is worth mentioning that the remaining decision variables x, b, w and tr are always included in the reduced problems solved throughout the method.

3. *Step 3 - Define sequence of buckets*
   To build the sequence of buckets, the variables of set $\overline{\Pi}$ are sorted according to their plant index. The main idea is to keep all the variables associated to a plant j in the same bucket. Therefore, for each variable $z_{ij}\in\overline{\Pi}$ the following order of the variables is defined to organize the sequence: $\{z_{ij}, y_{ij,1}, y_{ij,2}, \ldots, y_{ij,p}\}$. Next, the variables from $\overline{\Pi}$ are evenly distributed into nb buckets. Therefore, $\lceil|\overline{\Pi}|/nb\rceil$ binary variables are assigned to each bucket B_v, where $v\in\{1,\ldots,nb\}$. In this paper, the value of nb is set up to $\lceil|\overline{\Pi}|/|\Pi|\rceil$ as it is suggested in (Guastaroba et al., 2017).

4. *Step 4 - Solve the MIP of the MPCLSP-PFT restricted to* Π
   In the first attempt to improve the solution found by the MIP-based approach, the problem $(P3)$ is solved.

$$(\mathbf{P3}) \quad \min \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\left[sc_{ij}y_{ijt} + vc_{ij}x_{ijt} + hc_i s_{ijt} + \sum_{k\in\mathcal{K}}(tc_{ijk}tr_{ijkt})\right] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{itk}b_{itk}) \tag{26}$$

   subject to:

$$\text{Constraints } (2) - (7), \tag{27}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\left[sc_{ij}y_{ijt} + vc_{ij}x_{ijt} + hc_i s_{ijt} + \sum_{k\in\mathcal{K}}(tc_{ijk}tr_{ijkt})\right] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{itk}b_{itk}) < Z^* \tag{28}$$

$$x_{ijt} \geq 0,\ b_{itk} \geq 0,\ s_{ijt} \geq 0,\ tr_{ijkt} \geq 0, \qquad \forall (i,j,t,u,k) \tag{29}$$

$$y_{ijt} \in \{0,1\},\ z_{ij} \in \{0,1\}, \qquad \forall y_{ijt} \in \Pi,\ z_{ij} \in \Pi \tag{30}$$

$$y_{ijt} = 0,\ z_{ij} = 0, \qquad \forall y_{ijt} \in \overline{\Pi},\ z_{ij} \in \overline{\Pi} \tag{31}$$

   While constraint (28) imposes an upper bound (Z^*) on the objective function of the problem, constraints (31) ensure that the variables in $\overline{\Pi}$ are fixed at 0. The upper bound Z^* is equal to the value of the objective function of the solution $(x^*, y^*, z^*, b^*, w^*, tr^*)$ provided by the MIP-based approach as input to the KS heuristic.

Extension phase

In the Extension phase the solution space is investigated iteratively by solving reduced problems restricted to the kernel Π combined with the v-th bucket, where v represents the current iteration that starts from 1 and finishes at nb. Next, all the steps performed in the Extension phase are described.

1. *Step 1 - Solve subproblem restrict to* $\Pi \cup B_v$
   At each iteration v the method solves problem $(P4)^v$.

$$(\mathbf{P4}^v) \quad \min \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\left[sc_{ij}y_{ijt} + vc_{ij}x_{ijt} + hc_i s_{ijt} + \sum_{k\in\mathcal{K}}(tc_{ijk}tr_{ijkt})\right] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{itk}b_{itk}) \tag{32}$$

   subject to:

$$\text{Constraints } (2) - (7), \tag{33}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{F}}\sum_{t\in\mathcal{P}}\left[sc_{ij}y_{ijt} + vc_{ij}x_{ijt} + hc_i s_{ijt} + \sum_{k\in\mathcal{K}}(tc_{ijk}tr_{ijkt})\right] + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{P}}\sum_{k\in\mathcal{K}}(bc_{itk}b_{itk}) < Z^* \tag{34}$$

$$x_{ijt} \geq 0, b_{itk} \geq 0, s_{ijt} \geq 0, tr_{ijkt} \geq 0, \qquad \forall (i,j,t,u,k) \tag{35}$$

$$y_{ijt} = 0, z_{ij} = 0, \qquad \forall y_{ijt} \in \bigcup_{h=0,h\neq v}^{nb} B_h, z_{ij} \in \bigcup_{h=0,h\neq v}^{nb} B_h \tag{36}$$

$$\sum_{\{(ijt)|y_{ijt}\in B_v\}} y_{ijt} \geq 1, \tag{37}$$

$$\sum_{\{(ij)|z_{ij} \in B_v\}} z_{ij} \geq 1, \tag{38}$$

$$y_{ijt} \in \{0,1\}, z_{ij} \in \{0,1\}, \qquad \forall y_{ijt} \in (\Pi \cup B_v), z_{ij} \in (\Pi \cup B_v) \tag{39}$$

Constraints (36) have the same function as constraints (31). In order to ensure that the new solution provides promising variables to be added to the kernel, constraints (37) and (38) impose that at least one of the variables from the sets y and z in the bucket B_v belong to the solution of $(P4)^v$.

2. *Step 2 - Update the best solution, the set* Π *and the bucket* B_v
   If a feasible solution to the problem $(P4)^v$ is found by CPLEX within a stopping criteria, the method updates the solution $(x^*, y^*, z^*, b^*, w^*, tr^*)$, the upper bound Z^* and the set Π.
   Let B_v^+ be the set with the variable subsets $\{z_{ij}, y_{ij,1}, y_{ij,2}, \ldots, y_{ij,p}\}$, for all (i,j) such that $z_{ij} \in B_v$ is equal to 1 in the solution of $(P4)^v$. Then, to update the kernel the methods combines B_v^+ into Π, i.e., $\Pi := (\Pi \cup B_v^+)$. Moreover, the set B_v is updated by making $B_v := B_v n B_v^+$.
3. *Step 3 - Update* v *and check stopping criterion*
   After solving problem $(P4)^v$, v is updated by making $v = v + 1$. The Extension phase stops if $(v > nb)$ and returns solution $(x^*, y^*, z^*, b^*, w^*, tr^*)$, otherwise, if $v \leq nb$, the method returns to Step 1 from the Extension phase.
   If $v > nb$ and $(x^*, y^*, z^*, b^*, w^*, tr^*)$ is empty, then it returns that no feasible solution was found.

A pseudo-code of the proposed KS is displayed in Algorithm 2.

Algorithm 2: KS heuristic.

Data: Solution obtained by the MIP-based approach $(x^*, y^*, z^*, b^*, w^*, tr^*)$.
Result: $(x^*, y^*, z^*, b^*, w^*, tr^*)$.

```
/* Initialization phase                                                    */
Solve the problem (P2);
Build kernel Π, set Π̅;
Set up nb = ⌈|Π̅|/|Π|⌉;
Sort variables of set Π̅ and build sequence of buckets B_v, with v ∈ {1,...,nb};
(x^(P3), y^(P3), z^(P3), b^(P3), w^(P3), tr^(P3)) ← solution of (P3);
if (x^(P3), y^(P3), z^(P3), b^(P3), w^(P3), tr^(P3)) is feasible then
    (x*, y*, z*, b*, w*, tr*) ← (x^(P3), y^(P3), z^(P3), b^(P3), w^(P3), tr^(P3));
/* Extension phase                                                         */
for v = 1 to nb do
    (x^(P4)^v, y^(P4)^v, z^(P4)^v, b^(P4)^v, w^(P4)^v, , tr^(P4)^v) ← solution of (P4)^v;
    if (x^(P4)^v, y^(P4)^v, z^(P4)^v, b^(P4)^v, w^(P4)^v, , tr^(P4)^v) is feasible then
        (x*, y*, z*, b*, w*, tr*) ← (x^(P4)^v, y^(P4)^v, z^(P4)^v, b^(P4)^v, w^(P4)^v, , tr^(P4)^v);
        Update kernel Π and bucket B_v;
Return solution (x*, y*, z*, b*, w*, tr*).
```

5 Computational experiments

This section is divided in three parts. First, we compare the proposed mathematical models considering a set of small sized instances. Second, we compare the results of the best mathematical model with the heuristic results for some more difficult instances. Third, we present an analysis of the value of flexibility considering a set of small instances for which the optimization package finds the optimal solution or the solutions present small gaps.

The experiments were carried out on a computer with two Intel Core i5-9300H processor of 2.4 GHz, 16 GB DDR3 RAM . The algorithms are implemented in C++ language. To solve each instance, we used 1 thread and imposed a time limit of 3600 seconds to both the KS heuristic and CPLEX v. 12.10 commercial solver.

5.1 General data

In this section, we describe some general parameters that will be used in the next three sections. Among other parameters, we analyze the values for the parameter $Fmax$, which appear in constraints (7) and consists of the total budget available to invest in flexibility. We consider that $fc_{ij} = 1$, so that the total budget represents the maximum number of production-plant links.

The base case for the comparison is the case in which $Fmax$ is equal to the number of products (in the reminder, the $Fmax$ value representing this flexibility configuration is referred to as $Flex1$). The others possible values considered for the parameter $Fmax$ will be 50%, 75% and 100% (in the next tables, the $Fmax$ values representing these flexibility configurations are called, respectively, $Flex2$, $Flex3$ and $Flex4$), where 100% means that all plants can produce all products, that is, there is total flexibility, while the other values are percentages calculated relative to this maximum number of links, i.e., $n \times f$. For example, for the flexibility configuration with 50% ($Flex2$), it is considered that 50% of the total possible links can be used.

The possible choices of the capacity levels per period (constraints (5)) vary according to a factor μ, which multiplies the total average capacity $ACap$, which is calculated by $\left(\left(\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{P}} d_{ikt}vt_i + st_i\right)/\left(f \times p\right)\right)$. We consider values $\mu = 0.6, 0.7, 0.9, 1.1, 1.3$ in order to have a broad range of problems so that the solutions have different levels of backlog. Note that the data used are plant invariant, therefore, we did not consider the index related to the plant in the formula to indicate the production and setup time.

We also consider four different possible values for the transportation costs (tc_{ijk}) in the objective function (1):

- No transportation costs: where $tc_{ijk} = 0$, $\forall i, j, k$;
- Low transportation costs: where for each item i, tc_{ijk} is the Euclidean distance between plant j and client k multiplied by a scalar value $\beta = 0.3$;
- Medium transportation costs: similar to the Low transportation costs $\beta = 0.6$;
- High transportation costs: where tc_{ijk} is 100 times the value defined for Low transportation costs (hence $\beta = 30$);

Table 1 shows how the general parameters were defined for the MPCLSP-PFT, which are based on part of an instance set proposed in (Trigeiro et al., 1989). The remaining parameters will be specified in the next sections.

Table 1: Parameter definitions designed for the MPCLSP-PFT.

Parameters	All classes of instances
d_{itk}	U$[0, 180]$
st_{ij}	U$[10, 50]$
vt_{ij}	1
sc_{ij}	U$[400, 1000]$
hc_{ij}	U$[1, 5]$
bc_{jtk}	300
bc_{jpk}	600
vc_{ij}	0
fc_{ij}	1

Each row presents a set or value used to define their parameters where U$[a, b]$ represents the uniform distribution.

5.2 Comparison of the mathematical models

This section presents a comparison of the proposed mathematical models $M1$, $M2$ and $M3$ in terms of lower bounds, upper bounds, gap and computational time. For this, the instances have 4 and 5 plants (f); 5 and 8 items (n); 5 periods (p); 4 and 6 clients (c).

In this section, the choice of the flexibility levels are $Flex2 = 50\%$ and $Flex3 = 75\%$ and we consider low and high transportation costs (tc_{ijk}) according to the definition presented in the previous section. Moreover, in this section, the choice of the capacity levels per period varies with a factor $\mu = 0.6, 0.9, 1.3$.

Table 2 shows the lower bounds obtained by the linear relaxations of the three models. Each line of this table represents an average of 10 instances, being 5 instances with 4 clients and another 5 instances with 6 clients. It is possible to observe that the best LP lower bounds are obtained by model $M3$. Considering $\mu = 0.9$ and $\mu = 1.3$, model $M2$ obtains better LP lower bounds than model $M1$. However, when the capacity is very tight ($\mu = 0,6$) model $M1$ can obtain better LP lower bounds than model $M2$. In A we include a small example and a discussion about these lower bounds.

Table 2: Linear relaxation lower bounds obtained by each mathematical model.

μ	f	n	c	p	Low transportation cost					
					Flex2–50%			Flex3–75%		
					M1	M2	M3	M1	M2	M3
0,6	4	8	4/6	5	3749539	3825795	**3894188**	3749539	3825795	**3894188**
0,6	5	5	4/6	5	2776698	2442629	**2779972**	2776698	2442629	**2779972**
0,9	4	8	4/6	5	11188	24077	**24116**	11188	24068	**24107**
0,9	5	5	4/6	5	14470	15373	**16833**	14470	15372	**16833**
1,3	4	8	4/6	5	7995	23768	**23770**	7995	23763	**23766**
1,3	5	5	4/6	5	9932	15151	**15465**	9932	15150	**15465**
Averages					1094970	1057799	**1125724**	1094970	1057796	**1125722**
μ	f	n	c	p	High transportation cost					
					Flex2–50%			Flex3–75%		
					M1	M2	M3	M1	M2	M3
0,6	4	8	4/6	5	3860162	4036550	**4090468**	3860162	4036550	**4090467**
0,6	5	5	4/6	5	2842214	2572275	**2875256**	2842214	2572275	**2875292**
0,9	4	8	4/6	5	142934	206183	**206183**	142934	199112	**199112**
0,9	5	5	4/6	5	104230	131101	**131172**	104230	129581	**129621**
1,3	4	8	4/6	5	131958	183750	**183750**	131958	178055	**178055**
1,3	5	5	4/6	5	87915	117786	**117786**	87915	115987	**115987**
Averages					1194902	1207941	**1267436**	1194902	1205260	**1264756**

Next we fix the level of flexibility equal to 75% ($Flex3 = 75\%$) and analyze other aspects comparing the results of the mathematical models. Table 3 shows the best lower bound, upper bounds, the computational times and gap. In order to measure the solution quality, let *gap* be the distance between the best solution (Z_{UB}) and the best lower bound (Z_{LB}) found by CPLEX v.12.10 time limited to 3600 seconds. We calculate the *gap* as it is presented in Equation (40).

$$gap = 100 \times \frac{Z_{UB} - Z_{LB}}{Z_{LB}} \ \% \tag{40}$$

From Table 3 it is possible to observe that, considering low transportation cost, Model $M1$ obtained better lower bounds for four of the six considered configurations and on average the lower bounds are 1.11% and 0.21% better than models $M2$ and $M3$, respectively. On the other hand, when considering high transportation cost, the lower bounds found by the three models are more similar and the general average difference between the results of model $M1$ and the models $M2$ and $M3$ decreases to only

0.68% and 0.14%, respectively. Table 3 also shows that the values of the upper bounds are quite similar considering the three different models. However, again model $M1$ found better general average results (although the difference is less than 0.04% for all configurations and models). Finally, it is also interesting to see that both computational times and optimally gaps reduce considering high transportation cost when compared to the case with low transportation costs. We also observe that generally instances become easier to solve when the capacity is less tight.

Table 3: Lower bounds, upper bounds, computational times and gaps obtained by each mathematical model.

					Low transportation cost (Flex3–75%)											
μ	f	n	c	p	M1				M2				M3			
					LB	UB	GAP	Time	LB	UB	GAP	Time	LB	UB	GAP	Time
0,6	4	8	4/6	5	**3937580**	**3985694**	**1,2**	**3600**	3909431	3987682	2,0	**3600**	3925016	3988534	1,6	**3600**
0,6	5	5	4/6	5	**2806476**	**2822546**	**0,6**	**3600**	2759706	2822575	2,4	**3600**	2805370	2822554	0,7	**3600**
0,9	4	8	4/6	5	24452	24622	0,7	3600	**24508**	**24620**	**0,4**	**2692**	24387	24637	1,0	3251
0,9	5	5	4/6	5	**18272**	**18399**	**0,7**	**1800**	17733	18400	3,6	3022	18128	18400	1,4	2639
1,3	4	8	4/6	5	**23833**	**23835**	**0**	231	**23833**	**23835**	**0**	**12**	**23833**	**23835**	**0**	82
1,3	5	5	4/6	5	**15998**	**15999**	**0**	982	15975	**15999**	0,1	**955**	15977	**15999**	0,1	957
Averages					**1137769**	**1148516**	**0,5**	**2302**	1125198	1148852	1,4	2314	1135452	1148994	0,8	2355
					High transportation cost (Flex3–75%)											
μ	f	n	c	p	M1				M2				M3			
					LB	UB	GAP	Time	LB	UB	GAP	Time	LB	UB	GAP	Time
0,6	4	8	4/6	5	**4141558**	4168911	**0,7**	**3600**	4126407	**4168900**	1,1	**3600**	4132061	4170759	0,9	**3600**
0,6	5	5	4/6	5	**2896688**	2906011	**0,4**	**3295**	2859997	2907548	1,8	3600	2895446	**2906003**	**0,4**	3509
0,9	4	8	4/6	5	205011	**205068**	**0**	995	**205047**	**205068**	**0**	**443**	205017	**205068**	**0**	1222
0,9	5	5	4/6	5	132454	**132599**	**0,1**	1291	**132512**	**132599**	**0,1**	**862**	132470	132605	**0,1**	1452
1,3	4	8	4/6	5	**182325**	**182343**	**0**	63	**182325**	**182343**	**0**	**45**	**182325**	**182343**	**0**	90
1,3	5	5	4/6	5	**117530**	**117541**	**0**	24	**117530**	**117541**	**0**	**23**	117529	**117541**	**0**	39
Averages					**1279262**	**1285413**	**0,2**	1545	1270637	1285667	0,5	**1429**	1277475	1285720	**0,2**	1652

5.3 Evaluation of the proposed heuristic

This section presents a comparison of the proposed mathematical model $M1$ against the heuristic approach. We choose model $M1$ because, according to the results from the previous section, it show a good performance in terms of quality of upper bounds, gaps and computational times. Regarding the heuristic approach, we have tested different combinations of models and parameters. Analyzing these preliminary tests, the best results were obtained when considering DModel, so that to solve the problems DModel and $(P2)$ we time limited CPLEX v. 12.10 to 300 seconds, which is on average 100 times more than the necessary time for the instances considered. Moreover, to find a solution to $(P3)$ and $(P4)^v$ in the KS heuristic, CPLEX was time limited according to the amount of time spent (SpentTime^v) until the iteration v, the available time (AvailTime) and number of remaining iterations (RIt^v) at iteration v. Therefore, at each iteration v, the time given to CPLEX, called TimeMIP^v, is calculated as it is shown in Equation (41).

$$\text{TimeMIP}^v = (\text{AvailTime - SpentTime}^v)/\text{RIt}^v \tag{41}$$

Where AvailTime was set to 3600 seconds. It is important to highlight that the best results were obtained using model $M3$ to solve the linear relaxation $(P2)$ and model $M1$ in the other steps of the Kernel search.

The instances used in this section have 8, 10 and 12 plants (f); 8, 10 and 12 items (n); 8, 10 and 15 periods (p); 6 and 12 clients (c). The choice of the flexibility levels are $Flex3 = 75\%$ and $Flex4 = 100\%$ and we consider low transportation costs (tc_{ijk}) according to the definition presented in the previous section. Moreover, in this section, the choice of the capacity levels per period varies according to $\mu = 0.6, 0.9, 1.1, 1.3$.

In Table 4 we compare the results of the mathematical model and the heuristic solution for instances with Flex3- 75%. We see that the computational package reaches the time limit for all instances when solving the mathematical model while the heuristic solution presented a reduced computational time. The gaps of the heuristic presented in Table 4 are calculated using the lower bounds found by $M1$. Analysing the Gap, the mathematical model $M1$ obtains better gaps for instances up to 12 plants, 10 items, 6 clients and 10 periods. For instances bigger than this, the gaps obtained when solving the mathematical model are quite high, while the heuristic presents a smaller gap in most of the cases. Observe that on average, while the gap found by the heuristic is 12.29, the model presents a significant higher value (equal to 31.08).

Table 4: Mathematical model and heuristic solution (Flex3- 75%).

					Low transportation cost Flex3- 75%								
					M1					Kernel M1 + M3			
μ	f	n	c	p	LB linear	LB	UB	Time	Gap	Initial sol.	Kernel sol.	Time	Gap
0,6	8	8	6	8 and 10	15110259	15128755	**15278330**	3600	**0.99**	15705670	15351672	**1933**	1.48
0,6	8	10	6	8 and 10	18162430	18200448	**18376182**	3600	**0.98**	19004599	18381056	**1669**	1.00
0,9	8	8	6	8 and 10	39634	48286	**50160**	3600	**3.74**	59259	50300	**1029**	3.99
0,9	8	10	6	8 and 10	40268	58534	**63802**	3600	**8.11**	12040019	64884	**1423**	9.68
1,1	8	8	6	8 and 10	32510	46305	**47600**	3600	**2.77**	48106	47821	**1215**	3.22
1,1	8	10	6	8 and 10	33185	57362	**59642**	3600	**3.80**	9408708	60791	**1273**	5.57
1,3	8	8	6	8 and 10	27719	45545	**46822**	3600	**2.79**	47535	47260	**846**	3.74
1,3	8	10	6	8 and 10	28397	56832	**57332**	3600	**0.83**	6796277	58254	**1138**	2.34
0,6	10	12	6	8 and 10	88080144	88095492	88614425	3600	0.62	89559591	**88574293**	**3381**	**0.56**
0,6	12	12	6	8 and 10	89435478	89443062	**90303606**	3600	**1.01**	91262093	91045684	3600	1.65
0,9	10	12	12	10 and 15	77422	106566	48379917	3600	72.09	50914922	**29157543**	**2103**	**67.79**
0,9	12	12	12	10 and 15	88896	108960	207890297	3600	79.66	116889	**116819**	**2476**	**6.62**
1,1	10	12	12	10 and 15	64741	103840	54995327	3600	69.80	40909839	**159907**	**1673**	**27.31**
1,1	12	12	12	10 and 15	73906	101335	140378625	3600	91.89	116349	**116327**	**1895**	**12.59**
1,3	10	12	12	10 and 15	56145	100762	224347565	3600	79.88	30940009	**8458620**	**2745**	**33.34**
1,3	12	12	12	10 and 15	63482	98088	218967408	3600	78.24	116349	**116346**	**2882**	**15.30**
Averages					13213414	13237511	69241065	3600	31.08	22940388	**15737974**	**1955**	**12.26**

Table 5 shows the results of the mathematical model and the heuristic solution for instance with Flex4- 100%. It is possible to see that the behavior of the results is similar compared to the previous results presented in Table 4. In other words, for the instances with 8 and 10 items, although the upper bounds of the model are slightly smaller than those presented by the heuristic, the model always reaches the time limit of 3600 seconds while the solution times presented by heuristic are significantly smaller. On the other hand, considering the instances with 12 items, the upper bounds found by the heuristic are much better than the upper bounds found by the model and on average, the gap found by the model is greater than twice that found by the heuristic.

5.4 Analysis of flexibility

This section presents an analysis of the benefits of the flexibility in the integrated lot sizing and transportation problem. The focus of the analysis is concentrated on the chosen values for the parameter $Fmax$. In order to obtain good solutions allowing a better analysis of the value of the flexibility, reduced instances were used. The instances have 3 plants (f); 3 and 6 items (n); 5 periods (p); 4 and 6 clients (c). In this section, we consider values $\mu = 0.6, 0.7, 0.9, 1.1, 1.3$ in order to have a broad range of problems so that the solutions have different levels of backlog. Moreover, four different values for the transportation costs previously described are considered.

In Table 6 we present the average upper bounds (columns UB) found for all instances and flexibility configurations considering different levels of transportation costs. Note that the upper bounds found

Table 5: Mathematical model and heuristic solution (Flex4- 100%).

μ	f	n	c	p	Low transportation cost Flex4- 100%								
					M1					Kernel M1 + M3			
					LB linear	LB	UB	Time	Gap	Initial sol.	Kernel sol.	Time	Gap
0,6	8	8	6	8 and 10	15110259	15124683	**15277995**	3600	**1.02**	15705670	15338827	**2044**	1.42
0,6	8	10	6	8 and 10	18162430	18200916	**18376321**	3600	**0.98**	19004599	18395159	**1822**	1.08
0,9	8	8	6	8 and 10	39634	48015	**50162**	3600	**4.31**	59259	50376	**1095**	4.70
0,9	8	10	6	8 and 10	40268	58177	**64105**	3600	**9.11**	12040019	65265	**1292**	10.74
1.1	8	8	6	8 and 10	32510	46123	**47668**	3600	**3.22**	48106	47886	**921**	3.71
1,1	8	10	6	8 and 10	33185	57198	**59619**	3600	**4.02**	9408708	61156	**1455**	6.33
1,3	8	8	6	8 and 10	27719	45423	**46824**	3600	**3.01**	47535	47278	**920**	3.96
1,3	8	10	6	8 and 10	28397	56810	**57328**	3600	**0.84**	6796277	58395	**1253**	2.59
0,6	10	12	6	8 and 10	88080144	88094341	88651169	3600	0.63	89559591	**88551622**	**3413**	**0.54**
0,6	12	12	6	8 and 10	89435478	89444252	**90284573**	3600	**0.98**	91262093	91051251	3600	1.65
0,9	10	12	12	10 and 15	77422	105743	225969350	3600	76.24	50914922	**34401987**	**2759**	**61.96**
0,9	12	12	12	10 and 15	88896	108134	107713440	3600	79.41	116889	**116800**	**2704**	**7.38**
1,1	10	12	12	10 and 15	64741	102251	288008239	3600	78.50	40909839	**22419221**	**2582**	**63.32**
1,1	12	12	12	10 and 15	73906	100934	329445442	3600	83.50	116349	**116349**	**2860**	**13.01**
1,3	10	12	12	10 and 15	56145	98175	160148340	3600	61.39	30940009	**8459704**	**2672**	**35.86**
1,3	12	12	12	10 and 15	63482	95427	335676401	3600	91.97	116349	**116346**	**2893**	**17.52**
Averages					13213414	13236663	103742311	3600	31.20	22940388	**17456101**	**2143**	**14.74**

by the case in which each product can be made in exactly one plant ($Flex1$) were set to 100% and the other values are calculated relative to this. The overall computational results show that the benefits of flexibility depend on the amount of flexibility (number of links between plants and products), capacity level and transportation costs. In this analysis, the value of flexibility is defined as the possible relative cost reduction of a given flexibility level compared to the case with no flexibility. In Table 6, the value of flexibility is hence equal to 100% - UB(%).

Fiorotto et al. Fiorotto et al. (2018) analysed the case without transportation cost. The setting in their paper was an environment consisting of multiple parallel machines, which can be dedicated to produce only one type of product, or can be flexible to produce multiple types of products. This corresponds to our case with no transportation costs in Table 6. Fiorotto et al. Fiorotto et al. (2018) presented two main findings, which were in line with the observations made in Jordan and Graves (1995). However, when we analyse the case with (high) transportation cost, we see that these two observations do not hold anymore. We next discuss each of these two findings in detail and compare the case with and without transportation cost.

First, Fiorotto et al. (2018) observe that the value of flexibility is the highest when the overall capacity is roughly in line with overall demand, but this value decreases when capacity is much lower than demand, or when capacity is much higher than demand. In Table 6, we observe a similar trend for the case with no and low transportation cost, where the case with low capacity is represented by a low value of μ and the case with a high level of capacity is represented by a high value of μ. The reason is as follows. With a very low capacity level, the base case (in which each product is made in only one plant) has a lot of backlog and the capacity is already fully used, so flexibility will bring little benefit since it will not allow to satisfy more demand. With a high level of capacity, the base case with no flexibility already allows to satisfy all the demand with no backlog, and hence flexibility also has little value here.

However, when transportation costs are present and substantial, we observe a clear tendency that for cases with a high capacity level (corresponding to a high value of μ) the value of flexibility remains high. This can be explained by the additional trade-off with the transportation cost that is present in this problem. With a very low capacity level, flexibility will bring little benefit as explained for the case with no transportation cost. When capacity increases, flexibility will allow to first reduce the backlog and satisfy more of the demand and has hence more value. At a very high capacity level, the base case does not present any backlog, but flexibility still has value because it enables to reduce the transportation cost. The fact that products can be made in several plants allows to reduce the distance to the final customers. Of course, the exact value of flexibility also depends on other costs, such as the setup costs. Indeed, the number of overall setups will increase if we produce a specific type of item in several plants.

A second main difference that we observe when we introduce (high) transportation cost is that the difference between the value of a little flexibility ($Flex2$) and full flexibility ($Flex4$) becomes higher. In the case without transportation cost, as analysed by Fiorotto et al. (2018) and also observed in Jordan and Graves (1995), a low level of flexibility ($Flex2$) (if configured in a smart way) gives almost the same benefits as full flexibility ($Flex4$). We observe a similar trend for the case with no, low and medium transportation cost. Indeed, for these cases, the maximum difference between $Flex2$ and $Flex4$ is 2.28%. However, with high transportation costs, this difference becomes much higher for the cases with a high level of capacity ($\mu = 1.1$ and 1.3). For the case with $\mu = 1.3$, 6 products and six plants, this difference reaches 11.04%. For this case, even the difference between $Flex3$ and $Flex4$ is equal to 3.82%. This indicates that with high transportation cost, not all the benefits of full flexibility can be reached with a limited level of flexibility.

In order to better understand the results presented in Table 6, we show in Table 7 the characteristics of the solutions considering no transportation cost and high transportation cost for $Flex1$ and $Flex4$: the percentage of setup, backlog, transportation and holding cost (columns $SC\%$, $BC\%$, $TC\%$ and $IC\%$, respectively) in the objective function value. Considering no transportation cost, we observe that the biggest benefits of flexibility shown in Table 6 (capacity level equal to 0.9 and 0.7 for the problems with 3 and 6 items, respectively) are the result of decreasing the percentage of backlog cost in the objective function when considering flexibility. For example, considering the instance with 3 items, 4 clients and capacity level equal to 0.9, the percentage of backlog cost goes from 74.31% to 1.24% for $Flex1$ and $Flex4$, respectively. On the other hand, the percentage of setup cost goes from 24.01% to 93.36%. It shows that adding flexibility allowed a more adequate production planning where, although the number of setups increased, the total level of backlog decreased significantly. Regarding the case with high transportation cost, Table 6 showed high benefits of flexibility even for instances with high capacity levels. Table 7 shows that it happens because although there is no backlog in these cases, adding flexibility allows to reduce the transportation cost significantly. Note that for the instances with high capacity levels (1.1 and 1.3) by adding flexibility it is possible to decrease the percentage of transportation cost around 10%. Therefore, different from the case with no transportation cost (in which the benefits of flexibility comes only from reducing the backlog), in the studied problem, the benefits of flexibility comes from the backlog and also the number of items transported. However, for the same case, we also observe that the setup cost portion increased significantly when adding more flexibility, since one product can now be produced in many different plants.

Aiming to further analyse the effect of the flexibility considering transportation costs, in Figure 2 we present the behavior of the total setup, backlog, transportation and inventory costs when increasing the amount of flexibility. On the horizontal axis, zero stands for the case with no flexibility and the values of 1, 2, and 3 stand for Flex1, Flex2 and Flex3, respectively. The different colored lines within one graph indicate the various cases for the transportation cost, ranging from no transportation cost to high transportation cost. Figure 2 shows that while the decrease in the backlog cost is very similar for all levels of transportation costs, the behavior and especially the size of the decrease in the other costs are quite different. Note that with high transportation cost, the setup and inventory costs increase when increasing the amount of flexibility. On the other hand, the transportation cost decreases significantly by adding flexibility. It is important to see that for this problem, the value of the total transportation cost is much bigger than the setup and inventory costs which makes the increase in these latter costs insignificant when calculating the benefits of flexibility.

Regarding the solution gaps and times, Figure 3 shows that both values increases by adding flexibility and the biggest values are found for the problems without transportation costs. On the other

Table 6: Comparison of upper bounds considering three plants.

			No transportation costs				Low transportation cost				Medium transportation cost				High transportation cost			
I	C	μ	Flex1 UB(%)	Flex2 UB(%)	Flex3 UB(%)	Flex4 UB(%)	Flex1 UB(%)	Flex2 UB(%)	Flex3 UB(%)	Flex4 UB(%)	Flex1 UB(%)	Flex2 UB(%)	Flex3 UB(%)	Flex4 UB(%)	Flex1 UB(%)	Flex2 UB(%)	Flex3 UB(%)	Flex4 UB(%)
	4		100	100	97.72	97.72	100	97.73	97.71	97.71	100	97.71	97.67	97.66	100	95.75	95.2	95.09
	6	0.6	100	100	98.61	98.61	100	98.61	98.61	98.61	100	98.59	98.57	98.56	100	98.06	97.69	97.6
	4		100	96.33	96.33	96.33	100	96.33	96.32	96.32	100	96.28	96.21	96.2	100	97.75	95.2	95.09
	6	0.7	100	99.05	99.05	99.05	100	99.04	99.03	99.03	100	98.94	98.9	98.89	100	98.06	97.69	97.6
	4		100	29.86	29.81	29.81	100	31.58	31.55	31.53	100	43.79	42.74	42.55	100	70.86	64.86	63.29
3	6	0.9	100	52.37	52.37	52.37	100	55.52	55.4	55.36	100	68.48	67.87	67.67	100	77.09	70.55	67.74
	4		100	100	100	100	100	99.63	99.53	99.49	100	97.18	96.66	96.38	100	82.85	78.58	76.39
	6	1.1	100	100	100	100	100	99.37	99.25	99.21	100	93.22	92.59	92.37	100	79.34	74.14	70.61
	4		100	100	100	100	100	99.62	99.46	99.44	100	95.83	94.91	94.74	100	82.54	76.93	75.91
	6	1.3	100	100	100	100	100	99.27	99.08	99.06	100	92.4	91.81	91.64	100	78.17	73.76	69.9
	4		100	99.08	99.08	99.08	100	99.08	99.08	99.08	100	99.06	99.05	99.05	100	98.92	98.81	98.81
	6	0.6	100	98.83	98.83	98.83	100	98.83	98.82	98.82	100	98.81	98.78	98.78	100	98.64	98.44	98.44
	4		100	94.61	94.58	94.60	100	94.59	94.6	94.58	100	94.64	94.59	94.57	100	94.9	94.46	94.44
	6	0.7	100	95.43	95.29	95.29	100	95.44	95.29	95.29	100	95.44	95.26	95.25	100	95.4	94.92	94.89
	4		100	95.19	95.16	95.16	100	95.38	95.07	95.07	100	95.6	94.02	94	100	94.89	94.46	94.44
6	6	0.9	100	97.53	97.53	97.53	100	97.48	97.09	97.07	100	94.52	92.69	92.58	100	95.45	94.92	94.89
	4		100	99.56	99.56	99.56	100	99.15	98.98	98.98	100	96.79	95.71	95.66	100	87.13	79.13	77.42
	6	1.1	100	99.73	99.73	99.73	100	98.93	98.68	98.67	100	92.99	92.23	92.22	100	80.1	74.53	71.73
	4		100	99.92	99.92	99.92	100	99.45	99.3	99.3	100	97.47	96.92	96.91	100	89.55	80.87	80.22
	6	1.3	100	100	100	100	100	99.14	99.03	99.03	100	98.37	98.07	98.07	100	93.02	85.8	81.98

Table 7: Percentage of setup, backlog, transportation and holding cost in the objective function value.

			No transportation costs								High transportation cost							
			Flex1				Flex4				Flex1				Flex4			
I	C	μ	SC(%)	BC(%)	TC(%)	IC(%)	SC(%)	BC(%)	TC(%)	IC(%)	SC(%)	BC(%)	TC(%)	IC(%)	SC(%)	BC(%)	TC(%)	IC(%)
	4		0.61	99.38	0	0.01	0.63	99.36	0	0.01	0.59	95.66	3.74	0.01	0.62	96.21	3.16	0.01
	6	0.6	0.38	99.62	0	0	0.38	99.62	0	0	0.36	95.97	3.67	0	0.38	96.51	3.11	0
	4		1.22	98.71	0	0.07	1.33	98.63	0	0.05	1.10	89.65	9.18	0.06	1.23	90.74	7.99	0.04
	6	0.7	0.78	99.19	0	0.03	0.81	99.18	0	0.02	0.71	89.83	9.43	0.03	0.75	91.12	8.12	0.01
	4		24.01	74.31	0	1.69	93.36	1.24	0	5.4	6.88	21.31	71.33	0.48	17.33	0.24	80.62	1.81
3	6	0.9	42.80	54.81	0	2.38	94.66	0	0	5.34	5.23	6.70	87.78	0.29	14.51	0	83.40	2.09
	4		97.44	0	0	2.56	97.44	0	0	2.56	8.37	0	91.41	0.22	15.16	0	81.45	3.39
	6	1.1	96.60	0	0	3.40	96.60	0	0	3.40	5.40	0	94.41	0.19	13.46	0	83.87	2.68
	4		93.81	0	0	6.19	93.81	0	0	6.19	7.75	0	91.74	0.51	14.91	0	81.77	3.32
	6	1.3	93.63	0	0	6.37	93.63	0	0	6.37	5.10	0	94.55	0.35	12.75	0	84.13	3.12
	4		0.44	99.53	0	0.02	0.48	99.50	0	0.02	0.42	94.06	5.50	0.02	0.46	94.29	5.23	0.02
	6	0.6	0.28	99.71	0	0.01	0.31	99.69	0	0	0.26	94.73	5.00	0.01	0.29	95.07	4.64	0
	4		1.25	98.58	0	0.17	1.64	98.29	0	0.08	1.08	85.06	13.72	0.15	1.43	85.02	13.49	0.07
	6	0.7	0.80	99.14	0	0.05	0.91	99.07	0	0.02	0.71	86.54	12.70	0.05	0.80	86.77	12.41	0.02
	4		89.65	0	0	10.35	88.02	0	0	11.98	7.96	0	91.24	0.80	16.60	0	81.01	2.40
6	6	0.9	95.69	0	0	4.31	90.58	0	0	9.42	5.51	0	94.20	0.29	13.80	0	83.89	2.31
	4		85.90	0	0	14.10	85.17	0	0	14.83	7.41	0	91.66	0.93	15.09	0	81.30	3.61
	6	1.1	91.56	0	0	8.44	90.53	0	0	8.47	5.33	0	94.34	0.33	13.21	0	83.62	3.16
	4		81.16	0	0	18.84	81.23	0	0	18.77	7.77	0	91.15	1.08	14.30	0	81.63	4.07
	6	1.3	90.23	0	0	9.77	90.23	0	0	9.77	6.02	0	93.25	0.73	12.81	0	83.72	3.47

hands, the smallest values are found considering the problem with high transportation costs. However, note that solution gaps and times are on average always less than 0.3% and 600 seconds.

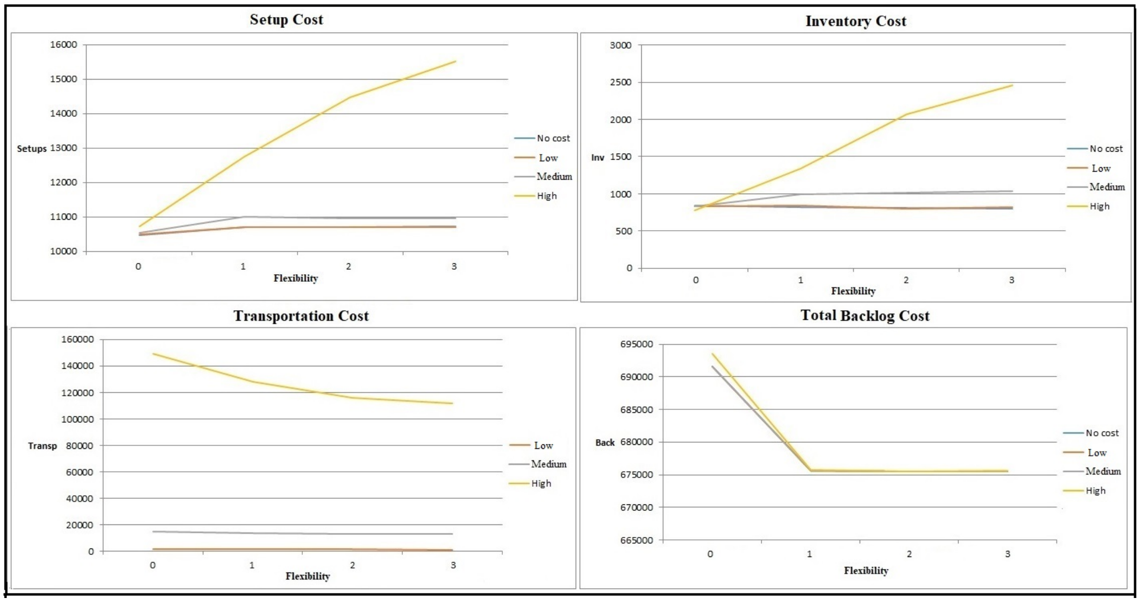

Figure 2: Averages setup, backlog, transportation and inventory costs.

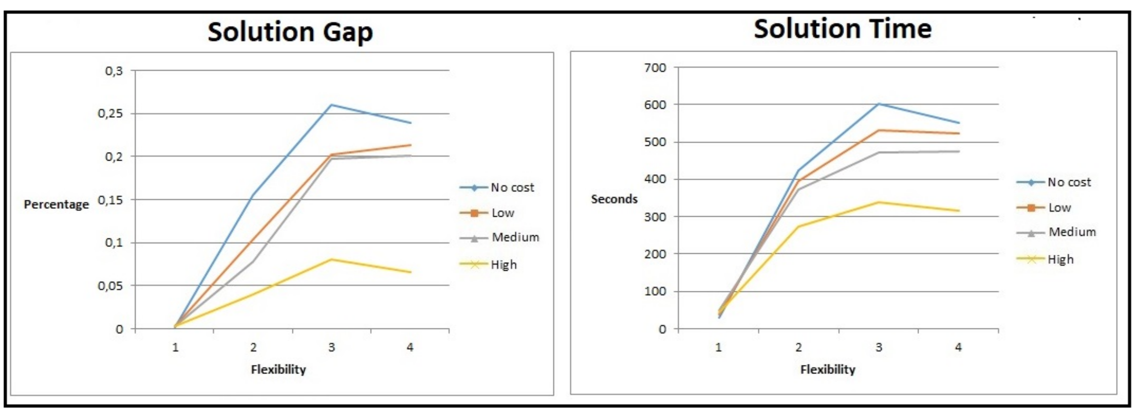

Figure 3: Average solution gaps and times.

6 Conclusion

In this paper the lot-sizing problem with process flexibility in a deterministic context was studied. Different from the standard problem with parallel resources, the studied problem considers a limited amount of flexibility so that each resource can produce only certain types of items. We extend a previous paper from the literature by considering a network of costumers dispersed around different plants obtaining an integrated lot sizing and transportation problem. The objective is to analyze the value of flexibility and develop solution methods for this problem. In order to overcome the difficulties in solving medium and especially big size instances, we propose a hybrid solution method that combines mixed integer programming-based approach and kernel search heuristic.

Our computational experiments show that, in terms of total costs, the proposed hybrid solution method presents on average better solutions with significantly lower computational times when compared with the results produced by a high-performance MIP software. Moreover, for several levels of capacity, the optimality gaps found by the proposed approach are significantly lower than those presented by the high-performance MIP software.

Our analyses indicate that some of the main managerial insights derived for the case without transportation cost are no longer valid when we introduce (high) transportation cost. More specifically, with transportation costs, we find that flexibility adds benefits in the case of high capacity levels because flexibility allows to lower the transportation costs. Furthermore, we found that with high transportation cost and a high level of capacity, a limited amount of flexibility does not provide similar benefits as the case with full flexibility. Therefore, considering that to invest in flexibility (adding new products to the production line) can be very expensive in practice, this study can be used as a guide for industries to carry out an adequate planning of investment in flexibility to decrease the total costs.

There are some interesting issues that can be explored as further research, for example, to solve the problem considering overtime instead of backlog costs and apply the proposed method to this new problem. It would also be interesting to analyse the value of flexibility in a stochastic setting in which demand is uncertain.

A Analysis of the lower bounds

Based on the results presented in Section 3, we can observe an unusual situation where, depending on the available capacity, the reformulation $M2$ of the problem does not provide better lower bounds than those obtained with the classical formulation. This fact motivated us to use a third formulation $M3$, which combines $M1$ and $M2$ in such a way that we always have the best lower bound independent of the available capacity.

To investigate what makes the classical formulation $M1$ providing better lower bounds to problems with tight capacity compared to the reformulation $M2$, we solved a small example. Consider two small instances to the MPCLSP-PFT with one item, one plant, one client, and two periods with the parameter values presented in Table 8. Note that, apart from the capacity values, all the parameter have the same values. In the first instance, the capacity in both periods is equal to 80 and in the second instance, the capacity in both periods is equal to 150.

Table 8: Parameter values predefined for a small example of the MPCLSP-PFT.

Parameter	Indices	Value	Parameter	Indices	Value
d_{it}	(1,1) (1,2)	108 107	Cap_{jt}	(1,1) (1,2)	{80,150} {80,150}
st_{ij}	(1,1)	40	sc_{ij}	(1,1)	1000
vt_{ij}	(1,1)	1	vc_{ij}	(1,1)	0
bc_{itk}	(1,1,1) (1,2,1)	300 600	hc_i	(1)	5
			tc_{jk}	(1,1)	0.2658
$Fmax$	-	1			

The solution and detailed information on the solution of the linear relaxation of both models $M1$ and $M2$ are presented in Tables 9 and 10. According to these tables, one can observe that the main difference in the results is the existence/absence of backlog in the last period of the planning horizon for the instance with the smallest/largest available capacity. Therefore, this indicates that the classical formulation provides better lower bounds than the reformulated problem when the available capacity is tight enough to lead to the existence of backlog (last period) in the solution of the linear relaxed problem. Moreover, it indicates that the opposite happens when there is no backlog (last period) in the solution of the linear relaxed problem.

Table 9: Solution values of the linear relaxation for both models and instances.

Cap_{jt}	$M1$		$M2$		Cap_{jt}	$M1$		$M2$	
	DV	Value	DV	Value		DV	Value	DV	Value
	y_{111}	1.00	y_{111}	0.67		y_{111}	0.98	y_{111}	1.00
	y_{112}	1.00	y_{112}	0.67		y_{112}	0.97	y_{112}	0.98
	z_{11}	1.00	z_{11}	1.00		z_{11}	1.00	z_{11}	1.00
	b_{111}	68.00	b'_{1211}	54.67	150	x_{111}	108.00	x_{11111}	108.00
80	b_{121}	135.00	b'_{1221}	53.67		x_{112}	107.00	x_{11121}	2.00
	x_{111}	40.00	x_{11111}	26.67		tr_{1111}	108.00	x_{11221}	105.00
	x_{112}	40.00	x_{11121}	26.67		tr_{1112}	107.00		
	tr_{1111}	40.00	x_{11211}	26.67					
	tr_{1112}	40.00	x_{11221}	26.67					

Table 10: Detailed information based on the results of the linear relaxation of the problem.

Cap_{jt}	Information	$M1$	$M2$	Cap_{jt}	Information	$M1$	$M2$
	Objective function	103421.26	90895.02		Objective function	2011.69	2048.46
	Used capacity	160.00	160.00		Used capacity	293.18	294.25
	Total capacity	160.00	160.00		Total capacity	300.00	300.00
	Number of setups	2.00	1.33		Number of setups	1.95	1.98
	Setup cost	2000.00	1333.33		Setup cost	1954.55	1981.31
80	Number of backlog	135.00	108.34	150	Number of backlog	0.00	0.00
	Backlog cost	101400.00	89400.00		Backlog cost	0.00	0.00
	#Units transported	80.00	106.67		#Units transported	215.00	215.00
	Transportation cost	21.26	28.35		Transportation cost	57.15	57.15
	Number of storage	0.00	26.67		Number of storage	0.00	2.00
	Storage cost	0.00	133.33		Storage cost	0.00	10.00

The solution and detailed information on the solution of the MIP of both models $M1$ and $M2$ are presented in Tables 11 and 12.

Table 11: MIP solution values for both models and instances.

Cap_{jt}	$M1$		$M2$		Cap_{jt}	$M1$		$M2$	
	DV	Value	DV	Value		DV	Value	DV	Value
	y_{111}	1.00	y_{111}	1.00		y_{111}	1.00	y_{111}	1.00
	y_{112}	1.00	y_{112}	1.00		y_{112}	1.00	y_{112}	1.00
	z_{11}	1.00	z_{11}	1.00		z_{11}	1.00	z_{11}	1.00
	b_{111}	68.00	b'_{1211}	28.00	150	x_{111}	108.00	x_{11111}	108.00
80	b_{121}	135.00	b'_{1221}	107.00		x_{112}	107.00	x_{11121}	107.00
	x_{111}	40.00	x_{11111}	40.00		tr_{1111}	108.00		
	x_{112}	40.00	x_{11121}	40.00		tr_{1112}	107.00		
	tr_{1111}	40.00							
	tr_{1112}	40.00							

Table 12: Detailed information based on the results of the MIP problem.

Cap_{jt}	Information	$M1$	$M2$	Cap_{jt}	Information	$M1$	$M2$
	Objective function	103421.26	103421.26		Objective function	2057.15	2057.15
	Used capacity	160.00	160.00		Used capacity	295.00	295.00
	Total capacity	160.00	160.00		Total capacity	300.00	300.00
	Number of setups	2.00	2.00		Number of setups	2.00	2.00
	Setup cost	2000.00	2000.00		Setup cost	2000.00	2000.00
80	Number of backlog	135.00	135.00	150	Number of backlog	0.00	0.00
	Backlog cost	101400.00	101400.00		Backlog cost	0.00	0.00
	#Units transported	80.00	80.00		#Units transported	215.00	215.00
	Transportation cost	21.26	21.26		Transportation cost	57.15	57.15
	Number of storage	0.00	0.00		Number of storage	0.00	0.00
	Storage cost	0.00	0.00		Storage cost	0.00	0.00

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