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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Great fish war with moratorium

Ilyass Dahmouni [a]

Elena M. Parilina [b]

Georges Zaccour [c, d]

[a] *Fisheries Economics Research Unit, Institute for the Oceans and Fisheries, University of British Columbia, Vancouver (BC), Canada, V6T 1Z4*

[b] *Saint Petersburg State University, Saint-Petersburg, 199034, Russia*

[c] *GERAD, Montréal (Qc), Canada, H3T 1J4*

[d] *HEC Montréal, Montréal (Qc), Canada, H3T 2A7*

i.dahmouni@oceans.ubc.ca
e.parilina@spbu.ru
georges.zaccour@gerad.ca

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Abstract : We consider a discrete-time version of the fish war model, where a regulator imposes a moratorium on fishing activities whenever the stock reaches a predetermined critical low value. The moratorium will be in place until the fish stock recovers, that is, attains a desirable value. We obtain conditions on the parameter values such that a moratorium could be avoided, and its optimal duration when its imposition is deemed necessary. We propose a coordinated harvesting strategy profile and determine when it matches the Nash equilibrium in linear-state strategies. Numerical examples show the significant influence of the fish reproduction rate on the length of a moratorium regime and the equilibrium properties of the coordinated strategy profile.

Keywords: Fish war, moratorium, regulation policy, dynamic games

Résumé : Nous considérons une version en temps discret du modèle de la guerre des poissons, où un régulateur impose un moratoire sur les activités de pêche lorsque le stock atteint une valeur basse critique prédéterminée. Le moratoire demeure en vigueur jusqu'à ce que le stock de poissons se rétablisse, c'est-à-dire jusqu'à ce qu'il atteigne une valeur souhaitable. Nous obtenons des conditions sur les valeurs des paramètres telles qu'un moratoire pourrait être évité, ainsi que sa durée optimale lorsque son imposition est jugée nécessaire. Nous proposons un profil de stratégie de récolte coordonnée et déterminons quand il correspond à l'équilibre de Nash dans les stratégies à état linéaire. Des exemples numériques montrent l'influence significative du taux de reproduction des poissons sur la durée d'un régime de moratoire et les propriétés d'équilibre du profil de stratégie coordonnée.

Mots clés : Guerre du poisson, moratoire, politique de régulation, jeux dynamiques

1 Introduction

We consider a fishery exploited by n firms under the eye of a regulator who would implement a moratorium on harvesting any time the stock x of the biomass reaches a pre-specified minimum level $\underline{x}$. Our objective is to deal with the following research questions:

1. Under what conditions, if any, a noncooperative harvesting equilibrium is moratorium-free during the entire (infinite) duration of the game?
2. If a moratorium is not avoidable, what should be its optimal duration, given that the regulator's objective is to bring back the stock to suitable level?
3. Can the players design a coordinated harvesting profile such that (i) the corresponding steady-state value is $\underline{x}$; and (ii) the outcome is a Nash equilibrium?

Using the classical fish-war model in Levhari and Mirman (1980), we answer these questions. In particular, we provide a simple condition on the parameter values under which no moratorium is imposed during the entire duration of the game. The condition involves the trigger moratorium value $\underline{x}$, the reproduction rate α of the species, the number of players and their intertemporal preferences. When the players cannot escape a moratorium, we show that its duration is given by a simple formula that only involves the initial stock x_0, $\underline{x}$, and α. Finally, although it is easy to determine a harvesting coordinated profile that avoids a moratorium, it is not always self-supporting, that is, this profile is a Nash equilibrium, only under some conditions.

1.1 Literature review

Effective fisheries regulations have been critical to meeting the commitments of local and international jurisdictions in their pledge for protecting the marine wildlife (Mora et al. (2009)). According to Ostrom et al. (1992), the adoption and enforcement of new regulations should be consistently architected around spatial and quantitative elements. This could entail the designation of marine protected areas and no-take marine zones, the allocation of landing quotas and the limitation of fishing effort. Still, the proven virtues of these actions as ecologically responsible mechanisms have been significantly sidetracked for a variety of reasons, including the high cost of rollout (McCrea-Strub et al. (2011)), inadequate tolerance of fishing communities (Sumaila (2021)), and myopia of political leaders (Sala et al. (2016)). This observation has prompted a number of scholars to explore alternative, and perhaps simpler, regulations that might be subject to further scrutiny by policymakers, among them the moratorium practice (Clark et al. (1979) and Munro (2010)). The distinctive feature of such a policy lies in its ability to target the protection of certain fish stocks for a given period of time, while the fishing area remains open to other fishing activities. Additionally, recognizing that most costs are incurred in implementing and enforcing other policies (Rosenberg (2007)), the moratorium is considerably lighter in cost (Ding et al. (2021)). It does not require monitoring of fishing areas, weighing of landings or auditing the operation costs reported by each fishing vessel. It simply requires compliance with the basic rule, that is, to ensure that the protected species is not landed or sold in the market if it has been mistakenly harvested.

In practice, the moratorium can take different forms, e.g.: (i) a seasonal fishing moratorium such as the one in the Pearl River estuary (Wang et al. (2015)); (ii) a partial moratorium similar to that applied for American shad in Virginia riverine fisheries (Olney and Hoenig (2001)); (iii) a logistical form akin to the moratorium on high seas transshipment studied in Ewell et al. (2017); (iv) an administrative form as implemented by the Indonesian government moratorium on fishing licenses for foreign and ex-foreign fishing vessels (Khan et al. (2018)); or (v) an operational form as imposed by the United Nations ban and enforced by the North Pacific Anadromous Fish Commission (NPAFC)[1] on all driftnets over 2.5 km in length (Hewison (1994)). Probably, one of the most prominent experiences

[1]The NPAFC includes Canada, the United States, the Russian Federation, the Republic of Korea and Japan. China is a co-operating non-party of this organization.

in the history of fishing restrictions is the moratorium on cod harvesting imposed by the Canadian government in 1992 (Frank et al. (2005) and Rose and Row (2015)). This occurred as a result of the stock being severely impoverished under the open-access regime in the 1960s and 1970s. In 1968, the total harvest was estimated at 820,000 metric tons of cod, of which 700,000 were harvested by foreigners (Baird et al. (1991)). It was not until 1977 that foreign vessels were banned from Canada's Exclusive Economic Zone (EEZ). Yet before the stock had fully rebounded, the number of Canadian vessels doubled, the number of registered fishers increased, and fish plants tripled their freezing potential (Schrank (2005)).[2] Thirty years later, this moratorium is still effective and scientists are observing an overall population growth of the species, with full recovery expected by the year 2030, although the stocks are still in a critical range (Castaneda et al. (2020)).

Clark and Munro (2017) indicate that while a decision to prohibit fishing altogether would have positive effects on the environment and the preservation of biodiversity, the recourse to a moratorium can have dramatic consequences. In other words, the recovery of a fishing stock, is only financially optimized under a very specific and restrictive set of circumstances. This leads us to believe that this trade-off between ecology and economy is at the heart of the reflection when it comes to declaring such a severe decision. The most desirable solution would therefore be to avoid such a situation at all costs and to act in a proactive manner (Munro (2010) and Bjorndal and Munro (2012)). Clark (1976) advocated that once the fish stock has fallen below a given equilibrium level, then it is ideal to institute a moratorium on fishing until the stock has increased to reach the desired steady-state level. Postmortem conduct is as important as its application, as the total harvest should not exceed the increase in the stock arising from the biological reproduction of the species.

In principle, the onset and length of the moratorium is ascertained on the basis of scientific observation of fish population dynamics (Frost et al. (1995)). For most species, surveys may be conducted during periods prior to the spawning season when scientists look for signs reflecting a temporal change in the structure of fish colonies, such as the predominance of relatively noncommercial, small, fast-growing species over larger commercial species with a prolonged overlap of generations. In Rice (2000), various community structure metric techniques used to assess stock dynamics are reviewed, namely: (i) diversity and similarity indices (Magurran (1988) and Warwick and Clarke (1995)); (ii) ordination methods (Jongman et al. (1987)); and (iii) metrics of aggregate community properties, such as abundance-biomass comparison of dominance curves (Warwick (1986) and Warwick and Pearson (1987)), and size spectra (Sheldon et al. (1972) and Lambshead et al. (1983)). Other methods exist for specific species such as temporal variation of species composition (Collins et al. (2000)), maturation rates (Flores et al. (2015) and Jokar (2021)), as well as the environmental DNA (eDNA) technique (Fukaya et al. (2021)). Each of these approaches has proven to be effective in informing the decisions of policymakers. Their scientific accuracy, however, relies on comparisons. Nonetheless, their unifying factor is that the hinge value of the stock level that triggers the start and/or end date decision is proportional to some stock biomass. Technically, this would be the maximum sustainable yield or the highest stock level ever measured. According to Warwick (1986) and Warwick and Pearson (1987), there are three critical stock levels, namely, undisturbed, moderately disturbed, and heavily disturbed. Therefore, throughout this paper, we consider the level of fish stock that triggers the moratorium to be proportional to the initial condition. This assumption reflects the situation where the policy maker begins this exercise by observing the reference fish stock level in the first period and regularly updates the assessments for the purpose of comparisons.

That said, any extensive review of the literature on fisheries economics recommends preventive actions over regulations. Nevertheless, the coordination of cooperation among fisherpersons may fail if no legal framework is put in place (Hardin (1968)). Typically, cooperative behavior must be stimulated by a central regulator; otherwise, coalition stability can only be achieved by a small subset of players (Breton and Keoula (2014) and Kwon (2006)). The cooperation in two-player fish war game can be supported by incentive strategies (see Mazalov and Rettieva (2010)). The nature of fishing practice,

[2] An estimated number of 30,000 fishermen experienced job losses as a result of this decision.

which typically involves a large number of players over an infinite time horizon, suggests the importance of dynamic game principles in designing preventive solutions. In this paper, the interaction between players and the dynamic nature of shared fish stocks drive our approach in designing the optimal solution.

Game theory has been widely applied to fisheries over the past four decades.[3] In particular, a considerable number of studies address regime shifts using dynamic game theory principles. For example, Polasky et al. (2011) analyze how the threat of a potential future regime shift affects optimal management. They used a simple general growth model to analyze four cases that involve combinations of stock collapse versus changes in system dynamics, and exogenous and endogenous probabilities of regime change. In Nævdal, (2003), the model for optimal regulation of a natural resource in the presence of irreversible threshold effects is proposed. The necessary conditions are presented for optimal regulation of these problems both when the threshold has a known location in the state space and when the location of the threshold is unknown. The process of eutrophication is modeled in Nævdal (2001), where the author takes threshold effects into account, and finds optimal policies for regulating eutrophication. In this paper, deterministic stock pollutants with threshold effects are examined. In general, the literature considers two approaches to dynamic threshold problems. The first is to treat the threshold as a constraint, that is, the threshold is a level that cannot be crossed (see, e.g., Perrings and Pearce (1994)). The second approach assumes that the threshold can be crossed (see, e.g., Farzin (1996) and Miller and Nkuiya (2016)). In Shin et al. (2019), the moratorium is considered as the instrument of a policy maker. The authors develop a bioeconomic model to evaluate the optimality of a moratorium when labor and capital costs are accounted for. In Long et al. (2017), a methodology exploring piecewise closed-loop equilibrium strategies in differential games with regime-switching actions is proposed. This involves the case of a game where two players choose actions that influence the evolution of a state variable, and decide the time to switch from one regime to another. The authors apply this methodology to a depletable resource extraction game. Gromov and Gromova (2017) consider a particular class of bimodal linear-quadratic differential games with two particular classes of switching rules, time-dependent and state-dependent. The main contribution of the paper is to formulate optimality conditions needed to determine optimal strategies in the cooperative and non-cooperative cases. A practically relevant hybrid differential pollution abatement game is considered to illustrate the results.

In the context of determining the optimal fishing behavior under the threat of a moratorium, the theoretical results we propose apply to the great fish war game introduced in Levhari and Mirman (1980). We extend the model by considering n symmetric players. Since we focus only on the noncooperative side of the solution, a version of the solution of this game with the Euler equation approach in Markov strategies could be obtained in González-Sánchez, D., Hernández-Lerma (2013). One argument in favor of using this model is that it has a proven quality such that there is a stationary equilibrium in this game that has a turnpike property, which results in higher steady states of stock and consumption than in the updated games (Nowak (2006)). Our line of inquiry raises some important questions related to organizational problems. We are interested in finding out how players decide to stop fishing at a given time, whether it is due to an external random effect or an endogenous decision. And after the moratorium is in place, how long and how often will it be observed.

The rest of the paper is organized as follows. In Section 2, we describe the model of the great fish war with moratorium. We examine the conditions under which the moratorium is not announced in Section 3. The second scenario of the game, when moratorium regime is applied, is considered in Section 4. We introduce the coordinated strategy profile to avoid moratorium regime and examine the conditions when this profile is the Nash equilibrium in Section 5. Section 6 briefly concludes.

[3] For extensive review on the application of game theory to fisheries management we refer the reader to Bailey et al. (2010); Hannesson (2011) and Gronbæk et al. (2018).

2 The model

In a fashion similar to the dynamic model of a fish war introduced in Levhari and Mirman (1980), we consider n players exploiting an open access common resource. Denote by $N = \{1, 2, \ldots, n\}$ the set of players. Player $i \in N$ at $t = 0, 1, \ldots$, harvests an amount of the resource using a level of effort $u_i(t) \in U_i \subset \mathbb{R}^{m_i}$. So, for each player $i \in N$ at any $t = 0, 1, \ldots$, we define a set of control (or decision) variables U_i. Let $u(t) = (u_1(t), \ldots, u_n(t)) \in \prod_{i\in N} U_i$ be the strategy profile at time t with $\prod_{i\in N} U_i$ being the product control sets.

The state variable $x(t) \in X = [0, 1]$ is the fish stock at time t, with initial value $x(0) = x_0 \in X$ at time $t = 0$. The state dynamics is given by

$$x(t + 1) = (x(t) - u_1(t) - \ldots - u_n(t))^{\alpha}, \qquad t = 0, 1, \ldots, \tag{1}$$

where $0 < \alpha < 1$ is the reproduction rate of the species and x_0 are given.

We assume that there exists a threshold for the state variable x denoted as $\underline{x}$, which is a common knowledge, and falling below this level means *the start of the moratorium on fish harvesting*. The state dynamics (1) correspond to the *regime without moratorium* or *normal regime*. If there exists a time period T for which $x(T - 1) > \underline{x}$ and $x(T) \leq \underline{x}$, then at time T the moratorium starts, and fishing activities are forbidden. Therefore, during a moratorium, the state dynamics (1) become

$$x(t + 1) = (x(t))^{\alpha}, \qquad t = T, \ldots, T + t' - 1, \tag{2}$$

where t' is the moratorium duration. The state dynamics (2) corresponds to the *moratorium regime*.
Remark 1. The steady-state value of the fish stock in the absence of human activities (due to a moratorium or any other reasons) is one, independently of the initial condition. For this reason, we assumed that $X = [0, 1]$, but nothing precludes the upper bound to be taken larger than one.

The duration of the moratorium can be defined in the two following ways:

1. Once the moratorium starts, it lasts for a specified number of time periods. Let the duration of the moratorium be t' periods. Therefore, moratorium periods are $T, T + 1, \ldots, T + t' - 1$. During these periods the state dynamics are given by equation (2). At time $T + t'$, the moratorium is cancelled and the system switches to the normal regime with state dynamics (1).
2. Once the moratorium starts, it lasts until the state variable reaches a given level $\overline{x} > \underline{x}$. The moratorium lasts until period $T + t' - 1$ inclusively. Time $T + t'$ can be found from (2) given that $x(T)$ is known. Again, at time $T + t'$ the moratorium stops, and the system follows the normal regime with state dynamics (1). For instance, $\overline{x}$ can be set equal to the initial level of the fish stock x_0.

In our approach, there is a direct relationship between t' and $\overline{x}$, that is, given the level of the stock that the regulator wants to reach through a moratorium, we can compute the time it would take. (This would not be true if t' and $\overline{x}$ are not linked to each other.)
Remark 2. In the model, there is an authority or government whose payoff is not taken into account directly, but its payoff is represented by an announced policy of moratorium. Two types of moratorium duration introduced above can be both adopted by government or natural resource authorities. A common example of regime 1 is the Chinese summer moratorium in South China Sea. Perceived as the most severe fishing moratorium in history (Ding et al. (2021)), this moratorium has a periodic form as it is automatically implemented each year from May 1 to August 16. Regime 2, on the other hand, is a biomass-based decision, similar to the Canadian cod moratorium alluded to earlier.

Player i maximizes the discounted sum of stage payoffs, that is,

$$J_i(x_0, u) = \sum_{t=0}^{\infty} \rho^t \phi_i(t, x(t), u(t)), \tag{3}$$

where $\rho \in (0,1)$ is a common discount factor and

$$\phi_i(t, x(t), u(t)) = \begin{cases} d + \ln u_i(t), & \text{in a normal regime,} \\ 0, & \text{in a moratorium regime,} \end{cases}$$

where d is a constant, $u(t) \in \prod_{i \in N} U_i$ such that the series in the right-handed side of (3) converges to a finite value. Player i aims to maximize her payoff (3) subject to state dynamics (1) and (2) and initial state $x(0) = x_0$.

Assumption 1. In a normal regime, we suppose that $d + \ln u_i(t) > 0$ for all positive values of $u_i(t)$.

This assumption ensures that fishers get larger revenues when the level of activity is positive than under a moratorium. Note that we could have specified the revenues as $\phi_i(t, x(t), u(t)) = d + \ln(1 + u_i(t))$, with $d > 0$, which would have implied that the revenues during a moratorium are given by the positive constant d. We stick to the $\ln u_i(t)$ to be in line with the literature in this area, e.g., Breton and Keoula (2014); Breton et al. (2019); Miller and Nkuiya (2016); Fesselmeyer and Santugini (2013).

We consider a feedback information structure where player i's strategy ψ_i is a function of time and state, i.e., $\psi_i = \psi_i(t, x)$. We denote the profile of feedback strategies at time t by $\psi(t, x) = (\psi_1(t, x), \ldots, \psi_n(t, x))$, and let $\psi_{-i}(t, x) = (\psi_1(t, x), \ldots, \psi_{i-1}(t, x), \psi_{i+1}(t, x), \ldots, \psi_n(t, x))$. For any time period, strategy $\psi_i(t, x)$ defines control $u_i(t)$ such that $u_i(t) = 0$ if t is a time period in a moratorium regime. We define this control value as zero for simplicity, because it does not influence the players' stage payoffs as $\phi_i(t, x(t), u(t)) \equiv 0$ for any $i \in N$ for all periods t in the moratorium regime.

Definition 1. A Nash equilibrium in a fish war game with moratorium is the profile of feedback strategies $\psi^{nc}(t, x) = (\psi_1^{nc}(t, x), \ldots, \psi_n^{nc}(t, x))$ if

$$J_i(x_0, \psi^{nc}(\cdot)) \geq J_i(x_0, \psi_i(\cdot), \psi_{-i}^{nc}(\cdot)),$$

for any admissible feedback strategy $\psi_i(\cdot)$ of player $i \in N$.

Let the initial state value $x(0) = x_0$ be larger than $\underline{x}$. Otherwise, the game starts in a moratorium regime and we can define a game starting from the initial time $t_1 > 0$ when the desirable stock level is reached, and the normal regime starts with the corresponding initial state given as a solution of (2) at time t_1.

The game starts in a normal regime and we consider two scenarios:

Scenario 1. The control variable $u_i(t)$ for all $i \in N$ is such that the corresponding state trajectory, given as a solution of equation (1), satisfies the properties $x(t) > \underline{x}$ for any $t = 1, 2, \ldots$ and $\lim_{t \to \infty} x(t) \geq \underline{x}$. In this case, the moratorium is never applied and the steady state would be equal to the threshold $\underline{x}$.

Scenario 2. The control variable $u_i(t)$ for all $i \in N$ is such that the corresponding state trajectory, given as a solution of equation (1), is such that, there exists a time T at which $x(t) > \underline{x}$ for any $t < T - 1$ and $x(T) = \underline{x}$, then at time T the moratorium starts and lasts for t' periods. Next, the normal regime starts, and so on.

In the last scenario, the moratorium could happen an infinite (but countable) number of times and we apply the "same pattern of strategies", the players' behavior will be "periodical".

In the following, we determine a condition on the parameter values under which Scenario 1 is part of a Nash equilibrium, and next characterize the equilibrium strategies in Scenario 2. Finally, we show the existence of a coordinated strategy profile that avoids a moratorium, and is a Nash equilibrium.

3 Always normal regime

We define the Nash equilibrium in the game without moratorium and find the conditions under which $\underline{x}$ is never reached. First, we provide auxiliary Propositions 1-2 defining the trajectory of the state variable and the steady-state values of the fish stock in the class of linear-feedback strategies in the great fish war game.

Let the strategy of player i consist of harvesting a positive share $\gamma_i \in [0,1]$ of the available stock x, that is,

$$u_i(t) = \gamma_i x(t), \qquad \forall t \geq 0. \tag{4}$$

Proposition 1. *When the players' strategies are defined by (4), the trajectory of the state variable is given by*

$$x(t) = x_0^{\alpha^t}\left(1 - \sum_{i\in N}\gamma_i\right)^{\left[\frac{\alpha(1-\alpha^t)}{1-\alpha}\right]}, \tag{5}$$

and the steady-state value by

$$x_\infty = (1 - \sum_{i\in N}\gamma_i)^{\left[\frac{\alpha}{1-\alpha}\right]}. \tag{6}$$

Proof. The trajectory of the state variable associated with controls $u_i(t)$, $i \in N$, is derived by substituting for $u_i(t)$ from (4) in (1), and solving for $x(t)$. Next, we have

$$\lim_{t\to\infty} x(t) = x_0^{\alpha^t}\left(1 - \sum_{i\in N}\gamma_i\right)^{\left[\frac{\alpha(1-\alpha^t)}{1-\alpha}\right]} = \left(1 - \sum_{i\in N}\gamma_i\right)^{\left[\frac{\alpha}{1-\alpha}\right]},$$

because $\lim_{t\to\infty}\alpha^t = 0$. □

Now, we find the Nash equilibrium in the fish war game and provide the conditions for never reaching the threshold $\underline{x}$ when the players implement their Nash equilibrium strategies (despite considering a game with a moratorium).

In a noncooperative setting, each player individually maximizes the sum of her discounted utility given in Equation (3). The feedback-Nash equilibrium strategies are derived by solving the following Hamilton-Jacobi-Bellman (HJB) equation, where $V_i(x)$ is the value function of player i:

$$V_i(x) = \max_{u_i\geq 0}\left(d + \ln u_i + \rho V_i\left((x - \sum_{i\in N}u_i)^\alpha\right)\right). \tag{7}$$

Proposition 2. *Assuming an interior solution and symmetric players, the unique feedback-Nash equilibrium is given by*

$$\gamma_i^{nc} = \frac{1-\alpha\rho}{n(1-\alpha\rho)+\alpha\rho}, \qquad \forall i \in N, \tag{8}$$

and the value function by

$$V_i^{nc}(x) = A_i^{nc}\ln x + B_i^{nc}, \qquad \forall i \in N,$$

where

$$A_i^{nc} = \frac{1}{1-\rho\alpha},$$
$$B_i^{nc} = \frac{\rho\alpha\ln(1-n\gamma_i^{nc}) + (1-\rho\alpha)(d+\ln\gamma_i^{nc})}{(1-\rho\alpha)(1-\rho)}.$$

Proof. See Appendix A. □

Evaluating the value function at x_0, we obtain the following total discounted payoff of any player i in the fish war game without moratorium:

$$V_i^{nc}(x_0) = \frac{d}{1-\rho} + \frac{\ln x_0}{1-\alpha\rho} + \frac{\alpha\rho\ln(\alpha\rho) + (1-\alpha\rho)\ln(1-\alpha\rho) - \ln(n(1-\alpha\rho)+\alpha\rho)}{(1-\rho)(1-\alpha\rho)}. \tag{9}$$

For a moratorium to never be implemented, we must have $x_0 > \underline{x}$, and $u_i(t)$, for all $i \in N$, is such that the solution of (1) satisfies the properties $x(t) > \underline{x}$ for all $t = 1, 2, \ldots$, and $\lim_{t\to\infty} x(t) \geq \underline{x}$. The following proposition provides a restriction on the parameter values for such scenario to materialize.

Proposition 3. *When symmetric players adopt the unique feedback-Nash equilibrium harvesting strategies, the moratorium is never applied if*

$$n \leq \frac{\alpha\rho(1-\underline{x}^{\frac{1-\alpha}{\alpha}})}{(1-\alpha\rho)\underline{x}^{\frac{1-\alpha}{\alpha}}}, \tag{10}$$

where $\underline{x}$ is the moratorium level.

Proof. See Appendix B. □

The function in the RHS of inequality (10) is defined and nonnegative for any $\underline{x} \in (0, 1]$, decreasing in $\underline{x}$ and α, and increasing in the discount factor ρ, that is, it is easier to satisfy the inequality when the players are more patient. The condition in (10) can never be met if $\underline{x} = 1$. Indeed, such value for $\underline{x}$ requires $n \leq 0$, which we knew already as $\underline{x} = 1$ is the steady-state value when harvesting cannot take place.

4 Normal and moratorium regimes

In this section, we look into the scenario where the inequality (10) is not satisfied. First, we specify the assumptions and the sequence of events.

1. The moratorium starts at period $t = T$ when the stock level is $x(T) = \underline{x}$, and the objective of the social planner is to bring back the stock level to x_0. (Note that other values than x_0 could be easily considered.) The moratorium regime lasts for t' periods (from $t = T$ till $t = T + t' - 1$ inclusively), where t' is found by solving

$$x(T + t') = x_0, \tag{11}$$

and $x(t)$ satisfies dynamics (2) during moratorium regime applied from T until $T + t' - 1$.

Lemma 1. *Given x_0 and $\underline{x}$, the duration of the moratorium regime is given by*

$$t' = \frac{\ln\left[\frac{\ln x_0}{\ln \underline{x}}\right]}{\ln\alpha}. \tag{12}$$

Proof. See Appendix C. □

Remark 3. We discussed how to define the moratorium level $\underline{x}$ in the introduction and Remark 2. If the moratorium level $\underline{x}$ is defined as a fraction of the initial fish stock, i.e., $\underline{x} = \theta x_0$, where $\theta \in (0, 1)$, then (12) becomes

$$t' = -\frac{\ln\left[1 + \frac{\ln\theta}{\ln x_0}\right]}{\ln\alpha}.$$

Clearly, t' is a decreasing function of $\theta \in (0, 1)$.

Time t' given by (12) may not be a natural number. In practice, it could be rounded to its closest integer number, that is, set $t' := \lceil t' \rceil$.

2. In this scenario, the sequence of events is as follows: In periods $t = [0, \dots, T-1]$, player i harvests a quantity $u_i^*(t)$, $i \in N$. At T, we have $x(T) = \underline{x}$, and a moratorium is implemented in periods $t = [T, \dots, T+t'-1]$, during which the players get zero payoffs. At $t = [T+t']$, the stock level is back to the desired level x_0, and the players can again harvest. Note that the moment T has yet to be determined. Player i's payoff in the game, at which the moratorium regime is firstly applied at $t = T$, is

$$J_i^T(x_0, u) = \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t))) + \rho^{T+t'} J_i^T(x_0, u), \tag{13}$$

from which we get

$$J_i^T(x_0, u) = \frac{1}{1-\rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t))), \tag{14}$$

or equivalently,

$$J_i^T(x_0, u) = \frac{(1-\rho^T)d}{(1-\rho)(1-\rho^{T+t'})} + \frac{1}{1-\rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t \ln u_i(t, x(t)), \tag{15}$$

subject to (11) and state dynamics (1) for $t = [0, T-1]$ and (2) for $t = [T, \dots, T+t'-1]$. Player $i \in N$ maximizes (14) with respect to $u_i \geq 0$ and $T \geq 0$.

Because $T > 0$ is a natural number, we can define the Nash equilibrium in the closed-loop strategies $u(t, x(t)) = (u_i(t, x(t)) : i \in N)$ when player i maximizes (14) for any given T and then finds the maximum over T.

The duration of the moratorium regime is uniquely defined as a solution of (11) when the moratorium level $\underline{x}$ is given. Therefore, we can vary the duration of the moratorium, that is, t', by changing $\underline{x}$. Further, the subgame starting at T is qualitatively the same as the initial game, with the only difference being the base date used to discount the stream of gains.

Proposition 4. *The symmetric Nash equilibrium in closed-loop strategies in a T-stage game of fish war, with $x(0) = x_0$ and $x(T) = \underline{x}$, is given as a unique solution $(u(t, x(t)) : t = [0, 1, \dots, T-1])$ of Bellman equation:*

$$\begin{aligned} V_i(t, x(t)) = d + \max_{u_i(t,x(t)) \in [0, x(t)]} \Big\{ &\ln u_i(t, x(t)) \\ &+ \rho V_i\Big(t+1, \big(x(t) - u_i(t, x(t)) - \sum_{j \in N, j \neq i} u_j(t, x(t))\big)^{\alpha}\Big)\Big\}, \end{aligned} \tag{16}$$

with terminal condition

$$V_i(T-1, x(T-1)) = d + \ln\left(\frac{x(T-1) - \underline{x}^{\frac{1}{\alpha}}}{n}\right),$$

such that

$$u(T-1, x(T-1)) = \frac{x(T-1) - \underline{x}^{\frac{1}{\alpha}}}{n}.$$

Proof. See Appendix D. □

Remark 4. It is difficult to write down a solution of Bellman equation (16) in an explicit form, but we can use the following approximation:

$$u_i(t, x(t)) = \frac{x(t) - \underline{x}^{1/\alpha^{T-t}}}{n + \sum\limits_{k=1}^{T-t-1} (\alpha\rho)^k},$$

which works well for low α or/and large number of players n. This approximation is obtained in the proof of Proposition 4.

The algorithm for finding a Nash equilibrium in this scenario, where T is not given, is as follows:

Step 1: Set $T = 1$. Using Proposition 4, find any player's equilibrium payoff $V(x_0, T)$, which is equal to $\sum\limits_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t)))$. Next, compute $J_i^1(x_0, u)$ using (14) for a given moratorium duration t'.

Step 2: Set $T := T + 1$ and repeat the calculations in Step 1. Compute $J_i^2(x_0, u)$ using (14) for a given moratorium duration t'.

Step 3: Find $\max\limits_{T>0} J_i^T(x_0, u)$.

5 How to avoid a moratorium?

As mentioned in the introduction, we are interested in checking if the players can agree on harvesting levels that result in avoiding a moratorium throughout the entire duration of the game. More specifically, assuming that the harvesting strategy of player i is of the form $u_i(x) = \gamma_i x$, we seek a γ_i^c, for all $i \in N$, such that the steady state computed with (6) satisfies the condition $x_\infty = \underline{x}$. If it exists, the constructed harvesting profile $(u_1^c(x), \ldots, u_n^c(x))$ will be referred to as *coordinated* profile.

Proposition 5. *For $i \in N$, the coordinated strategy is given by $u_i^c(x) = \gamma_i^c x$, where*

$$\gamma_i^c = \frac{1}{n}\left(1 - \underline{x}^{\frac{1-\alpha}{\alpha}}\right), \tag{17}$$

and the corresponding fish stock by

$$x^c(t) = x_0^{\alpha^t} \underline{x}^{1-\alpha^t}, \qquad t = 1, 2, \ldots, \tag{18}$$

with initial stock $x^c(0) = x_0$.

Player i's payoff is as follows:

$$J_i^c = J_i(x_0, u^c) = \frac{d}{1-\rho} + \frac{1}{1-\rho} \ln\left(\frac{\underline{x}(1 - \underline{x}^{\frac{1-\alpha}{\alpha}})}{n}\right) + \frac{1}{1-\alpha\rho} \ln\left(\frac{x_0}{\underline{x}}\right). \tag{19}$$

Proof. See Appendix E. □

Letting t goes to infinity in (18), we clearly obtain $x_\infty^c = \underline{x}$.

Corollary 1. *If inequality (10) is satisfied, then $\gamma_i^{nc} \leq \gamma_i^c$, and the fish stock in the Nash equilibrium is larger than in the coordinated strategy profile, $x^{nc}(t) > x^c(t)$ for all $t > 0$. Otherwise, $\gamma_i^{nc} > \gamma_i^c$ and $x^{nc}(t) < x^c(t)$ for any $t > 0$.*

Proof. It immediately follows from comparison of γ_i^{nc} and γ_i^c, and using expressions for $x^{nc}(t)$ and $x^c(t)$ from Propositions 2 and 5. □

When inequality (10) is satisfied, we know from Proposition 3 that a moratorium is never implemented in equilibrium. Consequently, there is no reason for seeking a coordinated solution to avoid it. However, if inequality (10) is not satisfied, which is equivalent to state that the moratorium level will be exceeded (see Proposition 2), then it becomes relevant to attempt to construct an alternative solution that avoids the moratorium. This is precisely what the above proposition is doing in the class of linear-state feedback strategies. Now, as the coordinated solution is not a Nash equilibrium, nothing ensures that the players will stick to the agreement. In the following proposition, we provide conditions for the coordinated profile to be a Nash equilibrium in the class of linear-state strategies.

Proposition 6. *Let inequality (10) be not satisfied. The coordinated profile $u^c(x) = (u_i^c(x) : i \in N)$, where $u_i^c(x) = \gamma_i^c x$, and γ_i^c is given by (17), is the Nash equilibrium in linear-state strategies in the game with moratorium if*

$$J_i^c \geq J_i',$$

where J_i^c is defined by (19), and

$$J_i' = \max_T \Big\{ \frac{1-\rho^T}{(1-\rho)(1-\rho^{T+t'})}\Big(d + \ln\gamma_i' + \frac{1}{1-\alpha^T}\ln \underline{x} - \frac{\alpha^T}{1-\alpha^T}\ln x_0\Big) + \frac{1-(\alpha\rho)^T}{(1-\alpha^T)(1-\alpha\rho)(1-\rho^{T+t'})}\ln\left(\frac{x_0}{\underline{x}}\right)\Big\}, \tag{20}$$

and

$$\gamma_i' = \frac{1}{n} + \frac{n-1}{n}\underline{x}^{\frac{1-\alpha}{\alpha}} - \left(\frac{\underline{x}}{x_0^{\alpha^T}}\right)^{\frac{1-\alpha}{\alpha(1-\alpha^T)}}.$$

Proof. See Appendix F. □

In Proposition 6, we find the conditions under which the individual deviation from the coordinated strategy profile is not profitable. The non-deviating players do not use trigger strategies but proceed using their coordinated strategies when the individual deviation is observed. In this case, the moratorium plays a trigger role, and the moratorium obviously starts at some period when the deviating player increases the harvesting level. The deviation is not profitable if the benefit from the deviation is smaller than the player's loss in a moratorium regime. The concept of ε-equilibrium can be also investigated for the model (see, e.g., Radner (1980), Mailath and Samuelson (2006)). Indeed, one can easily calculate any player's benefit from individual deviation using Propositions 5 and 6, and find the minimal level of ε, for which the coordinated strategy profile is an ε-equilibrium.

Corollary 2. *When $T \to \infty$, then $\gamma_i' \to \gamma_i^c$ and $J_i^T(x_0, (u_i, u_{-i}^c)) \to J_i^c$.*

Proof. This result can be easily obtained by finding the limits of γ_i' and $J_i^T(x_0, (u_i, u_{-i}^c))$ given in Proposition 6. □

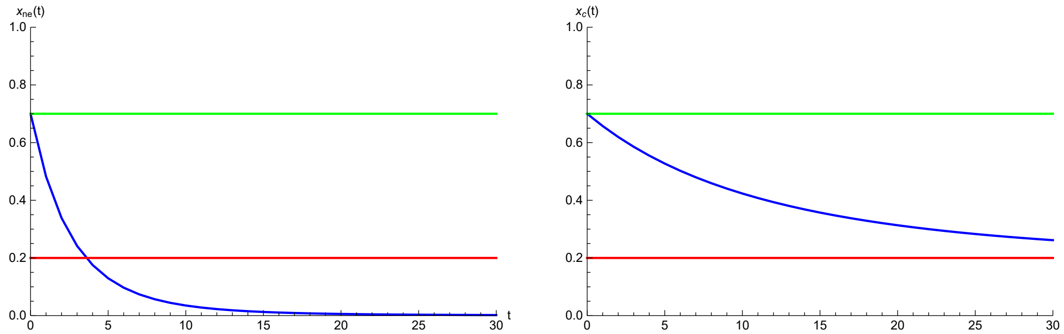
5.1 An illustrative example

Consider a three-player game with the following parameters: $x_0 = 0.7$, $\underline{x} = 0.2$, $\rho = 0.9$, $\alpha = 0.95$, $d = 5$.

Suppose there is no moratorium in this game and apply the results in Proposition 2. The Nash equilibrium in linear strategies is defined by $\gamma_i^{nc} = 0.112$, and player i's payoff is 46.872, for all $i \in N$. The state trajectory is depicted in Figure 1(a). The steady state corresponding to the Nash equilibrium profile is 0.0004. In Figure 1(a), we see that the moratorium level is crossed by the equilibrium state trajectory. So, if the players implement the Nash equilibrium strategies given in Proposition 2, then the moratorium starts at period $t = 4$. We can easily write condition (10), that is,

$$n \leq 0.521246,$$

and note that is not satisfied for $n = 3$. Further, to compute the duration of the moratorium, we use Equation (12) to get $t' = 30$.



(a) The fish stock trajectories with the Nash equilibrium (green — x_0, blue — $x^{nc}(t)$, red — $\underline{x}$)

(b) The fish stock trajectories with coordinated strategy profile (green — x_0, blue — $x^c(t)$, red — $\underline{x}$)

Figure 1: The game with moratorium when $\alpha = 0.95$

Now, using Proposition 5, we obtain the following coordinated harvesting strategy:

$$u_i^c(x) = \gamma_i^c x = 0.027x, \quad \text{for all } i \in N.$$

The corresponding state trajectory is shown in Figure 1(b). Player i's payoff in this coordinated solution is $J_i^c = 6.45316$, for all $i \in N$.

To verify if the coordinated strategy profile is the Nash equilibrium in linear-state strategies, we use Proposition 6. The profit of a deviating player $J_i^T(x_0, (u_i, u_{-i}^c))$ as a function of T, that is, the first moment starting from $t = 0$ when moratorium is applied, is represented in Figure 2. We can notice that the coordinated strategy profile is not the Nash equilibrium because a deviating player can benefit and gets the profit $J_i' = \max_T J_i^T(x_0, (u_i, u_{-i}^c))$, which is equal to 13.002, when $T = 11$. Moreover, in Figure 2 we see that

$$\lim_{T \downarrow \infty} J_i^T(x_0, (u_i, u_{-i}^c)) = J_i^c,$$

i.e., J_i^T converges from the right to J_i^c (see Corollary 2).

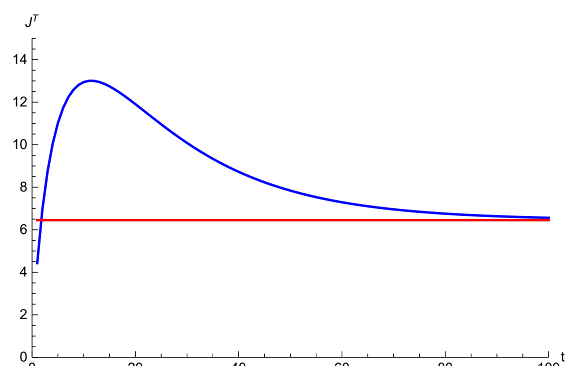
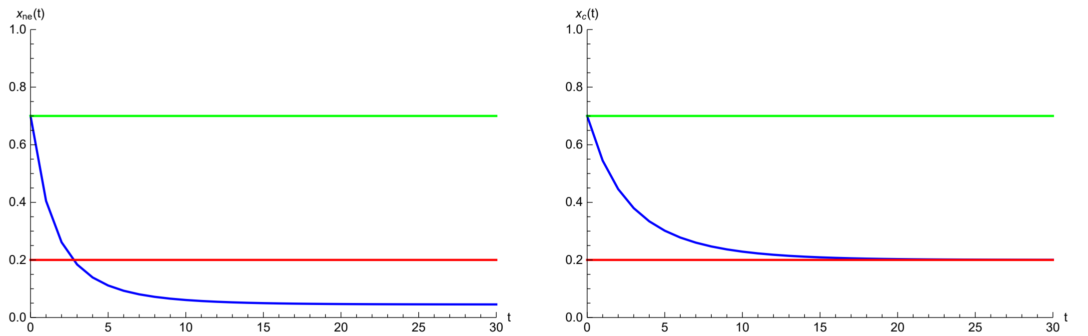


Figure 2: Blue: profit of a deviating player J_i^T as a function of T; red: profit in the coordinated strategy profile J_i^c, $\alpha = 0.95$

Consider a second run where we decrease α from 0.95 to 0.8, while keeping the same values for other parameters. The Nash equilibrium in linear strategies is defined by $\gamma_i^{nc} = 0.179487$, and player i's payoff is 47.689, for all $i \in N$. The state trajectory is depicted in Figure 3(a), and the steady state is 0.045. We see that the moratorium level is crossed by the equilibrium state trajectory. The condition in (10) reads

$$n \leq 1.274,$$

and is clearly not satisfied for $n = 3$.



(a) The fish stock trajectories with the Nash equilibrium (green — x_0, blue — $x^{nc}(t)$, red — $\underline{x}$)

(b) The fish stock trajectories with coordinated strategy profile (green — x_0, blue — $x^c(t)$, red — $\underline{x}$)

Figure 3: The game with moratorium when $\alpha = 0.8$

With this parameter constellation, the moratorium duration is $t' = 7$. The coordinated strategy profile is defined by

$$\gamma_i^c = 0.110, \qquad \forall i \in N.$$

The corresponding state trajectory is depicted in Figure 3(b), and player i's payoff is $J_i^c = 16.345$.

For this run, the condition in Proposition 6 is satisfied and the coordinated strategy profile is the Nash equilibrium. The profit of a deviating player $J_i^T(x_0, (u_i, u_{-i}^c))$ as a function of T is represented in Figure 4. For any $T \geq 1$, we have $J_i^T(x_0, (u_i, u_{-i}^c)) < J_i^c$. Moreover, in Figure 4 we see that

$$\lim_{T \uparrow \infty} J_i^T(x_0, (u_i, u_{-i}^c)) = J_i^c,$$

i.e., J_i^T converges from the left to J_i^c.

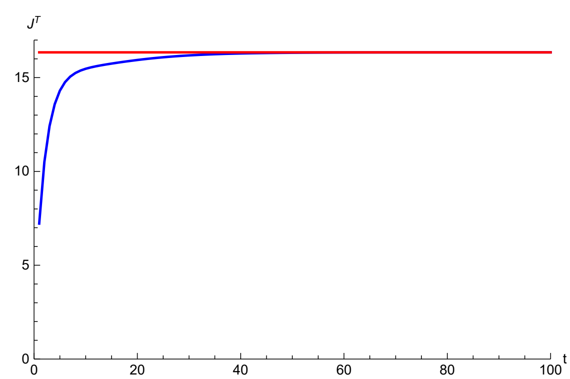


Figure 4: Blue: profit of a deviating player J_i^T as a function of T; red: profit in the coordinated strategy profile J_i^c, $\alpha = 0.8$

6 Conclusion

In this paper, we considered a great fish war game with moratorium, and determined the conditions under which a moratorium is never declared. When these conditions are not satisfied, we provided a simple formula that gives its optimal duration. Also, we showed that a coordinated strategy profile can be defined to avoid the moratorium and found the conditions under which it is Nash equilibrium in the class of linear-state strategies.

Few extensions to our work would be worth considering. First, having a fishery with multiple species biologically interacting with each other. Second, letting the central planner be an active player pursuing a certain objective. Third, allowing for asymmetries between the players. Finally, it would be interesting to analyze a case where the dynamics are stochastic. Clearly, all these extensions are computationally challenging, and most likely only a numerical solution approach could be implemented.

Appendix

Appendix A. Proof of Proposition 2

We write the HJB equation for player i, that is

$$V_i(x) = \max_{u_i \geq 0} \left(d + \ln u_i + \rho V_i \left((x - \sum_{j=1}^{n} u_j)^{\alpha} \right) \right). \tag{21}$$

Following Lehvari and Mirman (1980), we assume the value function is of the following form:

$$V_i^{nc}(x) = A_i^{nc} \ln x + B_i^{nc}, \qquad i = 1, \dots, n.$$

The above form yields

$$V_i^{nc}(x) = \max_{u_i^{nc} \geq 0} \left\{ d + \ln u_i^{nc} + \rho A_i^{nc} \ln \left((x - \sum_{j=1}^{n} u_j^{nc})^{\alpha} \right) + \rho B_i^{nc} \right\}. \tag{22}$$

Finally, maximizing the right-hand side of Equation (22) and assuming symmetric players give

$$\frac{1}{u_i^{nc}} - \frac{\rho \alpha A_i^{nc}}{x - n u_i^{nc}} = 0,$$

which yields the equilibrium stationary strategy

$$u_i^{nc}(x) = \frac{1 - \alpha\rho}{n(1 - \alpha\rho) + \alpha\rho} x = \gamma_i^{nc} x, \qquad i = 1, \dots, n.$$

Replacing u_i^{nc} by its value from Equation (4), we obtain

$$V_i^{nc}(x) = A_i^{nc} \ln x + B_i^{nc} = d + (1 + \rho\alpha A_i^{nc}) \ln x + \rho\alpha A_i^{nc} \ln(1 - n\gamma_i^{nc}) + \rho B_i^{nc} + \ln \gamma_i^{nc},$$

where

$$A_i^{nc} = \frac{1}{1 - \rho\alpha},$$
$$B_i^{nc} = \frac{\rho\alpha \ln(1 - n\gamma_i^{nc}) + (1 - \rho\alpha)(d + \ln \gamma_i^{nc})}{(1 - \rho\alpha)(1 - \rho)}.$$

Substituting for the initial condition $x(0) = x_0$ and the expressions of γ_i^{nc}, A_i^{nc}, and B_i^{nc} in $V_i^{nc}(x(0))$ given above, we get after simple algebraic transformations

$$V_i^{nc}(x_0) = \frac{d}{1 - \rho} + \frac{\ln x_0}{1 - \alpha\rho} + \frac{\alpha\rho \ln(\alpha\rho) + (1 - \alpha\rho) \ln(1 - \alpha\rho) - \ln(n(1 - \alpha\rho) + \alpha\rho)}{(1 - \rho)(1 - \alpha\rho)}.$$

Appendix B. Proof of Proposition 3

We substitute the values of γ_i^{nc} into the steady state expression of the stock given by (6), and requiring that $x_\infty \geq \underline{x}$, we obtain

$$x_\infty^{nc} = \left(1 - \frac{n(1-\alpha\rho)}{n(1-\alpha\rho)+\alpha\rho}\right)^{\frac{\alpha}{1-\alpha}} \geq \underline{x},$$

which can be rewritten as

$$n \leq \frac{\alpha\rho(1-\underline{x}^{\frac{1-\alpha}{\alpha}})}{(1-\alpha\rho)\underline{x}^{\frac{1-\alpha}{\alpha}}}.$$

Appendix C. Proof of Lemma 1

We can easily prove the lemma by solving Equation (11) taking into account the state dynamics Equation (2) with initial condition $x(T) = \underline{x}$. We obtain that

$$x(T+t') = x^{\alpha^{t'}} = x_0,$$

from which it follows that

$$\alpha^{t'} = \frac{\ln x_0}{\ln \underline{x}},$$

and finally,

$$t' = \frac{\ln\left[\frac{\ln x_0}{\ln \underline{x}}\right]}{\ln \alpha}.$$

Appendix D. Proof of Proposition 4

We find the feedback Nash equilibrium in finite-horizon symmetric game using Bellman equation by backward induction. We start with the game of one-stage duration and find $V(T-1, x(T-1))$. The terminal condition is $x(T) = \underline{x}$, and obviously, due to symmetric players, the equilibrium strategy of any player $i \in N$ is $u_i(T-1, x(T-1))$ such that

$$(x(T-1) - nu_i(T-1, x(T-1)))^\alpha = \underline{x}.$$

Solving this equation for $u_i(T-1, x(T-1))$, we obtain that

$$u_i(T-1, x(T-1)) = \frac{x(T-1) - \underline{x}^{1/\alpha}}{n}.$$

The value of Bellman function for $t = T-1$ is

$$V_i(T-1, x(T-1)) = d + \ln u_i(T-1, x(T-1)) = d + \ln\left[\frac{x(T-1) - \underline{x}^{1/\alpha}}{n}\right].$$

When $t = T-2$, then the Bellman equation is

$$\begin{aligned}V_i(T-2, x(T-2)) = d + &\max_{u_i(T-2,x(T-2))\in[0,x(T-2)]} \Big\{ \ln u_i(T-2, x(T-2)) \\ &+ \rho V_i\Big(T-1, \big(x(T-2) - u_i(T-2, x(T-2)) - \sum_{j\in N, j\neq i} u_j(T-2, x(T-2))\big)^\alpha\Big)\Big\}.\end{aligned}$$

Substituting $V_i(T-1, x(T-1))$ into the last equation, and using notation $u_i(T-2, x(T-2)) = u_i$ for simplicity, we solve the maximization problem in the RHS of the last equation. The strategy u_i should satisfy the equation:

$$\frac{1}{u_i} - \alpha\rho\frac{(x(T-2) - nu_i)^{\alpha-1}}{(x(T-2) - nu_i)^\alpha - \underline{x}^{1/\alpha}} = 0,$$

or equivalently,

$$(x(T-2)-nu_i)^\alpha - \alpha\rho u_i (x(T-2)-nu_i)^{\alpha-1} = \underline{x}^{1/\alpha}.$$

We can rewrite the equation as follows:

$$(x(T-2)-nu_i)^{\alpha-1}(x(T-2)-(n+\alpha\rho)u_i) = \underline{x}^{1/\alpha}. \tag{23}$$

This equation has a unique solution $u_i \in [0, x(T-2)]$, but it is difficult to write it in an explicit form. But we can use the following approximation of the solution when n is large and/or α is quite small, that is

$$u_i = u_i(T-2, x(T-2)) = \frac{x(T-2)-\underline{x}^{1/\alpha^2}}{n+\alpha\rho}.$$

If we substitute this approximated solution into (23), then we obtain that

$$\Big(\frac{\alpha\rho}{n+\alpha\rho}x(T-2)+\frac{n}{n+\alpha\rho}\underline{x}^{1/\alpha^2}\Big)^{\alpha-1}\underline{x}^{1/\alpha^2} = \underline{x}^{1/\alpha},$$

$$\Big(\frac{\alpha\rho}{n+\alpha\rho}\frac{x(T-2)}{\underline{x}^{1/\alpha^2}}+\frac{n}{n+\alpha\rho}\Big)^{\alpha-1}\underline{x}^{(\alpha-1)/\alpha^2}\underline{x}^{1/\alpha^2} = \underline{x}^{1/\alpha},$$

$$\Big(\frac{\alpha\rho}{n+\alpha\rho}\frac{x(T-2)}{\underline{x}^{1/\alpha^2}}+\frac{n}{n+\alpha\rho}\Big)^{\alpha-1} = 1,$$

which is approximately satisfied if $\frac{n}{n+\alpha\rho}$ close to one, which is reached when $\alpha\rho$ is small and n is large.

Proceeding with the backward induction we can obtain such an approximation for any t and any $x(t)$, that is

$$u_i(t, x(t)) = \frac{x(t)-\underline{x}^{1/\alpha^{T-t}}}{n+\sum\limits_{k=1}^{T-t-1}(\alpha\rho)^k}.$$

Appendix E. Proof of Proposition 5

By definition, the coordinated strategy profile is such that $x_\infty = \underline{x}$. From Equation (6), solving

$$(1-n\gamma_i^c)^{\frac{\alpha}{1-\alpha}} = \underline{x},$$

we find the harvesting share

$$\gamma_i^c = \frac{1}{n}\left(1-\underline{x}^{\frac{1-\alpha}{\alpha}}\right), \qquad\qquad i \in N.$$

Substituting this value into (5), we obtain an expression for the fish stock at any time, that is,

$$x^c(t) = x_0^{\alpha^t}\underline{x}^{1-\alpha^t},$$

for any $t > 0$.

To calculate any player i's payoff in the coordinated strategy profile, we substitute $u_i^c(x) = \gamma_i^c x^c$ into the payoff function and get

$$\begin{aligned} J_i^c = J_i(x_0, u^c) &= \sum_{t=0}^{\infty}\rho^t\Big(d+\ln u^c(x(t))\Big) = \frac{d}{1-\rho}+\sum_{t=0}^{\infty}\rho^t\ln\Big[\frac{x_0^{\alpha^t}\underline{x}^{1-\alpha^t}}{n}\Big(1-\underline{x}^{\frac{1-\alpha}{\alpha}}\Big)\Big] \\ &= \frac{d}{1-\rho}+\sum_{t=0}^{\infty}\rho^t\Big[\ln\Big(1-\underline{x}^{\frac{1-\alpha}{\alpha}}\Big)+\alpha^t\ln x_0+(1-\alpha^t)\ln\underline{x}-\ln n\Big] \\ &= \frac{d}{1-\rho}+\frac{1}{1-\rho}\ln\left(\frac{\underline{x}(1-\underline{x}^{\frac{1-\alpha}{\alpha}})}{n}\right)+\sum_{t=0}^{\infty}(\alpha\rho)^t(\ln x_0-\ln\underline{x}) \\ &= \frac{d}{1-\rho}+\frac{1}{1-\rho}\ln\left(\frac{\underline{x}(1-\underline{x}^{\frac{1-\alpha}{\alpha}})}{n}\right)+\frac{1}{1-\alpha\rho}\ln\left(\frac{x_0}{\underline{x}}\right), \end{aligned}$$

which finishes the proof.

Appendix F. Proof of Proposition 6

Player i's payoff, $i \in N$, in the game is $J_i^c = J_i(x_0, u^c)$ given by (19) if players adopt coordinated strategy profile. We calculate the payoff of player i, who individually deviates from strategy $u_i^c(x) = \gamma_i^c x$ to linear-state strategy $u_i(x) = \gamma_i' x$, where $\gamma_i' \neq \gamma_i^c$. We denote this profile by $(u_i(x), u_{-i}^c(x))$. Obviously, if $\gamma_i' < \gamma_i^c$, then his payoff decreases when all other players use their coordinated strategies γ_j^c, $j \in N$, $j \neq i$. It immediately follows from the form of the payoff function (3) and because the moratorium is not applied as the steady state stock in profile $(u_i(x), u_{-i}^c(x))$ is larger than in profile $u^c(x)$.

We consider the case when $\gamma_i' > \gamma_i^c$. In this case, the moratorium will be applied because the steady state stock in profile $(u_i(x), u_{-i}^c(x))$ will be lower than $\underline{x}$. Therefore, there exists a time period T, at which the moratorium starts and lasts for t' periods. The duration of moratorium regime t' is defined by (12). We calculate the payoff of player i in strategy profile $(u_i(x), u_{-i}^c(x))$ and compare it with J_i^c. The payoff of the deviating player i in the game is

$$J_i' = \max_T J_i^T(x_0, (u_i, u_{-i}^c)), \tag{24}$$

where $J_i^T(x_0, (u_i, u_{-i}^c))$ satisfies equation

$$J_i^T(x_0, (u_i, u_{-i}^c)) = \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t))) + \rho^{T+t'} J_i^T(x_0, (u_i, u_{-i}^c)), \tag{25}$$

where $u_i(t) = \gamma_i' x(t)$, s.t. state dynamics

$$\begin{aligned} x(t+1) &= (x(t) - \sum_{j \in N, j \neq i} u_j^c(t) - u_i(t))^\alpha, \\ &\qquad t = 0, 1, \ldots, T-1, T+t', \ldots, 2T+t'-1, 2T+2t', \ldots \\ x(t+1) &= (x(t))^\alpha, \\ &\qquad t = T, \ldots, T+t'-1, 2T+t', \ldots, 2T+2t'-1, \ldots \end{aligned} \tag{26}$$

Form (25) we can find $J_i^T(x_0, (u_i, u_{-i}^c))$, that is

$$J_i^T(x_0, (u_i, u_{-i}^c)) = \frac{1}{1 - \rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(x(t))). \tag{27}$$

As the game is considered in a discrete time, the player can deviate from strategy profile u_i^c by choosing strategy $u_i(x) = \gamma_i' x$ such that $\gamma_i' > \gamma_i^c$ and $x(T) = \underline{x}$ for a given $T = 1, 2, 3, \ldots$, where $x(t)$ satisfies dynamics (26).

First, we find such a strategy $u_i(x) = \gamma_i' x$ from the condition $x(T) = \underline{x}$. Substituting coordinated strategies γ_j^c of all the players $j \in N$, $j \neq i$ into (5), we obtain an equation

$$\underline{x} = x_0^{\alpha^T} \left(1 - \sum_{j \in N, j \neq i} \gamma_j^c - \gamma_i'\right)^{\left[\frac{\alpha(1-\alpha^T)}{1-\alpha}\right]},$$

$$\underline{x} = x_0^{\alpha^T} \left(1 - \frac{n-1}{n}\left(1 - \underline{x}^{\frac{1-\alpha}{\alpha}}\right) - \gamma_i'\right)^{\left[\frac{\alpha(1-\alpha^T)}{1-\alpha}\right]}.$$

Solving this equation, we can find the strategy of player i defined by γ_i' as follows:

$$\ln\left[\frac{\underline{x}}{x_0^{\alpha^T}}\right] = \frac{\alpha(1-\alpha^T)}{1-\alpha} \ln\left[1 - \frac{n-1}{n}\left(1 - \underline{x}^{\frac{1-\alpha}{\alpha}}\right) - \gamma_i'\right],$$

$$\left[\frac{\underline{x}}{x_0^{\alpha^T}}\right]^{\frac{1-\alpha}{\alpha(1-\alpha^T)}} = 1 - \frac{n-1}{n}\left(1 - \underline{x}^{\frac{1-\alpha}{\alpha}}\right) - \gamma_i',$$

$$\gamma_i' = \frac{1}{n} + \frac{n-1}{n}\underline{x}^{\frac{1-\alpha}{\alpha}} - \left[\frac{\underline{x}}{x_0^{\alpha^T}}\right]^{\frac{1-\alpha}{\alpha(1-\alpha^T)}}.$$

We can easily notice that γ_i' is a decreasing function of T.

Next, we calculate the duration of moratorium regime by formula (12), that is

$$t' = \frac{\ln\left[\frac{\ln x_0}{\ln \underline{x}}\right]}{\ln \alpha},$$

which does not depend on players' strategies. To be precise, as t' has to be a natural number, we round it up, and define $t' := \lceil t' \rceil$.

So, for any given $T = 1, 2, \ldots$, knowing the moratorium duration t', we can calculate the strategy $\gamma_i' > \gamma_i^c$ and payoff of the deviating player by (27), that is

$$J_i^T(x_0, (u_i, u_{-i}^c)) = \frac{1}{1-\rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t (d + \ln(\gamma_i' x'(t))),$$

where $x'(t)$ is defined by equation (5) from Proposition 1 substituting γ_i' defined above and coordinated strategies γ_j^c for $j \neq i$:

$$\begin{aligned} x'(t) &= x_0^{\alpha^t}\left(1 - \frac{n-1}{n}\left(1 - \underline{x}^{\frac{1-\alpha}{\alpha}}\right) - \frac{1}{n} - \frac{n-1}{n}\underline{x}^{\frac{1-\alpha}{\alpha}} + \left[\frac{\underline{x}}{x_0^{\alpha^T}}\right]^{\frac{1-\alpha}{\alpha(1-\alpha^T)}}\right)^{\frac{\alpha(1-\alpha^t)}{1-\alpha}} \\ &= \underline{x}^{\frac{1-\alpha^t}{1-\alpha^T}} x_0^{\frac{\alpha^t-\alpha^T}{1-\alpha^T}} \end{aligned}$$

for $t = 1, 2, \ldots, T$.

Substituting expressions of γ_i' and $x'(t)$ into $J_i^T(x_0, (u_i, u_{-i}^c))$ we obtain

$$\begin{aligned} J_i^T &= \frac{1}{1-\rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t (d + \ln \gamma_i' + \ln x'(t)) \\ &= \frac{1}{1-\rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t \left(d + \ln \gamma_i' + \ln\left(\underline{x}^{\frac{1-\alpha^t}{1-\alpha^T}} x_0^{\frac{\alpha^t-\alpha^T}{1-\alpha^T}}\right)\right) \\ &= \frac{(1-\rho^T)}{(1-\rho)(1-\rho^{T+t'})}\left(d + \ln \gamma_i' + \frac{1}{1-\alpha^T}\ln \underline{x} - \frac{\alpha^T}{1-\alpha^T}\ln x_0\right) \\ &+ \frac{1}{1-\rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t \alpha^t \frac{1}{1-\alpha^T} \ln\left(\frac{x_0}{\underline{x}}\right) \\ &= \frac{(1-\rho^T)}{(1-\rho)(1-\rho^{T+t'})}\left(d + \ln \gamma_i' + \frac{1}{1-\alpha^T}\ln \underline{x} - \frac{\alpha^T}{1-\alpha^T}\ln x_0\right) \\ &+ \frac{(1-(\alpha\rho)^T)}{(1-\alpha^T)(1-\alpha\rho)(1-\rho^{T+t'})} \ln\left(\frac{x_0}{\underline{x}}\right). \end{aligned}$$

We should mention that the last expression is an expression of player i's payoff if she individually deviates from the coordinated strategy profile.

Obviously, if $J_i^c \geq J_i' = \max_T J_i^T(x_0, (u_i, u_{-i}^c))$, then the coordinated strategy profile is the Nash equilibrium in the class of linear-state strategies (4).

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