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Adaptive simultaneous stochastic optimization of a gold mining complex: A case study

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Abstract : An innovative strategic mine planning approach is applied to a multi-mine and multi-process gold mining complex that simultaneously considers feasible capital investment alternatives and capacity management decisions that a mining enterprise may undertake. The simultaneous stochastic optimization framework determines the extraction sequence, stockpiling, processing stream, blending, waste management and capital investment decisions in a single mathematical model. A production schedule branches and adapts to uncertainty based on the likelihood of purchasing a number of feasible investment alternatives that may improve mill throughput, blending or increase the tailings capacity. Additionally, the mining rate is determined simultaneously by selecting the number of trucks and shovels required to maximize the value of the operation. The mining complex contains several sources – two open-pit gold mines and externally sourced ore material – stockpiles, waste dumps, tailings and three different processing streams. The simultaneous optimization framework integrates the blending of sulphates, carbonates, and organic carbon at the autoclave for refractory ore while managing acid consumption. The production schedule generated branches over an investment in the autoclave expansion; the first branch undertakes the capacity expansion at the autoclave resulting in a 6.4% increase in NPV, whereas the second branch results in a 27.5% increase in NPV without the investment. The adaptive approach is compared to a base case production schedule generated using a non-branching two-stage stochastic integer program.

Keywords: Mining complex, simultaneous stochastic optimization, capital investments

1 Introduction

Mining operations are capital-intensive ventures that require smart decisions to strategically time each investment and sustainably produce valuable products. The simultaneous stochastic optimization approach generates an optimal production schedule (Del Castillo & Dimitrakopoulos, 2019; Montiel & Dimitrakopoulos, 2018; Goodfellow & Dimitrakopoulos, 2016, 2017; Montiel & Dimitrakopoulos, 2015, 2017). The optimized production schedule defines the extraction sequence, stockpiling, processing stream, blending, waste management and capital investment decisions that maximize the net present value (NPV). These decisions are obtained by considering the interactions throughout the entire mining complex that for a mining complex, using a single mathematical formulation may consist of open pit and underground mines, several processing facilities, crushers, stockpiles, and waste destinations (Pimentel, Mateus, & Almeida, 2010). The stochastic approach also manages technical risk during the optimization by integrating a set of stochastic geostatistical simulations of the in-situ material supply, which reproduces the uncertainty and local variability of the material sourced from the mines. Selecting the appropriate time to undertake a capital investment during the life of mine is challenging due to a combination of supply uncertainty, high upfront costs and prolonged payback periods for each investment. Nevertheless, investments in shovels, trucks, crushers, process plant upgrades, and waste facilities are critical for maximizing the NPV of the long-term production schedule.

The uncertain aspects of mine planning and forecasting, which arise from supply uncertainty, indicate there is large risk of undertaking capital investments (Ajaka, Lilford, & Topal 2018; Asad & Dimitrakopoulos, 2013; Del Castillo & Dimitrakopoulos, 2014; Dowd, 1994; Githiria & Musingwini, 2019; Khan & Asad, 2019; Groeneveld & Topal, 2011; Dimitrakopoulos, 2018; Groeneveld, Topal, & Leenders, 2012; Mai et al. 2018; Dowd, 1994; Ravenscroft, 1992). In particular, supply uncertainty makes it challenging to produce an optimized production schedule with an investment plan that will satisfy the various futures that may unfold. The optimal investment decision for one future outcome may be very different from another scenario. This generates an interest in developing strategic mine plans that can adapt to uncertainty, by considering feasible investment alternatives that directly impact the production rate of certain components in the mining complex and manage technical risk.

Del Castillo and Dimitrakopoulos (2019) present an adaptive simultaneous stochastic optimization approach that considers a number of feasible investment alternatives and determines the optimal time to branch the production schedule to manage the potential risk of supply uncertainty. A set of orebody simulations are generated for each mine to quantify supply uncertainty. Then, an adaptive approach considers the probability of undertaking an investment in different groups of scenarios. If the decision is counterbalancing, where a representative group of simulations takes on an investment and another representative group does not, the production schedule splits or branches into alternative mine plans based on these investments. Each of these branching alternatives are fully optimized based on the investment that is undertaken, however, decisions made prior to the investment can not be changed once branching occurs. This prevents the optimization model from anticipating the investment decisions and changing the previous decisions that were made prior to choosing to invest, as the future investment choices remain uncertain until they are executed. The adaptive optimization approach integrates non-anticipativity constraints into the optimization formulation. The non-anticipativity constraints ensure that the same decisions are taken unless there is an investment alternative that branches the mine production schedule. If branching occurs, the resulting mine plan of each branch should be distinguishably different based on the investment choice. Otherwise, the non-anticipativity constraints are enforced and the same decision is taken over all the simulated scenarios. The single production schedule generated with feasible investment alternatives provides an advanced method for determining the optimal time to invest and identifies the risk of investing in new equipment, plant improvements, and other infrastructure purchases (Dixit & Pindyck, 1994). Evaluating feasible alternatives and the resulting mine plan creates opportunities to delay, pre-plan or undertake sizeable capital investments (De Neufville & Scholtes, 2011). Boland *et al.* (2008) also incorporates non-anticipativity constraints in a multistage optimization framework, however, this approach differs from the adaptive approach described above by Del Castillo & Dimitrakopoulos (2019). In the approach described by Boland et al. (2009), the simulated orebody scenarios are differentiated based on the spatial distribution of metal grades, which results in overfitting the production schedule to generate one mine plan per a simulated orebody scenario. This method does not lead to an optimal production schedule, given that a single scenario does not represent the uncertainty and local grade variability of the deposit, thus resulting in erroneous production and financial forecasts that misrepresent reality. Contrary to that, in the case study presented herein, the adaptive approach leverages the ability to branch over several capital investments instead of each block's simulated grades, leading to a practical production schedule with feasible investment alternatives.

Similar multistage frameworks have been applied to strategically time the purchase of capital investments and expand the production capacity in other industries (Ahmed, King, & Parija, 2003; Gupta & Grossmann, 2017; Li et al., 2008; Singh, Philpott, & Wood, 2009). These frameworks remain impractical for mine planning and design purposes as multistage frameworks lead to a production schedule with one plan per scenario, which misrepresents the ability to change capacities and is the major limitation of multistage approaches. Furthermore, when considering the execution of the long-term production schedule, operations can not proceed without fixed guidance for the current year of production. Groeneveld et al. (2012) suggest fixing the initial years of the mine production schedule, to address this limitation, ensuring that operations have the appropriate production guidance and lead time to consider different mining and plant options for the future.

The adaptive simultaneous stochastic optimization approach manages technical risk and delivers a mine production schedule that can identify synergies between different components of the mining complex. For example, in a Nevada type gold mining complex, the metal recovery of refractory ore is influenced by the composition of sulfates and carbonates in the material that is delivered to an autoclave processing facility (Montiel & Dimitrakopoulos, 2018; Thomas & Pearson, 2016). Blending the material from several sources in the mining complex to maximize recovery may lead to a higher NPV over the operating life and captures value that is unidentifiable using traditional sequential optimization methods (Gershon, 1983; Hustrulid & Kutcha, 2006; Whittle, 1999). Additionally, waste management considerations such as the production of acid generating waste and tailings can be integrated into the optimization to minimize environmental detriments and ensure permitting constraints are satisfied (Levinson & Dimitrakopoulos, 2019; Saliba & Dimitrakopoulos, 2018). These advancements are achieved by maximizing the value of the products sold (Goodfellow & Dimitrakopoulos, 2017; Montiel & Dimitrakopoulos, 2015), instead of the traditional approach that considers the economic value of a block determined a priori and sequentially optimizes the extraction sequence, cut-off grade and transportation of materials downstream (Hustrulid & Kutcha, 2006).

Furthermore, the proceeding case study strategically determines the optimal production rate during the mine production scheduling process using an adaptive simultaneous stochastic optimization. Several frameworks directly integrate investments into the optimization to achieve a certain level of production and increase the value of the operation (Goodfellow, 2014; Groeneveld & Topal, 2011; Groeneveld et al., 2012). These integrative frameworks allows the optimizer to decide the most suitable time to invest in capital investment overcoming limitations of defining the optimal mining and processing rates prior to optimizing the production schedule (Del Castillo & Dimitrakopoulos, 2014; Godoy & Dimitrakopoulos, 2004).

This work presents a major case study in a multi-mine and multi-process gold mining complex, where an adaptive simultaneous stochastic optimization approach strategically considers investment alternatives. The main contribution of this case study is the ability to simultaneously consider investments in process plant upgrades and the tailings management area, while allowing the model to adapt to uncertainty based on the corresponding investment decisions. In the following sections, the adaptive simultaneous stochastic optimization approach is outlined, followed by a comprehensive case study at a gold mining complex containing two open-pit mines, twelve material types, twelve stockpiles, three external sources (including an underground mine) and three processing stream alternatives. Subsequently, the conclusions and future work are presented.

2 Method

This section summarizes the method used for the adaptive simultaneous stochastic optimization approach proposed by Del Castillo and Dimitrakopoulos (2019), which allows the production schedule to branch on a set of feasible investment alternatives. All sets, parameters, and decision variables are defined in the following subsections and can be reviewed in Appendix A.

2.1 Definitions and notation

A mining complex is designed to include a set of open-pit and underground mines ($\mathcal{M}$), stockpiles ($\mathcal{S}$), processors ($\mathcal{P}$), and waste facilities ($\mathcal{W}$) (Goodfellow & Dimitrakopoulos, 2016, 2017; Montiel & Dimitrakopoulos, 2018; Montiel & Dimitrakopoulos, 2015, 2017). There can be many material types that are either extracted from the mine or generated through blending and processing. Each material has a set of attributes which can be transferred through the mining complex (i.e. mass, metal content, etc.). Attributes are further divided into two sub-categories; primary attributes that define the composition of the material passed between various locations in the mining complex; and hereditary attributes which are a derived through linear and non-linear expressions. Hereditary attributes track important information in the model including the costs incurred at different locations, revenues from the various processing streams, and metal grade. Two variables $v_{p,i,t,s}$ and $v_{h,i,t,s}$ quantify the value of primary ($p \in \mathbb{P}$) and hereditary ($h \in \mathbb{H}$) attributes at each location $i \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}$ in period $t \in \mathbb{T}$ under scenario $s \in \mathbb{S}$, respectively. Hereditary attributes allow both non-linear and linear functions to be incorporated into the model and are a function of the primary attributes, $f_h(p, i, k)$ for each primary attribute $p \in \mathbb{P}$ at location $i \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}$ and considering each available capital investment $k \in \mathbb{K}$. The primary source of material for the mining complex is obtained by extracting a set of mining blocks $b \in B_m$ from mine $m \in \mathcal{M}$. Every block b has a set of simulated primary attributes $\beta_{p,b,s}$ (Goovaerts, 1997; Boucher & Dimitrakopoulos, 2009) which are inputs into the optimization model. The recovery of each attribute p at location $i \in \mathcal{P}$ in each scenario s is defined as $r_{p,i,t,s}$ and are calculated using a non-linear recovery function (Del Castillo, 2018; Farmer, 2016; Goodfellow, 2014).

2.2 Decision variables

Considering a life-of-mine of $\mathbb{T}$ time periods, the adaptive simultaneous stochastic optimization approach aims to maximize the NPV of a mining complex and minimize deviations from the annual production targets. This is accomplished by simultaneously determining the optimal decisions for four decision variables: (i) the mining block extraction sequence; (ii) destination policy; (iii) processing stream; and (iv) capital investment decisions. The method uses a set of binary decision variables $x_{b,t,s}$ that denote whether a block b is extracted in period t, in simulation s. The destination policy is then defined by discretizing the range of metal grades into a set of bins to determine the cut-off grade policy during the optimization process (Menabde et al., 2007). Bins or groups $g \in \mathcal{G}$ are defined using k-means++ clustering algorithm for the primary block attributes $\beta_{p',b,s} \forall p' \subseteq \mathbb{P}, b \in \mathbb{B}_m, m \in \mathcal{M}, s \in \mathbb{S}$ of each material type (Goodfellow & Dimitrakopoulos, 2016). The destination policy decision variable $z_{g,j,t,s} \in \{0, 1\}$ determines if the blocks in group g are sent to destination $j \in O(g)$ in period t, where $O(g)$ is the set of locations where the group of materials can be delivered in scenario s. After the material reaches the first set of destinations, based on the extraction sequence decisions, the downstream material flow is controlled by the processing stream decision variables $y_{i,j,t,s} \in [0, 1]$. The processing stream variable defines the portion of product that is sent from destination $i \in \mathcal{S} \cup \mathcal{P}$ to destination $j \in O(i) \subseteq \mathcal{S} \cup \mathcal{P}$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. Lastly, the capital investment decision variable $\omega_{k,s,t}$ defines if a capital investment $k \in K$ is executed in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$. Subsequently explained in Section 2.3.

2.3 Branching the production schedule

Two different sets are used to describe the different types of investments branching (K^*) and non-branching ($K^=$), where $K^* \cup K^= = \mathbb{K}$. Branching alternatives are large capital investments decisions that are only purchased once during the life of the mining complex. For example, purchasing large crushers or constructing a new tailings facility. The non-branching investments may occur multiple times over the planning horizon, for instance truck and shovel purchases. The decision tree outlines the optimal timing of the branching investments and a new node n is created for each branching decision; this is defined as a stage. An optimized mine plan is produced for each branch that is created. The representative measure $R \in (0, 0.5)$ is a user defined parameter, which is used to describe the confidence interval for branching. R defines the probability threshold required to invest over all scenarios, branch the production schedule, or not invest in each capital investment (Equation 1).

$$\begin{cases} if\ the\ probability\ of\ investing\ in\ k^* < R & \rightarrow do\ not\ invest\ in\ k^*\ during\ t^\omega \\ if\ probability\ of\ investing\ in\ k^* \in [R, 1-R] & \rightarrow branch\ during\ t^\omega \\ if\ the\ probability\ of\ investing\ in\ k^* > 1-R & \rightarrow invest\ in\ k^*\ during\ t^\omega \end{cases} \tag{1}$$

The branching mechanism is described in the subsequent steps:

1. Calculate the probability of investing in all alternatives $k^* \in K^*$ in each time period t.
2. If there are a representative number of scenarios that choose to purchase the investment alternative, within an allotted time window, the solution branches and a new stage is created. Although, if the probability of investing is less than the threshold then the optimization will not branch, and the investment is not purchased. On the contrary, if the probability is greater than $(1 - R)$ there is no branching and the investment is made over all scenarios. This is mathematically described in Equation 1.
3. Given there are $\mathbb{S}_n \subseteq \mathbb{S}$ scenarios that belong to the root, these scenarios are partitioned into $\mathbb{S}_{n1}$ and $\mathbb{S}_{n2}$ when branching occurs. Therefore, when combined all the simulations from each branch are at the root ($\mathbb{S}_{n1} \cup \mathbb{S}_{n2} = \mathbb{S}_n$) and when the simulations are partitioned each simulation can only report to one of the two partitions ($\mathbb{S}_{n1} \cap \mathbb{S}_{n2} = \emptyset$).

A time window, $t_\omega = \{t - \omega, t + \omega\}$, is used to stabilize the solution as often there may be a representative number of scenarios between one or two consecutive periods making it more desirable to invest in one of those two years rather than completely ignoring the investment opportunity. ω is set as an integer value that allows the model to expand the time window of the branching mechanism. The branching or new stage will begin during the floor of the expected time period of investment k^* and is denoted as t^*. Lastly, N defines the minimum number of scenarios in a branch required to allow for further branching in periods $t + 1 \in \mathbb{T}$.

2.4 Capital investments

Capital investments are critical decisions that require a lead time (τ_k) to assemble or construct. For each investment alternative $k \in K$ there is a life expectancy (λ_k) and a unitary increase in capacity ($\kappa_{k,h}$) that comes at a discounted purchase cost $\left(p^K_{k,t}\right)$ for each period $t \in \mathbb{T}$. The periodicity (ψ_k) of the investment decisions is also incorporated into the optimization model to simplify the optimization process and ensure a practical plan. The number of investments undertaken is denoted by $\sigma_{k,t,s}$ for each investment$k \in K$ in period $t \in \mathbb{T}$ and scenario$s \in \mathbb{S}$.

2.5 Objective function and constraints

$$
\max \frac{1}{\|\mathbb{S}\|} \sum_{s\in\mathbb{S}} \sum_{t\in\mathbb{T}} \left\{ \begin{array}{c}
\underbrace{\sum_{i\in\mathcal{M}\cup\mathcal{S}\cup\mathcal{P}\cup\mathcal{W}} \sum_{h\in H} p_{h,t} v_{h,i,t,s}}_{\substack{\text{Profit of the mining complex} \\ \textbf{Part I}}} \\
\underbrace{- \sum_{k\in K^{=}} p_{k,t}\sigma_{k,t,s}}_{\substack{\text{Cost of Truck and Shovel Investments} \\ \textbf{Part II}}} \\
\underbrace{- \sum_{k\in K^{*}} p^{K^*}_{k^*,t}\sigma_{k^*,t,s}}_{\substack{\text{Cost of One Time Capital Investments} \\ \textbf{Part III}}} \\
\underbrace{- \sum_{i\in\mathcal{M}\cup\mathcal{S}\cup\mathcal{P}\cup\mathcal{W}} \sum_{h\in H} \left(c^{+}_{h,t} d^{+}_{h,i,t,s} + c^{-}_{h,t} d^{-}_{h,i,t,s}\right)}_{\substack{\text{Penalties for Deviations} \\ \textbf{Part IV}}}
\end{array} \right\} \tag{2}
$$

The objective function (Equation 2) maximizes the expected profit obtained by summing the revenues generated from the metal produced and subtracting the various costs, for example, transportation, mining, processing and refining costs (Part I). In addition, the objective aims to minimize the costs of investing in trucks and shovels (Part II), and one-time capital investments (Part III). Part IV minimizes the deviation from production targets, actively managing uncertainty. The adaptive optimization approach will only purchase investments when they lead to an increase in overall profitability and/or improve the capability to meet production targets in the mining complex.

Integrating the feasible investment alternatives into the optimization model changes the standard formulation of capacity constraints, from static upper ($U_{h,i,t}$) and lower ($L_{h,i,t}$) bounds in Equations 3 and 4, respectively, to dynamically changing capacities that are determined during the optimization. The capacities reflect changes in the corresponding investment decisions $\omega_{k,s,t}$. $\kappa_{k,h}$ represents the unitary increase in production capacity:

$$
v_{h,i,t,s} - d^{+}_{h,i,t,s} \leq U_{h,i,t} + \sum_{k\in K; t>\tau_k} \sum_{t'=t-\lambda_k-\tau_k}^{t-\tau_k} \kappa_{k,h} \cdot \omega_{k,s,t'} \tag{3}
$$

$$
v_{h,i,t,s} - d^{-}_{h,i,t,s} \leq L_{h,i,t} + \sum_{k\in K; t>\tau_k} \sum_{t'=t-\lambda_k-\tau_k}^{t-\tau_k} \kappa_{k,h} \cdot \omega_{k,s,t'} \tag{4}
$$

$$
\forall h \in H, i \in M \cup S \cup P \cup W, t \in \mathbb{T}, s \in \mathbb{S}, k \in K
$$

$$
d^{+}_{h,i,t,s}, d^{-}_{h,i,t,s} \geq 0 \tag{5}
$$

When investments are activated the capacity expansions and contractions can be explored allowing for changes to the extraction rate, processing capacity, and storage of waste materials.

In addition, non-anticipativity constraints (Equations 7, 8, and 9) ensure that all scenarios within the same branch must undertake the same decisions. The problem is initialized with the solution from a two-stage stochastic integer program and then non-anticaptivity constraints are enforced for the first period. Subsequently, the mechanism for branching iteratively solves a series of sub-problems to determine the optimal period to invest. The non-anticipativity constraints are then dynamically enforced over an iteratively increasing time frame T^{α} when a branching investment is undertaken. For example, once the first branching period is established non-anticipativity constraints become active for all periods up to t^{*}, the period a branching investment is undertaken. This ensures that the optimization framework will not change earlier decisions in anticipation of the investments made in future periods. A binary variable $u^{n}_{k^{*},t}$ equals one when the design branches over option $k^{*} \in K^{*}$ in node n in period $t \in \mathbb{T}$ and otherwise zero. Therefore, the variable A determines whether the non-anticipativity constraints are activated (0) or not (1) for a given partition of scenarios in a single branch:

$$A = \left\lceil \frac{\sum_{k^{*} \in K^{*}} u^{n}_{k^{*},t}}{|K^{*}|} \right\rceil = \{0, 1\} \tag{6}$$

When there is no branching all decision variables must be the same for all scenarios. However, when branching occurs the scenarios partition $\mathbb{S}_{n1} = \{s; w_{k^{*},t^{*},s} = 1, \forall s \in \mathbb{S}_{n}\}$, $\mathbb{S}_{n2} = \mathbb{S}_{n} \backslash \mathbb{S}_{n1}$. Examples of the non-anticipativity constraints are below:

$$(1 - A)\left(x_{b,(t+1),s} - x_{b,(t+1),s'}\right) = 0, \qquad \forall t \in T^{\propto}; b \in M \tag{7}$$

$$(1 - A)\left(z_{g,j,(t+1),s} - z_{g,j,(t+1),s'}\right) = 0, \qquad \forall t \in T^{\propto}; g \in G; j \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W} \tag{8}$$

$$(1 - A)\left(w_{k,(t+1),s} - w_{k,(t+1),s'}\right) = 0, \qquad \forall t \in T^{\propto}; k \in K \tag{9}$$

The destination policy, extraction sequence, and capital investment decisions are the same for all scenarios within each branch of the decision tree. Lastly, in order to ensure stochastic solution stability there must be a minimum number of simulated scenarios in each partition.

2.6 Solution method

A multi-neighbourhood simulated annealing metaheuristic is used to solve the optimization model. Metaheuristics are required as the number of decision variables are in the order of hundreds of millions when considering multi-mine long term production schedule. The metaheuristic used in this work explores a neighbourhood or class of perturbations that are used to change decision variables and achieve near optimal solutions in a short period of time (Goodfellow & Dimitrakopoulos, 2016, 2017; Montiel & Dimitrakopoulos, 2015, 2017). Del Castillo (2018) introduces perturbations that change capital investment decisions including adding or removing multiple investments in a period and swapping two investments between periods. The simulated annealing algorithm then uses an acceptance probability to determine whether the new solution is accepted or rejected to further explore the solution space (Kirkpatrick, Gelatt, & Vecchi, 1983). The modified simulated annealing approach, used in the subsequent case study, updates the probability of choosing a neighbourhood depending on its ability to improve the objective function (Goodfellow & Dimitrakopoulos, 2016).

3 Case study at a gold mining complex

The adaptive simultaneous stochastic optimization approach is applied to a gold mining complex that consists of two large open-pit mines with twelve different material types. These materials can be transported to a number of destinations; an autoclave processing facility, oxide mill, oxide leach, twelve stockpiles (one for each material type), waste facility, and a tailings management area. Each mine contains a mixture of sulphide ores, which must be pretreated at the autoclave before processing, and oxide ores that can be sent to the oxide processor or oxide leach. The mining complex, including each of its component, and the allowable material routing are presented in Figure 1. Sulphide materials, a refractory ore type, can be extracted from either of the open-pit mines and sent to the autoclave, stockpile or waste dump facility. Stockpiles are separated for each material type to provide accessibility to materials of certain chemistry compositions, shown in Table 1. Material that is sourced externally is used to supplement the ore feed that is produced at the two open-pit mines and sent to the autoclave to help meet blending requirements. The optimizer seeks opportunities to increase value and more effectively blend materials to obtain a satisfactory product quality for effectively running the autoclave. Sulphide or refractory ores must be blended to achieve the permissible operating criterion for the autoclave, by controlling the grades of sulphide sulphur (SS), carbonates (CO_3), organic carbon (OC), and the SS/CO_3 ratio. Therefore, these deleterious attributes must be managed within the optimization framework to ensure blending requirements will be meet. A constraint is added to the model to maintain the grade of SS and CO_3 from 3.8–4.2% and 4.5–6.5%, respectively. The deviations from these targets are penalized in the objective function to manage the risk similar to all the other production targets. Acid is used to pre-treat the ore by neutralizing CO_3 and ensuring the appropriate SS/CO_3 ratio (0.8–1.2) is entering the autoclave circuit. This becomes critical as there is variability in the material received from the different sources and often there are not enough materials with the desired qualities readily available. There is a maximum amount of acid (38,400 t) that can be used on an annual basis which introduces a constraint in the optimization process. The autoclave's target production is 2.5 Mt/y. Oxide materials can either report to the oxide mill, leach, or stockpile destinations and there are no constraints on the blending requirements for the oxide ore material. The oxide mill has a production target of 1.4 Mt/y and the leach pad is not constrained. After processing, the volume of mine tailings that are generated from the processing facilities are continuously examined to ensure there is a large enough containment area to continue mining, which then introduces a constraint on the available tailings capacity. Stockpiling facilities are used as intermediate locations to assist with blending and can be extracted from throughout the mine life. Lastly, any material that does not positively contribute to the NPV of the mining complex is sent to the corresponding waste dump facility.

Table 1: Material classification for blending and material routing.

Material	**Chemistry**			
Type	**CO3**	**SS**	**OC**	**Oxide**
1	Med-Low	Low	-	-
2	Med-Low	High	-	-
3	Low	Med	-	-
4	Low	Low	-	-
5	Low	Med-High	-	-
6	High	-	-	-
7	Med-High	Low	-	-
8	Low	High	-	-
9	Very High	-	-	-
10	High	-	Med-High	-
11	-	-	High	-
12	-	-	-	High

In this case study, there are three one-time feasible investment alternatives considered throughout the optimization process to test the adaptive optimization approach. First, the annual autoclave processing throughput may be expanded by investing in two additional positive displacement piston-diaphragm pumps (Eichhorn et al., 2014). Second, an investment in the process plant autoclave circuit is evaluated to increase the allowable acid consumption and manage blending. Third, an investment alternative that considers the construction of a new tailings storage area increases the life-of-mine by allowing for the processors to continue operating. The pump installation increases throughput at the autoclave by 25% which allows for more refractory ore to be processed. The capital cost of this expansion is minimal, however, the cost of implementation and loss of production during the

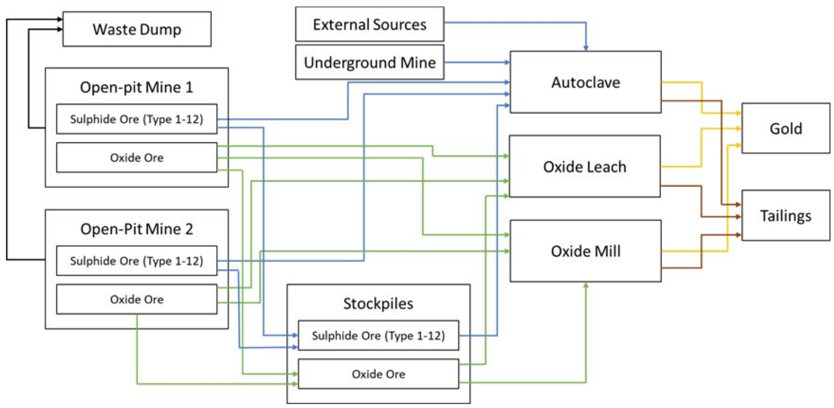

Figure 1: Mining complex and allowable material routing.

pump installation is also considered in the capital investment decision, resulting in a \$1M investment. Acid is ordered annually to satisfy production requirements, but storage areas and adaptations to the autoclave pre-treatment circuit are required to safely utilize the additional acid. The expected investment is \$0.2M. The most significant investment decision is related to the addition of a new tailings containment area which is expected to cost \$200M to construct completely. The new tailings area results in a 33% increase in tailings storage for the mining complex. Once any of the three investments are purchased, they can be continuously used for the remainder of the mine life. Additionally, these three capital investment decisions can potentially allow the production schedule to branch. In this case study, a representativity measure $R = 0.3$ is used based on the acceptable risk of investing in capital at this mining operation. Therefore, the production schedule branches when a representative number of scenarios, between 30 and 70 %, invest in one of these three feasible alternatives. The scenarios are then split, and further branching considerations are assessed in future periods. Further details on the parameters considered for each of the capital investments are described in Table 2.

Table 2: Parameters and cost of capital investments.

Parameters	Non-branching		Branching Expansions		
	Shovel	Truck	Tailings	Autoclave	Acid
Lead Time (years)	2	2	3	2	3
Capital Cost (M\$)	10	1.6	200	1	0.2
Life of Equipment (years)	7	7	13	13	13
Periodicity of Decision (years)	3	3	13	13	13
Increase in Capacity	Feed for 5 trucks/unit	1.4 Mt/unit	5.75 MCM	925 kt	9.6 kt

The mine initially begins with 30 haul trucks and 6 shovels that have two years remaining in their productive life before salvaging. The model dynamically considers the purchase of trucks and shovels throughout the thirteen-year production schedule. Truck and shovel purchases define the annual mine production rate. The cost per truck and shovel is \$1.6M and \$20M, respectively, which is accounted for in the annual cashflows. Allowing for the optimizer to decide on the appropriate time to invest in trucks and shovels throughout the mine life. The mining operation has an aging fleet and it is planning to replace the originally purchased haul trucks with a new fleet. The ability to consider the purchase of new equipment during the optimization provides an opportunity to re-establish the optimal mining rate to satisfy the processor requirements and maximize the value of the operation. The trucks and shovels have a corresponding lead time of two years to provide a suitable amount of time for purchasing equipment from the manufacturer, shipping, and on-site assembly. In addition, they have an expected equipment life of seven years and a purchase can be made every three years stabilizing the production rate.

3.1 Base case mine production schedule

A base case mine production schedule is defined herein using a simultaneous stochastic optimization approach that considers capital investment decisions within the optimization framework while managing uncertainty, however, branching is not considered. The base case mine production schedule can choose to invest in trucks, shovels, and the available expansions, but it can not branch and adapt to uncertainty by considering alternatives; it must either choose to invest or not invest. This is different then the adaptive simultaneous stochastic optimization that can be used to evaluate different alternatives and their corresponding value, as there is a fixed production schedule that must be executed in one way, which does not consider the value of having alternative options to manage uncertainty to a greater extent. The results from the base case mine production schedule are compared with the adaptive branching approach that considers feasible capital investment alternatives. Each method uses a set of multi-variate stochastic simulation of the orebody for each open-pit as input into the optimization model (Boucher & Dimitrakopoulos, 2009; Rossi & Deutsch, 2014). The external sources are simulated based on historical data associated with variability in the supply and quality of material received from other mines in the region. The variability and uncertainty of the material sources are accounted for directly in the optimization framework, unlike conventional frameworks that use a single estimated orebody model as input (Hustrulid & Kutcha, 2006). Lastly, the open-pit mines have a block size of 30 m x 30 m x 20 m, representing the selective mining unit and contain 296k and 172k blocks in Mine 1 and Mine 2, respectively.

The results from the base case production schedule including the extraction sequence, capital investments, stockpiling, blending, mining rate and processing decisions follow. Figure 2 defines the base case mining rate alongside the truck and shovel investment decisions. Noticeably the amount of equipment that is required is decreasing as the mine life proceeds and as the older equipment is approaching the end of its operational life. An opportunity arises to operate the two mines at a lower mining rate. Although a lower mining rate is utilized, the ability to satisfy the autoclave processor (Figure 3a) and oxide mill is fulfilled and a resulting NPV of \$3.65B is achieved in the 50^{th} percentile (P-50). The base case mine production schedule invests in both the expansion of the tailings management area and the additional acid storage facility. The investment in additional pumps do not contribute an increase in the mining complex's NPV when accounting for all scenarios, consequently the pumps are not purchased. The blending constraints are satisfied, between the upper (UB) and lower bounds (LB), in most years through the utilization of stockpiles and other available material (Figure 3b, Figure 3c). However, during the first year, the blending constraints are unachievable as the material that can be extracted during that year does not have the appropriate properties to meet the blending requirements. As the production schedule proceeds, stockpiles are established to help with blending in future years. The operational costs of stockpiling these materials are integrated into the optimization to ensure that the stockpiling decisions contribute to the profitability of the mining complex and help manage the technical risk.

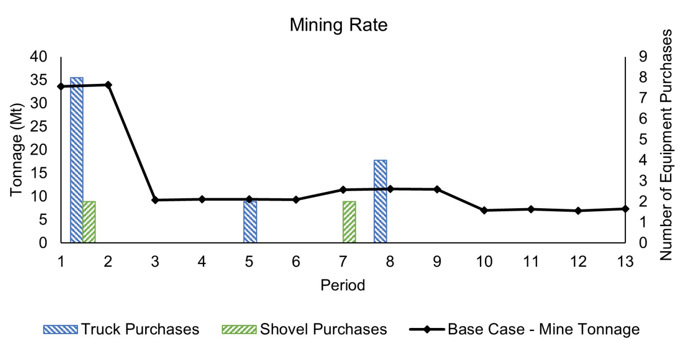

Figure 2: The mining rate and shovel/truck purchase plan for the base case production schedule with no branching.

Lastly, the base case production schedule invests in a tailings expansion in year 7. This investment increases the storage capacity and becomes available in year 10 (Figure 4). The increased tailings storage prolongs the mine life by three years and allows for 1–2 more years of gold production if the duration of this schedule was increased. This results in an additional \$0.7B in discounted cashflows generated. Waste management considerations, such as tailings disposal, are important to optimize directly in the mine production scheduling process in order to generate feasible life of mine designs. Additionally, the processor upgrade that allows for additional acid consumption was purchased in year 3 allowing for 20% increase in additional acid consumption in subsequent years (Figure 5). This controls the blending requirements at the autoclave processing stream.

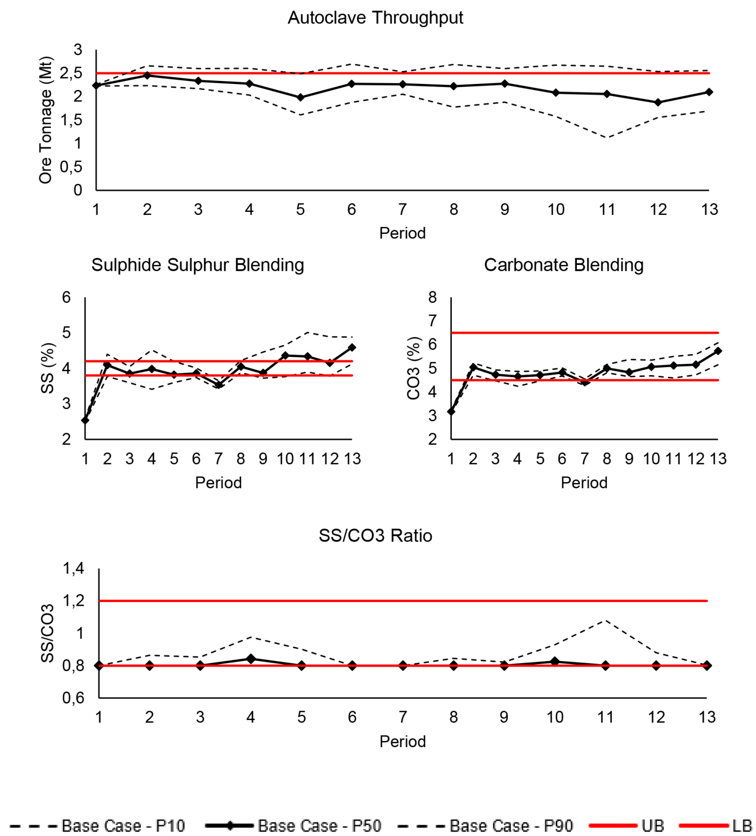

Figure 3: Base case autoclave throughput and blending (a) no expansion taken in the optimization for additional throughput; (b) blending of SS; (c) blending of CO_3; (d) maintaining the SS/CO_3 ratio for ideal operating conditions.

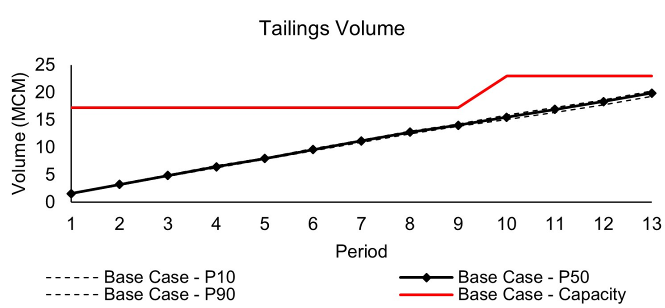

Figure 4: Tailings production over the long-term production schedule and the available capacity expanded in year 10.

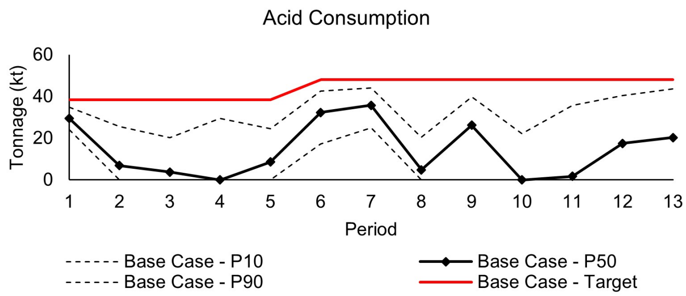

Figure 5: Annual acid consumption with additional capacity obtained in year 6.

3.2 Adapting to supply uncertainty in a gold mining complex

The previously mentioned results will be compared with the adaptive stochastic optimization that considers branching on feasible capital alternatives. During the adaptive simultaneous stochastic optimization groups of scenarios are optimized to determine if there is a beneficial time to invest in any of the one-time capital investments alternatives described previously. The scenarios that lead to a branching decision are separated based on those that invest and those that choose not to invest in the time window. The scenarios that choose not to invest maintain the ability to invest in the capital investment in future years, while the scenarios that invest lock-in that decision for that year activating the non-anticipativity constraints. The scenarios are grouped into separate branches and optimized to produce a feasible alternative for both investing and not investing in the solution. A representative number (over 30%) of scenarios must undertake the same decision for the solution to consider branching or investing in these alternatives, which reduces the number of branches and prevents overfitting the decision tree to each scenario. It is important to note that the scenarios in each branch all undertake the same decisions until a new branching decision is made.

Based on the available capital investments, it was first determined that the additional acid capacity was a suitable investment for greater than 70% of the scenarios leading to a non-branching investment decision. The first investment helped improve the ability to meet the quality requirements of the autoclave. After considering all the simulated scenarios (geostatistical simulations of each open-pit mine and an uncertain external source) and the branching mechanisms criterion, the first branching decision is undertaken allowing for the expansion of the autoclave throughput by installing two additional positive displacement pumps. This separates the number of scenarios into a group of 115 scenarios in branch 1 (B1) that invest and 205 scenarios in branch 2 (B2) that do not invest. After the branching occurs, the optimizer also decides to invest in the additional tailings capacity in more than 70% of the scenarios, for both branches, preventing further growth of the scenario tree. The resulting feasible alternatives both produce a higher NPV then the base case production schedule achieving a value of \$3.89B and \$4.66B in B1 and B2, respectively (Figure 6). This accounts for a 6.4% and 27.5% increase in NPV when comparing the P-50 of each alternative to the base case production schedule. Each of the branches or feasible alternatives perform better than the base case production schedule, however, this may not always be the case as there could be a group of scenarios that underperforms the base case production schedule. The method prevents overfitting by ensuring a number of scenarios do not become too few within each branch and that there is a significant difference in the number of scenarios that either invest or maintain the same operating conditions, hence the representativity parameter which ensures between 30-70% of the scenarios will be split and not a small group of outliers. This substantially reduces the number of branches and ensures feasible stable solutions. The changes in the investment decisions result in a very different response in the production scheduling process, as shown in Figure 7, when comparing the N-S cross sections. First observing, the solution is the exact same until branching occurs and then noticing the schedules change dramatically to take advantage of the new capital investments. There are a number of similarities between the base case and B2 in terms of depth and extents of the mine. However, in B1 there is a large portion of the mine that is no longer extracted in the north, when compared to the other two mine plans. This entails there is some high material variability and uncertainty in this section of the mine that leads to large changes in the resulting mine plan.

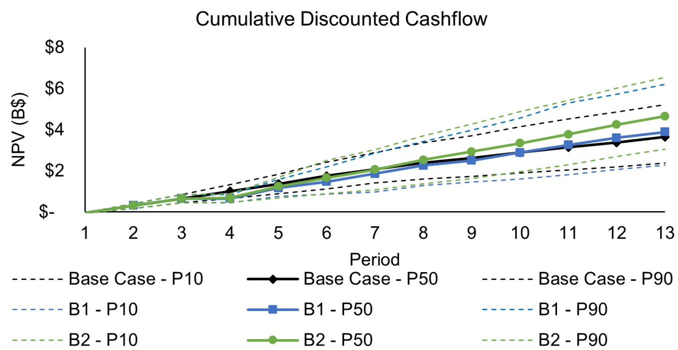

Figure 6: Comparison of the NPV from the adaptive branching and base case production schedule.

Figure 7: N-S cross section of production schedule Mine 1; a) base case (top left), b) branch 1 (top right), and c) branch 2 (bottom left).

B1 invests in the autoclave expansion (Figure 8), which can be fully utilized in year six, and has the lowest mining rate over the long-term production schedule. A comparison of the mining rates are reviewed in Figure 9, where the resulting production rates directly correlate to the amounts of trucks and shovels purchased. The autoclave expansion results in lower grade refractory ore material being processed and a higher throughput being used at the autoclave. Over the long-term production schedule, there is a 9% reduction in the number of gold ounces produced over the life of mine when compared with the P-50 of the base case scenario. However, the reduction in mining costs due to the lower mining rate overcomes the loss in revenue and results in a higher NPV. The lower mining rate is feasible as the throughput outweighs the grade of material through the autoclave changing the selectivity between ore and waste material. Lower utilization of the oxide processing facilities also decreased the operating costs. In B1, the optimizer has a challenging time meeting the blending constraints and is unable to provide the appropriate material to attain the blending targets, making the acid investment a critical decision for ensuring there is a suitable SS/CO_3 ratio.

B2 performs quite differently and instead increases the size of the truck and shovel fleet, which results in a higher extraction rate and ensures that higher-grade refractory ore is being sent to the processor. The oxide processing streams are utilized far more in B2 than in B1 and their target production is maintained during most years. A higher stripping ratio is required to move the additional waste between years five and nine (Figure 11), which is the reason for the additional truck and shovel requirements. Increasing the selectivity, between ore and waste, results in a substantially higher NPV, which B1 was unable to achieve even with the autoclave capacity expansion. The larger contribution in NPV is primarily due to the accessibility to oxide materials in the different groups of simulations and the uncertainty and variability in the gold, SS, CO_3, and OC grades. Here the adaptive approach is able to take advantage of understanding the inherent variability of the mineral deposits and identifies there is an important investigation to commence. This includes more information with regards to the mineralization of oxide materials and stricter guidelines in terms of the quality of material received from external sources before deciding on the autoclave expansion. B2 produces 10% more gold by fully utilizing all the processing stream capacities and better satisfying the blending constraints. The increased utilization of the oxide leach and mill contribute significantly more gold ounces.

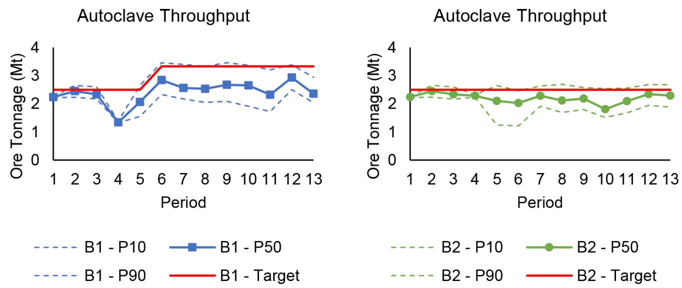

Figure 8: Autoclave throughput and targets a) B1 b) B2 with investments.

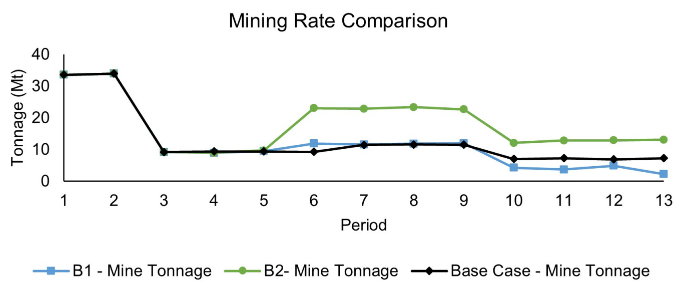

Figure 9: A comparison of the mining rates required to satisfy each production schedule.

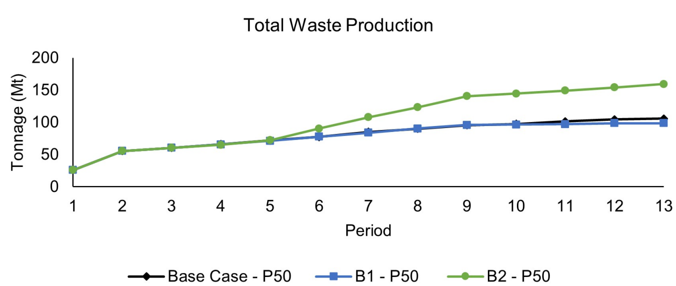

Figure 10: Total waste production over long-term production schedule.

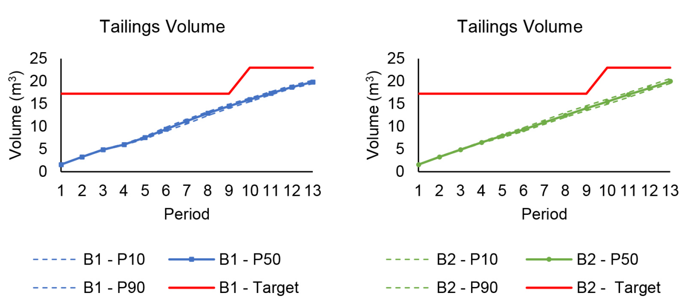

Figure 11: Total tailings production with investment decisions.

The feasible alternatives B1 and B2 invest in the additional tailings containment area in year seven and receive the capacity in year ten, similar to the base case. Had the tailings expansion not been considered during the optimization process, processing would have been required to stop in year ten and a loss of $1B and $1.3B of additional cashflow would be lost in B1 and B2, respectively. This would be a larger loss than the resulting $0.7B in the base case production schedule. The potential loss highlights the importance of simultaneously optimizing the entire mining complex to further understand the intrinsic value of each investment decision.

4 Conclusions

The simultaneous stochastic optimization of a gold mining complex is presented using an adaptive method that integrates feasible capital investment alternatives. The framework capitalizes on synergies and adapts to uncertainty resulting in a 6.4% and 27.5% increase in NPV in B1 and B2, respectively, while satisfying a wide array of production targets and managing supply uncertainty. Investments in trucks and shovel define a new mining rate that minimizes capital expenditures and satisfies each processors capacity. Additionally, an investment in a tailings facility expansion and additional acid consumption increase the life of the mining complex and manage variable material quality at the autoclave processor. Integrating tailings management into the optimization process increases the NPV by 0.7B in the base case production schedule and leads to an additional $1B and $1.3B in B1 and B2, respectively. This emphasizes the importance of considering waste and tailings management in the optimization process to capitalize on the available synergies. The optimizer chooses to branch the production schedule when the autoclave expansion is considered and identifies uncertainty and local variability associated with the supply of oxide and refractory ores sent to each processor. This leads to different mine plans and operating requirements for the processing streams and mining equipment, which is dependent on whether the investment alternative is purchased. The feasible investment alternatives provide a high-level insight on the appropriate attributes to investigate including highly variable areas of the deposit and large differences in the quantity of oxide materials being mined. The optimized production schedule does not branch for the first three years and provides the appropriate lead time to evaluate each alternative decision and gather the required information to make an informed final production schedule.

If either of the feasible alternatives are executed, the expected NPV increases substantially. The base case and adaptive approaches capitalize on the synergies that exist between the different components of the mining complex helping to manage the challenging blending constraints and determine the appropriate size of the mining fleet directly in the optimization. The results from this case study emphasize the importance of modelling the entire mining complex in a single optimization process. In addition, the branching mechanism and adaptive ability of the optimizer provides a method to easily evaluate several feasible alternatives and further understand the variability and uncertainty associated with the mining complex.

Appendix A

Adaptive simultaneous stochastic optimization sets, parameters, and decision variables

Sets and parameters	
$\mathcal{M}$	Set of open-pit and underground mines
$\mathcal{P}$	Set of processors
$\mathcal{W}$	Set of waste facilities
$\mathcal{S}$	Set of stockpiles
$\mathcal{G}$	Set of groups or bins for different cut-off grades $g \in \mathcal{G}$
$\mathbb{T}$	Set of scheduled time periods $t \in \mathbb{T}$
$\mathbb{S}$	Set of simulated orebody scenarios $s \in \mathbb{S}$ where there are $\mathbb{S}_n \subseteq \mathbb{S}$ scenarios that belong to the root, these scenarios are partitioned into $\mathbb{S}_{n1}$ and $\mathbb{S}_{n2}$ when branching occurs therefore $\mathbb{S}_{n1} \cup \mathbb{S}_{n2} = \mathbb{S}_n$ and $\mathbb{S}_{n1} \cap \mathbb{S}_{n2} = \emptyset$
$\mathbb{P}$	Set of primary attributes $p \in \mathbb{P}$
$\mathbb{H}$	Set of hereditary attributes $h \in \mathbb{H}$
$\mathbb{K}$	Set of available capital investments $k \in \mathbb{K}$. There are two different subsets used to describe the different types of investments branching (K^*) and non-branching ($K^=$), where $K^* \cup K^= = K$
$O(g)$	Set of locations where the groups of materials g can be delivered
B_m	Set of mining blocks $b \in B_m$ from mine $m \in \mathcal{M}$
$\beta_{p,b,s}$	Parameter that defines the set of simulated primary attribute p for block b in scenario s
$r_{p,i,t,s}$	Parameter that describes the recovery of each attribute p at location $i \in \mathcal{P}$ in each scenario s
R	Representativity measure that describes the confidence interval for branching $R \in (0, 0.5)$
t_ω	Time window used to stabilize solutions where ω represents the number of periods to search
N	Defines the minimum number of scenarios in a branch required for further branching periods $(t+1) \in \mathbb{T}$
τ_k	Lead time to assemble or construct a capital investment $k \in K$
λ_k	Life expectancy of each capital investment $k \in K$
$\kappa_{k,h}$	Unitary increase in capacity that each investment $k \in K$ leads to for each attribute $h \in \mathbb{H}$
$p^K_{k,t}$	Discounted purchase cost for each investment $k \in K$ for each period $t \in \mathbb{T}$
ψ_k	The periodicity of the investment $k \in K$
$L_{h,i,t}$, $U_{h,i,t}$	The static upper and lower bounds for each hereditary attribute $h \in \mathbb{H}$, location $i \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}$, and period $t \in \mathbb{T}$
Decision variables	
$v_{p,i,t,s}$, $v_{h,i,t,s}$	Quantify the value of primary (p) and hereditary (h) attributes at each location $i \in \mathcal{M} \cup \mathcal{S} \cup \mathcal{P} \cup \mathcal{W}$ in period t under scenario s, respectively
$x_{b,t,s}$	A set of binary extraction sequence decision variables that denotes if a block b is extracted in period t in scenario s as 1, otherwise 0
$z_{g,j,t,s}$	A destination policy decision variable that takes a value of 1 if blocks in group g are sent to destination $j \in O(g)$, in period $t \in \mathbb{T}$
$y_{i,j,t,s}$	A continuous processing stream decision variable that defines the portion of product that is sent from one destination $i \in \mathcal{S} \cup \mathcal{P}$ to destination $j \in O(i) \subseteq \mathcal{S} \cup \mathcal{P}$in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$, $y_{i,j,t,s} \in [0, 1]$
$\omega_{k,s,t}$	A capital investment decision variable that defines if a capital investment $k \in K$ is executed in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$
$\sigma_{k,t,s}$	The number of investments undertaken for each investment $k \in K$ in period $t \in \mathbb{T}$ and scenario $s \in \mathbb{S}$
$u^n_{k^*,t}$	A binary variable equals 1 when the design branches over option $k^* \in K^*$ in node n in period $t \in \mathbb{T}$, otherwise 0
A	A binary variable that activates the non-anticaptivity constraints taking on the value 0, 1 otherwise

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