

Corrigendum: A proximal quasi-Newton trust-region method for nonsmooth regularized optimization

Supplementary material

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Supplementary material

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Abstract : The purpose of the present note is to bring clarifications to certain concepts and surrounding notation of [1]. All results therein continue to hold. The clarifications ensure complete agreement between the paper and our implementations.

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In (3.2), we should have defined $\varphi(s; \mathbf{x}) := \varphi_{\text{cp}}(s; \mathbf{x}) + \frac{1}{2}\nu^{-1}\|s\|^2$ where φ_{cp} is the first order Taylor approximation

$$\varphi_{\text{cp}}(s; \mathbf{x}) = f(\mathbf{x}) + \nabla f(\mathbf{x})^\top s, \quad (1)$$

and “cp” stands for “Cauchy point”. In (3.4), $s_{k,1}$ could have been defined equivalently as the solution of

$$s_{k,1} \in \underset{s}{\operatorname{argmin}} \varphi_{\text{cp}}(s; \mathbf{x}_k) + \frac{1}{2}\nu^{-1}\|s\|^2 + \psi(s; \mathbf{x}_k) + \chi(s; \Delta_k). \quad (2)$$

The first step of the proximal-gradient algorithm applied to minimize $\varphi_{\text{cp}}(s; \mathbf{x}_k) + \psi(s; \mathbf{x}_k) + \chi(s; \Delta_k)$ with step size ν is then precisely $s_{k,1}$.

In addition to the definition of ξ in (3.6), we also define

$$\xi_{\text{cp}}(\Delta; \mathbf{x}_k, \nu) := f(\mathbf{x}_k) + h(\mathbf{x}_k) - \varphi_{\text{cp}}(s_{k,1}; \mathbf{x}_k) - \psi(s_{k,1}; \mathbf{x}_k). \quad (3)$$

Note that for $\varphi = \varphi_{\text{cp}}$,

$$\begin{aligned} \xi(\Delta_k; \mathbf{x}_k, \nu_k) &= (f + h)(\mathbf{x}_k) - (\varphi_{\text{cp}}(s_{k,1}; \mathbf{x}_k) + \frac{1}{2}\nu_k^{-1}\|s_{k,1}\|^2 + \psi(s_{k,1}; \mathbf{x}_k)) \\ &\leq (f + h)(\mathbf{x}_k) - (\varphi_{\text{cp}}(s_{k,1}; \mathbf{x}_k) + \psi(s_{k,1}; \mathbf{x}_k)) \\ &= \xi_{\text{cp}}(\Delta_k; \mathbf{x}_k, \nu_k). \end{aligned}$$

Thus $\xi_{\text{cp}}(\Delta_k; \mathbf{x}_k, \nu_k) = 0 \implies \xi(\Delta_k; \mathbf{x}_k, \nu_k) = 0$, and, by Proposition 3.3, \mathbf{x}_k is first-order stationary. Using the definition of $s_{k,1}$, we have

$$\varphi_{\text{cp}}(s_{k,1}; \mathbf{x}_k) + \frac{1}{2}\nu_k^{-1}\|s_{k,1}\|^2 + \psi(s_{k,1}; \mathbf{x}_k) \leq \varphi_{\text{cp}}(0; \mathbf{x}_k) + \psi(0; \mathbf{x}_k).$$

The above inequality, combined with the facts that $\varphi(0; \mathbf{x}_k) = f(\mathbf{x}_k)$ and $\psi(0; \mathbf{x}_k) = h(\mathbf{x}_k)$, leads to

$$\xi_{\text{cp}}(\Delta_k; \mathbf{x}_k, \nu_k) = f(\mathbf{x}_k) + h(\mathbf{x}_k) - (\varphi_{\text{cp}}(s_{k,1}; \mathbf{x}_k) + \psi(s_{k,1}; \mathbf{x}_k)) \geq \frac{1}{2}\nu_k^{-1}\|s_{k,1}\|^2. \quad (4)$$

In Step Assumption 3.1, (3.8b) should read

$$m_k(0; \mathbf{x}_k) - m_k(s_k; \mathbf{x}_k) \geq \kappa_{\text{mdc}} \xi_{\text{cp}}(\Delta_k; \mathbf{x}_k, \nu_k). \quad (5)$$

The justification of (3.8b) (below Model Assumption 3.2) is no longer relevant with the above change. However, the following proposition shows that (5) in Step Assumption 3.1 can still be satisfied if

$$\varphi(s; \mathbf{x}_k, \mathbf{B}_k) = f(\mathbf{x}_k) + \nabla f(\mathbf{x}_k)^\top s + \frac{1}{2}s^\top \mathbf{B}_k s = \varphi_{\text{cp}}(s; \mathbf{x}_k) + \frac{1}{2}s^\top \mathbf{B}_k s, \quad (6)$$

where $\{\mathbf{B}_k\}$ is bounded. In addition, we require the existence of a maximal trust-region radius $\Delta_{\text{max}} \geq \Delta_0$. This can be enforced in Algorithm 3.1 at the end of Step 11 with the following assignment:

$$\Delta_{k+1} \leftarrow \min(\Delta_{k+1}, \Delta_{\text{max}}). \quad (7)$$

Proposition 1. *Let Algorithm 3.1 be modified according to (7), and $\varphi(s; \mathbf{x}_k, \mathbf{B}_k)$ defined in (6) be used in Step 5. Let Model Assumption 3.2 be satisfied, and the sequence of Hessian approximations $\{\mathbf{B}_k\}$ generated by Algorithm 3.1 be bounded. Then, there exists $\kappa_{\text{mdc}} \in (0, 1)$ such that (5) holds for all k .*

Proof. By definition of s_k , computed in Step 8 of Algorithm 3.1 using (6) in Step 5,

$$\begin{aligned} \varphi(s_k; \mathbf{x}_k, \mathbf{B}_k) + \psi(s_k; \mathbf{x}_k) &= m(s_k; \mathbf{x}_k, \mathbf{B}_k) \leq m(s_{k,1}; \mathbf{x}_k, \mathbf{B}_k) = \varphi_{\text{cp}}(s_{k,1}; \mathbf{x}_k) + \frac{1}{2}s_{k,1}^\top \mathbf{B}_k s_{k,1} + \psi(s_{k,1}; \mathbf{x}_k) \\ &\leq \varphi_{\text{cp}}(s_{k,1}; \mathbf{x}_k) + \frac{1}{2}\|\mathbf{B}_k\| \|s_{k,1}\|^2 + \psi(s_{k,1}; \mathbf{x}_k). \end{aligned}$$

We multiply the above inequality by -1 and add $m(0; x_k, B_k) = m(0; x_k, \nu_k) = f(x_k) + h(x_k)$ to both sides to obtain

$$m(0; x_k, B_k) - m(s_k; x_k, B_k) \geq \xi_{\text{cp}}(\Delta_k; x_k, \nu_k) - \frac{1}{2} \|B_k\| \|s_{k,1}\|^2.$$

To satisfy (5), it is sufficient to show that there exists $\kappa_{\text{mdc}} \in (0, 1)$ such that

$$\xi_{\text{cp}}(\Delta_k; x_k, \nu_k) - \frac{1}{2} \|B_k\| \|s_{k,1}\|^2 \geq \kappa_{\text{mdc}} \xi_{\text{cp}}(\Delta_k; x_k, \nu_k),$$

i.e.,

$$(1 - \kappa_{\text{mdc}}) \xi_{\text{cp}}(\Delta_k; x_k, \nu_k) \geq \frac{1}{2} \|B_k\| \|s_{k,1}\|^2.$$

Using (4), it is also sufficient to show that there exists $\kappa_{\text{mdc}} \in (0, 1)$ such that

$$(1 - \kappa_{\text{mdc}}) \nu_k^{-1} \geq \|B_k\|. \quad (8)$$

If $\|B_k\| = 0$, (8) is satisfied for any $\kappa_{\text{mdc}} \in (0, 1)$. Otherwise, because (6) indicates that the Lipschitz constant $L(x_k)$ of $\nabla \varphi(\cdot; x_k, B_k)$ in Model Assumption 3.2 is $L(x_k) = \|B_k\|$, the definition of ν_k in Step 4 of Algorithm 3.1 yields

$$\|B_k\| \nu_k = \frac{\|B_k\|}{\|B_k\| + \alpha^{-1} \Delta_k^{-1}} \leq \frac{1}{1 + \alpha^{-1} \Delta_k^{-1} \|B_k\|^{-1}} \leq \frac{1}{\alpha^{-1} \Delta_{\max}^{-1} (\sup_{k \in \mathbb{N}} \|B_k\|)^{-1} + 1} \in (0, 1), \quad (9)$$

because $\{B_k\}$ is bounded by assumption. We deduce from (9) that (8), and therefore (5) hold. \square

In the proof of Theorem 3.4, (4) guarantees that the inequality

$$\xi_{\text{cp}}(\Delta_k; x_k, \nu_k) \geq \frac{1}{2} \nu_k^{-1} \|s_{k,1}\|^2 \geq \frac{1}{2} (L(x_k) + \alpha^{-1} \Delta_k^{-1}) \|s_{k,1}\|^2 \geq \frac{1}{2} \alpha^{-1} \Delta_k^{-1} \|s_{k,1}\|^2, \quad (10)$$

continues to hold, where $L(x_k)$ is the Lipschitz constant of the gradient of $\varphi(\cdot; x_k)$. Thus, the theorem remains valid.

In the simple case where $h = 0$ and $s_{k,1} = -\nu_k \nabla f(x_k)$ is inside the trust-region,

$$\xi_{\text{cp}}(\Delta_k; x_k, \nu_k) = -\nabla f(x_k)^T s_{k,1} = \nu_k \|\nabla f(x_k)\|^2, \quad (11)$$

which suggests that the appropriate criticality measure (analogous to the gradient norm in the smooth case) should be $\nu_k^{-1/2} \xi_{\text{cp}}(\Delta_k; x_k, \nu_k)^{1/2}$ instead of $\nu_k^{-1} \xi_{\text{cp}}(\Delta_k; x_k, \nu_k)^{1/2}$. The first-order optimality condition in (3.13) then becomes

$$\nu_k^{-1/2} \xi_{\text{cp}}(\Delta_{\min}; x_k, \nu_k)^{1/2} \leq \epsilon \quad (0 < \epsilon < 1). \quad (12)$$

This changes the bound in (3.15) to

$$|S(\epsilon)| \leq \frac{(f+h)(x_0) - (f+h)_{\text{low}}}{\eta_1 \kappa_{\text{mdc}} \nu_{\min} \epsilon^2} = O(\epsilon^{-2}), \quad (13)$$

i.e., ν_{\min}^2 becomes ν_{\min} , but does not alter the order of complexity.

Similarly, the limits considered in Corollary 3.9 and Theorem 3.11 should be $\liminf_{k \rightarrow \infty} \nu_k^{-1/2} \xi_{\text{cp}}(\Delta_k; x_k, \nu_k)^{1/2}$ and $\lim_{k \rightarrow \infty} \nu_k^{-1/2} \xi_{\text{cp}}(\Delta_k; x_k, \nu_k)^{1/2}$, respectively.

In addition to the definition of $\xi(\sigma; x)$ in (6.4), we should have also defined

$$\xi_{\text{cp}}(\sigma; x) := f(x) + h(x) - (\varphi(\sigma; x) + \psi(\sigma; x)), \quad (14)$$

where

$$s \in \arg \min_s m(s; x, \sigma). \quad (15)$$

However, (6.6) is still valid.

After (6.10), by analogy with Algorithm 3.1, the stationarity measure should be $\sigma_k^{1/2} \xi_{\text{cp}}(\sigma_{\max}; x_k)^{1/2}$.

In Section 7, all our numerical experiments use $\xi_{\text{cp}}(\Delta_k; x_k, \nu_k)^{1/2}$ and $\xi(\sigma_k; x_k)^{1/2}$ as proxies for the stationarity measures. Even though the factors $\nu_k^{-1/2}$ and $\sigma_k^{1/2}$, respectively, are not included in the proxies, the measures remain valid as long as $\nu_{\min} > 0$ and $\sigma_{\max} < \infty$, which is the case under our assumptions.

References

- [1] A. Y. Aravkin, R. Baraldi, and D. Orban. [A proximal quasi-Newton trust-region method for nonsmooth regularized optimization](#). *SIAM J. Optim.*, 32(2):900–929, 2022.