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# Long-term influence of priority order of short-term unit commitment in a complex hydroelectric system

Emmanuel Bignon <sup>a</sup>

Martin Gagnon <sup>b</sup>

Stéphane Alarie <sup>b,c</sup>

Antoine Tahan <sup>a</sup>

Michel Gamache <sup>c,d</sup>

<sup>a</sup> *École de technologie supérieure, Montréal (Québec) Canada, H3C 1K3*

<sup>b</sup> *Institut de recherche d'Hydro-Québec, Varennes (Québec) Canada, J3X 1S1*

<sup>c</sup> *GERAD, HEC Montréal, Montréal (Québec), Canada, H3T 2A7*

<sup>d</sup> *Department of Mathematics and Industrial Engineering, Polytechnique Montréal (Québec) Canada, H3C 3A7*

alarie.stephane@ireq.ca

michel.gamache@polymtl.ca

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**Abstract:** In the area of hydraulic power generation, there is a great deal of interest in two interdependent domains: operation and maintenance. This interdependence is usually simplified, or even ignored, to tackle the issue raised by the difference in planning horizons scales in each of these domains, causing conservative planning for the maintenance and missed opportunities in operation. This paper presents a new model for the simulation of the operation of a complex system that is a hydraulic power-generating grid, over an arbitrary long horizon, with a small step, in a reasonable time through a sequential resolution of the hourly operational plan of the system. The model is used to assess the influence of operation decisions (in this case, the start-up order of generating units) on the reliability of components through uptime, start-ups number and mode of operation over a medium horizon. A preliminary case study is presented using a standard operation strategy and its counterpart to validate the approach.

**Keywords:** Hydropower system management, complex systems, simulation, maintenance, reliability

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## 1 Introduction

Hydroelectricity is one of the main sources of renewable energy used today, and presents many advantages and benefits. In particular, these include the fact that energy can be stored through dams, and that the system provides a fast response time (under a minute) and is relatively robust to short climate fluctuations. The major cost factors associated with hydroelectricity are the setup costs (dam construction, equipment), which are usually irreducible, operation costs and maintenance costs (O&M costs) [1]. For example, Hydro-Québec (Canada), which possesses more than CAD 32 billion in assets, has maintenance costs of over CAD 400 million a year [2].

To limit the need for refurbishment, each generating unit must be operated within its functional limits. Each failure or lack of availability may generate significant financial losses. For these reasons, the prognostic of these machines has been extensively covered in research works (Heng et al. [3]). Froger et al. [4] report, however, that, in practice, current maintenance strategies for hydropower generating units are mostly based on systematic preventive planning, with corrective maintenance operations used to handle unexpected failures. This planning may span many decades.

Cost reduction is thus approached separately, and focused on two main objectives: operation cost optimization and maintenance cost optimization [5]. According to Bajestani et al. [6,7], combining these two objectives in a global optimization scheme yields better results.

A hydropower generating grid is a system that is difficult to model due to its structural complexity [8]. This complexity results from the heterogeneity of its elements (the system is organized in sets of plants containing generating units with independent characteristics), as well as the scale of the model (number of elements) and the interdependence of its elements.

Figure 1 illustrates such a system (at the plant scale, not in actual power generating units). The system is split over areas A1 to A5, each with specific demands. It has external interactions that can generate energy exchanges (E1 to E3). The energy produced can be transmitted to neighbouring areas through power lines that have capacity constraints. The plants (P1 to P10) are divided into several categories. Some plants may not rely on hydropower to produce energy; some may have their production planned separately (for example, in the case of a run-of-the-river plant). The hydropower plants are linked through waterways with different lengths and flows. Some may even share a common dam. The water level in each dam must satisfy minimum and maximum limits, which can vary throughout the year.

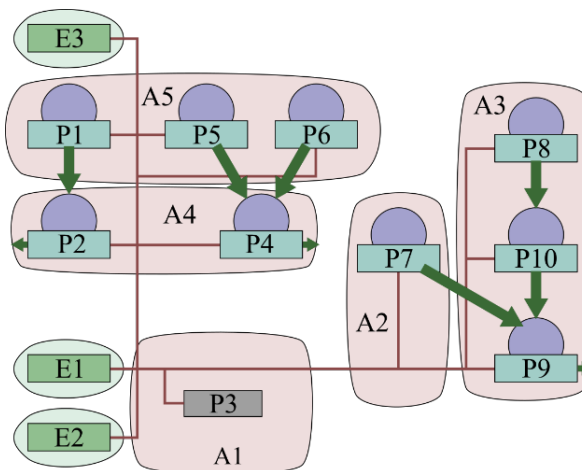


Figure 1: Hydropower system structure schematic example

The long-term planning problem for such a system is intrinsically difficult. Typically, the operation plan is subdivided into successive layers of shorter and shorter horizon, reaching the operational prob-

lem known as the Unit Commitment problem. Simply put, it consists in finding the generation units that must be used to fulfil the demand at the lowest cost. Research has provided methods to solve this problem in the case of thermal units, and to a lesser extent, for hydro generators [9]. However, as specified by Taktak and D'Ambrosio [10], the mathematical programming models used to solve this problem usually only consider local information, and not unit history.

The Hydraulic Unit Commitment problem, in particular, is hard to solve, and therefore requires long computational times. Simplifications can be made in the model to reduce this computational time, but, even then, this time would still be a matter of minutes (as reiterated by Van Ackooij et al. in [11]). As an example of such a model simplification, most of the methods used for large hydroelectric systems consider, for each plant, a preferential order of units (priority order). This allows modelling of the commitment in terms of the number of running units, instead of designating units individually (the economic dispatch is next solved at the plant level by extracting the identity of underlying units from the commitment solution).

The order of units is arbitrary. However, we are used to ordering units based on efficiency, and this allows the production of energy without wasting water and provides a convex piecewise linear production curve to the model, which is easy to solve and makes implicit the order of units [12].

That order does not take the state of units into consideration. Degradation of power-generating units typically occurs when operating conditions change or when they are maintained for overly long periods. For example, in the case of turbines, blade cracking due to material fatigue is closely linked to the number of unit start-ups. As Gagnon et al. indicated [13], a unit may operate for a long time without interruption and never degrade, while a turbine with a shorter operational time may degrade quickly if it is often used to adjust production; for instance, if it starts and stops several times a day.

Let us assume that one wants to limit the number of start-ups for a given unit. This may be necessary because the unit has a high risk of fatigue cracks or it is already scheduled for replacement in the coming years. Obviously, this is highly likely to be difficult if the units are only committed on the basis of their efficiency, and the unit considered is continuously selected for adjusting the production.

Obviously, unit commitment can easily interfere with, or simply compromise, previously established longer-term planning maintenance considerations.

This paper examines the feasibility of integrating long-term considerations into short-term ones to obtain unit commitments in accordance with scheduled repairs and replacements. To this end, it is proposed to order the units based on criteria other than just efficiency. To validate this, the reverse order in terms of efficiency is considered. If this is viable, then any other order should also be viable.

Since the complexity of the HUC problem is high (even recent studies such as Wang et al. [14] and Razavi et al. [15] only consider 1 and 8 hydro plants respectively), classical resolution approaches relying on optimization are impractical for models such as ours: 141 turbines into 25 plants considered over a long horizon of 3 years (26,280 hours). Therefore, the proposition is demonstrated by simulating hourly operations over three years by solving a unit commitment every 24 hours. The study does not aim to suggest a new formulation for the unit commitment problem or to improve its solution. We consider that the solver and the embedded formulation are given. The goal of the study is to assess whether margins are available to integrate maintenance considerations.

The paper is structured as follows. Section 2 describes the approach that is proposed, and introduce the reader to our simulator of a hydroelectric system operation as a solution to model and study arbitrary long horizons of operation. It is divided in 2 subsections: Section 2.1 briefly describes the Hydraulic Unit Commitment (HUC) problem, and the complexity of its model. Section 2.2 details the proposed simulation framework used for experiments. Section 3 presents the metrics that are considered to evaluate the system performance under the proposed orders. Numerical results for a standard operation policy and its counterpart are reported in Section 4, and finally, Section 5 ends this paper with a discussion of the results obtained and the future work this approach will allow.

## 2 Proposed approach

As we will see in the following subsection, the Hydraulic Unit Commitment problem is considering the operation of a complex system and is difficult to solve even for short-term planning (a day's plan on the hour).

Given the dimension of the problem, the planning of a mid-term or long-term plan through the classical methods, as reviewed by Parvez et al. [16] (MILP, Dynamic Programming, etc.), would be subject to combinatory explosion. We propose a simulation framework, relying on the repeated resolution of a daily planning over the horizon using the production data given by the previous days' resolutions.

In a short summary that will be explained in the following subsection, the required elements to reproduce our approach are the topology of the system, a set of data for the demand over the full horizon, the initial volume of reservoirs, the historical data of the rivers prior to the period, a water management policy and every other constraint required for a day's operation, over the full horizon. In terms of the process execution, our approach requires a solver for the daily (or short-term) unit commitment problem that allows choosing the starting order of units on each day. Finally, our framework should interface with the solver in a way such that it can create every day's input of the solver and store every output in its own historical data.

### 2.1 The hydraulic unit commitment problem

The unit commitment problem can be described as the challenge of determining the production unit to start, and in which operating mode, in order to satisfy a given demand under certain operating conditions. Many research works focus on the resolution of this problem, particularly in the field of thermal unit commitment problems, and we invite interested readers to consult Wood et al. [17] for a good introduction or to consult the review by Van Ackooij et al. in [11] to find papers related to this field. Interest in the hydraulic unit commitment problem has, however, been much less, and this is reflected in industry practice.

The HUC problem consists in finding the optimum of a specific function (maximum production, minimum deviation from the optimal production point of units, etc.). Other examples can also be found (see, for example, Paddy in [18]). This objective is related to the production of running units and the choice of such units.

To resolve an HUC problem, the following constraints must be dealt with:

1. Demand satisfaction.
2. Flow conservation in reservoirs, which links reservoir volumes with turbined water in upstream plants and, natural inflows and turbined water in the plant sitting on the dam.
3. Reservoir volume bounds.

Additionally, the network structure provides constraints on the inflows and outflows of reservoirs in sequence on a river, and the production of the plants sitting on a reservoir is directly linked to the actual outflow of the reservoir. Also, some constraints are usually added to reduce shifting units between periods of operation horizon.

As an example, considering the notations of Table 1, a simplified version of the flow conservation equation is that the volume of a reservoir  $d$  ( $V_d$ ) is gaining the natural inflow in the reservoir ( $\mathcal{N}_d$ ) and the turbined water from the upstream reservoir delayed by the duration of transition between reservoirs ( $O_{d'}(t - r_{d',d})$ ) and losing the turbined water ( $q_j^{k,l}$ ) from the plants on the reservoir ( $\omega_d(k,l)$ ), as given

by the following expressions:

$$\begin{aligned}
 V_d(t+1) &= V_d(t) + \mathcal{N}_d(t) - O_d(t) + I_d(t) \\
 I_d(t) &= \sum_{\substack{d'=1 \\ r_{d',d} < \infty}}^{N_R} O_{d'}(t - r_{d',d}) \\
 O_d(t) &= \sum_{k=1}^{N_A} \sum_{\substack{l=1 \\ \omega_d(k,l) > 0}}^{N_P^k} \sum_{j=1}^{N_G^{k,l}} q_j^{k,l}(t)
 \end{aligned} \tag{1}$$

**Table 1: Mathematical symbols used in the formulation of the HUC problem**

Symbol	Unit	Significance
$N_A$		Number of areas
$k$		Index of area
$N_P^k$		Number of plants in area $k$
$l$		Index of plant
$N_G^{k,l}$		Number of power units in plant $l$ of area $k$
$L_{k,k'}$	MW	Power transmitted from area $k$ to area $k'$ (negative if from $k'$ to $k$ )
$j$		Index of power unit
$\sigma_j^{k,l}$		State of the power unit $j$ of plant $l$ in area $k$ (0 for stopped, 1 for running)
$q_j^{k,l}$	m <sup>3</sup>	Turbined water by power unit $j$ of plant $l$ in area $k$
$D_k$	MW	Energy demand in area $k$
$P_j^{k,l}$	MW	Power output of the power unit $j$ of plant $l$ in area $k$
$\nu_l^k$		Number of units started in plant $l$ in area $k$
$P_{opt,j}^{k,l}$	MW	Optimal mode power output for unit $j$ of plant $l$ in area $k$
$\Delta P_{opt,j}^{k,l}$	MW	Distance of power output to $P_{opt,j}^{k,l}$
$N_R$		Number of reservoirs
$d$		Index of reservoir
$\omega_d(k,l)$		Function indicating if the plant $l$ in area $k$ is sitting on the reservoir $d$
$V_d$	m <sup>3</sup>	Volume of water in the reservoir $d$
$\bar{V}_d$	m <sup>3</sup>	Maximal volume of water that can be in reservoir $d$
$r_{d,d'}$	h	Delay for a turbined flow to go from a reservoir $d$ to a reservoir $d'$

A complete mathematical model is not the focus of this work, since our study assumes that the unit commitment problem is properly solved. However, the start-up strategies that result from the modelling play a direct role in degradation.

Some models, such as the one by Frangioni et al. [19], consider units in the system independently, while others consider a group of identical units, such as in Dal'Santo and Simões Costa [20]. To our knowledge, however, in order to cope with the combinatorial explosion of large-scale systems, most solutions on actual systems either assume that all units in a power plant are identical, that the starting order of the units inside a plant is predetermined, or consider a system without some of the constraints we have mentioned in this section (network constraints, for example).

Of those three approaches, the only one that does not cause the model to deviate from reality is the second one. The first approach changes the nature of the system (units are usually not identical); the third approach relaxes the system into an easier one (since some constraints can now be violated), and the second one reduces the feasible set.

Considering the parameter and variables as described in Table 1, the following describes the original model and the second modification.

In the original formulation, the demand satisfaction constraint is often expressed as follows:



For every area  $k \in \{1, 2, \dots, N_A\}$ :

$$D_k = \sum_{l=1}^{N_P^k} \sum_{j=1}^{N_G^{k,l}} \sigma_j^{k,l} P_j^{k,l} + \sum_{l=1}^{k-1} L_{l,k} - \sum_{l=k+1}^{N_A} L_{k,l} \quad (2)$$

where the generated power for every unit  $j \in \{1, 2, \dots, N_G^{k,i}\}$  of each plant  $i \in \{1, 2, \dots, N_P^k\}$  of each area  $k \in \{1, 2, \dots, N_A\}$  is a function of the turbined water:

$$P_j^{k,l} = \Phi_j^{k,l} \left( q_j^{k,l} \right) \quad (3)$$

The assumption that units are started in a given order in a plant, for example indicated by the index  $j$ , changes the demand satisfaction equation into the following formula:

$$D_k = \sum_{l=1}^{N_P^k} \sum_{j=1}^{\nu_l^k} P_j^{k,l} + \sum_{l=1}^{k-1} L_{l,k} - \sum_{l=k+1}^{N_A} L_{k,l} \quad (4)$$

There are now commitment decisions for each plant, rather than for each generating unit. Let us assume that for each plant  $l \in \{1, 2, \dots, N_P^k\}$ , the order in  $j \in \{1, 2, \dots, N_G^{k,l}\}$  refers to the units according to their decreasing efficiency, and the constraints  $D_k$  are piecewise convex functions. In this case, the problem becomes separable, and we can take advantage of this property to speed up the resolution (we refer the interested reader to Hillier and Lieberman [12] for a detailed proof and method to use this property).

We must recall that the order is arbitrary, and that it is possible and valid to change the order of the units (that is, not order them by decreasing efficiency). Doing so removes the separability property we just reiterated, but the function remains piecewise and monotonous. It is not necessary to add variables or constraints to handle this case; the HUC remains realistic, but will, however, be harder to solve.

The HUC formulation with a unit order is very sensitive to the starting order choice, and the resolution time becomes hard to predict (it depends on how close we are to the efficient-first order).

## 2.2 System midterm operation simulation

The consequences of operation decisions on the long-term degradation of the system are important for validating the operation and maintenance decision choices. However, the models are considered on completely different scales, and as such, the influence of one over the other is hard to model.

The costs associated with the degradation of units are not perceptible in the short term, and are completely dominated by the costs directly involved in production. Ideally, the validation of the joint consideration of maintenance and operations would require a planning model that takes into account both the production and degradation of the system. Generally, this implies creating a model that will consider the system with its smallest step and optimize it over the longest possible horizon.

The complexity of the system and the number of parameter data in the HUC problem make it difficult to scale over a long horizon; furthermore, in the case of a large hydroelectric system, the resolution of the hourly problem over a day already takes minutes to compute, as explained in [11].

The HUC problem scales badly in the time dimension, since for each considered instant, the decision variables are multiplied. Simply considering the pure commitment decision (power a turbine or not), the number of variables is given by the following equation:

$$t \times \sum_{k=1}^{N_A} \sum_{l=1}^{N_P^k} N_G^{k,l} \quad (5)$$

To our knowledge, very few tools or studies have been made to characterize the consequences of repeated use of an HUC solver to cover the long-term (with a comparatively very small step). Most of the recent works limit the resolution horizon of the HUC to a period that lasts around a day (see [14], for example).

Van den Bergh et al. [21] published a study to determine the long-term start-up costs in unit commitment. They present a methodology relying on the sequential resolution of smaller unit commitments over the long horizon. They propose to add an iterative resolution of that model with an evolving cost to determine the optimal cost model and associated strategy.

Their approach, however, is applied to a system without considering the hydraulic (flow and reservoir management) constraints. Even with this important simplification, the computation of the cost and operation takes more than a day for a one-year horizon. Furthermore, this approach only considers start-up costs, and not run-time costs or costs related to the mode of operation.

In this work, in order to emulate system operation, we use a simulator that will rely on an HUC problem solver to get each day's operation plan, with a one-hour step. The HUC problem solver uses a structural description of the system, including the constraints between the different plants, the values of the natural and past inflows, the expected demand on an hourly basis, the level of the dams, etc.

In this work, we chose to rely on a solver using a Mixed-Integer Linear Programming (MILP) formulation. It was chosen because it has been tailored by Hydro-Québec to perform well on their system (detailed in Section 5). It takes minutes to solve one day of operation. It also meets our requirement in the fact that it relies on a predetermined starting order for the units in a plant, which can be changed for each day (or each resolution).

The chosen HUC solver implements an operation strategy that attempts to minimize the difference between the optimal power output for the generating power unit and the actual output (where the optimal power output consists of the power output of the optimal functioning point, that is, where the unit is most efficient). This objective is also worthy in terms of degradation since, as Dörfler states [22], cavitation is more likely to occur when operating a turbine outside its optimal operation mode.

$$\min \sum_{k=1}^{N_A} \sum_{l=1}^{N_P^k} \sum_{j=1}^{N_G^{k,l}} \left( \frac{\Delta P_{opt,j}^{k,l}}{P_{opt,j}^{k,l}} \right)^2 \quad (6)$$

The midterm operation of the system can be simulated through the repetitive use of the HUC solver, by updating the input data with the output of the previous day, and possibly the operation policy, as illustrated in Figure 2.

The HUC solver computes day operational planning on a day-to-day basis, indicating which power units (turbines, in this case) are started, and when; what power is produced, how much water is flowing, and how much energy is transmitted through the power lines. We then use this data to update the information we have from our simulation's definition in order to begin the next day with a coherent initial state (and the updated history of the river flows). A new policy can also be chosen according to the data for this new day of operation.

A fatal drawback of this simulation would be the short-sightedness of the HUC solver, which generates local dryness by overusing more efficient plants. To tackle this problem, as is done in real life, a water management plan should be devised and imposed on the HUC solver. In our case, the solver accepts target levels for the reservoirs and an associated cost for the distance to this target, but it is also possible to force the target by adapting the bounds of the reservoir levels along the simulation. These constraints, if correctly estimated, are limiting the spillage in the reservoirs.

In this work, we are particularly interested in the influence of the choices in the start-up order for power units in the plants over the full horizon. To determine the impact of different choices, successive simulations of operations will be performed. Additionally, for a fair comparison between

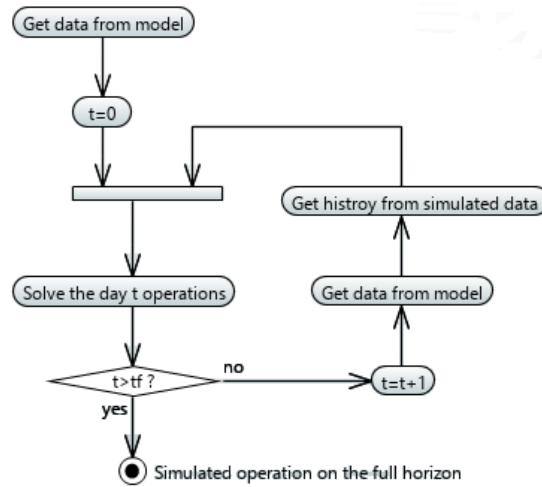


Figure 2: Simulator flow chart

different simulations of the system operation, we use a common history of demand and natural inflows. Furthermore, the same control function will be applied to the water reserve management.

A joint water management decision and start-up order choice study could provide more interesting results, but was deemed too complex for the purpose of this study.

### 3 Evaluation criteria

In order to evaluate the influence of operational planning for a given studied period (3 years, for this work) on maintenance, we consider three measures directly linked to the degradation of the power units:

1. The evolution and the final number of start-ups of each individual turbine.
2. The cumulative evolution and final value of the uptime (hours) of each power unit.
3. The distribution of the discrepancy (in MWh) between the optimal power of each power unit and their actual production on each hour.

These metrics were chosen because they are directly dependent on the decision variables in turbine and alternator usage that significantly affect degradation mechanisms. The first metric relates to the cracking mechanism in fatigue, as given by the reliability index computed in Gagnon et al. [22], while the second and third metrics relate to the cavitation [23]. These three parameters provide a good representation of the turbine runner.

An interesting approach would consist in relating these metrics to reliability index and deducing the evolution of this index with usage in order to find a Remaining Useful Life (RUL) function. However, this computation may be hard to obtain. As an example, in the case of the cracking of a turbine, the reliability index is computed using a probabilistic evaluation of the cracks size (since they are difficult to completely characterize). Furthermore, to compute the evolution of this index according to the number of start-ups, one would have to resolve a differential equation defined by parts (the Paris Law) with two of the inputs being probabilistic distribution (the crack size and the applied stress on the crack's area). For this reason, we limited our analysis to the metric rather than using a more precise RUL function that would take too long to compute.

As an additional measure, we consider a measure of the potential of production of the system: the final hydrologic volumes (proportion of the maximal volume) in the dams. This measure should be

considered to avoid strategies that perform well for the maintenance criteria but leaves the system in a state that prevents it from fulfilling its future production requirements.

As illustrated by the energy and instantaneous power production definitions (Equations (7) and (8)), the number of start-ups, the uptime, and the power discrepancy are linked to one another, and any improvement made to one may affect the others:

$$E = \int_t P(t) dt \quad (7)$$

with

$$P(t) = \sum_{k=1}^{N_A} \sum_{l=1}^{N_P^k} \sum_{j=1}^{N_G^{k,l}} \sigma_j^{k,l}(t) \left( P_{opt,j}^{k,l} + \Delta P_{opt,j}^{k,l}(t) \right) \quad (8)$$

Furthermore, power generation depends on the turbined water (as shown in Equation (2)), and therefore, it is linked to the reservoirs' final levels.

In the following sections, we use simplified indexes and notations for the formulas. These notations are indicated in Table 2.

**Table 2: Additional and alternate notations**

Symbol	Unit	Significance
$u$		Index for a unit in a set
$\Delta P_{opt,u}$	MW	Alternate notation for $\Delta P_{opt,j}^{k,l}$ where the unit is designated by the index $u$
$\delta_{\sigma_u}^+$		Number of start-ups of unit $u$
$T$	h	Duration of the simulation
$\sigma_u$		Alternate notation for $\sigma_j^{k,l}$ where the unit is designated by the index $u$
$d$		Index for a reservoir in a set
$V_d$	m <sup>3</sup>	Volume of water in the reservoir $d$
$\overline{V}_d$	m <sup>3</sup>	Maximal volume of water that can be in reservoir $d$

### 3.1 Start-ups

As expressed by Gagnon et al. [13], cracking occurs due to stress applied on the blades. Considering the profile of applied stress on turbine blades, it appears that damage can increase during the start-up and shut down phases.

When the turbine is stopped, no degradation occurs. When it is running, the amplitude of the stress applied is generally too small to affect the crack. If the crack is big enough to be affected, it will grow almost instantly.

In the Results section, we will graphically present the distribution of start-ups in each plant of the system, as well as the global distribution. To quantitatively compare different simulations, we also provide a numerical measure for this criterion: The mean number of start-ups for a set of units ( $S_U$ ), per unit and per year, as expressed by Equation (9), where Card is the cardinality function (number of elements of a set) and  $T$  corresponds to the number of instants in the simulation.

$$\mu_{\delta}(S_U) = \frac{365 \times 24}{T \times \text{Card}(S_U)} \times \sum_{u \in S_U} \delta_{\sigma_u}^+ \quad (9)$$

This measure may be applied to any subset of the system's power units in order to indicate local changes that may not be perceptible on a global scale.

### 3.2 Uptime

Cavitation may occur when the turbine is running. Differences in pressure in the flow of the water inside the turbine may lead to the creation of gas bubbles (cavities). For their part, these bubbles may cause erosions if they come in contact with the blade.

Furthermore, the uptime of a turbine indicates the part it played in satisfying the demand and the margin it gives to the system, in terms of production capabilities.

To compare different scenarios, we propose the numerical measure corresponding to the mean proportion of uptime in the lifetime of the unit, as expressed by Equation (10), where  $S_U$  is the set of units considered,  $T$  corresponds to the number of instants in the simulation and  $\int_t \sigma_u(t)dt$  corresponds to the uptime of a turbine  $u$ .

$$\mu_t(S_U) = \frac{\sum_{u \in S_U} \int_t \sigma_u(t)dt}{T \times \text{Card}(S_U)} \quad (10)$$

### 3.3 Distance to optimal production mode

Turbines are designed to avoid cavitation when they are used at the optimal production point (denoted  $P_{opt,j}^{k,i}$  for the unit  $j$  in plant  $i$  of area  $k$ ). The longer and further away the turbine runs from the optimal point, the more likely it is that some cavitation may occur.

This value's influence is considered to be relative to the previous metric (the uptime) because a turbine that runs for only two hours even at its worst rate is better than one that runs for 40 hours in a less-than-optimal regime.

As for the first measure, the exploitation result will be presented as a graphic of the distribution of difference in MW for each hour. We propose a measure that follows the same logic as in previous criteria and computes the mean distance to the optimal production mode power output of a unit over the simulation period, as expressed by Equation (11), where  $S_U$  is the set of considered units,  $T$  corresponds to the number of instants in the simulation and  $\int_t \Delta P_{opt,u}(t)dt$  is the overall deviation of  $u$  to the optimal production point.

$$\mu_{\Delta P}(S_U) = \frac{\sum_{u \in S_U} \int_t \Delta P_{opt,u}(t)dt}{T \times \text{Card}(S_U)} \quad (11)$$

### 3.4 Final hydrologic volumes

In a hydropower plant system, the level of water in the reservoir is directly linked to the potential production. It is also the insurance for the energy provider that demand will be met even in the case of several years with low natural inflows. As such, the level of water at the end of the simulation is a determinant factor when it comes to validating that an operation is acceptable.

We consider the proportion of the maximum volume of the reservoir as a direct measure of this criterion (for a set of reservoirs  $S_D$ ).

$$\mu_V(S_D) = \frac{\sum_{d \in S_D} V_d(t_{\text{final}})}{\sum_{d \in S_D} \overline{V}_d} \quad (12)$$

## 4 Case study

To illustrate the importance of ordering the power units differently on their respective and global usages, we use our simulator with the same situation, but two different orders. The first order is the one that will minimize the volume of water used to achieve production *a priori* – the order that

chooses the most efficient power unit first (within separated plants). We will call it efficient first, or EF. The second order is the inverse order (*a priori*, this order will use more water for the same result). It consists in choosing the most efficient unit last. We will call it efficient last, or EL.

As explained in Section 2.2, we use a realistic model for the operation system. The demand (Figure 3) and the constraints on the operation system are inspired by the Hydro-Québec system (see [13]) and contain similar complexities. Given the data available to derive this model, we limited the study to a 3-year horizon. This horizon gives a good representation of the climate cycles in Québec.

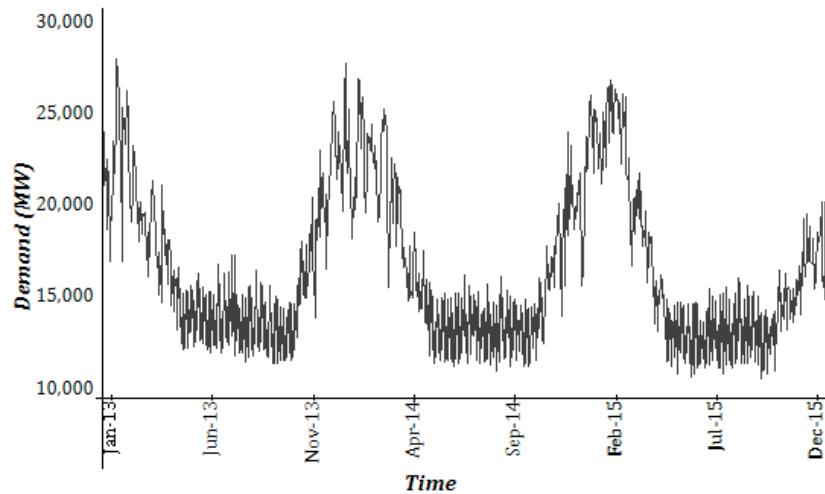


Figure 3: Energy demand on the first hour of each day

The system consists of 25 plants (A to Y) situated at seven rivers (A to H, I to K, L to N, P to U, X and Y), and with a demand divided into 19 areas. Each plant has an associated dam and reservoir, designated with the same respective letter (or set of letters if the reservoir is common to several plants), for a total of 25. They are grouped into 11 areas of demand, with an additional 8 areas outside the system, which that provide or request energy, but do not contain energy production plants managed by the system. There exist special plants that have a production plan defined in advance for longer periods. These elements will provide a portion of the energy that is known in advance, but can be simply integrated in the demand model by difference. The resulting system possesses 141 Francis turbines for a total capacity of 29 GW.

The demand is modelled from an altered sequence of real-life values to fit the modified model for the system. It presents large variations from day to day and an apparent periodicity due to the seasons. The actual demand varies from just over 11 GW in the spring and fall to almost 28 GW in the winter. The considered power generation system will be more solicited in winter due to the additional demand caused by heating.

The water management planning was adapted from the actual water levels over the same period. The ideal target level for the reservoir was derived from the real value achieved in the system, and a tolerance interval of 5% was added on that value before it was penalized by the HUC solver.

In this model, the HUC solver takes around 5 minutes to compute a daily operational planning on a computer with a 3.6 GHz 8-core processor, and thus, the whole simulation for a period of over 3 years takes 3 to 4 days.

The simulator attempts to satisfy the demand for each of the cases. However, as the system gives heuristic solutions, due to an early interruption of the optimization in the most complex cases, small discrepancies between the production and the demand will appear. In our case, there is a 0.5% and 0.3% production surplus in each case, respectively.

Table 3 presents the numerical values of each of the metrics provided in the previous section (in order, the mean number of start-ups per year, and the unit, the mean proportion of uptime per year and unit, the mean distance to the design point proportion and the final reserve of water relative to the maximum reserve). Each metric column is divided into two, and the measure in the case of the EF order appears in the first column, while that for the case of the EL appears in the second.

**Table 3: Global measures of criteria**

River/Plants	$\mu_\delta$		$\mu_t(\%)$		$\mu_{\Delta P}$		$\mu_V(\%)$	
	EF	EL	EF	EL	EF	EL	EF	EL
All	<b>172.2</b>	173.5	<b>64.3</b>	65.0	<b>0.161</b>	0.174	<b>73.6</b>	73.3
1/A to H	103.7	<b>101.9</b>	<b>75.2</b>	75.6	<b>0.127</b>	0.138	-	-
2/I, J, K	<b>331.6</b>	379.8	<b>45.4</b>	47.3	<b>0.197</b>	0.203	-	-
3/L, M, N	207.2	<b>203.0</b>	63.0	<b>62.9</b>	0.284	<b>0.282</b>	-	-
4/P to U	221.2	<b>211.6</b>	<b>51.9</b>	52.6	<b>0.185</b>	0.202	-	-
5/V, W	355.6	<b>355.2</b>	<b>42.7</b>	44.2	<b>0.214</b>	0.218	-	-
6/X	<b>223.1</b>	245.6	<b>64.5</b>	67.2	<b>0.168</b>	0.201	-	-
7/Y	<b>200.5</b>	208.5	<b>45.6</b>	45.8	<b>0.285</b>	0.309	-	-

The first line shows global values for the whole system, while the subsequent lines indicate the values of the criteria for the plants grouped according to the rivers on which they sit.

The criteria over the whole system seems to indicate that the EF order, that is expected to minimize the quantity of water used, indeed achieves better results. Globally, however, this improvement appears to be rather small: there are fewer than two start-ups a year, there is less than one percent of uptime gained, barely one percent of the power output distance to optimal functioning points and less than a one-percent difference in the water's total reserves.

Nonetheless, some parts of the system present significant changes, such as plants I, J and K, where the EF order shows almost 50 fewer start-ups per year for the power units, a 3% shorter uptime and a slightly shorter distance to optimal mode production.

Figure 4 shows a detailed distribution of the number of start-ups of plant power units in a year, for each plant. Each plant is shown as part of the river it sits on, the y-axis indicates the proportion of units of the plant that present a number of start-up under the corresponding x-axis's value and the x-axis the number of start-ups in a year. The green areas represent the part of the system where the EF order started fewer times than the EL order; the areas in red represent the opposite.

We can see that some plants present a similar distribution (A, B, M, N, T, V...), while some present no change in the total of start-ups in the plant, but present a different distribution of the start-ups between the units of the plant (S for example), and some present a significant change (H, I and U especially). The latter situation is due to the objective of the daily solver to use the generating unit in a mode closest to the optimal functioning point, combined with the actual production capacity of the units in a plant. This can shift the starting order from a unit in one plant to a unit in another.

We can notice that almost every unit in the River 2 (plants I, J and K) present similar gains to one another. This gain of around 12% in start-up numbers means, in practice, that a major maintenance operation, usually planned to occur every 6 years, can safely be delayed by half a year if the blade cracking is the main concern.

This is a complex system, and, as expected, the changes do not necessarily represent a global improvement. According to this criterion, plant U performs a lot better with the EL order, for example. However, this difference is balanced by an increased uptime and distance to the optimal functioning point.

On the same figure, we can see that although the EL order presents a higher mean number of start-ups globally, it also presents a lower maximum number of start-ups per year.

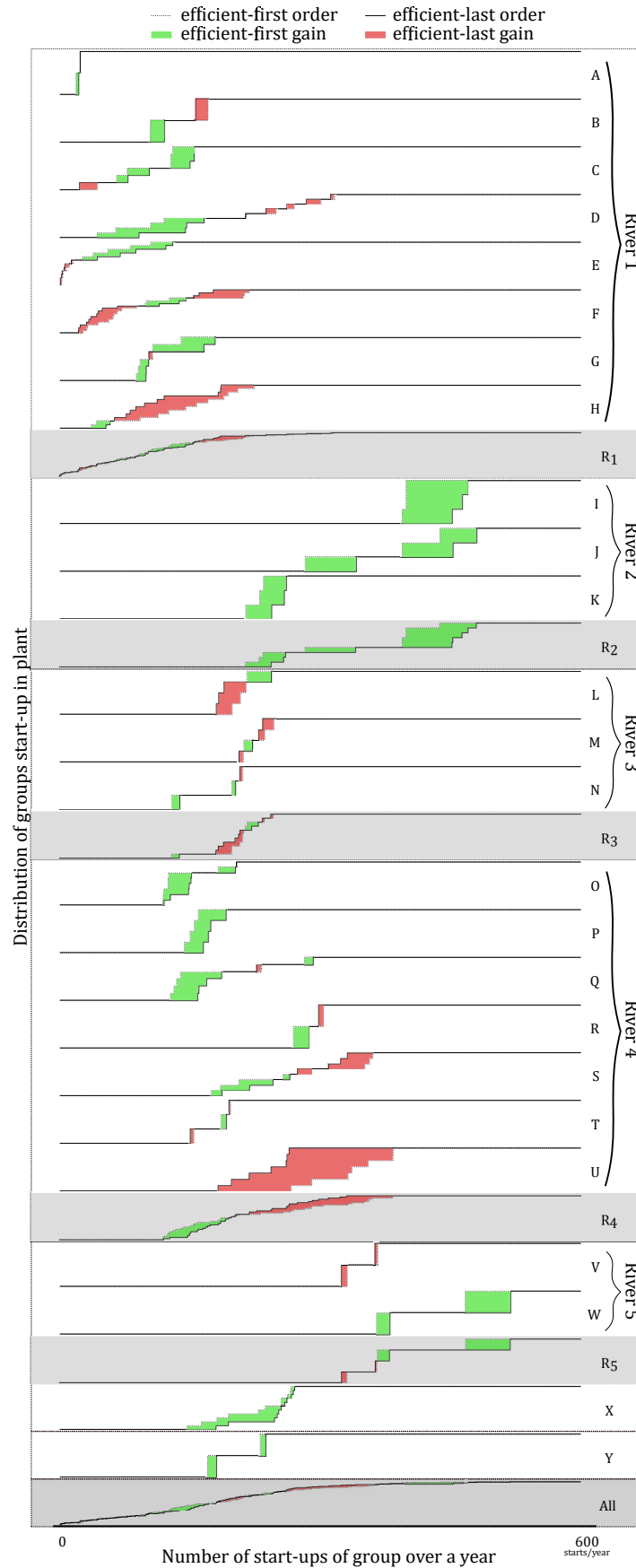


Figure 4: Repartitions of start-ups by plants



Plant E displays another interesting characteristic: some units have no start-up. Figure 4 indicates that some are actually running the entire time. This may prevent any degradation regarding the cracking mechanism for those units. Figure 5 shows the distribution of uptime during a year, in hours, for each plant (a year being 365 days, or 8760 hours). Following the same logic as for the start-ups, the x-axis is the proportion of units that present a shorter uptime than the value on the x-axis that is the uptime proportion in a year, green areas indicate units that run longer in the EL order and red areas indicate units that run longer in the EF order.

Globally, the difference is small for the mean, maximum and minimum uptime of a power unit. However, some plants show noticeable changes (mostly over the minimum and maximum uptime rather than the mean), such as plant R, where the maximal uptime is longer with the EL, but the minimal uptime is shorter with the same order.

As explained in Section 3, the uptime should be considered alongside the distance to the optimal efficiency operating condition. Figure 6 provides, for each plant, the distribution of the units' power output relative distance to the optimal functioning point's power output in percentages (y-axis) as density. In this figure, all plants don't have the span of value for the density, but the curves have been normalized since it is the shape, and not the actual values, that is interesting in a distribution's representation.

The resulting representation provides the distribution for all the units in the plant, which may not resemble the distribution for one individual plant. The smaller the span of the discrepancies for a plant, the more it can be identified to the individual unit's distribution (for example, in plant A, E or K).

Most plants have the same, or similar, distance distribution in both cases (efficient first and efficient last), but almost all the ones showing a difference perform better with the efficient-first order.

The actual power output is fully determined by the internal HUC solver, and very little can be done to change it except to force the output of the generating units to be the optimal functioning point's output. As such, we expect that most changes in a generating unit's distance to the optimal functioning point will actually be a trade-off decision between having it run and having another one run and present the discrepancy.

Contrary to the expected outcome, only three plants seem to be irrefutably performing better with the EF order (I, J and Y). Furthermore, as will be seen in Figure 7, these are also not the most important plants in the system.

Figure 7 presents the final volume for each dam, in percentage of the maximal volume of the dam. The left of the figure shows an indicator of the maximal volume of the reservoir: the longer the line, the larger the reservoir.

As can be seen, the final quantity of water is almost identical when the system is considered globally. The only cases where a significant change can be noticed (for example, with the reservoir of plant K, or those of plants T and U) appear to be on rather small reservoirs.

On the other hand, the reservoir with the biggest absolute difference (FG) is so large that this difference has very little influence over it (less than 4% of its maximum volume). This is due to the water management policy being defined by a volume target on each day of simulation. However, this indicates that both orders can be used indifferently without concern for water management satisfaction (assuming a valid policy has been established for reservoir targets). In our case, there was actually a surplus of water, and spillage occurred. In the simulation of the EF order policy, the spillage stood at 15.8% of the total water volume (initial reserve plus all inflows), and in the case of the EL order policy, the spillage was 15.5% of the total water volume. This indicates that the EF policy certainly consumes less water. However, these values are not significant when compared to the total volume of the reservoir. A less water-efficient policy may incur little additional cost in terms of water resources, but may have a significant influence on the degradation.

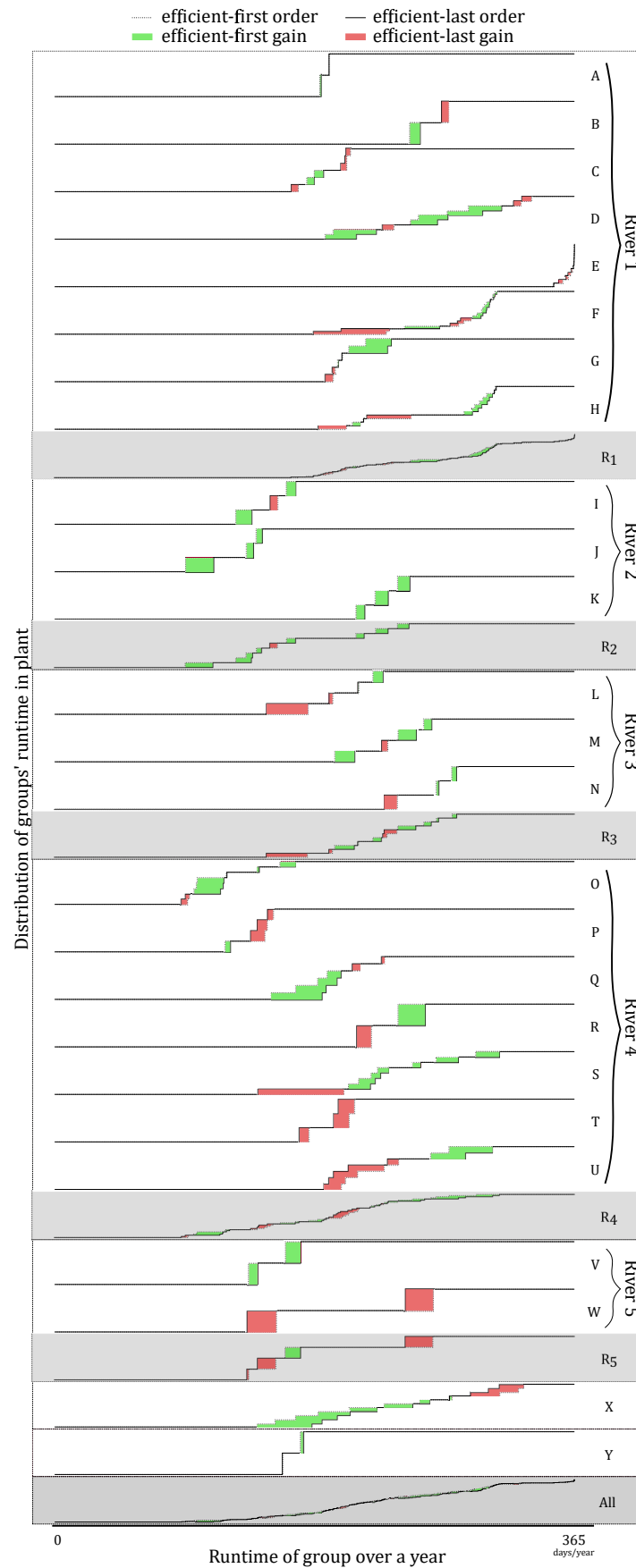


Figure 5: Repartitions of uptime of power units in plants

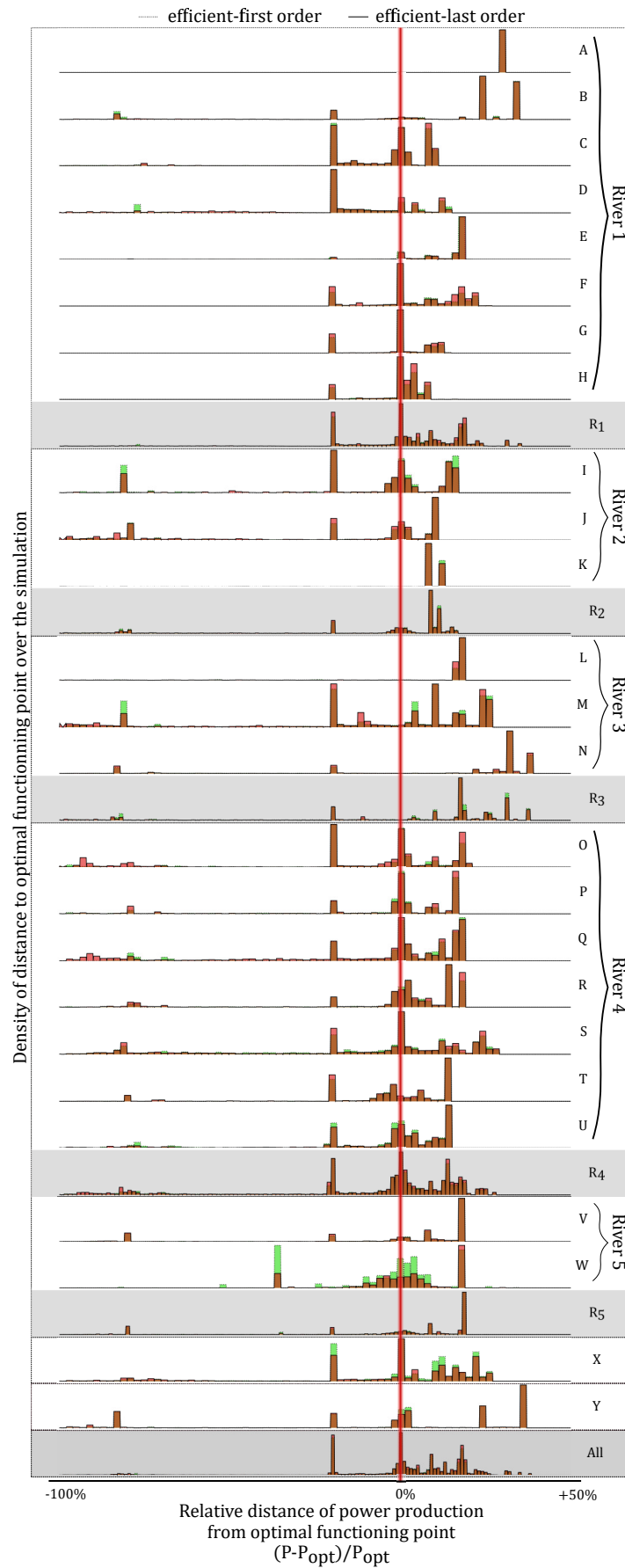


Figure 6: Power output repartition

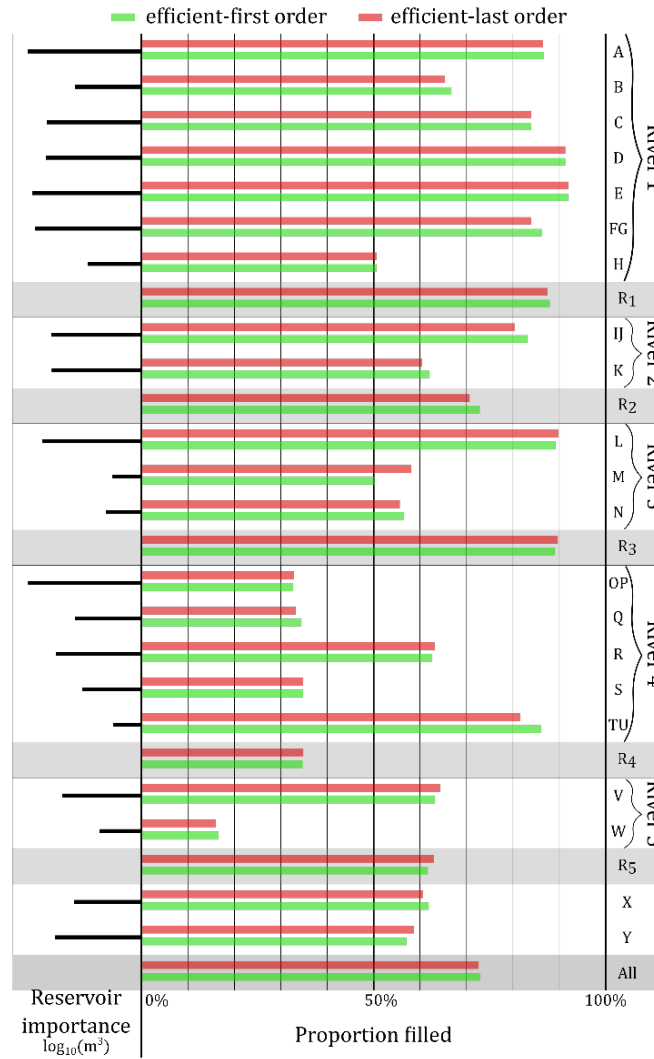


Figure 7: Proportion of the reservoir filled at the end of the simulation

## 5 Conclusions

In this work, we presented a model for simulating the operation of a complex electrical system over several years, and used this model to show that the starting order of the power unit inside a plant, that is, a modelling choice for the HUC solver, can have an influence on the actual number of start-ups in the system, as well as on the operating time and modes. A change in this order over time, that is to say between days in the simulation, is expected to have an even greater effect on maintenance schedules.

On the contrary we noticed that contrary to common belief, using an a priori bad operational decision (use the least efficient turbine first, thus using more of the resource than necessary) did not create a situation where operation became difficult or even significantly decrease the future potential of the system for production.

The efficiency of the model presented relies on the efficiency of the solver for the Hydraulic Unit Commitment (HUC) problem, the correctness of the model for natural inflows, and the practical feasibility of the operation. Given these three factors, our simulator provides a single possible management of the system operation that will be consistent.

The presented model offers the significant advantage of being linearly scalable in time, contrary to any approach relying on a global optimization of the system over the whole horizon.

We used the model to show the consequences of different order choices for the power units in their actual usage over the entire complex system, but it can be used to test a variety of other operation decisions, such as water-management constraints or objectives.

We also showed that different choices in the power units' start-up order can have a measurable influence on the system's global behaviour and an even greater influence on its local behaviour. To this end, we proposed three metrics linked to the degradation of the units (start-ups, uptime and functioning points) and one linked to the system performance (water reserves).

Future work will include the definition of order strategies to meet certain criteria, such as increasing the expected lifetime of a particular set of units by avoiding starting them as much as possible or reaching a defined number of start-ups for a particular unit necessary for a planned inspection to avoid inefficient maintenance operations (if the unit started only a few times, the probability of it being in a failure state is low if the last inspection was negative, but high if it has just been repaired).

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