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Stochastic short-term production scheduling and shovel allocation for mining complexes including stockpiling and operational alternatives

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Abstract: A new mathematical model for stochastic short-term optimization of mining complexes is presented that simultaneously optimizes the short-term extraction sequence, shovel allocation including costs of their relocation, stockpiling, and operational alternatives in processing streams. The optimization formulation accounts for metal and material type uncertainty, stemming from the geological reserve, as well as uncertain shovel production, reflecting the natural risk of underperformance of mining equipment. The method has been applied to a real-world gold mining complex. The resulting production schedule requires a minimum amount of shovel movements and shows an increased metal production of +1.23%, leading to a higher expected profit of +0.77%.

Keywords: Simultaneous short-term production scheduling, fleet management, stockpiling, operational alternatives in processing streams

1 Introduction

During the past years, methods for the simultaneous optimization of mining complexes have been developed for strategic planning, which have shown to increase NPV and manage technical risk by creating globally optimal solutions for long-term mine plans (Montiel and Dimitrakopoulos, 2015, 2018; Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel et al., 2016). As a next step, the simultaneous optimization of mining complexes is also sought for short-term planning. However, different operational components need to be integrated to create feasible short-term plans that meet production targets. Firstly, decisions related to the mining equipment, such as shovel allocation and repositioning, drilling and blasting activities, as well as the management of a heterogeneous truck fleet need to be integrated. Furthermore, detailed material access constraints have to be addressed, considering limited access from pit ramps and operational directions of mining. In addition, stockpiling and operational alternatives in processing facilities need to be considered for simultaneous short-term planning. Next to managing geological uncertainty, other pertinent sources of uncertainty stemming from these newly linked decisions, such as equipment performance uncertainty, uncertain cycle times, and others must be integrated to better meet production targets.

Short-term mine planning generally aims to make optimal decisions over a timeframe of days to months to best meet annual production targets given by the long-term mine plan (Wilke and Reimer, 1977; Fytas et al., 1987; Hustrulid et al., 2013). This task is conventionally accomplished in two separate steps, optimizing the physical short-term extraction sequence first, and then assigning the mining equipment (shovels and trucks) afterwards, which is summarized under the term fleet management. Recently, some effort has been made to combine these two planning aspects (Fioroni et al., 2008; L'Heureux et al., 2013; Torkamani and Askari-Nasab, 2015; Mousavi et al., 2016; Villalba Matamoros and Dimitrakopoulos, 2016; Quigley, 2016; Blom et al., 2017; Kozan and Liu, 2018; Upadhyay and Askari-Nasab, 2018).

Matamoros and Dimitrakopoulos (2016) simultaneously optimize a short-term production schedule of a single mine and a single processing facility next to truck-shovel allocation decisions while integrating metal uncertainty and equipment performance uncertainty into their optimization formulation. Several sets of equally likely uncertainty scenarios, including stochastic orebody simulations (Goovaerts, 1997), shovel and truck availability scenarios, and cycle time scenarios serve as an input to a stochastic integer programming (SIP) model, which is based on previous developments in stochastic mine planning (Ramazan and Dimitrakopoulos, 2005, 2013). Results show that the optimized short-term extraction sequence and equipment allocation decisions reduce costs and are more likely to meet planned production targets using stochastic inputs compared to average-type inputs. Quigley (2016) extends the formulation from Matamoros and Dimitrakopoulos (2016) by considering multiple pits, multiple processing streams, and access constraints related to hauling ramps inside the pits. Furthermore, availability scenarios of shovels and trucks are co-simulated, honoring historical data that show correlations of availability and utilization among shovel types and truck types.

A major limitation of the described approach of Matamoros and Dimitrakopoulos (2016) and Quigley (2016) is that they do not optimize a mining complex as a whole. Rather than solely minimizing costs, metal production and profit should be maximized, which honours the fact that material can be blended and possibly be sent to different available processing streams that are operated in different ways. Stockpiling should be included in the optimization and operational alternatives in processing streams should be considered, such as choosing the optimal mill performance of alternating throughput-recovery behavior (Wooller, 1999). The integration of operational alternatives into long-term planning has been discussed by Whittle (2014), who suggests that optimizers should include the natural flexibility of processing circuits to adapt their performance (e.g., throughput), leading to optimized metal production. Integrating this line of research can also help maximize metal production for short-term planning.

The newly developed optimization model herein aims to overcome the above-mentioned shortcomings by holistically optimizing a mining complex for short-term planning, while maximizing metal and profits. To date, the optimization model optimizes the short-term extraction sequence, destination of materials, shovel relocation, stockpiling, and the option of operational alternatives in processing streams. The optimization formulation takes into account geological uncertainty and uncertain shovel production, reflecting the natural risk of underperformance of mining equipment. Being part of the bigger picture, other pertinent components of short-term planning such as optimal allocation of a heterogeneous truck fleet, including their stochastic availabilities and cycle times, will be added later on. This is achieved by building upon previous advancements in simultaneous stochastic optimization of mining complexes (Goodfellow and Dimitrakopoulos, 2016) and extending their formulations for short-term planning.

In the following sections, the mathematical formulation of the developed stochastic programming model for short-term planning is presented first. A case study at a gold mining complex is presented afterwards to demonstrate the capability of the presented optimization framework for short-term mine planning, followed by conclusions and future work.

2 Method

The mathematical model for simultaneous stochastic short-term optimization of mining complexes is formulated as a stochastic integer programming model with fixed recourse (Birge and Louveaux, 2011). The formulation models a mining complex as a mineral value chain, where material is extracted from a set of mining areas $\mathbb{J}$ and sent to a set of destinations $\mathbb{D}= \mathcal{P}\cup\mathcal{S}\cup\mathcal{W}$, consisting of processors $\mathcal{P}$, stockpiles $\mathcal{S}$ or waste dumps $\mathcal{W}$. All blocks belonging to one area of the mining complex are summarised in the set $\mathbb{I}_j$, whereas the set $\mathcal{N}$comprises all blocks b that are to be extracted within the annual horizon of this optimization. The stochasticity of inputs is accounted for in different sets of scenarios. Orebody scenarios $s \in \mathbb{S}$ quantify geological uncertainty. Equipment performance scenarios $s_e \in \mathbb{S}_E$ quantify the uncertainty related to shovel production. Individual shovels are modelled with the index $l \in \mathbb{L}$.

2.1 Decision variables

Mine extraction variables ($x^t_{b,d} \in \{0,1\}$) define whether a block b is mined in period t and sent to destination d. Equipment-related decision variables are implemented in this model, which simultaneously allow for fleet management decisions in mining complexes. Shovel allocation variables ($\lambda^t_{l,j} \in \{0,1\}$) define if a shovel l is allocated to mining area j in period t. The shovel-movement variable ($\omega^t_{l,j,j'} \in \{0,1\}$) will be equal to 1 if a shovel l moved from area j to area j' between period $t-1$ and t, and 0 otherwise. An additional binary variable is added, which allows the integration and simultaneous optimization of operational alternatives for various processing streams within the mining complex. The binary decision variable $y^t_d \in \{0,1\}$ is equal to 1 if destination d in period t is active.

Several recourse or second-stage variables are in place. Target deviation variables ($d^{Grade-}_{d,e,t,s}$, $d^{Grade+}_{d,e,t,s}$) will be penalized in the objective function if grade targets of element e are violated in a period t in a scenario s in destination d. An additional recourse variable is defined for equipment-related constraints. Shovel production deviations f^t_{j,s_e} reflect the shortfall of shovel production in a mining area j for a period t, depending on equipment performance scenario s_e.

Table 1: Notations for indices in the mathematical model

Indices and sets	
$t \in \mathbb{T}$	Index of a period (bi-monthly) within the planning horizon (year)
$s \in \mathbb{S}$	Index of an orebody scenario
$s_e \in \mathbb{S}_E$	Index of an equipment performance scenario
$d \in \mathbb{D}$	Index of the set of destinations $\mathbb{D}$
$\mathbb{D}= \mathcal{P}\cup\mathcal{S}\cup\mathcal{W}$	The set of Destinations consists of Processors $\mathcal{P}$, Stockpiles $\mathcal{S}$ and Waste dumps $\mathcal{W}$
j $\in \mathbb{J}$	Index of a distinct area located at any pit of the mining complex among a set of areas
$i \in \mathbb{I}_j$	Index of a block i within area j among the set of blocks $\mathbb{I}_j$ belonging to the same area j
$b \in \mathcal{N}$	Index of block b in the set of all enumerated blocks $\mathcal{N}$.
$l \in \mathbb{L}$	Index of a loading equipment (shovel) l in the mining complex

Table 2: Notations for variables in the mathematical model

Variables	
$x_{b,d}^{t} \in \{0,1\}$	This binary decision variable is equal 1 if block b mined in period t and sent to destination d and 0 otherwise.
$\lambda_{l,j}^{t} \in \{0,1\}$	This binary decision variable is equal to 1 if shovel l is located at area j in period t and 0 otherwise
$y_{d}^{t} \in \{0,1\}$	This binary decision variable is equal to 1 if destination d in period t is chosen (representing one operational alternative)
$\omega_{l,j,j'}^{t} \in \{0,1\}$	This binary variable is equal to 1 if a shovel l has moved from area j to a different area j' between period $t-1$ and t and 0 otherwise. Associated movement costs are penalized in the objective function.
$f_{j,s_e}^{t} \in \mathbb{R}$	This continuous variable defines the deviation from expected shovel production in area j for equipment scenario s_e in period t. It is a recourse variable.
$d_{d,e,t,s}^{Grade-},\ d_{d,e,t,s}^{Grade+} \in \mathbb{R}$	These continuous variables represent excess and shortage of element e in destination d at time t in scenario s. They are recourse variables.

Table 3: Notations for parameters in the mathematical model

Parameters	
p_d	The unit selling price of the metal product in destination d (e.g., no smelting fees occur for heap leaching operation)
c_{d}^{proc}	The unit cost of processing material in destination d (\$/t)
$c_{d}^{Transport}$	Transportation cost from the excavation point to destination d (\$/t)
$c_{d,e,t}^{\text{Grade}+}, c_{d,e,t}^{\text{Grade}-}$	Cost for the excess/shortage of metal or deleterious element e in destination d at period t
$g_{b,s}^{metal}$	Metal grade of block b in geological scenario s
$\overline{g_{s}^{metal}}$	Average metal grade of stockpiled material in scenario s (approximated)
$P_{t,d}^{Min}\ P_{t,d}^{Max}$	Minimum and maximum processing capacity in destination d for period t
$G_{e,d}^{Min},\ G_{e,d}^{Max}$	Minimum and maximum grade of metal or deleterious element e in destination d
rec_d	Recovery (%) in destination d if d is a waste dump $d \in W$: rec_d= 0% if d is a processor $d \in P$: $rec_d > 0\%$
$\overline{g_{e,s}}$	Average grade of element e of stockpiled material in scenario s (approximated)
$CostTonnesNotProduced$	Cost for tonnage not produced regarding the expected shovel production
$CostMove_{l,j,j'}$	Cost (Matrix) of moving a shovel l from area j to area j' in period t
Ton_b	The tonnage of block b
L_{j}^{Max}	Maximum number of shovels l allowed in area j per period t
H^t	Scheduled working hours per period t
Q_{l,j,s_e}^{t}	Effective shovel production (t/h) of shovel l in area j for equipment scenario s_e in period t.
$\mathbb{U}_b = \mathbb{V}_b \cup \mathbb{H}_b$	Predecessor sets of block b unifying the vertical direction $\mathbb{V}_b$ and horizontal direction $\mathbb{H}_b$

2.2 Objective function

Other than most short-term optimization formulations, the goal is not only to minimize operational costs but to maximize the profit of the entire mining complex as a whole. The minimization of costs alone neglects the fact that material can be blended and possibly be sent to different available processing streams, which affects generated revenues from metal products, which should be maximized. The objective function (1) of the proposed mathematical model is shown.

$$
\begin{aligned}
\text{Maximize} \quad & \underbrace{\frac{1}{\parallel \mathbb{S} \parallel} \sum_{t\in\mathbb{T}} \sum_{s\in\mathbb{S}} \sum_{d\in\mathbb{D}} v_{t,s,d}}_{\text{All revenues and costs in the mining complex (I)}} \\
- & \underbrace{\frac{1}{\parallel \mathbb{S} \parallel} \sum_{t\in\mathbb{T}} \sum_{s\in\mathbb{S}} \sum_{d\in\mathbb{D}} \sum_{e\in\mathbb{E}} \left(c_{d,e,t}^{Grade+} \cdot d_{d,e,t,s}^{Grade+} + c_{d,e,t}^{Grade-} \cdot d_{d,e,t,s}^{Grade-} \right)}_{\text{Penalty term for shortfall or surplus of elements in processors (II)}} \\
- & \underbrace{\frac{1}{\parallel \mathbb{S}_E \parallel} \sum_{t\in\mathbb{T}} \sum_{s_e \in S_E} \sum_{j\in J} CostTonnesNotProduced * f_{j,s_e}^{t}}_{\text{Penalty for not achieving production target per shovel (III)}} \\
- & \sum_{t\in\mathbb{T}} \underbrace{\sum_{l\in L} \sum_{j\in J} \sum_{j' \in J\setminus\{j\}} CostMove_{l,j,j'} * \omega_{l,j,j'}^{t}}_{\text{Shovel movement cost (IV)}}
\end{aligned} \tag{1}
$$

Part (I) of the objective function summarizes revenues and costs that are generated in all modelled locations in the mining complex. Penalties for positive or negative deviations from production targets are accounted for in part (II). Parts (III) to (VI) are equipment-related, aiming to reduce the risk of not meeting short-term production targets that are related to the mining fleet. Stochastic shovel production targets are controlled in (III), whereas Part (IV) sums the cost related to move a shovel from one area to another. All constraints, including equipment constraints and constraints related to stockpiling and operational alternatives are listed in the appendix in the way they were implemented in the following case study.

3 Case study – Application at a gold mining complex

The gold mining complex considered comprises four spatially distinct mining areas (A1–A4) that can be accessed by specific ramp access points, which are depicted in Figure 1. This mining complex operates with a shared mining fleet, where material is excavated and hauled either to a mill, constrained by a maximum capacity of 116 kt per period, a stockpile connected to the mill, a heap leach facility or a single waste dump bearing unlimited capacity for the considered short-term planning horizon.

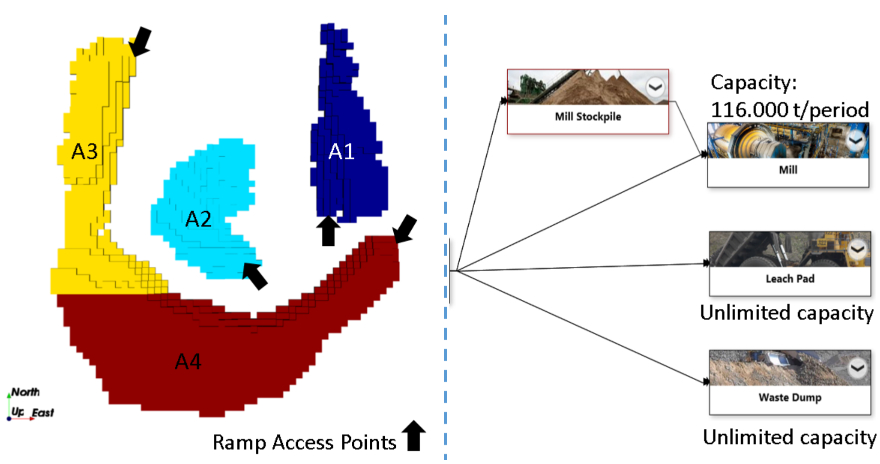

Figure 1: Mining areas and other components in the mining complex

The annual production horizon, to be scheduled in six bi-monthly periods, comprises 4,876 blocks (30x30x20 ft^3). Uncertainty of metal grade (Au) is accounted for by utilizing fifteen equally probable orebody scenarios using geostatistical simulation techniques (Goovaerts, 1997; Rossi and Deutsch, 2014). Economic and operational parameters for each destination in the mining complex are given in Table 4. The material to be scheduled equals to one year's planned production taken from a previously optimized life-of-mine production schedule. Other optimization parameters of the mining complex are summarized in Table 5. The expected production rates of the three shovels in the mining complex and their standard deviation are presented in Table 6. Costs due to shovel movements are provided in Table 7, which are calculated based on the expected travel time of shovels between areas and the associated operational costs of moving a shovel. The given mining directions per mining area, which have to be followed by the mining operation, are provided in Table 8.

Table 4: Economic and operational parameters for each destination in the mining complex

Parameter	Destination			
	Waste Dump	Mill (Operational Alternative 1)	Mill (Operational Alternative 2)	Heap Leach
Unit selling price of metal (Au) in destination	0$	1237 $/oz	1237 $/oz	1250 $/oz (no smelting fees paid for leaching operation)
Transportation cost to destination	0.05$/t	0.09 $/t	0.09$/t	0.14$/t
Processing cost of ore in destination	0$/t	7.84 $/t	8.84 $/t	2.30 $/t
Upper and lower limits of processing capacity for a 2-month period	Unlimited	$P^{Min} = 1.0\ Mt$ $P^{Max} = 1.3\ Mt$	$P^{Min} = 0.5\ Mt$ $P^{Max} = 0.8\ Mt$	$P^{Min} = 0.0\ Mt$ $P^{Max} = 2.0\ Mt$

Table 5: Optimization parameters for the mining complex

Parameter	Value
Number of shovels	3
Number of areas	4
Number of blocks	4876
Number of equipment performance scenarios	10
Number of orebody scenarios	15
Hours per period (one period = 2 months)	1440 h
Mining cost	1.10 $/t
Cost for scheduled tons of production that cannot be mined because of shortfall of shovel capacity	1.00 $/t
All-in costs that occur for relocating a shovel	950 $/h

Table 6: Expected production rate and standard deviation for loading equipment

Loading Equipment	Production	
	Mean (t/h)	Standard deviation (t/h)
Shovel 1 (large)	2240	145.5
Shovel 2 (small)	1120	61.5
Shovel 3 (small)	1120	113.0

Table 7: Costs caused by shovel movement

in '000 $	toArea1	toArea2	toArea3	toArea4
fromArea1	0.0	5.7	8.6	6.7
fromArea2	6.7	0.0	10.5	10.5
fromArea3	9.5	8.6	0.0	3.8
fromArea4	7.6	9.5	2.9	0.0

Table 8: Given mining directions for short-term planning per area

	toArea1	toArea2	toArea3	toArea4
Mining direction	North	North	South	West

Figure 2 shows the optimized extraction sequence that is obtained as a result of the simultaneous optimization of short-term production and fleet management. Figure 3 shows the simultaneously optimized shovel allocation decisions. It can be seen that the optimized extraction sequence follows the given mining directions, starting from the given ramp access points that are indicated for every area by white arrows in Figure 2. This shows the ability of the method to create well mineable sequences in the short-term. Furthermore, Figure 3 reveals that only two shovel moves are necessary to extract the annual scheduled production, a result of simultaneously matching shovel capacities and volumes of material by the optimization formulation. Logically, the largest shovel (shovel 1) is allocated to the largest mining volumes of area 4, whereas the smaller shovels will be located to area 2 first, before splitting to area 1 and area 3 respectively.

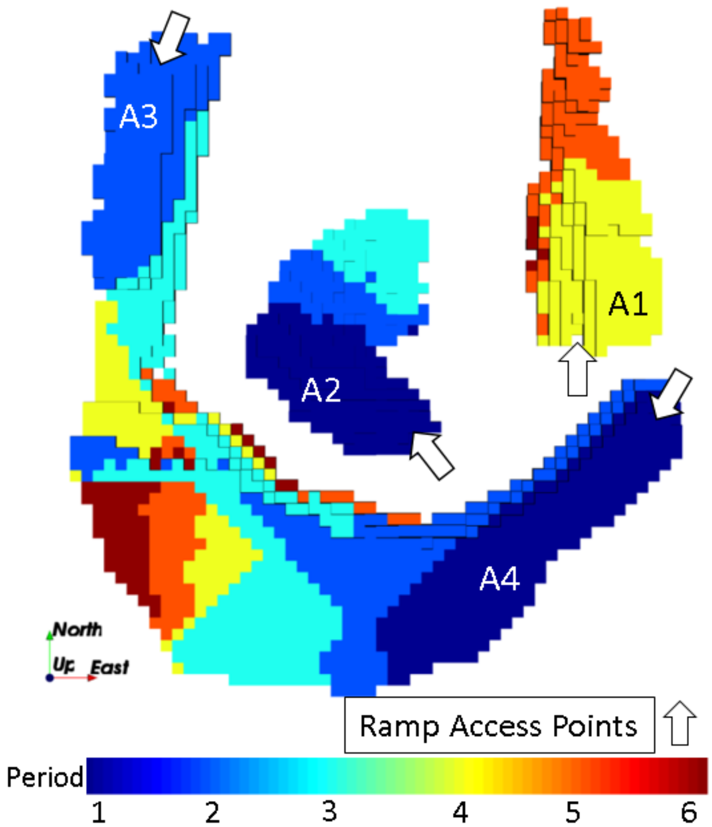

Figure 2: Extraction sequence obtained by simultaneous optimization of production schedule and fleet management

Period	Area1	Area2	Area3	Area4
1		Shovel 2 / Shovel 3		Shovel 1
2		Shovel 2	Shovel 3	Shovel 1
3		Shovel 2	Shovel 3	Shovel 1
4	Shovel 2		Shovel 3	Shovel 1
5	Shovel 2		Shovel 3	Shovel 1
6	Shovel 2		Shovel 3	Shovel 1

Figure 3: Shovel allocations obtained by simultaneous optimization of production schedule and fleet management

3.1 Stockpiling

As an additional result of simultaneous optimization, the amount of stockpiled material that is fed to the mill is included as well. Table 9 presents the optimized tonnages that are (i) sent to the stockpile (first column), (ii) sent from the stockpile to the mill (last column), and (iii) its resulting net inventory after each period (second column). It can be seen that mined material is mostly sent to the stockpile in period 4. The stockpiled material will then be sent to the mill in period 6. The reason for this is because a large amount of high-grade ore is extracted in period 4, mostly originating from mining area 1, which can be seen by observing the optimized destinations of material in Figure 4. The optimizer decided to stockpile 10 kt of ore until period 6 because the mill capacity would be exceeded otherwise. This surplus ore is thus stored for two periods and then fed to the mill in period 6 to fill this processing stream to full capacity when it is needed.

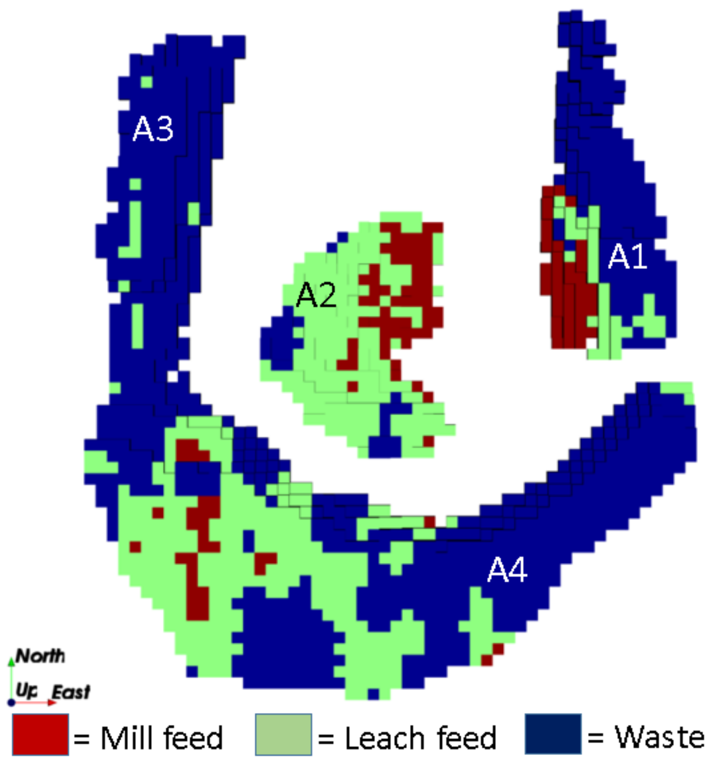

Figure 4: Destination of material obtained by simultaneous optimization of production schedule and fleet management

Table 9: Stockpile tonnages for mill stockpile

	Mill Stockpile		
Period	Sent to Stock (t)	Mill Stockpile Inventory (t)	Sent to Mill (t)
P1	0	0	0
P2	170	170	0
P3	190	360	0
P4	10,430	10,790	0
P5	0	10,790	0
P6	0	470	10,320

3.2 Operational alternatives

The introduction of operational alternatives for processing streams for simultaneous short-term planning of mining complexes is discussed herein. Rather than giving a continuous spectrum of mill throughput/recovery options, as presented by Wooller (1999), this optimization considers two distinct operational alternatives of how to operate the milling and grinding circuit. The changed parameter targets between both operational alternatives are noted in Table 4 and Table 10. Additional constraints ensure that only one operational alternative can be chosen per time period and that the concurrent alternative must receive no material, which is described in the appendix of this article. Note that operational alternative one was used as the default for all optimized results above (simultaneous optimization including stockpiling but excluding operational alternatives). The resulting optimized tonnages for including operational alternatives are presented in Table 10. The optimizer decided to operate the mill with maximized throughput (alternative two) in three out of six cases, whereas the concurrent alternative receives no material in these periods.

Table 10: Optimized tonnages sent to destinations, including an operational alternative for milling circuit

Period	Waste (kt)	Mill (Operational alternative 1) (kt)	Mill (Operational alternative 2) (kt)	Leach (kt)
P1	489	117	0	360
P2	590	0	150	360
P3	628	0	149	360
P4	366	0	150	359
P5	272	116	0	360
P6	400	117	0	359
Total	2,745	350	450	2,158

Alternative 1 (OA1): optimized recovery 95% , throughput 116kt
Alternative 2 (OA2): maximized throughput 150kt (+30%), recovery 90% (-5%)

Further insight on the optimizer's choice can be obtained by comparing the optimized production schedule when operational alternatives are given with the previously optimized production schedule. It can be noticed by comparing the schedule on the upper left to the lower left that the extraction sequence changes significantly. This is especially the case for areas of high-grade material, such as area 1. Here, the optimizer opted for the alternative of maximized recovery, seen in the lower right (destinations of material with operational alternatives) to benefit from additional produced metal. Medium grade material in areas two and four will be preferably mined by the alternative of maximized throughput (operational alternative one) to give way for the above-mentioned high-grade material.

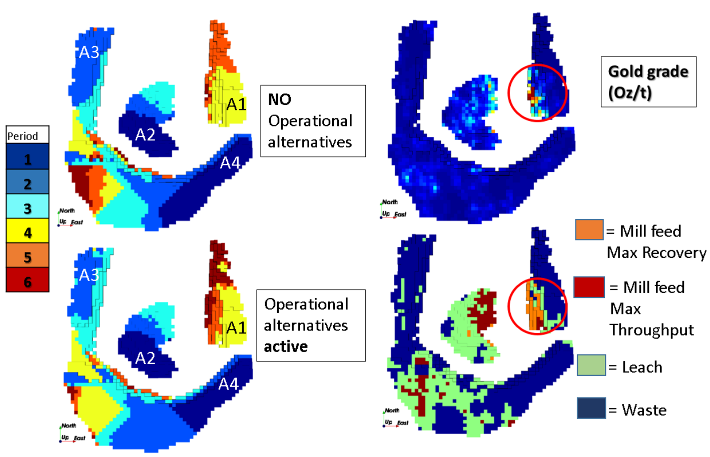

Figure 5: Comparison of optimized production schedules when operational alternatives are included (lower left) vs. no operational alternatives given (upper left)

The financial benefits of this method are given in Table 11, showing a higher expected metal production of 1.23% and higher expected profit of 0.77%. This result is due to the better utilization of the mill by balancing optimized throughput and optimized recovery for the received material, which is given by the simultaneously optimized production schedule.

Table 11: Financial comparison of including operational alternatives vs. no operational alternatives given

	No operational alternatives given	Operational alternatives are included into simultaneous optimization
P50 Cum. Cash Flow (M$)	44,30 100%	44.64 **+ 0.77%**
P50 Gold Produced (Oz)	51,376 100%	52,010 **+1.23%**

4 Conclusions and future work

A mathematical formulation for stochastic short-term optimization has been developed, which simultaneously optimizes short-term extraction sequence, shovel relocation, stockpiling, and operational alternatives in mining complexes. Several sources of uncertainty are integrated into the mathematical optimization formulation, including metal uncertainty stemming from the geological reserve and uncertainty in shovel production. The presented case study shows how simultaneous optimization can result in production schedules that require (i) a minimum amount of shovels relocation, (ii) optimize short-term stockpiling, and (iii) generate higher profit through increased metal production.

Future research aims to include additional aspects of the mining fleet, i.e., the truck hauling fleet and truck cycle times into the framework of simultaneous stochastic short-term optimization. Considering the large size of the stochastic integer programming model, a metaheuristic solution method will be developed that has potential to decrease runtime, overcome linear assumptions, and opens the possibility to include more complex geometallurgical considerations into short-term production scheduling.

5 Appendix

Detailed mathematical model, including calculation of revenues and costs in objective function part (I) and constraints used for the linear program.

5.1 Objective function, part (I)

$$v_{t,s,d} = \underbrace{\sum_{b\in\mathcal{N}} x^t_{b,d} * Ton_b \ * \left(p_d * g^{metal}_{b,s} * \ rec_d - \ c^{proc}_d - c^{Transport}_d\right)}_{Mined\ material\ sent\ to\ processor\ d}$$
$$+ \underbrace{(k^{t+}_d - k^{t-}_d) * \left(p_d * \overline{g^{metal}_s} * \ rec_d - \ c^{proc}_d - \ \ c^{rehandle}_d\right)}_{rehandled\ \ material\ sent\ to\ and\ received\ from\ stockpile}$$

5.2 Constraints for multiple destinations

The reserve constraint (2) ensures that a block needs to be mined once (need to mine all scheduled blocks), and sent only to one destination. Constraint (3) is a predecessor constraint that ensures that preceding blocks $\mathbb{U}_b$ have been extracted before not only in vertical direction $\mathbb{V}_b$, but also in horizontal direction $\mathbb{H}_b$ so that full access can be guaranteed for shovels from the ramp in a certain mining direction.

$$\sum_{d\in\mathrm{D}}\sum_{t\in\mathbb{T}} x^t_{b,d} = 1 \qquad \forall\ b\in\mathcal{N} \tag{2}$$

$$\sum_{d\in\mathrm{D}} x^\tau_{b,d} \leq \sum_{d\in\mathrm{D}}\sum_{t=1}^{\tau} x^t_{k,d} \qquad \forall\ k\in\mathbb{U}_b, t\in\mathbb{T}, b\in\mathcal{N} \tag{3}$$

The tonnage constraints (4) and (5) ensure that the minimum and maximum processing capacities are not exceeded in any processing destination $d \in \mathcal{P}$ in period t. The stockpile variables k_d^{t-} k_d^{t+} consider material sent to and from the corresponding stockpile of destination d. The maximum processing capacity $P_{t,d}^{Max}$ varies with the operational alternative implemented in processing destination d in period t.

$$\sum_{b \in \mathcal{N}} x_{b,d}^t * Ton_b - k_d^{t+} + k_d^{t-} \leq P_{t,d}^{Max} * y_d^t \qquad \forall t \in \mathbb{T}, \mathrm{d} \in \mathbb{D} \tag{4}$$

$$\sum_{b \in \mathcal{N}} x_{b,d}^t * Ton_b - k_d^{t+} + k_d^{t-} \geq P_{t,d}^{Min} * y_d^t \qquad \forall t \in \mathbb{T}, \mathrm{d} \in \mathbb{D} \tag{5}$$

Constraints (6) and (7) impose lower and upper grade limits on several elements for every processing destination d$\in\mathcal{P}$ in the mining complex. The reason for these constraints is twofold. First, it is ensured that processing facilities can process the material. Second, the quality of the products is ensured this way. The grade limits for each element e in each destination d$\in\mathcal{P}$ are defined a priori by $G_{e,d}^{Min}$ and $G_{e,d}^{Max}$. Surplus $d_{d,e,t,s}^{Grade+}$, or shortage $d_{d,e,t,s}^{Grade-}$ respectively, is penalized in the second term of the objective function.

$$\sum_{b \in \mathcal{N}} x_{b,d}^t * Ton_b * \left(g_{e,b,s} - G_{e,d}^{Min}\right) + \left(k_d^{t+} - k_d^{t-}\right) * \left(\overline{g_{e,s}} - G_{e,d}^{Min}\right) + d_{d,e,t,s}^{Grade-} \geq 0$$
$$\forall t \in \mathbb{T}, s \in \mathbb{S}, d \in \mathcal{P}, e \in \mathbb{E} \tag{6}$$

$$\sum_{b \in \mathcal{N}} x_{b,d}^t * Ton_b * \left(g_{e,b,s} - G_{e,d}^{Max}\right) + \left(k_d^{t+} - k_d^{t-}\right) * \left(\overline{g_{e,s}} - G_{e,d}^{Min}\right) - d_{d,e,t,s}^{Grade+} \leq 0$$
$$\forall t \in \mathbb{T}, s \in \mathbb{S}, d \in \mathbb{D}, e \in \mathbb{E} \tag{7}$$

5.3 Equipment-related constraints

Constraint (9) ensures that shovel $l \in \mathbb{L}$ can only be assigned at most to one area in period t. However, it is still possible that two or more shovels can operate in the same area. Constraint (9) guarantees that a mining block in sector j is mined only if a shovel is allocated to area j.

$$\sum_{j \in J} \lambda_{l,j}^t \leq 1 \qquad \forall t \in \mathbb{T}, l \in \mathbb{L} \tag{8}$$

$$x_{b,d}^t - \sum_{l \in L} \lambda_{l,j}^t \leq 0 \qquad \forall t \in \mathbb{T}, b \in \mathbb{I}_j, d \in \mathbb{D}, j \in \mathbb{J} \tag{9}$$

Constraint (10) links the theoretical shovel capacity allocated to an area to the scheduled production in that area. If shovel capacity is less than the scheduled production, the shortage is stored by the variable f_{j,s_e}^t for every stochastic equipment performance scenario $s_e \in \mathbb{S}_E$, which is penalized in the objective function. Surplus shovel capacity remains feasible. The recourse variable f_{j,s_e}^t is utilized in such a way that first stage shovel allocation decisions shall be found to minimize production losses with respect to every equipment performance scenario, i.e. the more reliable shovels will be allocated to strategically important excavation points.

$$\underbrace{\sum_{l \in L} Q_{l,j,s_e}^t * \lambda_{lj}^t * H^t}_{\substack{Effective\ shovel \\ capacity\ per\ area}} - \underbrace{\sum_{b \in \mathbb{I}_j} \sum_{d \in \mathrm{D}} x_{b,d}^t * Ton_b}_{\substack{Scheduled\ production \\ per\ area}} + f_{j,s_e}^t \geq 0 \quad \forall t \in \mathbb{T}, j \in \mathbb{J}, s_e \in \mathbb{S}_E \tag{10}$$

Constraint (11) ensures that, in every period, the maximum available number of shovels in any area j cannot be exceeded. The shovel-movement variable $\omega_{l,j,j'}^t$ is thus a direct result of shovel allocation variables $\lambda_{l,j}^{t}$. Constraints (12) ensure the correct calculation of movement costs.

$$\sum_{l\in L}\lambda_{l,j}^{t}\leq L_{j}^{Max} \qquad \forall t\in\mathbb{T}, j\in\mathbb{J} \tag{11}$$

$$\lambda_{l,j'}^{t+1}+\lambda_{l,j}^{t}-\omega_{l,j,j'}^{t}\leq 1 \qquad \forall t\in\mathbb{T}, l\in\mathbb{L}, j\in\mathbb{J}, j^{'}\in\mathbb{J},\quad j^{'}\neq j \tag{12}$$

5.4 Stockpile constraints

All modelled stockpiles in this linear programming model are strictly bound to their respective processing stream. This means that each processing stream can have exactly one stockpile. This modelling approach has also been applied by Lamghari and Dimitrakopoulos (2016, 2018). Constraints (13) ensure the mass balance in the stockpile at the end of each period by balancing incoming and outgoing material.

$$\underbrace{h_{d}^{t-1}}_{\substack{\text{Tonnage of}\\ \text{stockpile d}\\ \text{in previous period}}} - \underbrace{k_{d}^{t+}}_{\substack{\text{Tonnage sent}\\ \text{from stockpile d to}\\ \text{processor in}\\ \text{current perod}}} + \underbrace{k_{d}^{t-}}_{\substack{\text{Tonnage sent}\\ \text{to stockpile d}\\ \text{in current period}}} = \underbrace{h_{d}^{t}}_{\substack{\text{Tonnage of}\\ \text{stockpile d}\\ \text{in current period}}}$$

$$\forall t\in\mathbb{T}, \mathrm{d}\in\mathbb{D} \tag{13}$$

The capacity of the stockpile may be limited for each stockpile in stockpile destination d$\in\mathcal{S}$ by the stockpile capacity $Stock_{d}^{Max}$ (14). An initial amount of stockpiled material $Stock_{d}^{Init}$ is defined in (15).

$$h_{d}^{t}\leq Stock_{d}^{Max} \qquad \forall t\in\mathbb{T}, \mathrm{d}\in\mathbb{D} \tag{14}$$

$$h_{d}^{0}= Stock_{d}^{Init} \qquad \forall t=0, \mathrm{d}\in\mathbb{D} \tag{15}$$

The amount of material that can be taken from the stockpile cannot be more than the available amount at the stockpile at the end of the previous period. If constraint (16) is not imposed, material that has been sent to the stockpile in period t might be sent to processing in the same period. This shall not happen since material might not be available yet when scheduled.

$$k_{d}^{t+}-h_{d}^{t-1}\leq 0 \qquad \forall t\in\mathbb{T}, \mathrm{d}\in\mathbb{D} \tag{16}$$

5.5 Integration of operational alternatives in destinations

For modelling operational alternatives for processors in the mining complex, several additional constraints are introduced. To demonstrate an example of operational alternatives, destinations d=1 and d=2 can be considered the same processing facility, e.g., a mill that can be operated in two different ways bearing different throughput, recovery, and costs. The binary variable y_{d}^{t} must be set to 1 for destinations bearing only one operational alternative. However, for destinations bearing two or more operational alternatives, constraints (17) are imposed Subsets $\Lambda\subseteq\mathbb{D}$ represent destinations d that can be operated using different operational alternatives.

$$\sum_{d\in\Lambda}y_{d}^{t}\leq 1 \qquad \forall t\in\mathbb{T} \tag{17}$$

All extraction variables of blocks that are sent to ambiguous destinations need to be forced to zero if the operational alternative is not chosen. The following constraints (18) realize this requirement.

$$x_{b,d}^{t}\leq y_{d}^{t} \qquad \forall t\in\mathbb{T}, b\in\mathcal{N}, d\in\Lambda \tag{18}$$

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