Les Cahiers du GERAD ISSN: 0711–2440

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G–2019–95

December 2019

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2019
– Bibliothèque et Archives Canada, 2019

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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Integrated lot sizing and blending problems

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Abstract: The standard blending problem consists of combining components to produce a final product with a given demand, while satisfying specific criteria with respect to the global blend and minimizing the total cost. The Bill-Of-Material (BOM) (or recipe) indicates which components are used and in which proportion. Typically, there is some flexibility in the planning process with respect to the proportion used for each of the components, where it may vary between a minimum and a maximum level instead of being fixed. This problem has been widely studied in a single period setting. However, the problem becomes more complex when we take into account a longer time frame. In such a case, demand for the final product occurs in several time periods, and both the final product and the components can be held in stock. In the integrated lot sizing and blending problem, the decisions relate to the production of the final product via the blending process, and the production (or procurement) of the components over an extended time horizon.

We propose mathematical formulations for this integrated problem. In a computational experiment, we analyse the impact of important parameters such as the level of flexibility in the BOM, the variance in the procurement cost among the components, and the variance of the proportion of the components in the total mix. Furthermore, we analyse the value of integration by comparing the solutions of the integrated models to the solutions of approaches that do not fully capture this integration such as a lot-for-lot approach, just-in-time models without inventory for the final product or components, and a hierarchical approach.

Keywords: Lot sizing, blending problem, bill of material, product flexibility

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Acknowledgments: This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) (Process numbers 2016/01860–1, 2018/18754–5) and through the Professorship in Operations Planning provided by HEC Montreal and the Foundation HEC Montreal. This support is gratefully acknowledged.

1 Introduction

In an assembly process, end products are produced using several components and subassemblies. In many cases, the Bill-of-Material is fixed, meaning that for each final product, it is known in advance how many units of each component will be needed. However, in some cases there might be some flexibility with respect to the use of the components. One case of flexibility that has already been studied in the lot sizing literature is the case of alternative components, where the company has the choice between different versions of a component. This is the case of lot sizing with component substitution (see e.g., Balakrishnan and Geunes 2000). In these cases, there are preferred components to meet a specific demand which may be replaced by alternative components, leading to a replacement cost. Practical applications for lot sizing problems with component substitution can be found, for example, in the electronics and metallurgical industries (Denton and Gupta, 2004, Gallego et al., 2006). In certain industries (e.g., food, steel), on the other hand, there is some flexibility with respect to the amount or volume that is needed of each component (sometimes referred to as ingredients). The proportion of the different components in the final mix may vary as long as certain constraints are satisfied. In a single-period setting, this problem is well-known and is referred to as the blending problem. However, in a medium-term planning perspective, this problem does not only require a solution for the blending problem in each production period, but also the planning of the production of the end items and components. Therefore, the integrated problem results in a two-level lot sizing and blending problem, in which the first level is related to the production plan of the end product and the second level is related to the production plan of the components. In this paper, we will focus on this integrated problem.

The aim of this paper is: (1) to propose and compare different formulations for the combined lot sizing and blending problem; (2) to analyse the benefits obtained considering the BOM flexibility; (3) to determine the value of the integration of the lot sizing and blending problem compared to approaches where this integration is not fully taken into account.

The remaining sections of this paper are organized as follows. In Section 2, we present some papers that are related to this work. In Section 3, we formally introduce the problem and propose three different formulations for the combined lot sizing and blending problem. Section 4 contains the computational results comparing the proposed formulations and the benefits of the BOM flexibility. Section 5 presents the value of the integration in relation to four different approaches and finally in Section 6, we present our conclusions.

2 Literature review

The integration of lot sizing into more global models has increasingly attracted the interest of researchers in recent years. Indeed, with the fast progress in optimization theory, software, hardware and, a better understanding of the individual problems as well as the dependencies among decisions observed in practical cases, more attention has been paid to the integration of different problems (Jans and Degraeve, 2008). There are several studies considering the integration of lot sizing and other related problems. For example, Adulyasak et al. (2015) discuss models for integrated production, routing and inventory planning. Studies of combined lot sizing and scheduling problems appear in the glass container industry (Almado-Lobo et al., 2007), the animal feed supplements industry (Toso et al., 2009) and the soft drink industry (Ferreira et al., 2010). Further, the integration of lot sizing and cutting stock problems has also aroused the interest of several researchers. For example, recently, Silva et al. (2015), Poldi and de Araujo (2016) and Vanzela et al. (2017) address this integration in the context of textile, paper and furniture industry, respectively and Melega et al. (2018) provide a literature review on this problem. Finally, the concept of process flexibility in lot sizing problems has been explored in Fiorotto et al. (2018). In this paper, we analyse the concept of product flexibility in a lot sizing context by extending the classical single-period blending problem to a multi-period setting.

Although there are, to the best of our knowledge, no studies analyzing the value of BOM flexibility and the benefits of integration in the context of a multi-period combined lot sizing and blending problem, there are relevant studies discussing separate aspects that are related to this such as lot sizing with substitution, flexibility in blending problems and two-level lot sizing problems. Some of the most relevant works are discussed next.

Balakrishnan and Geunes (2003) study the problem with component substitution. The computational results using real data from an aluminum-tube manufacturer show that substitution can save, on average, 8.7% of manufacturing costs. Ram et al. (2006) propose the concept of flexibility in bills-of-materials to deal with unexpected shortages when using material requirements planning by allowing the substitution of items for one another. Using a LP model and minimizing the deviation from a standard BOM, they state that while the BOM flexibility leads to some complexity in the logic that is used in scheduling computations, this flexibility provides alternatives to deal with future uncertainties in the availability of materials. Lang and Domschke (2010) develop a simple plant location reformulation and valid inequalities for the lot sizing with product substitution and their computational results show that this reformulation has a better performance than the classical formulation for this problem.

Blending models appear in several practical applications, for example gasoline, coal and beef cattle ration industry (Singh et al., 2000; Jia and Ierapetritou, 2003; Lyu et al., 1995; Liu and Sherali, 2000; Glen, 1980) and are related to product substitution, as the input components of the blending process are to some extent substitutable by each other. Lang (2010) state that blending models are especially appropriate for applications with continuous productions and an infinite number of feasible BOMs. Furthermore, he points out that partial substitution of divisible goods has an analogy to the blending models because frequently multiple input components are blended that partly contain the same substances, which can be interpreted as a partial substitution.

Considering the blending problem with alternative recipes, Rutten and Bertrand (1998) state that there are different situations which lead to variations in the recipes used for manufacturing a product (for example, costs and variation in quality of the components) and using simulations they study the trade-off between the safety stock costs and flexible recipe costs in order to establish under which conditions recipe flexibility should be used to achieve minimal total costs. The results show that the use of recipe flexibility will be profitable when a high service level is demanded and a long lead time for raw materials exists.

Crama et al. (2001) present an overview of the distinctive features of process industries with a focus on the concept of a recipe. They present a classification of the literature on blending models and recipe optimization and state that in process industries alternative recipes arise in one of two ways: 1) a finite collection of fixed admissible recipes is established; 2) the final product is characterized by a set of attribute values and any production plan yielding these attribute values is considered admissible (blending models).

Crama et al. (2004) study the problem of purchasing decisions faced by a multi-plant company in which there are some different recipes to produce each final product. The problem consist of determining which recipe(s) should be used for each end product and, simultaneously, which quantity of each component should be purchased from each supplier. Three models are proposed to solve a real-world problem from a large chemical company.

Finally, our research is also related to the two-level lot sizing problem with an assembly structure, since we consider production decisions both for the end item and the raw materials. A general discussion on multi-level lot sizing problems can be found in Pochet and Wolsey (2006). Some recent work focuses on solving complex multi-level problems with a Bill-of-Material structure (see, for example, Helber and Sahling, 2010; Wu et al., 2011; and Almeder et al., 2015).

3 Mathematical formulations

In the next three sections, we develop three alternative formulations for the integrated lot sizing and blending problem with a single end item.

3.1 Combined lot sizing and blending problems

To analyse the benefits of the BOM flexibility we focus on the integrated lot sizing and blending problem considering only one single end item which is produced by blending different components. Contrary to the classical blending models, we do not consider specific attributes for the end products for which there is some flexibility (such as nutritive restrictions with respect to the calcium, protein or fat content). We assume that the proportion of each component used to produce the end product can vary in an interval (minimum and maximum proportion). In the production process there are different inventory holding costs for the components and end items. Furthermore, the blending capacity is limited, and as in the classical lot sizing problem (see Trigeiro et al. 1989, Degraeve and Jans 2007), we assume a setup cost and setup time in case the end item, i.e. the blend, is produced. There is also a setup cost for the individual components.

For the mathematical formulation of the combined lot sizing and blending problem with a single end item, we consider the following sets and input parameters:

$I = \{1, ..., n\}$:	set of components;
$T = \{1, ..., m\}$:	set of periods;
LB_i:	lower bound on the proportion of the component i in the blend;
UB_i:	upper bound on the proportion of the component i in the blend;
d^E_t:	demand of the end product in period t;
$sd^E_{t\tau}$:	the sum of the demand of the end product from period t until period τ ($\tau \geq t$);
hc^E_t:	unit inventory cost of the end product in period t;
sc^E_t:	setup cost of the end product in period t;
st^E_t:	setup time of the end product in period t;
vc^E_t:	unit production cost of the end product in period t;
vt^E_t:	unit production time of the end product in period t;
Cap^E_t:	capacity (in terms of time) to produce the end product in period t;
hc_{it}:	unit inventory cost of component i in period t;
sc_{it}:	setup cost for component i in period t;
vc_{it}:	unit production cost of component i in period t;

The decision variables are then defined as follows:

x^E_t:	number of units produced of the end product in period t;
y^E_t:	binary setup variable, indicating the production or not of the end product in period t;
s^E_t:	quantity of inventory of the end product at the end of period t;
p_{it}:	amount of component i used in the blend (end product) in period t;
x_{it}:	number of units produced of component i in period t;
y_{it}:	binary setup variable, indicating the production or not of component i in period t;
s_{it}:	quantity of inventory of component i at the end of period t;

The first mathematical formulation of the problem $(F1)$ is then as follows:

$$Min \sum_{t=1}^{m} (sc^E_t y^E_t + vc^E_t x^E_t + hc^E_t s^E_t) + \sum_{t=1}^{m} \sum_{i=1}^{n} (sc_{it} y_{it} + vc_{it} x_{it} + hc_{it} s_{it}) \tag{1}$$

Subject to:

$$s^E_{t-1} + x^E_t = d^E_t + s^E_t \qquad \forall t \in T \tag{2}$$

$$x^E_t \leq sd^E_{tm} y^E_t \qquad \forall t \in T \tag{3}$$

$$st^E_t y^E_t + vt^E_t x^E_t \leq Cap^E_t \qquad \forall t \in T \tag{4}$$

$$s_{i,t-1} + x_{it} = p_{it} + s_{it} \qquad \forall i \in I, t \in T \tag{5}$$

$$x_{it} \leq UB_i \sum_{l=t}^{T} d^E_l y_{it} \qquad \forall i \in I, t \in T \tag{6}$$

$$p_{it} \leq UB_i x^E_t \qquad \forall i \in I, t \in T \tag{7}$$

$$p_{it} \geq LB_i x^E_t \qquad \forall i \in I, t \in T \tag{8}$$

$$\sum_{i=1}^{n} p_{it} = x_t^E \qquad \forall t \in T \qquad (9)$$

$$y_t^E \in \{0,1\}, \quad x_t^E \geq 0, \quad s_t^E \geq 0, \qquad \forall t \in T \qquad (10)$$

$$y_{it} \in \{0,1\}, \quad x_{it} \geq 0, \quad p_{it} \geq 0, \quad s_{it} \geq 0 \qquad \forall i \in I, t \in T \qquad (11)$$

The objective function (1) minimizes the total setup, production and inventory costs of the end product and components. The constraints (2) guarantee the inventory balance of the end product in each period. Next, are the setup constraints (3) and the capacity limits (4) of the end product. Constraints (5) guarantee the inventory balance of each component in each period. The setup constraints (6) do not allow any production of the components in period t unless a setup is done. Constraints (7) and (8) impose that the amount used of a specific component in the current blend falls within the admissible upper and lower bounds. As such, they guarantee that the end product satisfies all the component specifications. Constraints (9) impose that the sum of the quantities of the components used is equal to the total quantity of the blend produced. Finally, constraints (10) and (11) define the variables domains.

It is worth comparing this new model with the classical two-level lot sizing problem with an assembly structure (see e.g. Section 13.2 in Pochet and Wolsey, 2006). In the latter model, the inventory balance constraints for the ingredients (components) look slightly differently since the amount of each component needed (i.e., the dependent demand) is directly determined by the amount of the end product produced (i.e., the independent demand) via a fixed utilization factor (i.e. the number of items of a specific component needed to produce one end item). In the integrated model, there is some flexibility in this utilization factor. Therefore, we need an additional decision variable p_{it} indicating the amount of component i used in the end product in period t. The flexibility is then modeled in constraints (7) and (8).

3.2 Transportation formulation

Next we present a reformulation of the model (1)–(11) using the transportation approach. For the reformulation the following parameters are defined:

cs_{iktl}: unit cost of production and inventory holding to produce a component i in period k to be mixed in period t to satisfy the demand of period l;

$$cs_{iktl} = vc_{ik} + vc_t^E + \sum_{s=k}^{t-1} hc_{is} + \sum_{j=t}^{l-1} hc_j^E$$

We also define new variables for the model:

z_{iktl} : Amount of component i produced in period k, mixed in period t to satisfy the demand of period l.

The reformulation based on the transportation problem ($F2$) is as follows:

$$Min \sum_{t=1}^{m} sc_t^E y_t^E + \sum_{t=1}^{m}\sum_{i=1}^{n} sc_{it} y_{it} + \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{t=k}^{m}\sum_{l=t}^{m} cs_{iktl} z_{iktl} \qquad (12)$$

Subject to:

$$\sum_{i=1}^{n}\sum_{k=1}^{l}\sum_{t=k}^{l} z_{iktl} = d_l^E \qquad \forall l \in T \qquad (13)$$

$$\sum_{i=1}^{n}\sum_{k=1}^{t} z_{iktl} \leq d_l^E y_t^E \qquad \forall t, l \in T, l \geq t \qquad (14)$$

$$st_t^E y_t^E + \sum_{i=1}^{n}\sum_{k=1}^{t}\sum_{l=t}^{m} vt_t^E z_{iktl} \leq Cap_t^E \qquad \forall t \in T \tag{15}$$

$$\sum_{t=k}^{l} z_{iktl} \leq UB_i d_l^E y_{ik} \qquad \forall i \in I, k, l \in T, l \geq k \tag{16}$$

$$\sum_{k=1}^{t}\sum_{l=t}^{m} z_{iktl} \leq UB_i \sum_{j=1}^{n}\sum_{k=1}^{t}\sum_{l=t}^{m} z_{jktl} \qquad \forall i \in I, t \in T \tag{17}$$

$$\sum_{k=1}^{t}\sum_{l=t}^{m} z_{iktl} \geq LB_i \sum_{j=1}^{n}\sum_{k=1}^{t}\sum_{l=t}^{m} z_{jktl} \qquad \forall i \in I, t \in T \tag{18}$$

$$y_t^E \in \{0,1\}, y_{it} \in \{0,1\}, z_{iktl} \geq 0 \qquad \forall i \in I, k \in T, t \in T, l \in T \tag{19}$$

The objective function (12) minimizes the total cost, which consist of the setup cost of the end product, the setup cost of the components and the aggregated production and holding costs. The constraints (13) ensure that the demand of the end product is met for each period. The setup constraints (14) do not allow any production of the end product in period t unless a setup is done. The capacity constraints (15) limit the sum of the total setup and production times of the end product. Next, the constraints (16) are the setup constraints for the components. Constraints (17) and (18) guarantee the component specifications of the end product and constraints (19) define the variable domains.

3.3 Multicommodity formulation

Next we present a reformulation of the model (1)–(11) using the multicommodity approach (see Pochet and Wolsey (2006)). We adapted some ideas proposed by Cunha and Melo (2016) to our problem. For the reformulation the following variables are defined:

w_{kt}^{0i} : amount produced in period k of component i to satisfy the demand of end product in period t;
σ_{kt}^{0i} : amount stocked of the component i at the end of period k to satisfy the demand in period t;
w_{kt}^{1i} : amount blended of component i in period k to satisfy the demand of end product in period t;
σ_{kt}^{1} : amount stocked of the end product at the end of period k to satisfy the demand in period t;

The reformulation based on the multicommodity problem ($F3$) is as follows:

$$Min \sum_{t=1}^{m} sc_t^E y_t^E + \sum_{k=1}^{m}\sum_{t=k}^{m} hc_k^E \sigma_{kt}^1 + \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{t=k}^{m} vc_k^E w_{kt}^{1i} + \sum_{i=1}^{n}\sum_{t=1}^{m} sc_{it} y_{it} + \sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{t=k}^{m} (hc_{ik}\sigma_{kt}^{0i} + vc_{ik} w_{kt}^{0i}) \tag{20}$$

Subject to:

$$\sigma_{k-1,t}^1 + \sum_{i=1}^{n} w_{kt}^{1i} = \delta_{kt} d_t^E + (1-\delta_{kt})\sigma_{kt}^1 \qquad \forall t \in T, k \in T, k \leq t \tag{21}$$

$$\sum_{i=1}^{n} w_{kt}^{1i} \leq d_t^E y_k^E \qquad \forall t \in T, k \in T, k \leq t \tag{22}$$

$$\sigma_{k-1,t}^{0i} + w_{kt}^{0i} = w_{kt}^{1i} + \sigma_{kt}^{0i} \qquad \forall i \in I, t \in T, k \in T, k \leq t \tag{23}$$

$$w_{kt}^{0i} \leq UB_i d_t^E y_{ik} \qquad \forall i \in I, t \in T, k \in T, k \leq t \tag{24}$$

$$st_k^E y_k^E + \sum_{i=1}^{n}\sum_{t=k}^{m} vt_k^E w_{kt}^{1i} \leq Cap_k^E \qquad \forall k \in T \tag{25}$$

$$\sum_{t=k}^{m} w_{kt}^{1i} \leq UB_i \sum_{t=k}^{m}\sum_{j=1}^{n} w_{kt}^{1j} \qquad \forall i \in I, k \in T \tag{26}$$

$$\sum_{t=k}^{m} w_{kt}^{1i} \geq LB_i \sum_{t=k}^{m}\sum_{j=1}^{n} w_{kt}^{1j} \qquad \forall i \in I, k \in T \tag{27}$$

$$y_t^E \in \{0,1\}, \quad y_{it} \in \{0,1\} \qquad \forall i \in I, t \in T \tag{28}$$

$$w_{kt}^{1i} \geq 0, \sigma_{kt}^{1} \geq 0, w_{kt}^{0i} \geq 0, \sigma_{kt}^{0i} \geq 0 \qquad \forall i \in I, t \in T, k \in T, k \geq t \tag{29}$$

We define δ_{kt} to be equal to one if $k = t$ and zero otherwise. The objective function (20) minimizes the total costs. Constraints (21) and (22) are the balance and setup constraints for the end product. Next, constraints (23) and (24) are the balance and setup constraints for the components. Constraints (25) ensure that the total capacity consumed during a period for production and setup of the end product is less than or equal to the available capacity. Constraints (26) and (27) guarantee that the end product satisfies all the component specifications. Finally, constraints (28) and (29) define the variable domains.

4 Computational results

The models described in the previous sections were tested on a total of 1620 instances with a number of components equal to 2, 5 and 10 and 30 periods. The 1620 instances are divided into different types of classes according the following factors: capacity utilization, number of components, Time Between Orders of the components (TBO_i), setup cost of the end product and production cost of the components. For each combination of these five parameters, 10 instances were generated. We assume that all the parameters, except the demand, are time-invariant. Most of the parameters were generated in intervals $[a, b]$ with a uniform distribution called $U[a, b]$:

- setup cost end product ($sc_t^E = sc^E$) $U[5000, 15000]$;
- production time end product ($vt_t^E = vt^E$) $U[1, 5]$;
- setup time end product ($st_t^E = st^E$) $U[10, 50]$;
- demand end product (d_t^E) $U[500, 1500]$;
- production cost components ($vc_{it} = vc_i$) $U[50, 150]$;

Furthermore, we have fixed the production cost of the end product (vc_t^E) equal to zero and calculated the inventory holding cost as a fraction of the production cost of the components, i.e., $hc_i = 0.0030 \times U[50, 150]$. The holding cost of the end product is usually larger than the holding cost of the components. In order to ensure this, the value that we choose is $hc^E = 0.0045 \times U[50, 150]$.

The Time Between Order of the components (TBO_i) is usually larger than that of end products. Using the ideas proposed by Brahimi et al. (2015), we cover several possible cases by setting it equal to:

$$TBO_i = \left\{ \begin{array}{c} 1 \times TBO \\ 2 \times TBO \\ 3 \times TBO \end{array} \right\}$$

where TBO is the time between order of the end product (it is given by $TBO = \frac{\sqrt{\frac{2sc_t^E AVD}{hc_t^E}}}{AVD}$, where AVD is the average of the demand). Given the TBO_i and the inventory cost of the components, we can calculate the setup cost of the components sc_i as follows:

$$sc_i = \frac{(TBO_i)^2 hc_t \sum_{t=1}^{m} d_t^E}{2m}$$

The setup costs of the end product and the production costs of the components can be low (L), normal (N) and high (H). To generate instances with high setup and production costs, the costs were multiplied by 5. On the other hand, to generate instances with low setup and production costs the costs were divided by 5.

To generate the capacity, we use as base the average capacity needed to produce all the items in a lot-for-lot strategy (i.e., produce every period), i.e., $Cap_t^E = \frac{\sum_{t=1}^{m}(vt_t^E d_t^E + st_t^E)}{m}$. Furthermore, to generate the instances with normal (N) and tight (T) capacity we multiply this by 2 and 1.3, respectively. Therefore, considering that we have 10 instances for each configurations of the 5 factors, the total number of instances is equal to $10 \times 2 \times 3 \times 3 \times 3 \times 3 = 1620$.

The formulations $F1$, $F2$ and $F3$ were modeled in C++ using the concert technology and CPLEX 12.7.1 as solver. The tests were done on a computer with 2 Intel(R) Xeon(R) X5675 processors, 3.07GHz with 96GB of RAM and the Linux operating system. Moreover, we have limited the computational time for each instance to 1800 seconds.

4.1 Comparison of the formulations

In this section we compare the three proposed formulations in relation to the value of the linear relaxation (LP values) and the integer solution (IP values). In Table 1 we consider different levels of BOM flexibility and give the average of the LP values found for all classes and flexibility levels. In the base case, the proportion of each component is fixed and equal for all components, i.e., 50%, 20% and 10% for the instances with 2, 5 and 10 components, respectively. There is hence no flexibility in the component usage. Next, we vary the BOM flexibility by allowing the actual proportion used to be within a range, which is determined by setting the upper and lower bounds as a specified percentage deviation (i.e. 5%, 10%, 20%, 40% and 80%) around the original proportion. For example, if the original fixed proportion for a component is 20% (for the case with 5 components), then a 10% flexibility will allow this proportion to vary between 18% ($= 0.9 * 20\%$) and 22% ($= 1.1 * 20\%$). Furthermore, we set the LP value found by the formulation $F1$ to 100% and calculate the LP values of the formulations $F2$ and $F3$ relative to this.

Table 1: Comparison of the linear relation (LP values)

		base			5%			10%			20%			40%			80%		
		F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
Cap	N	100	103.3	103.3	100	103.3	103.3	100	103.4	103.4	100	103.4	103.4	100	103.6	103.6	100	104.0	104.0
	T	100	106.0	106.0	100	106.1	106.1	100	106.1	106.1	100	106.3	106.3	100	106.6	106.6	100	107.3	107.3
	2	100	104.9	104.9	100	104.9	104.9	100	105.0	105.0	100	105.1	105.1	100	105.3	105.3	100	105.7	105.7
Comp.	5	100	104.6	104.6	100	104.7	104.7	100	104.7	104.7	100	104.8	104.8	100	105.1	105.1	100	105.6	105.6
	10	100	104.4	104.4	100	104.5	104.5	100	104.6	104.6	100	104.7	104.7	100	104.9	104.9	100	105.6	105.6
	1	100	104.8	104.8	100	104.9	104.9	100	104.9	104.9	100	105.0	105.0	100	105.2	105.2	100	105.7	105.7
TBO	2	100	104.4	104.4	100	104.5	104.5	100	104.6	104.6	100	104.7	104.7	100	104.9	104.9	100	105.5	105.5
	3	100	104.7	104.7	100	104.7	104.7	100	104.8	104.8	100	104.9	104.9	100	105.2	105.2	100	105.7	105.7
S. cost	H	100	109.3	109.3	100	109.4	109.4	100	109.5	109.5	100	109.8	109.8	100	110.3	110.3	100	111.5	111.5
End	N	100	103.1	103.1	100	103.1	103.1	100	103.2	103.2	100	103.2	103.2	100	103.4	103.4	100	103.7	103.7
	L	100	101.6	101.6	100	101.6	101.6	100	101.6	101.6	100	101.6	101.6	100	101.6	101.6	100	101.7	101.7
P. cost	H	100	101.6	101.6	100	101.6	101.6	100	101.6	101.6	100	101.6	101.6	100	101.6	101.6	100	101.7	101.7
Ing.	N	100	103.1	103.1	100	103.1	103.1	100	103.2	103.2	100	103.2	103.2	100	103.4	103.4	100	103.7	103.7
	L	100	109.2	109.2	100	109.4	109.4	100	109.5	109.5	100	110.3	110.3	100	110.3	110.3	100	111.5	111.5
Av.		100	104.6	104.6	100	104.7	104.7	100	104.8	104.8	100	104.9	104.9	100	105.1	105.1	100	105.6	105.6

In all tables, we analyse the following five factors separately: capacity level (Cap), number of components (Comp.), time between orders (TBO), setup cost for the end product (S. cost End) and the production cost of the ingredients (P. cost Ing.). The first column of each table indicates the factor analysed and the second column indicates its level. Finally, we also compute the global average (Av.). Table 1 shows that the LP values found by formulations $F2$ and $F3$ are larger than the values found by formulation $F1$ for all classes of instances. Furthermore, we observe that the biggest differences are found for the instances with tight capacity, high setup cost for the end product and low production cost of the components. Comparing the LP values found by formulations $F2$ and $F3$, we see that the LP relaxation of these two formulations are equivalent, i.e., these formulations have the same performance for all levels of BOM flexibility.

In Table 2 we show the averages of the CPU times in seconds for the LP results obtained in Table 1. We see that formulation $F1$ is much faster to solve the LP model than the other two formulations for all levels of BOM flexibility. We also observe that formulation $F2$ is significantly slower than formulation $F3$. Furthermore, considering the formulations $F2$ and $F3$, the factor which has biggest impact on the CPU times to solve the LP model is the number of components. Note that the CPU times increase significantly for the instances with 10 components.

Table 2: Average CPU times to solve the LP model (in seconds)

		base			5%			10%			20%			40%			80%		
		F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
Cap	N	0.01	24.92	4.92	0.01	28.17	4.98	0.01	27.52	4.92	0.01	27.47	4.99	0.01	27.00	4.95	0.01	29.01	5.01
	T	0.01	26.37	3.58	0.01	32.86	3.97	0.01	34.31	4.00	0.01	32.59	4.04	0.01	29.64	4.13	0.01	25.60	3.90
	2	0.01	0.58	0.22	0.01	0.60	0.24	0.01	0.61	0.23	0.01	0.67	0.23	0.01	0.56	0.22	0.01	0.49	0.19
Comp.	5	0.01	7.66	2.01	0.01	8.87	2.24	0.01	9.05	2.26	0.01	9.12	2.31	0.01	6.29	2.37	0.01	6.28	2.15
	10	0.01	68.70	10.52	0.01	82.07	10.94	0.01	83.08	10.88	0.01	80.30	11.00	0.01	78.10	11.04	0.01	75.12	11.02
	1	0.01	17.35	3.70	0.01	19.73	3.80	0.01	19.43	3.73	0.01	18.00	3.75	0.01	16.07	3.62	0.01	12.96	3.34
TBO	2	0.01	27.62	4.46	0.01	32.22	4.67	0.01	32.54	4.64	0.01	31.51	4.67	0.01	29.51	4.68	0.01	28.09	4.43
	3	0.01	31.98	4.59	0.01	39.59	4.93	0.01	40.77	5.00	0.01	40.58	5.10	0.01	39.37	5.31	0.01	40.86	5.57
S. cost	H	0.01	29.17	5.24	0.01	38.78	5.21	0.01	38.86	5.28	0.01	36.41	5.34	0.01	39.08	5.44	0.01	35.49	5.53
End	N	0.01	28.54	4.30	0.01	32.69	4.78	0.01	32.66	4.70	0.01	32.57	4.74	0.01	28.06	4.79	0.01	29.40	4.65
	L	0.01	19.23	3.21	0.01	20.07	3.41	0.01	21.22	3.38	0.01	21.11	3.44	0.01	17.81	3.38	0.01	17.02	3.16
P. cost	H	0.01	15.90	3.54	0.01	21.54	3.38	0.01	20.69	3.41	0.01	19.09	3.43	0.01	19.02	3.39	0.01	16.60	3.39
Ing.	N	0.01	28.49	4.46	0.01	31.65	4.76	0.01	34.40	4.75	0.01	32.24	4.74	0.01	29.55	4.80	0.01	29.92	4.73
	L	0.01	32.51	4.68	0.01	38.38	5.26	0.01	37.63	5.22	0.01	38.78	5.36	0.01	36.40	5.44	0.01	35.42	5.23
AV.		0.01	25.64	4.25	0.01	30.51	4.47	0.01	30.91	4.46	0.01	30.03	4.51	0.01	28.32	4.54	0.01	27.30	4.45

Table 3 presents a comparison of the upper bounds (IP values) found for the three formulations within the fixed time limit (1800 seconds). In this table, we set the upper bound found by formulation $F1$ to 100% and calculate the other values relative to this. The three formulations found very similar IP values for all classes of instances and level of BOM flexibility. The biggest difference is only 0.58% found by formulation $F2$ for the instances with 10 components and a level of BOM flexibility equal to 80%. We observe that these small differences occur because the formulations (especially $F2$ and $F3$) have not found the optimal solutions for several instances within the fixed time limit. However, the optimality gaps found by these formulations are on average very small.

Tables 4 and 5 show the averages of the CPU times and the percentage of instances solved to optimality by the three proposed formulations, respectively. Note that the fixed time limit is equal to 1800 seconds and for several instances, especially with 5 and 10 components, the formulations do not find the optimal solution within this fixed time. The results show that formulation $F2$ is the slowest formulation for all classes of problems and levels of BOM flexibility. Moreover, considering formulation $F2$, the CPU times are very high and the percentage of instances solved to optimality is very small for instances with 10 components. Comparing the formulations $F1$ and $F3$ we see that when there is no BOM flexibility (base case) formulation $F3$ is faster than formulation $F1$ for most classes of instances. However, by increasing the level of BOM flexibility formulation $F3$ becomes slower than formulation $F1$ and this difference is very significant for most classes of instances when the level of BOM flexibility is equal to 80%. We observe that, for this level of flexibility, the percentage of instances solved to optimality by formulation $F1$ is, on average, around 22% larger than formulation $F3$. Finally, in relation to the CPU times and the level of BOM flexibility, we see that by increasing the level of flexibility, the CPU times generally increase for all formulations. On the other hand, by

Table 3: Comparison of upper bounds (IP values) of the formulations

		base			5%			10%			20%			40%			80%		
		F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
Cap	N	100	100	100	100	100.02	100.01	100	100.03	100.01	100	100.07	100.01	100	100.06	100.02	100	100.24	100.02
	T	100	100	100	100	100.01	100	100	100.02	100	100	100.02	100.01	100	100.07	100.01	100	100.19	100.01
	2	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
Comp.	5	100	100	100	100	100.01	100	100	100.01	100	100	100.02	100	100	100.04	100	100	100.06	100
	10	100	100.01	100	100	100.04	100.01	100	100.05	100.01	100	100.12	100.02	100	100.16	100.04	100	100.58	100.03
	1	100	100.01	100	100	100.02	100	100	100.03	100	100	100.03	100.01	100	100.09	100.02	100	100.21	100.02
TBO	2	100	100	100	100	100.02	100	100	100.03	100.01	100	100.08	100.01	100	100.07	100.02	100	100.32	100.01
	3	100	100	100	100	100.01	100.01	100	100.01	100	100	100.02	100.01	100	100.04	100.01	100	100.11	100.01
S. cost	H	100	100	100	100	100.01	100	100	100.01	100	100	100.02	100.01	100	100.05	100.01	100	100.26	100.01
End	N	100	100	100	100	100.02	100	100	100.02	100.01	100	100.08	100.01	100	100.06	100.01	100	100.17	100.01
	L	100	100.01	100	100	100.02	100	100	100.03	100	100	100.04	100.01	100	100.09	100.02	100	100.21	100.01
P. cost	H	100	100.01	100	100	100.02	100	100	100.03	100	100	100.08	100.01	100	100.09	100.02	100	100.31	100.01
Ing.	N	100	100	100	100	100.02	100	100	100.02	100	100	100.03	100.01	100	100.07	100.02	100	100.18	100.01
	L	100	100	100	100	100.01	100	100	100.01	100	100	100.02	100.01	100	100.05	100.01	100	100.15	100.01
AV.		100	100	100	100	100.02	100	100	100.02	100	100	100.05	100.01	100	100.07	100.01	100	100.21	100.01

increasing the level of BOM flexibility, the percentage of instances solved to optimality generally decreases significantly for most instances.

Table 4: Average CPU times to solve the formulations (in seconds)

		base			5%			10%			20%			40%			80%		
		F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
Cap	N	249.9	586.1	147.2	490.6	873.9	315.9	510.6	977.1	511.8	574.7	1043.6	636.0	601.8	1124.2	727.0	578.3	1162.4	696.4
	T	67.7	498.9	109.0	187.9	805.4	282.9	208.9	900.6	458.6	287.6	948.2	609.7	360.8	992.4	680.4	235.2	1100.8	651.8
	2	8.0	20.1	4.1	57.6	38.4	7.6	62.3	63.5	12.0	65.4	132.3	24.1	53.3	249.8	46.8	54.1	342.4	51.8
Comp.	5	158.9	313.6	101.4	393.9	862.5	210.9	410.5	1084.0	440.3	507.9	1186.9	730.2	573.1	1249.1	915.1	526.8	1338.6	858.9
	10	309.4	1293.8	278.9	566.4	1618.0	679.8	606.5	1669.1	1003.3	720.3	1668.5	1113.2	817.6	1675.9	1149.1	639.5	1713.8	1111.5
	1	181.4	604.1	193.8	402.1	856.7	425.5	425.0	966.5	637.3	508.5	1042.7	805.0	550.6	1135.1	905.5	451.1	1227.4	854.5
TBO	2	162.6	557.3	111.6	347.8	867.6	284.6	366.7	957.5	494.2	454.5	1000.1	596.8	523.4	1045.8	702.0	405.7	1100.0	686.5
	3	132.4	466.1	78.9	268.2	794.7	188.2	287.6	892.6	324.1	330.5	944.9	465.7	369.9	994.0	503.5	363.5	1067.5	481.3
S. cost	H	75.5	582.6	98.1	189.3	825.2	167.7	225.6	902.1	280.2	284.7	917.2	347.6	320.5	946.4	405.6	257.8	987.0	339.3
End	N	181.0	596.0	147.8	409.5	942.3	358.4	406.3	1036.3	588.1	484.6	1094.0	726.7	556.1	1155.2	807.7	474.2	1214.9	747.9
	L	219.8	448.9	138.4	419.0	751.5	372.3	447.4	878.2	587.3	524.3	976.4	793.3	567.3	1073.3	897.7	488.3	1192.9	935.0
P. cost	H	214.4	444.3	132.1	419.0	743.8	365.5	449.2	882.4	582.9	527.1	971.3	787.0	581.1	1075.5	897.7	489.8	1195.3	931.1
Ing.	N	185.4	601.3	148.2	394.1	936.9	362.4	401.5	1031.3	580.7	490.4	1079.2	719.9	537.0	1148.3	804.2	473.3	1210.2	739.7
	L	76.6	581.9	104.0	204.7	838.2	170.3	228.7	902.9	292.0	275.9	937.1	360.7	325.8	951.0	409.2	257.2	989.4	351.5
AV.		158.8	542.5	128.1	339.3	559.8	299.4	359.8	938.9	485.2	431.2	995.9	622.9	481.3	1058.3	703.7	406.8	1131.6	674.1

4.2 Analysis of BOM flexibility

In this section we use formulation $F1$ to analyse the value of the BOM flexibility in relation to the base case without flexibility. Note that we have chosen formulation $F1$ because this formulation was the most computationally efficient when there is BOM flexibility (see Table 4).

In Table 6 we consider different levels of BOM flexibility and give the average upper bounds (UB) and the capacity utilization (Ut.) found for all classes of problems and flexibility levels. For the upper bounds, we set the upper bounds found by the base case to 100% and calculate the other values relative to this. Table 6 shows that the value of BOM flexibility depends on the characteristics of the instances. For all levels of BOM flexibility, we observe that the benefit, which is measured as the percentage decrease in the objective function value compared to the base case, is the highest for the instances with 10 components, low setup cost for the end product and high production cost of

Table 5: Percentage of instances solved to optimality

		base			5%			10%			20%			40%			80%		
		F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
Cap	N	89.5	81.4	96.9	77.9	58.9	83.6	75.6	52.8	73.6	72.2	49.4	64.7	68.8	44.7	56.5	70.9	38.5	56.1
	T	99.3	84.3	94.2	98.1	65.7	85.9	93.9	57.3	77.5	90.1	55.4	67.5	86.2	53.1	62.8	91.5	48.1	62.0
	2	100	100	100	98.0	100	99.8	98.0	100	99.0	98.1	100	98.7	98.5	97.7	97.1	98.0	93.1	96.5
Comp.	5	95.5	97.1	98.5	85.2	70.4	87.8	83.5	53.7	78.0	79.6	46.3	60.2	73.9	38.8	47.7	76.5	30.5	46.7
	10	87.7	51.6	94.1	74.4	16.7	66.7	72.6	11.6	49.5	65.9	11.2	39.4	60.2	10.2	34.2	69.8	6.4	33.7
	1	94.9	80.0	95.6	82.7	59.7	77.5	83.3	52.8	67.6	78.4	49.0	55.9	74.0	44.2	50.2	78.9	38.0	50.8
TBO	2	93.2	82.2	99.0	86.0	61.4	85.8	84.2	54.0	74.1	80.8	52.6	68.0	76.0	50.3	59.8	83.3	45.9	59.1
	3	95.1	87.6	98.9	88.9	66.2	91.0	86.6	58.6	84.8	84.5	55.9	74.3	82.5	49.8	69.0	82.4	46.1	67.3
S. cost	H	98.2	82.0	98.1	92.8	64.8	89.3	90.6	59.0	85.1	87.6	58.5	80.6	85.1	55.7	76.9	88.7	51.6	76.9
End	N	93.3	81.7	96.9	82.7	56.3	81.6	82.2	49.9	69.5	77.6	46.4	59.0	74.2	43.4	53.5	78.3	38.5	53.2
	L	91.7	85.0	97.6	82.0	66.0	83.4	81.3	56.4	71.9	78.5	52.6	58.8	73.2	47.7	49.1	77.6	40.0	46.9
P. cost	H	92.7	85.0	97.0	82.4	65.7	83.2	79.6	56.5	72.0	77.1	52.4	58.7	73.5	47.9	49.4	77.2	40.3	46.3
Ing.	N	92.5	80.2	96.8	83.5	57.2	82.0	83.1	49.9	71.5	78.1	47.7	60.2	74.0	44.2	53.7	78.3	38.8	55.6
	L	97.9	83.5	98.7	91.7	64.2	89.1	91.5	59.0	83.1	88.4	57.4	79.6	85.1	54.6	75.9	89.1	50.8	75.2
AV.		94.4	82.9	95.6	88.0	62.3	84.8	84.7	55.1	75.6	81.2	52.4	66.1	77.5	48.9	59.7	81.2	43.3	59.1

the components and the benefits reach approximately 16.5% for these classes of instances. The BOM flexibility exploits individual differences in production costs of the ingredients in order to lower the total cost. In the three cases mentioned (i.e., a higher number of components, a lower level for the setup cost of the end product and a higher level for the production cost of the ingredients) there is an increased dominance of the ingredient production cost in the total cost function. This hence explains the impact of these three factors. Furthermore, by increasing the number of components, the probability of having components with a high cost difference (i.e., some of them cheap and others very expensive) increases, which also makes the benefits of BOM flexibility more significant. Finally, we see that the time between orders for the components (TBO) has no significant impact on the benefits of the BOM flexibility.

Table 6: General results for formulation F1

		base		5%		10%		20%		40%		80%	
		UB	Ut.	UB	Ut.	UB	Ut.	UB	Ut.	UB	Ut.	UB	Ut.
Cap	N	100	49.8	99.1	49.8	98.2	49.8	96.3	49.8	92.6	49.8	85.1	49.8
	T	100	76.9	99.1	76.9	98.3	76.9	96.6	76.9	93.2	76.9	86.3	76.9
	2	100	63.4	99.3	63.4	98.5	63.4	97.1	63.4	94.1	63.4	88.1	63.4
Comp.	5	100	63.4	99.1	63.4	98.2	63.4	96.5	63.4	92.9	63.4	85.8	63.4
	10	100	63.4	99.0	63.4	97.9	63.4	95.8	63.4	91.6	63.4	83.2	63.4
	1	100	63.4	99.1	63.4	98.2	63.4	96.4	63.4	92.8	63.4	85.6	63.4
TBO	2	100	63.4	99.1	63.4	98.2	63.4	96.5	63.4	92.9	63.4	85.7	63.4
	3	100	63.4	99.1	63.4	98.2	63.4	96.5	63.4	92.9	63.4	85.8	63.4
S. cost	H	100	63.4	99.3	63.4	98.6	63.4	97.2	63.4	94.3	63.4	88.5	63.4
End	N	100	63.4	99.1	63.4	98.2	63.4	96.3	63.4	92.6	63.4	85.0	63.4
	L	100	63.4	99.0	63.4	98.0	63.4	95.9	63.4	91.8	63.4	83.5	63.4
P. cost	H	100	63.4	99.0	63.4	98.0	63.4	95.9	63.4	91.8	63.4	83.5	63.4
Ing.	N	100	63.4	99.1	63.4	98.2	63.4	96.3	63.4	92.6	63.4	85.0	63.4
	L	100	63.4	99.3	63.4	98.6	63.4	97.2	63.4	94.3	63.4	88.5	63.4
AV.		100	63.4	99.1	63.4	98.2	63.4	96.5	63.4	92.9	63.4	85.7	63.4

In Table 7 we analyse the value of the BOM flexibility for the combined lot sizing and blending problem without considering the production capacity constraints for the end product. In other words, we consider model (1)–(11) without the constraints (4). Table 7 shows the average upper bounds and computational times found for all classes of problems and flexibility levels. We observe that the value of BOM flexibility increases for all classes of problems compared to the capacitated model. At 80% of BOM flexibility, the difference is on average equal to 2.0%. Regarding the computational times, we see that the uncapacitated model is on average faster, and by increasing the level of flexibility the difference of the computational times decreases.

Table 7: General results for formulation F1 without capacity

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Comp.	2	100	0.4	99.2	1.2	98.3	1.5	96.7	1.7	93.3	2.4	86.5	1.7
	5	100	5.9	99.0	174.3	98.0	244.4	96.0	287.9	91.9	348.4	83.7	381.7
	10	100	196.6	98.8	319.7	97.7	350.6	95.3	407.9	90.6	527.7	81.1	692.0
TBO	1	100	1.9	99.0	9.4	97.9	8.3	95.9	12.5	91.8	67.3	83.5	262.6
	2	100	103.1	99.0	210.7	98.0	260.4	96.0	319.2	92.0	412.3	83.9	414.4
	3	100	97.9	99.0	275.1	98.0	327.7	96.0	365.8	92.0	399.0	83.9	398.3
S. cost	H	100	0.6	99.1	2.6	98.1	3.7	96.2	16.4	92.4	49.4	84.8	85.4
End	N	100	100.8	99.0	208.5	98.0	230.8	95.9	272.5	91.9	337.3	83.6	407.3
	L	100	101.5	98.9	284.1	97.9	362.0	95.8	408.5	91.5	491.8	83.0	582.6
P. cost	H	100	105.4	98.9	296.3	97.9	242.4	95.8	405.1	91.5	494.2	83.0	597.0
Ing.	N	100	96.8	99.0	194.5	98.0	247.6	95.9	266.8	91.9	338.1	83.6	394.2
	L	100	0.8	99.1	4.3	98.1	6.5	96.2	25.5	92.4	46.3	84.8	84.2
AV.		100	67.6	99.0	165.1	98.0	198.8	96.0	232.5	91.9	292.8	83.8	358.5

Aiming to further analyse the effect of the benefits of BOM flexibility, we consider in Table 8 different base cases for the instances with 5 components. In the base case explored in Tables 6 and 7, we assumed that each component had the same base weight relative to the total blend. In Table 8 we analyse three additional scenarios with respect to the base weight of the components, in order to consider a bigger variation of the base weights among the components. Whereas in the base case, each of the 5 components has an equal weight of 20% in the total blend, we have three new cases which put more weight on one particular component, while adjusting the weights for the other components. In case 2, the first component has a weight of 70%, whereas the other components have a weight of 10%, 10%, 5% and 5%, respectively. For case 3 the weight distribution for the five components is 50%, 15%, 15%, 10% and 10% and for case 4 it is 30%, 20%, 20%, 15% and 15%.

We note that the benefits of the BOM flexibility are bigger for the instances in which the distribution of the base case are less concentrated (case 4 in Table 8). Furthermore, we see that with an increasing level of flexibility the difference between the benefits of the more concentrated (case 2) and less concentrated (case 4) configurations increases significantly. Indeed, this difference is on average only 0.40% with 5% of flexibility and it increases to 6.8% with 80% of flexibility. The reason is that when there is a high concentration, the big imbalance in the weights limits the possibilities for substituting one component by another. We take case 2 with a 10% BOM flexibility to illustrate this. The first component has a total weight of 70% and there is flexibility to increase this up to 77% or down to 63% (i.e., plus or minus 10%). However, even if the first component is the cheapest, we cannot increase the total amount of this component to 77%, since the four other components only allow a global decrease from 30% to 27% (i.e., a 10% decrease). When there is a low concentration as in case 4, the possibilities for substitution are less limited, and therefore, the benefits of BOM flexibility increase.

5 Analysis of the value of integration

In this section we analyse the value of the integration of the lot sizing and blending problem compared to four different approaches which do not take into account the (full) integration aspect.

Throughout this section, we set the upper bounds found considering the integrated model (1)–(11) to 100% and calculate the upper bound found by the other approaches that will be analysed in each section relative to this.

Table 8: General upper bounds for different base case with 5 components

		Case 2 (70% 10% 10% 5% 5%)						Case 3 (50% 15% 15% 10% 10%)						Case 4 (30% 20% 20% 15% 15%)					
		base	5%	10%	20%	40%	80%	base	5%	10%	20%	40%	80%	base	5%	10%	20%	40%	80%
Comp.	class	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB	UB
Cap	N	100	99.5	99.0	98.1	96.1	92.2	100	99.2	98.5	97.0	93.9	87.7	100	99.1	98.2	96.4	92.7	85.3
	T	100	99.5	99.1	98.1	96.3	92.5	100	99.3	98.5	97.1	94.1	88.2	100	99.1	98.2	96.5	93.0	85.9
	1	100	99.5	99.0	98.1	96.2	92.3	100	99.2	98.5	97.0	94.0	87.8	100	99.1	98.2	96.4	92.8	85.5
TBO	2	100	99.5	99.0	98.1	96.2	92.3	100	99.3	98.5	97.0	94.0	87.9	100	99.1	98.2	96.4	92.8	85.6
	3	100	99.5	99.1	98.1	96.3	92.5	100	99.3	98.5	97.0	94.1	88.1	100	99.1	98.2	96.4	92.9	85.7
S. cost	H	100	99.6	99.2	98.5	97.0	93.9	100	99.4	98.8	97.6	95.2	90.4	100	99.3	98.6	97.1	94.2	88.4
End	N	100	99.5	99.0	98.0	96.0	92.0	100	99.2	98.4	96.9	93.7	87.4	100	99.1	98.1	96.3	92.5	84.9
	L	100	99.4	98.9	97.8	95.6	91.1	100	99.1	98.3	96.6	93.1	86.1	100	99.0	97.9	95.9	91.8	83.4
P. cost	H	100	99.5	98.9	97.8	95.6	91.2	100	99.1	98.3	96.6	93.1	86.1	100	99.0	97.9	95.9	91.7	83.4
Ing.	N	100	99.5	99.0	98.0	96.0	92.0	100	99.2	98.4	96.9	93.8	87.4	100	99.1	98.1	96.3	92.5	84.9
	L	100	99.6	99.2	98.5	97.0	93.9	100	99.4	98.8	97.6	95.2	90.3	100	99.3	98.6	97.1	94.3	88.4
AV.		100	99.5	99.0	98.1	96.2	92.4	100	99.2	98.5	97.0	94.0	87.9	100	99.1	98.2	96.4	92.8	85.6

5.1 Lot-for-lot approach

In this section we analyse the value of integration, compared to the lot-for-lot approach (no looking ahead) for the lot sizing and blending problem. In this approach we have to produce the end product and the components every period in which we have a positive demand for the end product. In other words, we consider that there is no inventory of the end product and components for all periods ($s_t^E = 0$ and $s_{it} = 0 \, \forall t, i$.) This approach is motivated by the observation that the blending problem is typically considered as a one-period problem and as such, no integration over time is considered.

Figure 1 shows an example with 7 periods and the characteristics of the lot-for-lot approach in terms of the setup schedule for the end products and components. Note that in this figure we suppose $d_t^E > 0$ for all periods.

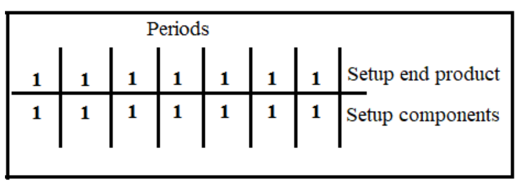

Figure 1: Characteristic of the lot-for-lot approach

In Table 9 we give the average upper bounds (UB) and the computational times in seconds T(s) of the lot-for-lot approach, found for all classes and flexibility levels. The results show that this approach makes the problem easy to solve since the computational times are almost zero for all instances.

We note that this approach found results considerably worse than the integrated model, especially for the instances with a high TBO for the components, high setup cost for the end product and low production cost of the components. Observe that the capacity level also has a significant impact in the value of the integrated model on relation to the lot-for-lot approach. Moreover, in some cases, the lot-for-lot approach results in a total cost which is approximately 2.3 times higher than with the integrated approach. This occurs because in these instances the setup costs have a big impact on the value of the objective function and in the lot-for-lot approach there is a setup for the end product and components in all periods. On the other hand, the better results found by this approach are found for the instances in which the setup costs are not a large component of the objective function (instances with low setup cost of the components and high production cost of the components). Finally, observe that by increasing the level of flexibility the benefits of the integrated model increase.

Table 9: General results for the lot-for-lot approach

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	170.8	0.1	171.3	0.1	171.4	0.1	173.1	0.1	175.5	0.1	180.5	0.1
	T	156.2	0.1	156.7	0.1	157.0	0.1	157.8	0.1	161.0	0.1	163.1	0.1
Comp.	2	162.8	0.1	163.1	0.1	163.4	0.1	164.4	0.1	166.1	0.1	169.5	0.1
	5	164.0	0.1	164.6	0.1	164.6	0.1	166.1	0.1	168.2	0.1	172.6	0.1
	10	163.7	0.1	164.3	0.1	164.8	0.1	166.0	0.1	170.4	0.1	173.5	0.1
TBO	1	124.3	0.1	124.5	0.1	124.7	0.1	125.0	0.1	126.3	0.1	127.6	0.1
	2	172.9	0.1	173.4	0.1	173.8	0.1	174.9	0.1	177.9	0.1	182.1	0.1
	3	193.4	0.1	194.1	0.1	194.2	0.1	196.5	0.1	200.6	0.1	207.7	0.1
S. cost End	H	216.1	0.1	216.8	0.1	216.8	0.1	219.1	0.1	223.6	0.1	227.6	0.1
	N	160.4	0.1	161.1	0.1	161.4	0.1	162.7	0.1	165.5	0.1	170.5	0.1
	L	114.0	0.1	114.2	0.1	114.4	0.1	114.7	0.1	115.7	0.1	117.3	0.1
P. cost Ing.	H	114.0	0.1	114.2	0.1	114.4	0.1	114.7	0.1	115.6	0.1	117.3	0.1
	N	160.5	0.1	161.1	0.1	161.4	0.1	162.7	0.1	165.6	0.1	170.5	0.1
	L	216.0	0.1	216.7	0.1	216.8	0.1	219.0	0.1	223.6	0.1	227.7	0.1
AV.		163.5	0.1	164.0	0.1	164.2	0.1	165.5	0.1	168.3	0.1	171.8	0.1

5.2 JIT for the components

In this section we analyse the value of integration, compared to the just-in-time approach for the components. In this approach we do not have to produce the end product in every periods (no lot-for-lot) but we have to produce the components in the same periods in which there is production of the end product. In other words, we consider the possibility of inventory of the end product but not for the components ($s_t^E \geq 0$ and $s_{it} = 0 \ \forall t, i$). Figure 2 shows an example of this approach.

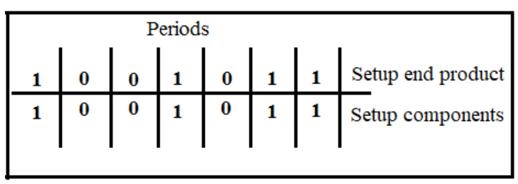

Figure 2: Characteristic of the JIT for the components approach

Table 10 presents the general results considering the JIT for the components approach. Note that the computational times are much bigger than for the lot-for-lot approach. Considering the quality of the solutions, as expected, this approach is, in general, much better than the lot-for-lot approach. However, there is still a big difference compared to the integrated model and as in the lot-for-lot approach, this difference increases with an increasing level of flexibility. We see that for the maximum level of flexibility considered (80%) the best performance of this approach is still 10.0% worse than the integrated model (for the case with high production costs for the components). On the other hand, the worst performance is equal to 73.0% (for the case with high setup costs for the end product and low production costs for the components), which indicates that there is a significant benefit in considering the integrated model compared to the JIT for the components approach.

5.3 JIT for the end product

In this section we analyse the value of integration, compared to the just-in-time approach for the end product. In this approach we produce the end product every day (lot-for-lot) but it is not necessary to produce the components every day. In other words, we consider the possibility of inventory of the components but not for the end product ($s_t^E = 0$ and $s_{it} \geq 0 \ \forall t, i$). Figure 3 shows an example with the characteristics of this approach.

Table 10: General results for the JIT for components approach

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	128.7	1.5	128.9	983.1	129.2	985.4	129.7	979.4	130.7	979.5	133.2	975.8
	T	142.7	0.1	143.0	670.6	143.4	665.4	144.2	654.1	145.7	647.6	149.5	640.9
	2	133.6	1.2	133.8	586.5	134.0	587.1	134.5	576.9	135.5	575.5	137.8	574.3
Comp.	5	134.1	0.2	134.4	861.8	134.6	850.6	135.2	844.7	136.3	835.7	139.0	819.7
	10	139.4	1.1	139.8	1033.8	140.2	1038.6	141.1	1030.1	142.9	1029.9	147.2	1030.9
	1	110.7	0.5	110.8	518.5	110.9	517.2	111.1	512.6	111.5	503.9	112.6	499.0
TBO	2	141.8	0.6	142.1	886.2	142.4	886.8	143.0	875.0	144.4	877.1	147.7	873.6
	3	154.6	1.3	155.1	1077.4	155.6	1072.2	156.6	1064.1	158.7	1059.6	163.5	1052.3
S. cost	H	164.4	1.7	164.8	1271.0	165.3	1265.2	166.3	1253.5	168.3	1246.1	173.0	1236.9
End	N	134.8	0.7	135.1	836.3	135.5	834.7	136.1	826.5	137.6	827.5	141.0	819.0
	L	107.9	0.1	108.0	374.8	108.1	376.3	108.3	371.7	108.8	367.0	110.0	369.1
P. cost	H	107.9	0.1	108.0	377.7	108.1	376.8	108.8	369.5	109.3	368.6	110.6	365.0
Ing.	N	134.8	0.7	135.1	839.4	135.5	837.3	136.1	835.0	137.6	825.6	140.7	818.5
	L	164.4	1.7	164.8	1264.9	165.3	1262.2	166.3	1247.2	168.3	1246.4	173.0	1241.4
AV.		135.7	0.8	136.0	826.9	136.3	825.4	137.0	816.8	138.2	813.6	141.4	808.4

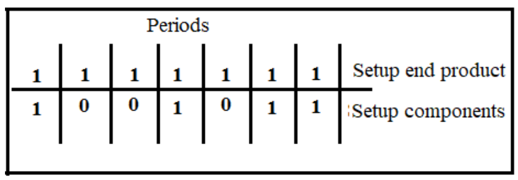

Figure 3: Characteristic of the JIT for the end product approach

In Table 11 we give the general results considering the JIT for the end product approach. In relation to the computational times, as in the lot-for-lot approach, they are almost zero for all instances. We observe that the solutions found by this approach are significantly better than the two previous approaches considered. Compared to the integrated model, we see that the biggest and smallest difference, compared to the solution of the integrated model, is equal to 18.7% and 1.3%, respectively.

Table 11: General results for produce end product every day

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	111.7	0.2	111.8	0.2	111.5	0.3	112.0	0.4	112.3	0.5	113.1	0.4
	T	104.3	0.2	104.3	0.2	104.5	0.3	104.5	0.2	105.0	0.3	104.8	0.2
	2	108.3	0.1	108.3	0.1	108.3	0.1	108.5	0.1	108.7	0.1	109.1	0.1
Comp.	5	108.2	0.2	108.3	0.2	108.1	0.3	108.5	0.3	108.7	0.4	109.2	0.3
	10	107.5	0.3	107.6	0.4	107.6	0.5	107.8	0.5	108.6	0.6	108.6	0.4
	1	108.3	0.3	108.3	0.2	108.2	0.3	108.5	0.3	108.9	0.4	109.2	0.2
TBO	2	107.8	0.2	107.9	0.2	107.8	0.3	108.1	0.4	108.4	0.4	108.8	0.3
	3	108.0	0.1	108.0	0.2	108.0	0.2	108.1	0.2	108.6	0.3	108.9	0.2
S. cost	H	117.0	0.1	117.1	0.1	116.8	0.2	117.4	0.2	118.2	0.2	118.7	0.2
End	N	105.7	0.2	105.8	0.2	105.8	0.3	105.9	0.4	106.3	0.4	106.6	0.3
	L	101.4	0.3	101.4	0.3	101.4	0.3	101.4	0.3	101.4	0.4	101.6	0.3
P. cost	H	101.3	0.3	101.4	0.3	101.4	0.3	101.4	0.4	101.4	0.4	101.6	0.3
Ing.	N	105.7	0.2	105.8	0.2	105.7	0.3	105.9	0.3	106.3	0.4	106.6	0.3
	L	117.0	0.1	117.1	0.1	116.8	0.2	117.4	0.2	118.2	0.2	118.6	0.2
AV.		108.0	0.2	108.1	0.2	108.0	0.3	108.2	0.3	108.6	0.4	108.9	0.3

5.4 Solving the models sequentially

In this section we analyse the value of the integration of the lot sizing and blending problem considering the approach in which the models are solved sequentially. In other words, we first solve the lot sizing problem for the end product (using the appropriate terms of the objective function (1) and constraints (2), (3), (4) and (10)) and with the values of the variables x_t^E that were found we solve the problem for the components (using the appropriate terms of the objective function and constraints (5)–(9) and (11)).

In Table 12 we see the general results considering solving the models sequentially. We observe that the results of this approach is similar to the integrated model for most instances and the biggest and smallest difference, compared to to the solutions of the integrated model, is equal to 1.8% and 0.2%, respectively. In relation to the computational times, we see that they are very small for almost instances.

Table 12: General results considering solving the models sequentially

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	101.0	1.1	101.1	1.0	101.0	1.0	101.2	1.0	100.3	1.0	101.4	1.0
	T	100.3	0.2	100.3	0.4	100.4	0.4	100.3	0.4	100.1	0.4	100.5	0.5
	2	100.5	0.5	100.4	0.5	100.6	0.6	100.5	0.6	100.1	0.6	100.7	0.5
Comp.	5	100.7	0.4	100.8	0.5	100.7	0.5	100.8	0.5	100.3	0.4	101.2	0.6
	10	100.8	1.1	100.9	1.1	100.8	1.0	100.9	1.0	100.2	1.1	101.0	1.1
	1	100.7	0.6	100.7	0.8	100.7	0.8	100.7	0.8	100.1	0.8	101.0	0.8
TBO	2	100.6	0.7	100.7	0.7	100.7	0.7	100.7	0.7	100.2	0.7	100.9	0.7
	3	100.7	0.5	100.7	0.6	100.7	0.6	100.8	0.6	100.2	0.6	101.0	0.7
S. cost	H	101.4	1.1	101.4	1.2	101.7	1.2	101.5	1.2	100.3	1.2	101.8	1.2
End	N	100.4	0.5	100.4	0.5	100.5	0.5	100.4	0.6	100.1	0.6	100.6	0.5
	L	100.2	0.4	100.2	0.5	100.2	0.4	100.2	0.3	100.5	0.4	100.3	0.4
P. cost	H	100.2	0.4	100.2	0.3	100.2	0.4	100.2	0.4	100.4	0.4	100.4	0.3
Ing.	N	100.4	0.5	100.4	0.5	100.5	0.5	100.4	0.6	100.1	0.5	100.6	0.5
	L	101.4	1.1	101.4	1.3	101.5	1.2	101.5	1.1	100.3	1.2	101.8	1.3
Av.		100.7	0.7	100.7	0.7	100.7	0.7	100.7	0.7	100.3	0.7	100.9	0.7

After some additional computational tests, we observed that the value of the integrated model in relation to the approach in which the models are solved sequentially appears especially for instances in which the setup and inventory holding costs of the components are relatively bigger compared to the costs of the end product.

Note that when we decide on the production plan for the components in the sequential approach (levels of inventory and number of setups), we consider a fixed demand (the variables x_t^E are fixed) and there is no possibility of changing the levels of production and inventory of the end product, in order to try to reduce the total production costs. On the other hand, in the integrated approach, where the decisions regarding the end product and its components are taken simultaneously, it is possible to increase a bit the cost to produce the end product in order to reduce the cost of components, and consequently, reduce the total production cost. Therefore, in this section, to stress the value of the integration, we also generate instances with high setup and inventory costs of the components. To do so, we multiply the setup costs of the components by 5 and 10 and the inventory costs of the components by 10 and 100, resulting in four different classes of problems.

Tables 13 and 14 show the results solving the models sequentially considering the component setup cost multiplied by 5 and the component inventory costs multiplied by 10 (Table 13) and 100 (Table 14). The global analysis shows that the value of the integration is significant for the instances with normal capacity, high setup cost of the end product and low production cost of the components. Furthermore, we see that, in general, the value of the integration decreases a bit when increasing the level of flexibility.

Table 13 contains the results for the instances in which the component setup and inventory costs are multiplied by 5 and 10, respectively. We see that on average, the highest value of the integration is equal to 2.95% which is found for the settings with small amount of flexibility (up to 10%). Note that the value of the integration decreases to 2.10% for the instances with 80% of flexibility.

Table 13: General results with component setup costs multiplied by 5 and component inventory cost multiplied by 10

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	104.3	1.1	104.3	1.3	104.3	1.4	104.2	1.6	104.0	1.7	103.1	1.7
	T	101.6	0.4	101.1	3.9	101.6	4.5	101.6	5.4	101.4	5.7	101.1	4.6
	2	102.9	1.2	102.9	1.1	102.9	1.1	102.8	1.1	102.7	1.2	102.1	1.1
Comp..	5	103.0	0.6	103.0	1.5	103.0	1.6	102.9	2.0	102.8	2.2	102.2	2.1
	10	103.0	0.6	103.0	5.4	103.0	6.2	102.9	7.2	102.7	7.7	102.1	6.2
	1	101.7	0.9	101.7	4.1	101.7	3.6	101.6	3.7	101.5	3.3	101.1	3.8
TBO	2	103.3	0.7	103.3	2.2	103.3	3.5	103.2	4.0	102.9	4.6	102.2	2.4
	3	103.9	0.8	103.9	1.6	103.9	1.8	103.8	2.8	103.7	3.2	103.0	3.2
S. cost	H	104.0	1.1	104.0	4.1	104.0	3.9	103.9	4.4	103.7	4.1	102.9	2.9
End	N	103.2	0.6	103.2	2.6	103.2	3.4	103.1	3.8	102.9	3.9	102.3	2.8
	L	101.7	0.6	101.7	1.2	101.7	1.6	101.6	2.1	101.5	30	101.1	3.8
P. cost	H	101.7	0.5	101.7	1.5	101.7	1.9	101.6	2.6	101.6	3.1	101.1	3.5
Ing.	N	103.2	0.6	103.2	3.7	103.2	3.6	103.1	4.3	102.9	4.1	102.3	3.0
	L	104.0	1.2	104.0	2.8	104.0	3.5	103.9	3.5	103.7	3.9	102.9	2.9
Av.		103.0	0.8	103.0	2.7	103.0	2.5	102.9	3.5	102.7	3.7	102.1	3.1

Table 14 contains the results for the instances in which the component setup and inventory costs are multiplied by 5 and 100, respectively. We observe that the value of the integration increases significantly compared to the results of Table 13. In this setting, the highest value of the integration is, on average, equal to 5.3%. Moreover, different from the results of Table 13, the computational times are significantly larger for the instances with high levels of flexibility.

Table 14: General results with component setup costs multiplied by 5 and component inventory cost multiplied by 100

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	108.0	1.2	108.0	4.4	107.9	11.2	107.8	11.9	107.6	14.3	106.8	44.0
	T	102.6	0.6	102.6	93.9	102.6	162.3	102.6	218.3	102.5	270.6	102.2	415.1
	2	105.2	1.3	105.2	1.2	105.1	1.2	105.1	1.3	104.9	1.3	104.4	1.3
Comp.	5	105.3	0.7	105.3	18.3	105.3	40.1	105.2	64.2	105.1	119.5	104.6	310.8
	10	105.4	1.7	105.3	127.9	105.3	219.0	105.2	279.8	105.1	306.4	104.5	376.7
	1	102.3	0.8	102.3	18.5	102.3	34.6	102.3	52.7	102.3	80.8	102.1	153.6
TBO	2	105.8	0.9	105.8	48.4	105.8	81.8	105.7	101.5	105.6	115.6	104.9	251.2
	3	107.7	0.9	107.7	80.4	107.6	143.9	107.5	191.1	107.2	230.8	106.6	284.0
S. cost	H	108.3	1.4	108.2	90.2	108.2	152.3	108.1	201.0	107.8	237.6	106.9	307.0
End	N	105.3	0.8	105.3	47.4	105.3	86.4	105.2	111.6	105.0	137.7	104.6	246.5
	L	102.4	4.5	102.3	9.7	102.3	21.5	102.3	32.7	102.3	51.9	102.1	135.2
P. cost	H	102.3	0.5	102.3	10.2	102.3	18.2	102.3	32.6	102.3	51.4	102.1	136.0
Ing.	N	105.3	0.8	105.3	54.6	105.3	89.6	105.2	112.2	105.0	140.0	104.6	242.5
	L	108.3	1.5	108.2	82.6	108.2	152.5	108.1	200.6	107.8	235.8	106.9	310.2
Av.		105.3	0.9	105.3	49.1	105.2	86.8	105.2	115.1	105.1	142.5	104.5	229.6

Tables 15 and 16 present the results solving the models sequentially considering the setup cost of the components multiplied by 10 and the inventory costs of the components multiplied by 10 (Table 15) and 100 (Table 16). The overall results confirms the insights of the previous experiments. The integration is more important for the instances with a small level of flexibility and for the instances with normal capacity, high setup cost of the end product and low production cost of the components.

In Table 15 we present the results for the instances in which the component setup and inventory costs are multiplied by 10 and 10, respectively. We see that, similar to the instances with the setup costs multiplied by 5 (Table 13), the computational times are small for all classes of problems. Furthermore, the value of the integration is bigger compared to Table 13 in which the highest value is on average equal to 3.0%.

Table 15: General results with component setup costs multiplied by 10 and component inventory cost multiplied by 10

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	105.2	1.3	105.1	1.5	105.1	1.6	105.0	1.7	104.8	1.8	103.8	1.9
	T	102.1	0.4	102.1	1.8	102.1	2.9	102.0	4.7	101.9	7.9	101.5	4.4
	2	103.5	1.3	103.5	1.2	103.5	1.2	103.4	1.2	103.2	1.2	102.6	1.2
Comp.	5	103.6	0.6	103.6	1.6	103.6	2.0	103.5	2.5	103.3	3.1	102.7	2.5
	10	103.8	0.6	103.8	2.1	103.7	3.6	103.7	5.8	103.5	10.2	102.7	5.7
	1	102.3	0.9	102.3	1.7	102.3	1.8	102.2	2.9	102.2	5.4	101.6	3.3
TBO	2	103.7	0.8	103.7	1.5	103.6	2.1	103.6	2.7	103.5	4.5	102.9	2.8
	3	105.0	0.7	105.0	1.8	104.9	2.9	104.8	3.9	104.4	4.6	103.4	3.3
S. cost	H	104.4	1.2	104.3	1.7	104.3	1.9	104.2	2.2	104.0	4.3	103.4	2.3
End	N	103.9	0.7	103.9	1.7	103.9	2.3	103.8	3.2	103.6	5.0	102.9	2.8
	L	102.7	0.6	102.7	1.6	102.6	2.6	102.6	4.1	102.4	5.2	101.7	4.3
P. cost	H	102.6	0.6	102.6	1.5	102.6	1.9	102.6	3.2	102.3	6.6	101.7	4.5
Ing.	N	103.9	0.7	103.9	1.7	103.9	2.9	103.8	3.8	103.6	4.9	102.9	2.8
	L	104.4	1.3	104.3	1.8	104.3	2.0	104.2	2.5	104.0	3.0	103.4	2.1
Av.		103.7	0.9	103.6	1.7	103.6	2.3	103.5	3.2	103.4	4.9	102.7	3.2

Table 16 contains the results for the instances in which the component setup and inventory costs are multiplied by 10 and 100, respectively. Table 15 shows that for some classes of problems the value of the integration is larger than 11% for a small level of flexibility and the smallest value at these levels of flexibility is 4%, which shows that considering the instances with high setup and inventory cost of the components, the benefits obtained by solving the integrated model can be significant.

Table 16: General results with component setup costs multiplied by 10 and component inventory cost multiplied by 100

		base		5%		10%		20%		40%		80%	
		UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)	UB	T(s)
Cap	N	111.3	1.2	111.2	4.8	111.2	11.1	111.0	13.1	110.6	12.2	109.3	48.5
	T	103.8	0.6	103.8	101.4	103.8	176.3	103.7	242.9	103.6	290.7	103.2	411.2
	2	107.4	1.3	107.4	1.2	107.4	1.2	107.3	1.3	107.0	1.3	106.1	1.3
Comp.	5	107.6	0.6	107.6	19.8	107.5	42.6	107.4	66.3	107.2	133.6	106.4	320.0
	10	107.7	0.7	107.6	138.3	107.6	237.4	107.4	316.5	107.2	319.4	106.3	368.3
	1	104.0	0.9	104.0	43.1	103.9	66.4	103.9	91.1	103.7	100.7	103.3	191.2
TBO	2	108.6	0.9	108.5	55.5	108.4	100.2	108.2	130.8	107.8	169.5	106.8	233.0
	3	110.1	0.9	110.1	60.7	110.1	114.6	110.0	162.1	109.8	184.1	108.7	265.3
S. cost	H	111.2	1.3	111.1	83.4	111.0	137.0	110.8	187.5	110.3	199.8	109.0	224.6
End	N	107.5	0.8	107.5	49.2	107.4	97.3	107.3	130.8	107.2	168.1	106.4	272.2
	L	104.0	0.6	104.0	26.7	104.0	46.9	103.9	65.7	103.8	86.4	103.4	192.7
P. cost	H	104.0	0.5	104.0	21.9	104.0	45.3	103.9	68.5	103.8	85.4	103.4	198.8
Ing.	N	107.5	0.8	107.5	51.1	107.4	93.2	107.4	130.6	107.2	173.3	106.4	269.1
	L	111.2	1.3	111.1	82.3	111.0	142.6	110.8	184.8	110.3	195.5	109.0	221.7
Av.		107.6	0.9	107.5	53.1	107.5	93.7	107.4	127.9	107.1	151.5	106.3	229.9

6 Conclusions

In this paper, the integrated lot sizing and blending problem is studied. Three different new formulations have been proposed. Furthermore, the value of the BOM flexibility is analysed, i.e., the proportion imposed for each of the components can vary between a minimum and a maximum level instead of being fixed. Finally, the value of the integration of these two problems is also analysed compared to four different approaches that do not fully capture this integration. Our computational experiments show that there is a significant difference in terms of the LP values for the proposed formulations. The formulation using the transportation approach and the multi-commodity formulation both found better lower bounds, especially for instances with a high level of flexibility. However, the IP values are similar for all classes of instances considering the fixed time limit of 1800 seconds. The results also show that the value of BOM flexibility depends on the characteristics of the instances and this value is highest for the instances with 10 components, low setup cost for the end product and high production cost of the components in which the benefits of BOM flexibility reach 16.5%. Finally, we also see that there is a significant value of considering the integrated model compared to the lot-for-lot, just-in-time and sequential approach.

There are several interesting issues that can be explored as further research. For example, we can focus on devising specific heuristics algorithms to solve the integrated models. It would also be interesting to study some extensions of the problem, such as, multiple end products and production capacity constraints for the components. Another relevant extension would be the inclusion of storage constraints, e.g., different types of animal food can be stored in silos, which each have their own storage limits. Finally, it is also interesting to analyse if standard cutting planes developed for the basic lot sizing model can be adapted for this problem.

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