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A Progressive approximation approach for the exact solution of sparse large-scale binary interdiction games

Supplementary material

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Appendix

A Detailed results

In this online appendix we report detailed results of our algorithm, this is the lower bound, upper bound, and CPU time takes by the algorithm. Whenever the time limit of 1 day or the memory limit of 16GB are exceeded, we rather report it as TL or ML, respectively. For the former, lower, upper bounds and gaps are reported, while for the latter no log information is available.

A.1 Shortest path interdiction

In this section we report the detailed results of our method on the shortest pat instances considered in our study. The results are grouped by budget size and reportted in Tables 9–26. The results for small budgets ($\Delta \in \{1, 3, 5\}$) are reported in separate tables as those for large budgets ($\Delta \in \{10, 15, 20\}$).

Table 9: Detailed results for road network DC for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	8,349	8,349	0.00	18.9
1	2	4,559	4,559	0.00	19.5
1	3	19,774	19,774	0.00	19.3
1	4	21,575	21,575	0.00	19.6
1	5	10,438	10,438	0.00	19.6
1	6	9,638	9,638	0.00	20.0
1	7	14,028	14,028	0.00	20.0
1	8	10,262	10,262	0.00	19.5
1	9	15,928	15,928	0.00	18.4
3	1	8,990	8,990	0.00	19.4
3	2	4,883	4,883	0.00	19.5
3	3	24,002	24,002	0.00	19.6
3	4	22,167	22,167	0.00	20.6
3	5	11,369	11,369	0.00	20.2
3	6	10,033	10,033	0.00	19.6
3	7	14,937	14,937	0.00	19.8
3	8	10,838	10,838	0.00	20.1
3	9	16,443	16,443	0.00	20.4
5	1	9,394	9,394	0.00	20.2
5	2	5,146	5,146	0.00	20.2
5	3	24,731	24,731	0.00	21.4
5	4	22,592	22,592	0.00	20.3
5	5	12,222	12,222	0.00	20.6
5	6	10,403	10,403	0.00	20.4
5	7	15,367	15,367	0.00	20.2
5	8	11,366	11,366	0.00	19.7
5	9	16,785	16,785	0.00	21.3

Table 10: Detailed results for road network DC for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	10,209	10,209	0.00	20.1
10	2	5,620	5,620	0.00	20.3
10	3	25,683	25,683	0.00	22.9
10	4	23,369	23,369	0.00	21.2
10	5	13,157	13,157	0.00	21.0
10	6	11,220	11,220	0.00	20.0
10	7	16,276	16,276	0.00	20.2
10	8	12,319	12,319	0.00	20.5
10	9	17,447	17,447	0.00	23.3
15	1	10,869	10,869	0.00	20.6
15	2	5,897	5,897	0.00	21.8
15	3	26,198	26,198	0.00	28.0
15	4	24,003	24,003	0.00	22.3
15	5	13,728	13,728	0.00	20.8
15	6	11,893	11,893	0.00	20.3
15	7	17,066	17,066	0.00	20.4
15	8	13,016	13,016	0.00	20.8
15	9	18,028	18,028	0.00	29.5
20	1	11,342	11,342	0.00	21.9
20	2	6,082	6,082	0.00	25.1
20	3	26,607	26,607	0.00	26.7
20	4	24,485	24,485	0.00	23.0
20	5	14,183	14,183	0.00	21.5
20	6	12,464	12,464	0.00	21.3
20	7	17,770	17,770	0.00	20.3
20	8	13,605	13,605	0.00	21.4
20	9	18,506	18,506	0.00	41.0

Table 11: Detailed results for road network RI for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	21,935	21,935	0.00	20.5
1	2	21,175	21,175	0.00	19.9
1	3	49,077	49,077	0.00	20.6
1	4	40,630	40,630	0.00	20.2
1	5	55,838	55,838	0.00	21.2
1	6	33,536	33,536	0.00	20.8
1	7	36,853	36,853	0.00	20.4
1	8	59,960	59,960	0.00	21.7
1	9	21,158	21,158	0.00	20.2
3	1	22,388	22,388	0.00	21.7
3	2	24,649	24,649	0.00	21.1
3	3	53,252	53,252	0.00	23.7
3	4	43,008	43,008	0.00	23.2
3	5	61,145	61,145	0.00	25.1
3	6	35,303	35,303	0.00	23.0
3	7	42,303	42,303	0.00	21.8
3	8	62,102	62,102	0.00	25.2
3	9	23,333	23,333	0.00	22.5
5	1	25,523	25,523	0.00	23.1
5	2	25,488	25,488	0.00	23.5
5	3	57,321	57,321	0.00	25.9
5	4	44,546	44,546	0.00	25.9
5	5	64,646	64,646	0.00	27.9
5	6	36,151	36,151	0.00	25.3
5	7	43,872	43,872	0.00	25.6
5	8	63,368	63,368	0.00	28.2
5	9	24,374	24,374	0.00	24.3

Table 12: Detailed results for road network RI for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	27,407	27,407	0.00	26.3
10	2	26,148	26,148	0.00	26.3
10	3	60,325	60,325	0.00	40.8
10	4	48,401	48,401	0.00	31.9
10	5	68,532	68,532	0.00	44.4
10	6	38,358	38,358	0.00	31.8
10	7	46,712	46,712	0.00	27.0
10	8	66,102	66,102	0.00	94.6
10	9	26,023	26,023	0.00	30.9
15	1	31,037	31,037	0.00	30.5
15	2	26,462	26,462	0.00	27.3
15	3	61,627	61,627	0.00	252.7
15	4	49,833	49,833	0.00	40.3
15	5	70,346	70,346	0.00	276.0
15	6	40,041	40,041	0.00	42.7
15	7	48,191	48,191	0.00	50.5
15	8	67,785	67,785	0.00	232.2
15	9	27,214	27,214	0.00	42.5
20	1	32,113	32,113	0.00	45.3
20	2	26,694	26,694	0.00	33.0
20	3	63,360	63,360	0.00	226.1
20	4	51,115	51,115	0.00	181.2
20	5	71,561	71,561	0.00	630.7
20	6	41,214	41,214	0.00	53.6
20	7	49,159	49,159	0.00	34.3
20	8	69,491	69,491	0.00	638.1
20	9	28,190	28,190	0.00	61.7

Table 13: Detailed results for road network NJ for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	77,813	77,813	0.00	29.8
1	2	83,666	83,666	0.00	31.7
1	3	50,274	50,274	0.00	30.7
1	4	79,280	79,280	0.00	30.7
1	5	65,396	65,396	0.00	29.1
1	6	52,189	52,189	0.00	28.1
1	7	22,578	22,578	0.00	24.4
1	8	39,951	39,951	0.00	27.0
1	9	65,442	65,442	0.00	27.4
3	1	82,919	82,919	0.00	35.5
3	2	93,666	93,666	0.00	60.8
3	3	51,863	51,863	0.00	46.5
3	4	84,356	84,356	0.00	54.4
3	5	69,821	69,821	0.00	33.6
3	6	53,616	53,616	0.00	37.2
3	7	28,329	28,329	0.00	31.7
3	8	44,056	44,056	0.00	33.6
3	9	69,416	69,416	0.00	45.2
5	1	85,480	85,480	0.00	57.2
5	2	99,401	99,401	0.00	64.6
5	3	52,449	52,449	0.00	65.9
5	4	86,135	86,135	0.00	73.6
5	5	72,316	72,316	0.00	47.5
5	6	57,448	57,448	0.00	46.6
5	7	31,059	31,059	0.00	41.7
5	8	45,260	45,260	0.00	40.1
5	9	71,113	71,113	0.00	53.0

Table 14: Detailed results for road network NJ for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	91,831	91,831	0.00	77.4
10	2	105,113	105,113	0.00	202.0
10	3	56,063	56,063	0.00	196.8
10	4	89,778	89,778	0.00	373.3
10	5	78,050	78,050	0.00	125.2
10	6	61,334	61,334	0.00	123.7
10	7	33,130	33,130	0.00	61.5
10	8	47,604	47,604	0.00	91.4
10	9	76,754	76,754	0.00	112.3
15	1	95,172	95,172	0.00	372.2
15	2	109,198	109,198	0.00	841.1
15	3	59,262	59,262	0.00	485.5
15	4	92,792	92,792	0.00	986.0
15	5	82,891	82,891	0.00	234.7
15	6	63,391	63,391	0.00	210.4
15	7	33,858	33,858	0.00	88.3
15	8	49,598	49,598	0.00	179.8
15	9	79,911	79,911	0.00	339.1
20	1	96,910	96,910	0.00	421.3
20	2	111,090	111,090	0.00	4,493.3
20	3	61,546	61,546	0.00	785.7
20	4	95,110	95,110	0.00	3,786.5
20	5	85,345	85,345	0.00	454.7
20	6	64,862	64,862	0.00	425.7
20	7	34,570	34,570	0.00	130.8
20	8	50,697	50,697	0.00	164.9
20	9	81,970	81,970	0.00	1,059.4

Table 15: Detailed results for road network NY for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	175,225	175,225	0.00	65.5
1	2	80,937	80,937	0.00	44.1
1	3	84,957	84,957	0.00	53.8
1	4	97,566	97,566	0.00	53.7
1	5	113,724	113,724	0.00	46.2
1	6	79,085	79,085	0.00	58.1
1	7	74,401	74,401	0.00	42.2
1	8	114,827	114,827	0.00	51.8
1	9	131,558	131,558	0.00	65.2
3	1	183,534	183,534	0.00	103.1
3	2	87,183	87,183	0.00	85.9
3	3	90,262	90,262	0.00	68.3
3	4	105,954	105,954	0.00	90.1
3	5	123,724	123,724	0.00	57.6
3	6	81,198	81,198	0.00	91.9
3	7	84,401	84,401	0.00	84.5
3	8	124,827	124,827	0.00	91.5
3	9	138,156	138,156	0.00	106.6
5	1	187,873	187,873	0.00	139.6
5	2	92,753	92,753	0.00	123.4
5	3	93,055	93,055	0.00	93.4
5	4	108,713	108,713	0.00	100.2
5	5	130,287	130,287	0.00	82.4
5	6	82,515	82,515	0.00	128.0
5	7	91,439	91,439	0.00	101.9
5	8	128,258	128,258	0.00	130.7
5	9	143,676	143,676	0.00	127.5

Table 16: Detailed results for road network NY for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	196,082	196,082	0.00	1,053.6
10	2	99,829	99,829	0.00	379.7
10	3	98,901	98,901	0.00	203.2
10	4	113,631	113,631	0.00	189.9
10	5	143,724	143,724	0.00	110.9
10	6	87,457	87,457	0.00	554.9
10	7	97,226	97,226	0.00	153.3
10	8	133,675	133,675	0.00	639.9
10	9	151,458	151,458	0.00	747.2
15	1	202,008	202,008	0.00	2,581.2
15	2	106,431	106,431	0.00	655.9
15	3	102,614	102,614	0.00	453.0
15	4	117,866	117,866	0.00	385.8
15	5	148,721	148,721	0.00	818.9
15	6	91,198	91,198	0.00	1,243.1
15	7	99,745	99,745	0.00	438.7
15	8	140,560	140,560	0.00	794.8
15	9	154,341	154,341	0.00	6,917.2
20	1	204,989	206,996	0.98	TL
20	2	109,453	109,453	0.00	1,272.6
20	3	104,677	104,677	0.00	713.3
20	4	121,505	121,505	0.00	604.5
20	5	152,981	152,981	0.00	2,514.6
20	6	93,747	93,759	0.10	TL
20	7	101,126	101,126	0.00	3,729.9
20	8	145,622	145,622	0.00	729.2
20	9	156,736	158,446	1.90	TL

Table 17: Detailed results for road network CA for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	136,846	136,846	0.00	128.8
1	2	71,710	71,710	0.00	93.0
1	3	141,670	141,670	0.00	103.4
1	4	102,215	102,215	0.00	102.0
1	5	72,666	72,666	0.00	92.1
1	6	106,241	106,241	0.00	118.1
1	7	127,194	127,194	0.00	111.7
1	8	43,223	43,223	0.00	106.0
1	9	110,300	110,300	0.00	104.6
3	1	137,628	137,628	0.00	240.5
3	2	76,297	76,297	0.00	151.6
3	3	151,484	151,484	0.00	188.7
3	4	114,799	114,799	0.00	92.0
3	5	82,666	82,666	0.00	178.3
3	6	116,241	116,241	0.00	152.6
3	7	139,424	139,424	0.00	211.2
3	8	43,996	43,996	0.00	189.3
3	9	113,095	113,095	0.00	209.9
5	1	140,537	140,537	0.00	370.9
5	2	79,505	79,505	0.00	290.5
5	3	153,733	153,733	0.00	247.1
5	4	124,799	124,799	0.00	194.4
5	5	86,896	86,896	0.00	260.7
5	6	120,158	120,158	0.00	192.0
5	7	144,158	144,158	0.00	273.3
5	8	44,688	44,688	0.00	228.4
5	9	118,176	118,176	0.00	325.8

Table 18: Detailed results for road network CA for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	147,628	147,628	0.00	1,023.5
10	2	84,821	84,821	0.00	455.0
10	3	161,484	161,484	0.00	1,157.8
10	4	135,862	135,862	0.00	226.6
10	5	94,665	94,665	0.00	358.3
10	6	126,316	126,316	0.00	541.8
10	7	153,852	153,852	0.00	731.1
10	8	45,841	45,841	0.00	321.3
10	9	123,669	123,669	0.00	864.8
15	1	152,818	152,818	0.00	2,590.3
15	2	87,581	87,581	0.00	830.4
15	3	165,612	165,612	0.00	1,363.8
15	4	138,792	138,792	0.00	488.8
15	5	99,159	99,159	0.00	588.9
15	6	130,846	130,846	0.00	2,014.8
15	7	159,407	159,407	0.00	1,646.0
15	8	47,153	47,153	0.00	491.0
15	9	129,270	129,270	0.00	1,695.7
20	1	155,602	155,657	0.40	TL
20	2	89,006	89,006	0.00	1,857.9
20	3	169,687	169,687	0.00	5,271.3
20	4	141,414	141,414	0.00	801.2
20	5	102,132	102,132	0.00	1,290.5
20	6	134,401	134,401	0.00	11,850.0
20	7	162,027	162,027	0.00	22,522.0
20	8	48,378	48,378	0.00	452.4
20	9	132,098	132,099	0.00	TL

Table 19: Detailed results for road network FL for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	106,366	106,366	0.00	70.3
1	2	41,519	41,519	0.00	55.6
1	3	100,298	100,298	0.00	78.2
1	4	42,619	42,619	0.00	53.9
1	5	89,912	89,912	0.00	63.4
1	6	123,956	123,956	0.00	82.5
1	7	107,570	107,570	0.00	77.8
1	8	126,256	126,256	0.00	82.9
1	9	26,265	26,265	0.00	52.0
3	1	112,532	112,532	0.00	185.4
3	2	45,249	45,249	0.00	58.0
3	3	106,836	106,836	0.00	113.0
3	4	43,938	43,938	0.00	83.2
3	5	94,429	94,429	0.00	137.6
3	6	129,784	129,784	0.00	177.2
3	7	113,473	113,473	0.00	171.5
3	8	131,371	131,371	0.00	168.9
3	9	27,763	27,763	0.00	100.5
5	1	119,703	119,703	0.00	214.6
5	2	45,928	45,928	0.00	114.4
5	3	112,016	112,016	0.00	198.2
5	4	45,366	45,366	0.00	90.8
5	5	99,327	99,327	0.00	169.3
5	6	132,598	132,598	0.00	225.1
5	7	119,235	119,235	0.00	149.1
5	8	138,038	138,038	0.00	195.6
5	9	28,621	28,621	0.00	135.7

Table 20: Detailed results for road network FL for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	126,006	126,006	0.00	771.5
10	2	48,971	48,971	0.00	208.1
10	3	117,904	117,904	0.00	550.1
10	4	47,875	47,875	0.00	131.5
10	5	104,672	104,672	0.00	527.5
10	6	138,444	138,444	0.00	1,079.4
10	7	125,125	125,125	0.00	447.0
10	8	147,658	147,658	0.00	403.3
10	9	30,934	30,934	0.00	196.5
15	1	130,224	130,224	0.00	2,439.2
15	2	51,667	51,667	0.00	284.7
15	3	123,265	123,265	0.00	1,670.7
15	4	49,014	49,014	0.00	124.7
15	5	107,805	107,805	0.00	2,260.8
15	6	142,062	142,062	0.00	4,531.6
15	7	129,098	129,098	0.00	1,235.0
15	8	152,851	152,851	0.00	1,014.7
15	9	31,950	31,950	0.00	323.7
20	1	134,457	134,457	0.00	42,592.0
20	2	52,566	52,566	0.00	337.4
20	3	126,521	126,521	0.00	4,681.5
20	4	50,126	50,126	0.00	135.2
20	5	110,581	110,581	0.00	12,665.4
20	6	145,560	145,599	0.30	TL
20	7	131,168	131,168	0.00	7,066.6
20	8	156,409	156,409	0.00	9,912.6
20	9	32,650	32,650	0.00	279.5

Table 21: Detailed results for road network TX for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	152,895	152,895	0.00	134.0
1	2	62,247	62,247	0.00	78.0
1	3	171,331	171,331	0.00	136.9
1	4	154,008	154,008	0.00	93.0
1	5	185,548	185,548	0.00	132.2
1	6	105,812	105,812	0.00	121.8
1	7	143,700	143,700	0.00	139.8
1	8	67,570	67,570	0.00	83.2
1	9	182,579	182,579	0.00	116.9
3	1	169,929	169,929	0.00	123.7
3	2	68,271	68,271	0.00	198.1
3	3	183,220	183,220	0.00	174.5
3	4	165,306	165,306	0.00	112.3
3	5	205,548	205,548	0.00	135.6
3	6	115,812	115,812	0.00	207.6
3	7	148,847	148,847	0.00	198.4
3	8	75,765	75,765	0.00	128.9
3	9	186,332	186,332	0.00	305.4
5	1	177,628	177,628	0.00	266.5
5	2	69,612	69,612	0.00	366.5
5	3	193,220	193,220	0.00	277.0
5	4	175,306	175,306	0.00	182.1
5	5	219,536	219,536	0.00	250.8
5	6	123,809	123,809	0.00	249.7
5	7	153,700	153,700	0.00	274.9
5	8	78,974	78,974	0.00	237.7
5	9	192,579	192,579	0.00	312.9

Table 22: Detailed results for road network TX for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	191,369	191,369	0.00	328.6
10	2	76,920	76,920	0.00	546.3
10	3	203,280	203,280	0.00	720.7
10	4	187,317	187,317	0.00	410.4
10	5	232,314	232,314	0.00	349.3
10	6	136,651	136,651	0.00	304.2
10	7	163,888	163,888	0.00	995.3
10	8	86,107	86,107	0.00	363.8
10	9	200,801	200,801	0.00	599.0
15	1	198,556	198,556	0.00	867.0
15	2	79,200	79,200	0.00	941.2
15	3	209,228	209,228	0.00	6,196.7
15	4	192,154	192,154	0.00	1,185.5
15	5	239,665	239,665	0.00	966.0
15	6	143,364	143,364	0.00	389.2
15	7	172,270	172,270	0.00	771.9
15	8	88,561	88,561	0.00	645.8
15	9	205,166	205,179	0.10	TL
20	1	201,474	201,474	0.00	12,185.6
20	2	81,959	81,959	0.00	1,153.5
20	3	214,308	214,331	0.10	TL
20	4	199,114	199,114	0.00	3,835.1
20	5	246,356	246,356	0.00	4,698.6
20	6	147,412	147,412	0.00	738.0
20	7	177,083	177,083	0.00	1,710.0
20	8	90,771	90,771	0.00	496.9
20	9	212,632	212,632	0.00	2,118.4

Table 23: Detailed results for road network USE for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	174,195	174,195	0.00	168.0
1	2	303,889	303,889	0.00	146.1
1	3	247,938	247,938	0.00	278.3
1	4	166,443	166,443	0.00	141.1
1	5	282,113	282,113	0.00	230.9
1	6	198,639	198,639	0.00	167.6
1	7	260,480	260,480	0.00	127.4
1	8	358,531	358,531	0.00	147.3
1	9	320,375	320,375	0.00	141.6
3	1	183,930	183,930	0.00	236.4
3	2	317,614	317,614	0.00	309.8
3	3	255,613	255,613	0.00	234.8
3	4	173,069	173,069	0.00	245.9
3	5	302,113	302,113	0.00	291.6
3	6	204,614	204,614	0.00	307.0
3	7	274,093	274,093	0.00	352.6
3	8	369,053	369,053	0.00	325.5
3	9	340,375	340,375	0.00	354.0
5	1	190,440	190,440	0.00	481.7
5	2	329,364	329,364	0.00	407.4
5	3	265,956	265,956	0.00	654.3
5	4	181,665	181,665	0.00	363.9
5	5	311,958	311,958	0.00	439.7
5	6	209,719	209,719	0.00	568.9
5	7	284,093	284,093	0.00	425.8
5	8	379,053	379,053	0.00	902.6
5	9	355,599	355,599	0.00	434.7

Table 24: Detailed results for road network USE for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	202,798	202,798	0.00	1,331.3
10	2	349,253	349,253	0.00	1,096.1
10	3	283,452	283,452	0.00	1,354.2
10	4	196,203	196,203	0.00	952.8
10	5	331,713	331,713	0.00	694.3
10	6	220,384	220,384	0.00	1,983.4
10	7	303,014	303,014	0.00	929.5
10	8	396,066	396,066	0.00	1,242.6
10	9	379,405	379,405	0.00	780.6
15	1	208,913	209,095	0.90	TL
15	2	362,338	362,338	0.00	2,657.6
15	3	298,389	298,389	0.00	3,007.5
15	4	209,605	209,605	0.00	1,473.2
15	5	341,713	341,713	0.00	1,340.0
15	6	226,662	226,685	0.10	TL
15	7	311,262	311,262	0.00	7,596.1
15	8	406,645	406,645	0.00	43,186.2
15	9	395,609	395,609	0.00	3,904.0
20	1	210,486	217,514	3.34	TL
20	2	ML	ML	ML	ML
20	3	307,382	310,262	0.94	TL
20	4	217,986	217,986	0.00	5,152.5
20	5	351,065	351,065	0.00	20,974.7
20	6	234,231	234,241	0.00	TL
20	7	315,460	320,168	1.49	TL
20	8	414,769	417,956	0.77	TL
20	9	393,692	413,764	5.10	TL

Table 25: Detailed results for road network USW for small budgets

Δ	s	LB	UB	Gap	CPU
1	1	391,616	391,616	0.00	351.7
1	2	297,824	297,824	0.00	232.7
1	3	349,856	349,856	0.00	309.2
1	4	132,629	132,629	0.00	192.4
1	5	291,721	291,721	0.00	387.1
1	6	151,992	151,992	0.00	246.9
1	7	291,890	291,890	0.00	393.0
1	8	332,881	332,881	0.00	239.4
1	9	497,970	497,970	0.00	492.9
3	1	401,616	401,616	0.00	972.1
3	2	317,824	317,824	0.00	556.0
3	3	361,869	361,869	0.00	429.0
3	4	142,629	142,629	0.00	427.2
3	5	305,088	305,088	0.00	681.8
3	6	161,992	161,992	0.00	622.3
3	7	308,250	308,250	0.00	574.6
3	8	352,881	352,881	0.00	359.8
3	9	517,970	517,970	0.00	675.4
5	1	411,616	411,616	0.00	899.3
5	2	337,824	337,824	0.00	653.5
5	3	371,869	371,869	0.00	1,185.1
5	4	147,786	147,786	0.00	769.2
5	5	319,391	319,391	0.00	1,006.4
5	6	170,498	170,498	0.00	928.4
5	7	318,250	318,250	0.00	683.7
5	8	362,881	362,881	0.00	1,060.3
5	9	529,710	529,710	0.00	1,051.9

Table 26: Detailed results for road network USW for large budgets

Δ	s	LB	UB	Gap	CPU
10	1	429,797	429,797	0.00	2,079.2
10	2	380,010	380,010	0.00	1,062.1
10	3	395,213	395,213	0.00	2,003.4
10	4	167,734	167,734	0.00	1,443.4
10	5	344,186	344,186	0.00	1,622.7
10	6	182,314	182,314	0.00	2,301.5
10	7	341,431	341,431	0.00	1,512.1
10	8	383,612	383,612	0.00	1,307.8
10	9	555,972	555,972	0.00	6,312.6
15	1	445,719	445,719	0.00	2,929.9
15	2	404,890	404,890	0.00	1,319.2
15	3	412,928	412,928	0.00	3,776.6
15	4	178,184	178,184	0.00	2,787.8
15	5	361,645	361,645	0.00	3,067.6
15	6	192,712	192,712	0.00	3,548.5
15	7	354,651	354,651	0.00	1,914.1
15	8	395,638	395,638	0.00	7,793.4
15	9	ML	ML	ML	ML
20	1	457,266	457,661	0.90	TL
20	2	422,084	422,084	0.00	4,013.4
20	3	427,443	427,631	0.40	TL
20	4	186,594	186,594	0.00	28,692.2
20	5	373,758	373,758	0.00	7,956.0
20	6	198,901	200,216	0.66	TL
20	7	370,476	370,476	0.00	3,098.0
20	8	410,254	410,254	0.00	5,728.4
20	9	571,123	590,648	3.42	TL

A.2 0-1 Knapsack interdiction

In this section, we report detailed results of our method on the 0-1 knapsack instances considered in this study. The results are given separately for every value of $N, \kappa, \alpha^f, \alpha^l$. The results for the 9 problems generated for every choice of these parameters are given in Tables 27–62.

Table 27: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.125$, $\alpha^l = 5$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	60,896	60,896	0.00	28.2
2	61,037	61,037	0.00	25.3
3	59,636	59,636	0.00	27.4
4	57,959	57,959	0.00	26.0
5	63,405	63,405	0.00	26.6
6	64,231	64,231	0.00	35.6
7	60,746	60,746	0.00	25.6
8	65,931	65,931	0.00	27.6
9	62,295	62,295	0.00	23.3

Table 28: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.125$, $\alpha^l = 5$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	103,150	103,150	0.00	50.6
2	107,638	107,638	0.00	49.5
3	105,588	105,588	0.00	51.2
4	106,232	106,232	0.00	35.9
5	107,606	107,606	0.00	42.1
6	101,920	101,920	0.00	32.7
7	111,386	111,386	0.00	42.8
8	101,571	101,571	0.00	42.8
9	101,765	101,765	0.00	58.9

Table 29: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.125$, $\alpha^l = 10$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	62,201	62,201	0.00	32.9
2	59,757	59,757	0.00	29.9
3	61,239	61,239	0.00	43.2
4	60,787	60,787	0.00	33.1
5	62,630	62,630	0.00	50.9
6	62,048	62,048	0.00	36.7
7	58,835	58,835	0.00	38.1
8	62,011	62,011	0.00	31.2
9	63,219	63,219	0.00	29.6

Table 30: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.125$, $\alpha^l = 10$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	105,844	105,844	0.00	163.8
2	102,603	102,603	0.00	58.4
3	101,905	101,905	0.00	58.6
4	100,884	100,884	0.00	90.3
5	104,213	104,213	0.00	67.1
6	106,431	106,431	0.00	161.1
7	107,572	107,572	0.00	83.0
8	104,299	104,299	0.00	141.2
9	101,258	101,258	0.00	124.9

Table 31: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.125$, $\alpha^l = 20$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	61,405	61,405	0.00	67.5
2	58,416	58,416	0.00	47.3
3	59,080	59,080	0.00	132.1
4	62,893	62,893	0.00	89.0
5	63,927	63,927	0.00	39.9
6	59,992	59,992	0.00	73.8
7	60,234	60,234	0.00	43.6
8	62,360	62,360	0.00	44.7
9	57,273	57,273	0.00	62.9

Table 32: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.125$, $\alpha^l = 20$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	100,062	100,062	0.00	239.3
2	108,966	108,966	0.00	360.2
3	105,324	105,324	0.00	161.4
4	102,745	102,745	0.00	307.4
5	108,363	108,363	0.00	171.2
6	102,292	102,292	0.00	249.0
7	104,403	104,403	0.00	265.0
8	100,114	100,114	0.00	158.0
9	103,654	103,654	0.00	462.1

Table 33: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.25$, $\alpha^l = 5$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	56,857	56,857	0.00	30.6
2	60,949	60,949	0.00	25.0
3	58,435	58,435	0.00	28.4
4	55,561	55,561	0.00	27.5
5	63,921	63,921	0.00	26.5
6	59,146	59,146	0.00	35.8
7	61,213	61,213	0.00	28.4
8	61,131	61,131	0.00	32.3
9	63,511	63,511	0.00	29.2

Table 34: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.25$, $\alpha^l = 5$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	102,905	102,905	0.00	76.6
2	105,931	105,931	0.00	55.8
3	104,949	104,949	0.00	39.4
4	104,810	104,810	0.00	42.4
5	100,545	100,545	0.00	54.5
6	101,264	101,264	0.00	73.5
7	100,055	100,055	0.00	40.1
8	106,178	106,178	0.00	70.0
9	99,505	99,505	0.00	91.7

Table 35: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.25$, $\alpha^l = 10$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	59,135	59,135	0.00	35.5
2	65,893	65,893	0.00	40.4
3	63,561	63,561	0.00	31.9
4	57,975	57,975	0.00	38.9
5	64,456	64,456	0.00	35.2
6	62,467	62,467	0.00	53.5
7	64,238	64,238	0.00	34.4
8	60,033	60,033	0.00	48.9
9	64,081	64,081	0.00	61.5

Table 36: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.25$, $\alpha^l = 10$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	103,259	103,259	0.00	199.5
2	100,406	100,406	0.00	133.5
3	100,335	100,335	0.00	504.8
4	98,992	98,992	0.00	300.2
5	100,254	100,254	0.00	107.5
6	101,075	101,075	0.00	127.4
7	96,748	96,748	0.00	76.9
8	96,467	96,467	0.00	214.2
9	99,234	99,234	0.00	83.7

Table 37: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.25$, $\alpha^l = 20$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	55,344	55,344	0.00	247.8
2	58,843	58,843	0.00	53.6
3	59,925	59,925	0.00	152.3
4	58,058	58,058	0.00	110.4
5	57,914	57,914	0.00	657.8
6	61,660	61,660	0.00	123.6
7	60,089	60,089	0.00	82.5
8	59,154	59,154	0.00	204.1
9	60,693	60,693	0.00	143.8

Table 38: Results for 0-1 knapsack instance with N = 10000, $\kappa = 0.25$, $\alpha^l = 20$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	99,287	99,287	0.00	1,421.8
2	100,384	100,384	0.00	740.3
3	101,551	101,551	0.00	728.6
4	106,355	106,355	0.00	1,189.2
5	100,166	100,169	0.00	TL
6	100,796	100,798	0.00	TL
7	98,441	98,441	0.00	999.8
8	97,151	97,151	0.00	190.6
9	92,534	92,534	0.00	315.4

Table 39: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.125$, $\alpha^l = 5$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	140,060	140,060	0.00	53.0
2	140,749	140,749	0.00	59.4
3	147,956	147,956	0.00	58.3
4	144,451	144,451	0.00	54.8
5	131,698	131,698	0.00	58.5
6	135,021	135,021	0.00	56.2
7	144,009	144,009	0.00	57.2
8	144,590	144,590	0.00	46.8
9	139,361	139,361	0.00	51.7

Table 40: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.125$, $\alpha^l = 5$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	214,823	214,823	0.00	76.5
2	208,238	208,238	0.00	64.5
3	217,726	217,726	0.00	154.7
4	222,164	222,164	0.00	76.5
5	214,099	214,099	0.00	92.5
6	199,284	199,284	0.00	68.8
7	215,593	215,593	0.00	80.5
8	215,440	215,440	0.00	137.8
9	213,736	213,736	0.00	79.4

Table 41: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.125$, $\alpha^l = 10$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	139,619	139,619	0.00	78.1
2	134,010	134,010	0.00	68.7
3	142,692	142,692	0.00	72.5
4	141,291	141,291	0.00	72.7
5	149,119	149,119	0.00	68.9
6	148,163	148,163	0.00	108.1
7	147,292	147,292	0.00	67.2
8	141,382	141,382	0.00	61.7
9	140,828	140,828	0.00	75.6

Table 42: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.125$, $\alpha^l = 10$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	213,225	213,225	0.00	160.0
2	204,291	204,291	0.00	341.8
3	215,264	215,264	0.00	367.1
4	220,989	220,989	0.00	484.4
5	218,539	218,539	0.00	115.4
6	220,269	220,269	0.00	144.3
7	216,189	216,189	0.00	114.3
8	220,249	220,249	0.00	185.4
9	211,341	211,341	0.00	130.3

Table 43: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.125$, $\alpha^l = 20$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	141,408	141,408	0.00	169.0
2	154,594	154,594	0.00	262.2
3	139,519	139,519	0.00	133.6
4	141,317	141,317	0.00	212.9
5	145,990	145,990	0.00	158.7
6	139,568	139,568	0.00	348.9
7	140,685	140,685	0.00	167.5
8	148,522	148,522	0.00	116.6
9	138,618	138,618	0.00	144.4

Table 44: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.125$, $\alpha^l = 20$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	206,416	206,416	0.00	251.9
2	218,242	218,242	0.00	612.3
3	203,238	203,238	0.00	233.6
4	210,597	210,597	0.00	1,232.4
5	205,447	205,447	0.00	354.5
6	207,856	207,856	0.00	865.6
7	211,134	211,134	0.00	230.8
8	216,618	216,618	0.00	536.9
9	205,618	205,618	0.00	1,543.3

Table 45: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.25$, $\alpha^l = 5$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	132,747	132,747	0.00	44.7
2	135,946	135,946	0.00	49.4
3	140,686	140,686	0.00	49.3
4	139,239	139,239	0.00	83.3
5	141,921	141,921	0.00	49.9
6	138,655	138,655	0.00	64.6
7	137,429	137,429	0.00	47.5
8	143,028	143,028	0.00	59.1
9	137,592	137,592	0.00	43.3

Table 46: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.25$, $\alpha^l = 5$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	215,844	215,844	0.00	199.0
2	218,653	218,653	0.00	92.9
3	200,480	200,480	0.00	118.7
4	213,602	213,602	0.00	166.3
5	210,844	210,844	0.00	82.8
6	218,694	218,694	0.00	114.2
7	216,168	216,168	0.00	81.5
8	224,094	224,094	0.00	108.8
9	215,088	215,088	0.00	128.8

Table 47: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.25$, $\alpha^l = 10$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	132,071	132,071	0.00	80.3
2	131,662	131,662	0.00	96.1
3	147,317	147,317	0.00	97.3
4	141,610	141,610	0.00	88.2
5	139,800	139,800	0.00	146.5
6	150,124	150,124	0.00	100.9
7	137,928	137,928	0.00	93.1
8	146,551	146,551	0.00	77.3
9	145,486	145,486	0.00	89.4

Table 48: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.25$, $\alpha^l = 10$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	211,177	211,177	0.00	217.3
2	203,481	203,481	0.00	487.0
3	215,158	215,158	0.00	247.3
4	222,308	222,308	0.00	149.9
5	200,625	200,625	0.00	551.8
6	213,710	213,710	0.00	266.8
7	207,824	207,824	0.00	242.6
8	212,851	212,851	0.00	631.7
9	214,580	214,580	0.00	223.4

Table 49: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.25$, $\alpha^l = 20$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	141,348	141,348	0.00	205.4
2	136,483	136,483	0.00	275.3
3	135,975	135,975	0.00	695.1
4	143,899	143,899	0.00	178.5
5	137,323	137,323	0.00	319.3
6	136,102	136,102	0.00	223.1
7	145,754	145,754	0.00	184.3
8	133,013	133,013	0.00	380.5
9	151,758	151,758	0.00	196.4

Table 50: Results for 0-1 knapsack instance with N = 100000, $\kappa = 0.25$, $\alpha^l = 20$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	212,819	212,819	0.00	5,824.0
2	213,382	213,382	0.00	1,210.0
3	220,149	220,149	0.00	2,330.8
4	217,938	217,938	0.00	1,537.2
5	213,822	213,822	0.00	2,322.1
6	205,095	205,095	0.00	1,692.9
7	216,475	216,475	0.00	1,079.7
8	204,269	204,269	0.00	680.8
9	211,911	211,911	0.00	877.7

Table 51: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.125$, $\alpha^l = 5$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	449,131	449,131	0.00	218.6
2	461,727	461,727	0.00	232.9
3	460,979	460,979	0.00	196.7
4	480,508	480,508	0.00	267.3
5	476,949	476,949	0.00	222.1
6	471,821	471,821	0.00	293.0
7	462,813	462,813	0.00	264.6
8	444,178	444,178	0.00	226.6
9	471,440	471,440	0.00	245.5

Table 52: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.125$, $\alpha^l = 5$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	607,598	607,598	0.00	314.9
2	627,920	627,920	0.00	304.8
3	620,674	620,674	0.00	432.1
4	645,212	645,212	0.00	441.6
5	619,161	619,161	0.00	281.5
6	637,675	637,675	0.00	480.9
7	629,186	629,186	0.00	436.1
8	635,042	635,042	0.00	261.4
9	632,482	632,482	0.00	351.4

Table 53: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.125$, $\alpha^l = 10$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	465,407	465,407	0.00	427.5
2	484,464	484,464	0.00	433.9
3	481,724	481,724	0.00	509.1
4	462,348	462,348	0.00	287.2
5	480,426	480,426	0.00	1,202.2
6	469,724	469,724	0.00	442.4
7	479,074	479,074	0.00	433.4
8	465,615	465,615	0.00	295.4
9	467,571	467,571	0.00	304.0

Table 54: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.125$, $\alpha^l = 10$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	642,016	642,016	0.00	913.4
2	651,265	651,265	0.00	569.5
3	633,880	633,880	0.00	460.7
4	623,772	623,772	0.00	376.4
5	656,737	656,737	0.00	661.8
6	637,635	637,635	0.00	939.8
7	614,466	614,466	0.00	703.6
8	618,463	618,463	0.00	540.8
9	614,508	614,508	0.00	677.2

Table 55: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.125$, $\alpha^l = 20$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	449,575	449,575	0.00	700.4
2	483,146	483,146	0.00	945.2
3	459,556	459,556	0.00	740.8
4	467,401	467,401	0.00	640.4
5	451,916	451,916	0.00	1,063.7
6	481,795	481,795	0.00	633.0
7	461,210	461,210	0.00	417.8
8	486,720	486,720	0.00	1,350.4
9	457,971	457,971	0.00	1,325.4

Table 56: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.125$, $\alpha^l = 20$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	627,525	627,525	0.00	1,876.4
2	607,355	607,355	0.00	2,948.2
3	626,309	626,309	0.00	2,017.8
4	663,047	663,047	0.00	1,584.2
5	617,285	617,285	0.00	1,520.9
6	601,189	601,189	0.00	1,721.7
7	621,825	621,825	0.00	1,356.0
8	599,561	599,561	0.00	2,848.3
9	619,260	619,260	0.00	1,710.8

Table 57: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.25$, $\alpha^l = 5$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	461,314	461,314	0.00	335.6
2	467,614	467,614	0.00	230.6
3	458,924	458,924	0.00	231.3
4	469,911	469,911	0.00	244.3
5	481,566	481,566	0.00	241.8
6	466,905	466,905	0.00	351.6
7	485,951	485,951	0.00	310.8
8	464,944	464,944	0.00	239.2
9	489,914	489,914	0.00	168.1

Table 58: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.25$, $\alpha^l = 5$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	611,258	611,258	0.00	714.2
2	620,324	620,324	0.00	816.1
3	617,994	617,994	0.00	499.1
4	621,540	621,540	0.00	1,345.9
5	635,260	635,260	0.00	350.8
6	605,841	605,841	0.00	445.3
7	634,493	634,493	0.00	382.0
8	622,171	622,171	0.00	367.2
9	630,532	630,532	0.00	389.7

Table 59: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.25$, $\alpha^l = 10$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	463,187	463,187	0.00	1,272.5
2	458,985	458,985	0.00	1,412.8
3	457,749	457,749	0.00	624.2
4	481,440	481,440	0.00	452.3
5	460,927	460,927	0.00	327.5
6	457,200	457,200	0.00	714.0
7	489,749	489,749	0.00	388.9
8	480,903	480,903	0.00	357.2
9	477,564	477,564	0.00	440.1

Table 60: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.25$, $\alpha^l = 10$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	614,986	614,986	0.00	923.5
2	611,189	611,189	0.00	1,112.2
3	625,453	625,453	0.00	764.3
4	613,827	613,827	0.00	1,461.9
5	613,341	613,341	0.00	759.4
6	605,321	605,321	0.00	868.2
7	605,458	605,458	0.00	1,376.8
8	638,763	638,763	0.00	6,883.8
9	627,978	627,978	0.00	1,005.2

Table 61: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.25$, $\alpha^l = 20$, $\alpha^f = 20$

s	LB	UB	Gap	CPU
1	470,074	470,074	0.00	3,396.2
2	455,770	455,770	0.00	3,053.3
3	473,507	473,507	0.00	1,175.2
4	477,873	477,873	0.00	1,175.4
5	459,963	459,963	0.00	2,496.7
6	454,724	454,724	0.00	3,302.3
7	457,526	457,526	0.00	689.8
8	470,765	470,765	0.00	921.3
9	462,172	462,172	0.00	3,861.6

Table 62: Results for 0-1 knapsack instance with N = 1000000, $\kappa = 0.25$, $\alpha^l = 20$, $\alpha^f = 40$

s	LB	UB	Gap	CPU
1	624,832	624,832	0.00	2,767.5
2	648,759	648,759	0.00	1,834.6
3	608,793	608,793	0.00	3,685.7
4	647,504	647,504	0.00	3,592.4
5	609,511	609,511	0.00	3,749.2
6	608,582	608,582	0.00	6,897.3
7	601,179	601,179	0.00	3,357.9
8	620,676	620,676	0.00	6,196.9
9	598,703	598,703	0.00	11,133.3

A.3 Facility location

In this section we report detailed results of our method for solving the facility location instances considered in our study. In Tables 63–102 we report the results for every choice of N, F and Δ, and for erach of the 9 instances generated for that choice of parameters.

Table 63: Results for UFLP with interdiction for $N = 500, F = 50, \Delta = 1$

s	LB	UB	Gap	CPU
1	132,734	132,734	0.00	77.6
2	136,770	136,770	0.00	124.4
3	133,829	133,829	0.00	51.1
4	134,854	134,854	0.00	59.3
5	132,189	132,189	0.00	154.1
6	132,055	132,055	0.00	61.5
7	131,091	131,091	0.00	56.9
8	135,312	135,312	0.00	87.4
9	132,338	132,338	0.00	82.4

Table 64: Results for UFLP with interdiction for $N = 500, F = 50, \Delta = 2$

s	LB	UB	Gap	CPU
1	133,509	133,509	0.00	128.0
2	137,437	137,437	0.00	226.6
3	134,321	134,321	0.00	231.9
4	135,196	135,196	0.00	130.1
5	133,233	133,233	0.00	138.8
6	133,128	133,128	0.00	210.2
7	132,940	132,940	0.00	120.2
8	136,182	136,182	0.00	122.9
9	132,870	132,870	0.00	210.0

Table 65: Results for UFLP with interdiction for $N = 500, F = 50, \Delta = 3$

s	LB	UB	Gap	CPU
1	134,095	134,095	0.00	129.7
2	138,158	138,158	0.00	347.5
3	134,847	134,847	0.00	712.3
4	135,914	135,914	0.00	404.3
5	134,242	134,242	0.00	319.6
6	134,074	134,074	0.00	237.2
7	133,974	133,974	0.00	144.3
8	136,942	136,942	0.00	261.1
9	133,691	133,691	0.00	372.9

Table 66: Results for UFLP with interdiction for $N = 500, F = 50, \Delta = 4$

s	LB	UB	Gap	CPU
1	134,781	134,781	0.00	270.4
2	138,466	138,466	0.00	677.1
3	135,339	135,339	0.00	821.6
4	136,469	136,469	0.00	555.9
5	134,680	134,680	0.00	506.5
6	135,116	135,116	0.00	620.7
7	134,489	134,489	0.00	438.6
8	137,590	137,590	0.00	224.2
9	134,140	134,140	0.00	249.4

Table 67: Results for UFLP with interdiction for $N = 500, F = 50, \Delta = 5$

s	LB	UB	Gap	CPU
1	135,136	135,136	0.00	320.9
2	138,885	138,885	0.00	638.2
3	135,511	135,511	0.00	940.2
4	137,181	137,181	0.00	381.4
5	134,956	134,956	0.00	633.8
6	135,276	135,276	0.00	568.3
7	135,134	135,134	0.00	291.2
8	138,104	138,104	0.00	443.9
9	135,015	135,015	0.00	295.1

Table 68: Results for UFLP with interdiction for $N = 500, F = 100, \Delta = 1$

s	LB	UB	Gap	CPU
1	130,239	130,239	0.00	298.8
2	130,274	130,274	0.00	336.6
3	131,439	131,439	0.00	329.5
4	131,114	131,114	0.00	186.4
5	131,410	131,410	0.00	238.1
6	131,814	131,814	0.00	415.9
7	131,411	131,411	0.00	195.5
8	131,574	131,574	0.00	277.7
9	129,858	129,858	0.00	166.0

Table 69: Results for UFLP with interdiction for $N = 500, F = 100, \Delta = 2$

s	LB	UB	Gap	CPU
1	130,638	130,638	0.00	1,159.2
2	130,649	130,649	0.00	1,668.8
3	131,755	131,755	0.00	1,184.5
4	132,097	132,097	0.00	349.7
5	132,144	132,144	0.00	1,136.9
6	132,462	132,462	0.00	1,245.5
7	131,523	131,523	0.00	790.0
8	132,444	132,444	0.00	584.2
9	130,638	130,638	0.00	548.8

Table 70: Results for UFLP with interdiction for $N = 500, F = 100, \Delta = 3$

s	LB	UB	Gap	CPU
1	131,335	131,335	0.00	1,779.8
2	131,177	131,177	0.00	2,345.6
3	132,133	132,133	0.00	2,021.6
4	132,655	132,655	0.00	706.5
5	132,424	132,424	0.00	1,624.3
6	133,166	133,166	0.00	2,026.6
7	131,986	131,986	0.00	3,463.9
8	132,589	132,589	0.00	1,380.4
9	131,361	131,361	0.00	668.3

Table 71: Results for UFLP with interdiction for $N = 500, F = 100, \Delta = 4$

s	LB	UB	Gap	CPU
1	131,772	131,772	0.00	2,465.8
2	131,402	131,402	0.00	4,595.1
3	132,795	132,795	0.00	3,054.1
4	133,375	133,375	0.00	1,045.7
5	132,971	132,971	0.00	2,216.6
6	133,833	133,833	0.00	2,421.3
7	132,696	132,696	0.00	2,379.6
8	133,767	133,767	0.00	1,505.8
9	131,893	131,893	0.00	1,305.0

Table 72: Results for UFLP with interdiction for $N = 500, F = 100, \Delta = 5$

s	LB	UB	Gap	CPU
1	132,058	132,058	0.00	2,444.5
2	131,759	131,759	0.00	5,690.8
3	133,345	133,345	0.00	2,528.3
4	133,868	133,868	0.00	1,218.2
5	133,252	133,252	0.00	2,707.1
6	134,174	134,174	0.00	2,617.1
7	132,776	132,776	0.00	2,463.7
8	133,933	133,933	0.00	980.4
9	132,161	132,161	0.00	2,595.3

Table 73: Results for UFLP with interdiction for $N = 1000, F = 50, \Delta = 1$

s	LB	UB	Gap	CPU
1	213,806	213,806	0.00	168.0
2	212,033	212,033	0.00	135.5
3	213,992	213,992	0.00	181.4
4	215,872	215,872	0.00	287.4
5	215,127	215,127	0.00	954.7
6	212,313	212,313	0.00	1,110.3
7	214,200	214,200	0.00	1,255.3
8	216,698	216,698	0.00	544.2
9	211,052	211,052	0.00	543.8

Table 74: Results for UFLP with interdiction for $N = 1000, F = 50, \Delta = 2$

s	LB	UB	Gap	CPU
1	215,144	215,144	0.00	402.9
2	213,455	213,455	0.00	529.5
3	215,067	215,067	0.00	643.4
4	216,750	216,750	0.00	933.2
5	215,541	215,541	0.00	1,754.2
6	213,386	213,386	0.00	2,602.8
7	215,083	215,083	0.00	2,056.5
8	217,047	217,047	0.00	2,828.6
9	212,204	212,204	0.00	916.8

Table 75: Results for UFLP with interdiction for $N = 1000, F = 50, \Delta = 3$

s	LB	UB	Gap	CPU
1	216,217	216,217	0.00	1,239.5
2	214,738	214,738	0.00	604.8
3	215,827	215,827	0.00	1,795.0
4	217,240	217,240	0.00	3,661.1
5	215,740	215,740	0.00	4,221.4
6	214,025	214,025	0.00	3,661.2
7	215,547	215,547	0.00	2,513.3
8	218,270	218,270	0.00	3,015.7
9	213,304	213,304	0.00	1,834.3

Table 76: Results for UFLP with interdiction for $N = 1000, F = 50, \Delta = 4$

s	LB	UB	Gap	CPU
1	217,238	217,238	0.00	1,197.0
2	215,972	215,972	0.00	854.2
3	216,902	216,902	0.00	1,844.1
4	217,900	217,900	0.00	2,998.1
5	216,422	216,422	0.00	6,399.8
6	215,039	215,039	0.00	4,298.4
7	216,476	216,476	0.00	6,318.7
8	218,588	218,588	0.00	3,071.0
9	214,018	214,018	0.00	4,769.2

Table 77: Results for UFLP with interdiction for $N = 1000, F = 50, \Delta = 5$

s	LB	UB	Gap	CPU
1	218,073	218,073	0.00	1,619.9
2	216,657	216,657	0.00	467.3
3	217,513	217,513	0.00	1,836.6
4	218,570	218,570	0.00	4,780.0
5	216,910	216,910	0.00	6,505.1
6	215,678	215,678	0.00	4,146.8
7	217,070	217,070	0.00	7,705.5
8	219,757	219,757	0.00	3,633.1
9	214,792	214,792	0.00	2,939.7

Table 78: Results for UFLP with interdiction for $N = 1000, F = 100, \Delta = 1$

s	LB	UB	Gap	CPU
1	206,686	206,686	0.00	1,662.3
2	209,270	209,270	0.00	919.8
3	212,371	212,371	0.00	1,139.2
4	208,173	208,173	0.00	859.7
5	206,549	206,549	0.00	1,396.2
6	207,488	207,488	0.00	1,819.9
7	212,392	212,392	0.00	1,481.9
8	207,685	207,685	0.00	277.2
9	205,030	205,030	0.00	1,682.7

Table 79: Results for UFLP with interdiction for $N = 1000, F = 100, \Delta = 2$

s	LB	UB	Gap	CPU
1	207,203	207,203	0.00	4,465.1
2	209,746	209,746	0.00	2,155.2
3	213,117	213,117	0.00	2,488.9
4	209,088	209,088	0.00	2,084.5
5	207,847	207,847	0.00	3,632.4
6	208,213	208,213	0.00	3,831.7
7	212,830	212,830	0.00	6,725.0
8	208,931	208,931	0.00	743.4
9	205,759	205,759	0.00	8,291.3

Table 80: Results for UFLP with interdiction for $N = 1000, F = 100, \Delta = 3$

s	LB	UB	Gap	CPU
1	207,790	207,790	0.00	12,012.2
2	210,835	210,835	0.00	6,649.2
3	213,754	213,754	0.00	6,304.6
4	209,964	209,964	0.00	4,146.8
5	208,412	208,412	0.00	6,239.2
6	209,062	209,062	0.00	8,954.5
7	213,632	213,632	0.00	8,579.0
8	209,432	209,432	0.00	1,269.7
9	206,682	206,682	0.00	3,930.9

Table 81: Results for UFLP with interdiction for $N = 1000, F = 100, \Delta = 4$

s	LB	UB	Gap	CPU
1	208,307	208,307	0.00	14,586.5
2	211,331	211,331	0.00	7,650.6
3	214,108	214,108	0.00	11,270.5
4	210,772	210,772	0.00	7,639.8
5	209,020	209,020	0.00	6,989.3
6	209,412	209,412	0.00	5,776.0
7	214,099	214,099	0.00	10,245.4
8	209,808	209,808	0.00	5,850.6
9	207,043	207,043	0.00	7,611.4

Table 82: Results for UFLP with interdiction for $N = 1000, F = 100, \Delta = 5$

s	LB	UB	Gap	CPU
1	208,702	208,702	0.00	29,031.9
2	211,853	211,853	0.00	12,696.4
3	214,732	214,732	0.00	11,022.0
4	211,596	211,596	0.00	4,294.4
5	209,817	209,817	0.00	5,310.0
6	210,196	210,196	0.00	15,458.9
7	214,458	214,458	0.00	12,446.7
8	210,193	210,193	0.00	9,669.6
9	207,993	207,993	0.00	10,843.3

Table 83: Results for SSCFLP with interdiction for $N = 500, F = 50, \Delta = 1$

s	LB	UB	Gap	CPU
1	132,905	132,905	0.00	149.4
2	138,037	138,037	0.00	762.0
3	134,727	134,727	0.00	391.6
4	135,364	135,364	0.00	366.5
5	132,522	132,522	0.00	286.4
6	132,229	132,229	0.00	191.1
7	133,378	133,378	0.00	1,604.9
8	136,892	136,892	0.00	1,053.5
9	133,642	133,642	0.00	353.1

Table 84: Results for SSCFLP with interdiction for $N = 500, F = 50, \Delta = 2$

s	LB	UB	Gap	CPU
1	134,266	134,266	0.00	234.4
2	139,075	139,075	0.00	1,372.1
3	135,111	135,111	0.00	407.7
4	135,488	135,488	0.00	1,104.1
5	133,531	133,531	0.00	747.3
6	133,998	133,998	0.00	260.9
7	134,448	134,448	0.00	5,776.7
8	137,574	137,574	0.00	4,959.4
9	134,713	134,713	0.00	552.8

Table 85: Results for SSCFLP with interdiction for $N = 500, F = 50, \Delta = 3$

s	LB	UB	Gap	CPU
1	135,307	135,307	0.00	353.6
2	139,435	139,435	0.00	1,701.4
3	135,745	135,745	0.00	2,349.8
4	136,818	136,818	0.00	1,099.5
5	134,518	134,518	0.00	882.2
6	135,132	135,132	0.00	1,479.7
7	134,933	134,933	0.00	11,226.2
8	138,157	138,157	0.00	5,039.6
9	135,388	135,388	0.00	1,206.4

Table 86: Results for SSCFLP with interdiction for $N = 500, F = 50, \Delta = 4$

s	LB	UB	Gap	CPU
1	135,758	135,758	0.00	1,184.8
2	139,725	139,725	0.00	7,997.1
3	136,129	136,129	0.00	4,142.6
4	136,961	136,961	0.00	3,720.3
5	135,527	135,527	0.00	1,108.5
6	135,535	135,535	0.00	2,823.8
7	135,763	135,763	0.00	9,338.1
8	138,756	138,756	0.00	6,705.6
9	135,781	135,781	0.00	3,578.1

Table 87: Results for SSCFLP with interdiction for $N = 500, F = 50, \Delta = 5$

s	LB	UB	Gap	CPU
1	136,685	136,685	0.00	2,350.7
2	140,044	140,044	0.00	10,379.5
3	136,721	136,721	0.00	3,935.7
4	137,805	137,805	0.00	4,172.8
5	135,576	135,576	0.00	3,471.2
6	136,504	136,504	0.00	3,414.6
7	136,318	136,318	0.00	13,370.8
8	139,209	139,209	0.00	8,933.1
9	136,059	136,059	0.00	11,629.3

Table 88: Results for SSCFLP with interdiction for $N = 500, F = 100, \Delta = 1$

s	LB	UB	Gap	CPU
1	131,377	131,377	0.00	3,227.2
2	130,990	130,990	0.00	2,126.4
3	132,053	132,053	0.00	6,054.3
4	132,980	132,980	0.00	3,523.7
5	133,239	133,239	0.00	5,169.4
6	132,639	132,639	0.00	2,044.9
7	132,510	132,510	0.00	4,171.5
8	133,147	133,147	0.00	2,318.1
9	131,547	131,547	0.00	12,517.8

Table 89: Results for SSCFLP with interdiction for $N = 500, F = 100, \Delta = 2$

s	LB	UB	Gap	CPU
1	131,768	131,768	0.00	6,005.2
2	131,177	131,177	0.00	6,377.3
3	132,294	132,294	0.00	9,583.8
4	133,669	133,669	0.00	3,831.0
5	134,075	134,075	0.00	19,265.9
6	132,986	132,986	0.00	6,888.1
7	133,120	133,120	0.00	23,290.1
8	133,567	133,567	0.00	5,633.9
9	132,217	132,217	0.00	7,009.4

Table 90: Results for SSCFLP with interdiction for $N = 500, F = 100, \Delta = 3$

s	LB	UB	Gap	CPU
1	132,263	132,263	0.00	13,664.9
2	131,482	131,482	0.00	20,131.4
3	132,632	132,632	0.00	23,823.0
4	134,378	134,378	0.00	17,023.0
5	ML	ML	ML	ML
6	133,894	133,894	0.00	9,812.2
7	133,414	133,414	0.00	51,189.4
8	ML	ML	ML	ML
9	132,609	132,609	0.00	25,946.0

Table 91: Results for SSCFLP with interdiction for $N = 500, F = 100, \Delta = 4$

s	LB	UB	Gap	CPU
1	133,161	133,161	0.00	17,430.3
2	131,982	131,982	0.00	29,799.0
3	133,473	133,473	0.00	15,227.1
4	ML	ML	ML	ML
5	134,532	134,532	0.00	35,982.2
6	134,068	134,068	0.00	17,829.1
7	133,832	133,832	0.00	55,934.4
8	134,663	134,663	0.00	12,961.6
9	ML	ML	ML	ML

Table 92: Results for SSCFLP with interdiction for $N = 500, F = 100, \Delta = 5$

s	LB	UB	Gap	CPU
1	133,204	133,204	0.00	28,659.1
2	132,260	132,260	0.00	50,118.0
3	133,659	133,659	0.00	36,374.7
4	135,085	135,085	0.00	35,543.6
5	ML	ML	ML	ML
6	134,714	134,714	0.00	18,536.5
7	134,154	134,154	0.00	76,777.5
8	134,766	134,766	0.00	20,402.4
9	133,381	133,381	0.00	79,470.4

Table 93: Results for SSCFLP with interdiction for $N = 1000, F = 50, \Delta = 1$

s	LB	UB	Gap	CPU
1	213,806	213,806	0.00	238.2
2	212,033	212,033	0.00	182.9
3	213,992	213,992	0.00	314.3
4	215,872	215,872	0.00	729.8
5	215,128	215,128	0.00	2,899.0
6	212,313	212,313	0.00	1,745.0
7	214,200	214,200	0.00	2,476.2
8	216,698	216,698	0.00	1,062.9
9	211,052	211,052	0.00	946.6

Table 94: Results for SSCFLP with interdiction for $N = 1000, F = 50, \Delta = 2$

s	LB	UB	Gap	CPU
1	215,144	215,144	0.00	626.2
2	213,455	213,455	0.00	745.8
3	215,067	215,067	0.00	1,003.8
4	216,750	216,750	0.00	2,512.8
5	215,541	215,541	0.00	4,760.2
6	213,386	213,386	0.00	4,545.5
7	215,083	215,083	0.00	3,207.5
8	217,047	217,047	0.00	4,576.5
9	212,204	212,204	0.00	1,105.2

Table 95: Results for SSCFLP with interdiction for $N = 1000, F = 50, \Delta = 3$

s	LB	UB	Gap	CPU
1	216,217	216,217	0.00	1,349.0
2	214,738	214,738	0.00	830.7
3	215,827	215,827	0.00	2,594.4
4	217,240	217,240	0.00	7,284.3
5	215,740	215,740	0.00	11,888.8
6	214,025	214,025	0.00	8,997.8
7	215,547	215,547	0.00	8,413.6
8	218,270	218,270	0.00	2,770.6
9	213,304	213,304	0.00	1,883.9

Table 96: Results for SSCFLP with interdiction for $N = 1000, F = 50, \Delta = 4$

s	LB	UB	Gap	CPU
1	217,238	217,238	0.00	2,038.9
2	215,972	215,972	0.00	1,340.8
3	216,902	216,902	0.00	2,190.6
4	217,900	217,900	0.00	14,182.8
5	216,423	216,423	0.00	13,142.7
6	215,039	215,039	0.00	7,190.9
7	216,476	216,476	0.00	15,810.2
8	218,588	218,588	0.00	7,335.0
9	214,018	214,018	0.00	4,560.7

Table 97: Results for SSCFLP with interdiction for $N = 1000, F = 50, \Delta = 5$

s	LB	UB	Gap	CPU
1	218,073	218,073	0.00	1,492.1
2	216,668	216,668	0.00	1,789.1
3	217,513	217,513	0.00	3,615.2
4	218,570	218,570	0.00	14,747.2
5	216,910	216,910	0.00	16,394.7
6	215,678	215,678	0.00	7,654.2
7	217,070	217,070	0.00	23,622.3
8	219,757	219,757	0.00	7,125.3
9	214,792	214,792	0.00	10,195.3

Table 98: Results for SSCFLP with interdiction for $N = 1000, F = 100, \Delta = 1$

s	LB	UB	Gap	CPU
1	206,686	206,686	0.00	7,190.9
2	209,270	209,270	0.00	4,212.2
3	212,371	212,371	0.00	2,769.5
4	208,173	208,173	0.00	2,671.7
5	206,549	206,549	0.00	5,225.4
6	207,488	207,488	0.00	8,329.3
7	212,392	212,392	0.00	6,788.2
8	207,685	207,685	0.00	2,110.0
9	205,030	205,030	0.00	5,045.2

Table 99: Results for SSCFLP with interdiction for $N = 1000, F = 100, \Delta = 2$

s	LB	UB	Gap	CPU
1	207,203	207,203	0.00	21,715.4
2	209,746	209,746	0.00	9,857.1
3	213,117	213,117	0.00	7,067.9
4	209,088	209,088	0.00	5,538.9
5	207,847	207,847	0.00	17,967.5
6	208,213	208,213	0.00	20,613.6
7	212,830	212,830	0.00	32,304.7
8	208,931	208,931	0.00	6,210.4
9	205,759	205,759	0.00	18,690.5

Table 100: Results for SSCFLP with interdiction for $N = 1000, F = 100, \Delta = 3$

s	LB	UB	Gap	CPU
1	207,790	207,790	0.00	39,913.2
2	210,835	210,835	0.00	25,668.4
3	213,754	213,754	0.00	21,233.7
4	209,964	209,964	0.00	12,753.8
5	208,412	208,412	0.00	19,516.6
6	209,062	209,062	0.00	25,472.0
7	213,632	213,632	0.00	51,788.5
8	209,432	209,432	0.00	7,022.2
9	206,682	206,682	0.00	11,647.4

Table 101: Results for SSCFLP with interdiction for $N = 1000, F = 100, \Delta = 4$

s	LB	UB	Gap	CPU
1	208,307	208,307	0.00	51,088.8
2	211,331	211,331	0.00	31,255.3
3	214,108	214,108	0.00	32,394.4
4	210,772	210,772	0.00	17,439.5
5	209,020	209,020	0.00	23,443.3
6	209,412	209,412	0.00	39,497.7
7	214,099	214,099	0.00	60,765.2
8	209,808	209,808	0.00	24,489.9
9	207,043	207,043	0.00	29,450.2

Table 102: Results for SSCFLP with interdiction for $N = 1000, F = 100, \Delta = 5$

s	LB	UB	Gap	CPU
1	208,702	208,702	0.00	75,711.9
2	211,853	211,853	0.00	53,405.7
3	214,732	214,732	0.00	47,055.9
4	211,596	211,596	0.00	18,722.2
5	209,817	209,817	0.00	20,408.5
6	210,196	210,196	0.00	80,734.3
7	214,458	214,459	0.00	TL
8	210,193	210,193	0.00	53,943.7
9	207,993	207,993	0.00	27,690.4

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