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An exact optimization approach for an integrated process configuration, lot-sizing, and scheduling problem

Supplementary material

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Abstract: We study an integrated process configuration, lot-sizing, and scheduling problem, which appears in a real production environment in the packaging industry. Products are produced by alternative process configurations. The production quantities and capacity consumption depend on which process configurations are used, how long they are used for, and in which sequence. For the particular case studied here, configuration decisions are generated at the same time as lot-sizing and sequencing decisions, which involve sequence-dependent setup costs and times. Due to dependency of these decisions, the model is nonlinear. Even though a linearization technique can be applied, the problem is still difficult to solve by a mixed integer programming (MIP) solver due to its complexity. This paper aims to develop efficient solution methods to deal with this integrated production planning problem. An exact branch-and-check (B&Ch) algorithm based on a relaxed formulation and using logic-based Benders cuts is proposed to find optimal solutions. In addition, symmetry-breaking constraints are applied to strengthen the formulations. Results show that in general, the B&Ch outperforms the linearized models solved by an MIP solver. To efficiently solve large instances, an MIP-based heuristic is then proposed to find good quality solutions in shorter computing times. Although the problem studied here is based on the packaging industry, the logic of the B&Ch and the proposed heuristic can be adapted to other applications where lot-sizing must be determined simultaneously with process configuration decisions.

Keywords: Lot-sizing and scheduling, process configuration, logic-based Benders decomposition, branch-and-check, molded pulp industry.

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A Linearization approach for the MIP formulations

The mathematical models proposed in this paper have some nonlinear constraints, specifically constraints (11)–(12) and (14). Constraints (11)–(12) involve the multiplication of the integer variables β_{ks} and the binary variables y_{ipks}. To linearize these constraints, we replaced the product of these variables by the continuous variables δ_{ipks}, so that $\delta_{ipks} = \beta_{ks} y_{ipks}$. Thus, we replace constraints (11)–(12) by the following inequalities:

$$\sum_{f \in F_k} x_{ifs} \leq \sum_{p \in P} \delta_{ipks} + B_k \sum_{i \in N} \alpha_{iks} \qquad \forall s \in S;\, i \in N;\, k \in K \tag{1}$$

$$\sum_{f \in F_k} x_{ifs} \geq \sum_{p \in P} \delta_{ipks} - B_k \sum_{i \in N} \alpha_{iks} \qquad \forall s \in S;\, i \in N;\, k \in K \tag{2}$$

$$\delta_{ipks} \leq B_k y_{ipks} \qquad \forall s \in S;\, i \in N;\, k \in K; p \in P \tag{3}$$

$$\delta_{ipks} \leq \beta_{ks} \qquad \forall s \in S;\, i \in N;\, k \in K; p \in P \tag{4}$$

$$\delta_{ipks} \geq \beta_{ks} - B_k(1 - y_{ipks}) \qquad \forall s \in S;\, i \in N;\, k \in K; p \in P \tag{5}$$

$$\delta_{ipks} \in \mathrm{R}^+ \qquad \forall s \in S;\, i \in N;\, k \in K; p \in P \tag{6}$$

Similarly, in constraints (14), the integer variables x_{iks} are multiplied by the continuous variables w_s. To linearize these constraints, the integer variables x_{ifs} are substituted by a binary representation and additional constraints, as follows:

$$x_{ifs} = \sum_{\hat{r} \in \hat{R}} 2^{\hat{r}-1} \theta_{\hat{r}ifs}$$

where $\hat{r} \in \hat{R}$ represents each bit of the notation and $\theta_{\hat{r}ifs} \in \{0,\, 1\}$.

Thus, constraints (14) are substituted by the following set of linear constraints.

$$I^+_{i(t-1)} + \sum_{s \in S_t} \sum_{f \in F} \sum_{\hat{r} \in \hat{R}} R(2^{\hat{r}-1} \Theta_{\hat{r}ifs}) + I^-_{it} = I^-_{i(t-1)} + d_{it} + I^+_{it} \quad \forall i \in N;\, t \in T \tag{7}$$

$$\Theta_{\hat{r}ifs} \leq w_s \quad \forall s \in S;\, f \in F;\, i \in N;\, \hat{r} \in \hat{R} \tag{8}$$

$$\Theta_{\hat{r}ifs} \leq Q_t Y_{is} \quad \forall t \in T;\, s \in S_t;\, f \in F;\, i \in N;\, \hat{r} \in \hat{R} \tag{9}$$

$$\Theta_{\hat{r}ifs} \leq Q_t \theta_{\hat{r}ifs} \quad \forall t \in T;\, s \in S_t;\, f \in F;\, i \in N;\, \hat{r} \in \hat{R} \tag{10}$$

$$\Theta_{\hat{r}ifs} \geq w_s - Q_t(2 - \theta_{\hat{r}ifs} - Y_{is}) \quad \forall t \in T;\, s \in S_t;\, f \in F;\, i \in N;\, \hat{r} \in \hat{R} \tag{11}$$

$$\theta_{\hat{r}ifs} \in \{0, 1\}; \Theta_{\hat{r}ifs} \in \mathrm{R}^+ \quad \forall s \in S;\, i \in N; \hat{r} \in \hat{R} \tag{12}$$

B Description of the 3-phase MIP-based heuristic

As presented in Section 5.3, the heuristic developed comprises three different stages: (i) defining a set of feasible process configurations; (ii) finding a feasible solution using the choices defined in phase (i); and (iii) applying an improvement procedure to the solution obtained in phase (ii). The techniques used for each phase are described in the following subsections.

B.1 Approaches for Phase 1

In this phase, two relax-and-fix (R&F) heuristics are performed in order to create a list of possible molding patterns to use in Phase 2. As the relax-and-fix heuristics use the mathematical formulation of the original problem, the results obtained after this first phase are indeed feasible solutions for the original problem. In this phase, the R&F in Algorithm 1 is performed once, then the process configurations which appear in the solution are appended to the list ξ. Local branching inequalities (13) are added to the formulation used by

Algorithm 2, in order to avoid that configurations already in list ξ appear in the heuristic solutions. Then, Algorithm 2 is performed, and the process configurations in the solution are also appended to the list ξ.

The first R&F set partitions for the variables according to the main decisions of the original problem. Firstly decisions about arrangements in conveyors are made, then the set of products to be produced are decided, then feasible molding patterns to produce these products are defined. The detailed procedure is described in Algorithm 1.

Algorithm 1 R&F by decisions

Consider the relaxed formulation presented in Section 5.2.1 with all the variables set as continuous;
Step 1 (*defining the arrangement of each conveyor*): Set variables z_{kes} and $b_{ee'ks}$ as binary and solve the relaxed formulation;
Step 2 (*defining the types of products to be produced*): Fix the values of the variables z_{kes} and $b_{ee'ks}$ obtained after **Step 1**.
Set the variables Y_{is} as binary and solve the relaxed formulation using CPLEX;
Step 3 (*defining the molding patterns to be used*): Fix the values of the variables Y_{is} obtained after **Step 2**;
Set the remaining variables of the model as defined in the relaxed formulation presented in Section 5.2.1 and solve it using CPLEX;
if the problem in **Step 3** is infeasible **then**
    Unfix the variables Y_{is} and solve the problem using CPLEX;
end if
Step 4 (*adjusting molding patterns according to original demand*): Fix the values of Y_{is}. Fix also α_{iks}, β_{ks}, and x_{ifs} only for the cases $s \in S$; $i \in N$; $k \in K, f \in F_k : \alpha_{iks} = 1$,
Solve the problem using the B&Ch;
return A feasible solution for the original problem

The second relax-and-fix heuristic set partitions of variables according to each subperiod $s \in S$. This procedure is described in Algorithm 2.

Algorithm 2 R&F by subperiods

Consider the original formulation presented in Section 5.2.1 in addition to constraints (13);
for all $s \in S$ **do**
    Keep the integrality of the variables z_{kes}, y_{ipks}, Y_{is}, x_{ifs}, α_{iks}, β_{ks}, v_s, and $b_{ee'ks}$ $\forall i \in N$; $k \in K$; $e, e' \in E$; $p \in P$;
    Relax the integer variables z_{kes^*}, y_{ipks^*}, Y_{is^*}, x_{ifs^*}, α_{iks^*}, β_{ks^*}, v_{s^*}, and $b_{ee'ks^*}$ $\forall s^* > s : s^* \in S$; $i \in N$; $k \in K$; $e, e' \in E_k$; $p \in P_k$;
    Solve the problem using the B&Ch;
    Fix the values of the variables z_{kes}, y_{ipks}, Y_{is}, x_{ifs}, α_{iks}, β_{ks}, v_s, $b_{ee'ks}$, u_{iks}, and β_{ks} for the next iterations;
end for
return A feasible solution for the original problem

Inequalities (13) are the local branching constraints added to the problem formulation, before Algorithm 2 is performed. It considers $\hat{z}_{\epsilon ke}$ and $\hat{y}_{\epsilon ipk}$ as the values of variables z and y of the original problem which generates each process configuration ϵ in the list ξ. These constraints ensure that the configurations in list ξ will not be generated again in the second relax-and-fix.

$$\sum_{k \in K} \sum_{\substack{e \in E: \\ \hat{z}_{\epsilon ke}=1}} (1 - z_{kes}) + \sum_{k \in K} \sum_{\substack{e \in E: \\ \hat{z}_{\epsilon ke}=0}} z_{kes} + \sum_{p \in P} \sum_{i \in N} \sum_{\substack{k \in K: \\ \hat{y}_{\epsilon ipk}=1}} (1 - y_{ipks}) + \sum_{p \in P} \sum_{i \in N} \sum_{\substack{k \in K: \\ \hat{y}_{\epsilon ipk}=0}} y_{ipks} \geq 1 \quad \forall \epsilon \in \xi;\ s \in S \tag{13}$$

B.2 Approach for Phase 2

For this stage, the MIP model presented below is built using the output of Phase I and solved using CPLEX. This formulation is also based on the classical model for the GLSP and uses as input parameter the list ξ of process configurations.

Sets and indices

T	set of time periods (indexed by t);
S_t	set of subperiods belonging to period t;
S	set of subperiods, i.e., $\bigcup_{t\in T} S_t$;
ξ	set of possible process configurations (or patterns) (indexed by ϵ);
N	set of types of products (indexed by i);
K	set of set of conveyors (indexed by k);
E	set of possible arrangements for conveyors (indexed by e).

Parameters

$X_{\epsilon i}$	number of molds for product i according to pattern ϵ;
$\tilde{R}_{i\epsilon}$	production rate of product i if pattern ϵ is used (units per hour);
$Z_{\epsilon ke}$	configuration of conveyor k according to the pattern ϵ; equals 1, if conveyor k is set up to arrangement e in pattern ϵ;
d_{it}	demand of product i in period t;
Q_t	total time capacity in period t;
st^I	setup time for stopping/restarting the production line (Setup I);
st_i^{II}	setup time for attaching/detaching one mold for product i (Setup II);
$st_{ee'}^{III}$	setup time for changeovers from arrangement e to e' (Setup III);
sc^I	cost of Setup I;
sc_i^{II}	cost of attaching/detaching one mold for product i (Setup II);
$sc_{ee'}^{III}$	cost of changeovers from arrangement e to e' (Setup III);
h_i^+	unit inventory holding cost for product i;
h_i^-	unit backlogging cost for product i.

Decision variables

$y_{\epsilon s}$	1, if pattern ϵ is used in subperiod s; 0, otherwise;
$w_{\epsilon s}$	production time of pattern ϵ in subperiod s;
I_{it}^+	inventory units of product i at the end of period t;
I_{it}^-	backlogged units of product i at the end of period t;
v_s	1, if the production line is stopped for setting operations in subperiod s; 0, otherwise;
u_{is}	number of molds for product i attached or detached in subperiod s;
$b_{ee'ks}$	1, if there is a changeover from arrangement e to e' in conveyor k, in subperiod s; 0, otherwise.

$$\text{Min} \sum_{s\in S} sc^I v_s + \sum_{s\in S}\sum_{i\in N} sc_i^{II} u_{is} + \sum_{s\in S}\sum_{k\in K}\sum_{e,\,e'\in E} sc_{ee'}^{III} b_{ee'ks} + \sum_{t\in T}\sum_{i\in N}(h_i^+ I_{it}^+ + h_i^- I_{it}^-) \tag{14}$$

$$\sum_{\epsilon\in\xi} y_{\epsilon s} = 1 \qquad \forall s\in S \tag{15}$$

$$w_{\epsilon s} \le Q_t y_{\epsilon s} \qquad \forall t\in T;\ s\in S_t;\ \epsilon\in\xi \tag{16}$$

$$\sum_{s\in S_t}\sum_{\epsilon\in\xi} w_{\epsilon s} + \sum_{s\in S_t} v_s st^I + \sum_{s\in S_t}\sum_{i\in N} u_{is} st_i^{II} + \sum_{s\in S_t}\sum_{k\in K}\sum_{e,e'\in E} b_{ee'ks} st_{ee'}^{III} \le Q_t \qquad \forall t\in T \tag{17}$$

$$I_{i(t-1)}^+ + \sum_{s\in S_t}\sum_{\epsilon\in\xi} w_{\epsilon s}\tilde{R}_{i\epsilon} + I_{it}^- = d_{it} + I_{i(t-1)}^- + I_{it}^+ \qquad \forall t\in T;\ i\in N \tag{18}$$

$$u_{is} \le M_i v_s \qquad \forall i\in N;\ s\in S \tag{19}$$

$$u_{is} \ge \sum_{\epsilon\in\xi} y_{\epsilon s} X_{\epsilon i} - \sum_{\epsilon\in\xi} y_{\epsilon(s-1)} X_{\epsilon i} \qquad \forall s\in S;\ i\in N \tag{20}$$

$$u_{is} \ge \sum_{\epsilon\in\xi} y_{\epsilon(s-1)} X_{\epsilon i} - \sum_{\epsilon\in\xi} y_{\epsilon s} X_{\epsilon i} \qquad \forall s\in S;\ i\in N \tag{21}$$

$$\sum_{e\in E} b_{ee'ks} = \sum_{\epsilon\in\xi} Z_{\epsilon ke'} y_{\epsilon s} \qquad \forall s\in S;\ k\in K;\ e'\in E \tag{22}$$

$$\sum_{e'\in E} b_{ee'ks} = \sum_{\epsilon\in\xi} Z_{\epsilon ke} y_{\epsilon(s-1)} \qquad \forall s\in S;\ k\in K;\ e\in E \tag{23}$$

$$v_s,\ y_{\epsilon s},\ b_{ee'ks} \in \{0,1\}; \quad w_{\epsilon s},\ u_{is},\ I_{it}^+,\ I_{it}^- \in \mathbf{R}^+ \qquad \forall t\in T;\ s\in S;\ \epsilon\in\xi;\ i\in N;\ k\in K;\ e,e'\in E \tag{24}$$

B.3 Approach for Phase 3

The improvement technique used in this phase is the fix-and-optimize (F&O) approach described in Algorithm 3. The first step in this approach consists of transforming the solution obtained in Phase 2 into a feasible solution for the original problem. This is done by fixing the molding patterns and the lot sizes of the solution in Phase II to the original formulation. It can be accomplished by adding constraints (25) to the original formulation, fixing the w_s variables to the values $\sum_{\epsilon \in \xi} \hat{w}_{\epsilon s}$, and solving the original formulation using the B&Ch or CPLEX to determine the value of the remaining variables, where $\hat{y}_{\epsilon s}$ and $\hat{w}_{\epsilon s}$ are the values of $y_{\epsilon s}$ and $w_{\epsilon s}$ after solving the model (14)–(24) in Phase II, respectively.

$$\sum_{f \in F} x_{ifs} = \sum_{\epsilon \in \xi} X_{\epsilon i} \hat{y}_{\epsilon s} \qquad \forall i \in N;\ s \in S \tag{25}$$

Algorithm 3 F&O by periods

```
Consider the original formulation presented in Section 3.2;
Transform the solution obtained in Phase 2 to be feasible for the original formulation. Consider this as the initial solution S_0;
for all t* ∈ T do
    Unfix all the variables indexed to period t* and s ∈ S_{t*}, except the variables z_{kes} and b_{ee'ks},   ∀s ∈ S_t; k ∈ K; e, e' ∈ E; p ∈ P;
    Solve the problem using the B&Ch;
    if the current solution is better than S_0 then
        Update S_0
    end if
    Fix the variables indexed to t* and s ∈ S_{t*} as defined in S_0
end for
return A feasible solution for the original problem
```

C Detailed results for the best performing case of the solution approaches

Table 1: Detailed results of the best case performing solution approaches for Model 1 reported in Table 3

		Linearized MIP model solved by CPLEX [a]					Reformulation solved by the B&Ch[a]					
Group	Inst.	Upper B.	Lower B.	Gap	Time (s)	# nodes	Upper B.	Lower B.	Gap	Time (s)	# nodes	# cuts
T2P5_F6K2A1	1	25334.3	25331.8	0.01%	1886.6	89530	25334.3	25332.6	0.01%	38.8	2387	214
	2	15766.8	13672.8	13.28%	10805.9	339360	15766.8	15766.8	0.00%	71.2	2702	254
	3	16563.6	13672.8	17.45%	10801.4	161715	16563.6	16563.5	0.00%	47.7	2156	331
	4	16836.2	14133.0	16.06%	10800.3	398101	16836.2	16836.2	0.00%	46.2	2276	303
	5	16148.2	13746.6	14.87%	10800.3	267884	16148.2	16148.2	0.00%	49.8	2305	332
	6	16407.1	13730.2	16.32%	10800.4	283341	16407.1	16407.1	0.00%	115.7	4985	357
	7	16477.3	13857.2	15.90%	10800.2	275577	16477.3	16477.3	0.00%	120.8	6211	286
	8	16960.7	13866.3	18.24%	10800.0	230834	16960.7	16960.7	0.00%	68.1	3142	287
	9	16362.0	13672.8	16.44%	10800.0	350427	16362.0	16362.0	0.00%	42.3	1411	266
	10	16503.2	13672.8	17.15%	10800.3	194256	16503.2	16503.2	0.00%	45.9	1810	307
Avg.		**17336.0**	**14935.6**	**14.57%**	**9909.5**	**259102.5**	**17336.0**	**17335.8**	**0.00%**	**64.7**	**2938.5**	**293.7**
T2P8_F6K2A1	1	26378.0	17279.4	34.49%	10800.0	13156	26378.0	26375.5	0.01%	6924.7	39530	311
	2	26365.8	20289.5	23.05%	10800.2	10495	26365.8	26363.2	0.01%	10387.4	67569	390
	3	26528.0	20803.8	21.58%	10800.0	11295	26736.3	26734.6	0.01%	3183.2	20316	318
	4	26838.2	16450.2	38.71%	10800.4	11855	26838.2	26723.4	0.43%	10800.4	71519	273
	5	27162.5	19069.0	29.80%	10800.6	16722	26974.8	26972.3	0.01%	10470.5	95582	405
	6	27005.8	21393.9	20.78%	10800.5	11838	27005.8	27003.3	0.01%	5668.3	38579	342
	7	26212.0	16672.4	36.39%	10800.2	11769	26212.0	26209.5	0.01%	3840.3	36345	364
	8	27473.9	21361.2	22.25%	10800.3	20266	27473.9	26893.1	2.11%	10800.0	56577	346
	9	26731.1	21257.9	20.48%	10800.2	17193	26731.1	26729.0	0.01%	5261.8	35025	303
	10	26174.1	20808.8	20.50%	10800.2	13044	26174.1	26171.6	0.01%	5731.7	72166	309
Avg.		**26686.9**	**19538.6**	**26.80%**	**10800.3**	**13763.3**	**26689.0**	**26617.6**	**0.26%**	**7306.8**	**53320.8**	**336.1**
T2P8_F6K3A1	1	23328.4	23327.6	0.00%	3918.7	15332	23328.4	23328.4	0.00%	844.1	7726	636
	2	23252.6	23252.6	0.00%	8835.5	16028	23252.6	23251.2	0.01%	2225.7	25691	762
	3	23703.0	23701.2	0.01%	6373.3	22883	23703.0	23700.8	0.01%	1454.0	20661	610
	4	23494.8	23493.0	0.01%	7460.1	21605	23494.8	23492.5	0.01%	1190.8	17319	518
	5	23828.3	23827.7	0.00%	6759.4	19592	23828.3	23828.3	0.00%	2087.0	18008	770
	6	23846.4	23844.9	0.01%	7302.6	20194	23846.4	23846.3	0.00%	2165.9	14679	722
	7	23724.8	23724.8	0.00%	5258.3	12760	23724.8	23724.8	0.00%	1671.8	14835	782
	8	23604.9	23603.7	0.00%	8969.4	22848	23604.9	23604.9	0.00%	1273.9	10983	628
	9	23497.4	23495.7	0.01%	6584.1	18266	23497.4	23497.4	0.00%	1091.1	8723	602
	10	23269.8	23267.6	0.01%	5898.2	15218	23269.8	23268.1	0.01%	6212.3	36080	840
Avg.		**23555.0**	**23553.9**	**0.00%**	**6736.0**	**18472.6**	**23555.0**	**23554.3**	**0.00%**	**2021.6**	**17470.5**	**687**

a Results of the best performing case of the solution approaches reported in Table 3.

Table 2: Detailed results of the best case performing solution approaches for Model 2 reported in table 5 (Groups T2P5_F6K2A2, T2P8_F6K2A2, and T2P8_F6K3A3)

		Linearized MIP model solved by CPLEX[a]					Reformulation solved by B&Ch[a]					
Group	Inst.	Upper B.	Lower B.	Gap	Time (s)	# nodes	Upper B.	Lower B.	Gap	Time (s)	# nodes	# cuts
T2P5_F6K2A2	1	21926.9	19089.4	12.94%	10800.0	131852	21926.9	21926.9	0.00%	45.1	4142	343
	2	21252.9	21250.8	0.01%	5983.6	287167	21252.9	21252.9	0.00%	44.8	4173	254
	3	22825.5	20424.4	10.52%	10800.0	196917	22825.5	22824.4	0.00%	110.4	11961	295
	4	23079.1	21044.2	8.82%	10800.0	365258	23079.1	23079.1	0.00%	188.9	25648	310
	5	21004.4	20139.6	4.12%	10800.0	403450	21004.4	21004.4	0.00%	45.1	4790	313
	6	21245.1	20161.2	5.10%	10800.0	311657	21245.1	21243.1	0.01%	88.1	11230	248
	7	22751.1	20642.0	9.27%	10800.0	240436	22751.1	22750.4	0.00%	104.3	14378	353
	8	23434.0	22426.3	4.30%	10800.0	338141	23434.0	23434.0	0.00%	77.9	7612	313
	9	22235.8	22233.6	0.01%	9192.1	385238	22235.8	22234.1	0.01%	42.8	3558	291
	10	22956.5	21027.1	8.40%	10800.0	241497	22956.5	22954.4	0.01%	110.1	12838	402
Avg.		**22271.1**	**20843.9**	**6.35%**	**10157.6**	**290161.3**	**22271.1**	**22270.4**	**0.00%**	**85.7**	**10033**	**312.2**
T2P8_F6K2A2	1	27671.9	27671.9	0.00%	1477.6	37471	27671.9	27671.9	0.00%	145.2	4773	372
	2	28005.3	28005.3	0.00%	1045.3	38019	28005.3	28005.3	0.00%	61.7	1300	378
	3	28811.4	28811.4	0.00%	1082.0	30705	28811.4	28811.4	0.00%	96.3	4158	268
	4	28477.6	28477.6	0.00%	5103.5	440075	28477.6	28477.6	0.00%	136.9	3352	375
	5	28082.1	28082.1	0.00%	8113.9	702180	28082.1	28082.1	0.00%	43.7	2011	307
	6	28836.1	28836.1	0.00%	1805.1	70458	28836.1	28833.6	0.01%	64.4	2336	322
	7	28383.9	28383.9	0.00%	5470.9	46379	28383.9	28383.9	0.00%	52.9	1441	281
	8	29323.2	29323.2	0.00%	896.4	32297	29323.2	29323.2	0.00%	54.5	2018	371
	9	27768.4	27767.4	0.00%	2880.3	62016	27768.4	27768.4	0.00%	52.1	1417	354
	10	27274.3	27274.3	0.00%	1497.2	29716	27274.3	27273.3	0.00%	70.0	2421	290
Avg.		**28263.4**	**28263.3**	**0.00%**	**2937.2**	**148931.6**	**28263.4**	**28263.1**	**0.00%**	**77.8**	**2522.7**	**331.8**
T2P8_F6K3A3	1	33137.3	33134.3	0.01%	6552.2	25624	33137.3	33134.1	0.01%	10085.1	191741	833
	2	33356.3	33355.3	0.00%	5528.1	20142	33356.3	33353.6	0.01%	9007.9	89125	707
	3	34224.3	34221.3	0.01%	10371.7	38989	34224.3	34220.9	0.01%	5174.1	67218	756
	4	33711.6	32256.4	4.32%	10800.0	32740	33711.6	33708.2	0.01%	8984.5	140545	944
	5	33627.9	30657.7	8.83%	10800.0	41064	33627.9	33625.1	0.01%	7322.2	110876	918
	6	34297.1	34294.7	0.01%	8293.3	25155	34297.1	32365.9	5.63%	10800.0	95934	810
	7	33712.4	30944.6	8.21%	10800.0	27770	33712.4	32721.2	2.94%	10800.4	134114	1008
	8	35031.5	33491.3	4.40%	10800.0	33131	35031.5	35028.1	0.01%	6982.9	112459	764
	9	33282.1	32932.9	1.05%	10800.0	40827	33282.1	33278.8	0.01%	7027.4	142819	803
	10	32623.7	32621.7	0.01%	6830.9	26429	32623.7	32620.8	0.01%	3771.1	69770	783
Avg.		**33700.4**	**32791.0**	**2.68%**	**9157.6**	**31187.1**	**33700.4**	**33405.7**	**0.86%**	**7995.6**	**115460.1**	**832.6**

a Results of the best performing case of the solution approaches reported in Table 5

Table 3: Detailed results of the best case performing solution approaches for Model 2 reported in Table 6 (Groups T4P5_F6K2A2, T4P5_F6K3A3, T4P8_F6K2A2, and T4P8_F6K3A3)

		Linearized MIP model solved by CPLEX[a]					Reformulation solved by the B&Ch[a]					
Group	Inst.	Upper B.	Lower B.	Gap	Time (s)	# nodes	Upper B.	Lower B.	Gap	Time (s)	# nodes	# cuts
T4P5_F6K2A2	1	67714.7	21190.9	68.71%	10800.0	50582	50872.4	29886.2	41.25%	10800.0	311539	662
	2	86995.4	62044.0	28.68%	10800.0	70161	86596.4	73511.2	15.11%	10800.0	306471	670
	3	139362.2	113278.4	18.72%	10800.0	40975	136799.5	124719.5	8.83%	10800.0	310156	500
	4	142566.2	105277.9	26.16%	10800.0	93104	141245.7	111892.0	20.78%	10800.0	264362	776
	5	91051.1	64399.8	29.27%	10800.0	71112	85276.6	75869.9	11.03%	10800.0	275100	503
	6	102518.0	97127.8	5.26%	10800.0	72240	102518.0	102507.8	0.01%	2368.5	137049	303
	7	157262.4	145580.7	7.43%	10800.0	51509	157262.4	157246.7	0.01%	7587.7	328328	422
	8	76233.9	61953.0	18.73%	10800.0	43632	76233.9	76226.5	0.01%	6547.7	321567	378
	9	111072.7	93608.4	15.72%	10800.0	73060	111072.7	104398.7	6.01%	10800.0	300469	537
	10	74255.4	68200.2	8.15%	10800.0	109064	74255.4	74248.6	0.01%	1913.8	99900	373
Avg.		**104903.2**	**83266.1**	**22.68%**	**10800.0**	**67543.9**	**102213.3**	**93050.7**	**10.31%**	**8321.8**	**265494.1**	**512.4**
T4P5_F6K3A3	1	58557.1	25051.4	57.22%	10800.0	29519	53048.5	27059.4	48.99%	10800.0	114425	2560
	2	94836.1	61798.6	34.84%	10800.0	54918	94836.1	57044.7	39.85%	10800.0	170877	1402
	3	126338.7	97864.0	22.54%	10800.0	54550	131776.9	88528.7	32.82%	10800.0	161500	1711
	4	142087.5	84245.9	40.71%	10800.0	59723	137011.9	87274.1	36.30%	10800.0	197542	2529
	5	91282.2	49561.0	45.71%	10800.0	27394	87670.0	57575.1	34.33%	10800.0	156703	1063
	6	99961.6	99952.1	0.01%	4561.4	24520	99961.6	99952.4	0.01%	735.3	26421	558
	7	108545.8	90647.8	16.49%	10800.0	41586	108280.1	93136.9	13.99%	10800.0	137000	1664
	8	73859.8	66556.2	9.89%	10800.0	41915	73859.8	73852.6	0.01%	4145.2	98696	1000
	9	113036.3	95964.5	15.10%	10800.0	35982	113036.3	97056.6	14.14%	10800.0	152000	1560
	10	60637.9	50343.7	16.98%	10800.0	47752	60047.1	56011.4	6.72%	10800.0	163871	628
Avg.		**96914.3**	**72198.5**	**25.95%**	**10176.1**	**41785.9**	**95952.8**	**73749.2**	**22.72%**	**9128.1**	**137903.5**	**1467.5**
T4P8_F6K2A2	1	177714.9	122738.5	30.94%	10800.0	9463	170352.1	131490.0	22.81%	10800.0	52754	868
	2	215940.7	143118.2	33.72%	10800.0	16538	209516.0	149892.2	28.46%	10806.8	85800	548
	3	187676.2	116506.2	37.92%	10800.0	14468	188990.3	119967.3	36.52%	10800.0	125264	552
	4	181605.0	107665.7	40.71%	10800.0	15460	188836.3	117892.3	37.57%	10800.0	111497	794
	5	226631.5	98062.1	56.73%	10800.0	10620	198865.8	126880.4	36.20%	10800.0	97775	581
	6	171127.9	122502.4	28.41%	10800.0	14907	172562.5	147079.1	14.77%	10800.0	64360	484
	7	125077.5	104369.3	16.56%	10800.0	15314	124608.0	118112.9	5.21%	10800.0	99336	405
	8	133539.5	108536.7	18.72%	10800.0	21522	131411.7	119181.9	9.31%	10800.0	78200	487
	9	151566.6	94018.5	37.97%	10800.0	10243	128828.1	109311.4	15.15%	10800.0	58969	739
	10	71575.4	42413.4	40.74%	10800.0	31174	69899.9	49586.8	29.06%	10800.0	130820	577
Avg.		**164245.5**	**105993.1**	**34.24%**	**10800.0**	**15970.9**	**158387.1**	**118939.4**	**23.51%**	**10800.7**	**90477.5**	**603.5**
T4P8_F6K3A3	1	176199.6	125483.4	28.78%	10800.0	12143	173261.9	125569.9	27.53%	10800.0	90023	1758
	2	186454.3	129110.0	30.76%	10800.0	13966	169822.7	131170.8	22.76%	10800.0	59792	1051
	3	176423.3	94787.0	46.27%	10800.0	12174	165116.0	94918.9	42.51%	10800.0	63903	1440
	4	165719.8	96791.2	41.59%	10800.0	16950	168530.3	95506.8	43.33%	10800.0	55868	1393
	5	177420.4	106202.8	40.14%	10800.0	10676	174515.1	110693.3	36.57%	10800.0	81947	787
	6	179481.1	120510.7	32.86%	10800.0	11686	171367.7	119162.5	30.46%	10800.0	44200	790
	7	134907.6	96188.3	28.70%	10800.0	12492	122554.4	104397.7	14.82%	10800.0	39581	679
	8	128385.2	104029.3	18.97%	10800.0	19260	128122.8	114965.4	10.27%	10800.0	63500	765
	9	162176.5	100090.5	38.28%	10800.0	13867	160970.8	101784.8	36.77%	10800.0	57334	935
	10	69411.1	45790.2	34.03%	10800.0	19345	66655.1	51363.3	22.94%	10800.0	78801	905
Avg.		**155657.9**	**101898.3**	**34.04%**	**10800.0**	**14255.9**	**150091.7**	**104953.3**	**28.80%**	**10800.0**	**63494.9**	**1050.3**

a Results of the best performing case of the solution approaches reported in Table 6

Table 4: Detailed results of the 3-phase heuristic reported in Table 7 (groups T2P5_F6K2A2, T2P8_F6K2A2, and T4P8_F6K3A3)

Group	Inst.	Phase 1			Phase 2			Phase 3		
		Obj. Val.	Time(s)	Dev.[a]	Obj. Val.	Time(s)[b]	Dev.[a]	Obj. Val.	Time(s)[b]	Dev.[a]
T2P5_F6K2A2	1	30876.9	10.3	40.82%	21926.9	10.3	0.00%	21926.9	18.1	0.00%
	2	29818.6	9.8	40.30%	21252.9	9.9	0.00%	21252.8	16.9	0.00%
	3	32072.2	11.4	40.51%	22825.5	11.4	0.00%	22825.4	20.9	0.00%
	4	31430.6	28.4	36.19%	23079.1	28.5	0.00%	23079.0	36.3	0.00%
	5	28335.4	12.7	34.90%	21004.4	12.7	0.00%	21004.4	20.6	0.00%
	6	29766.9	12.5	40.11%	21245.1	12.6	0.00%	21245.1	21.4	0.00%
	7	31105.9	11.9	36.72%	22751.1	12.0	0.00%	22751.0	20.8	0.00%
	8	36850.1	13.1	57.25%	23434.0	13.2	0.00%	23434.0	19.9	0.00%
	9	30123.8	10.9	35.47%	22235.8	10.9	0.00%	22235.8	18.8	0.00%
	10	31165.4	15.2	35.76%	22956.5	15.2	0.00%	22956.5	23.3	0.00%
Avg.		**31154.6**	**13.6**	**39.80%**	**22271.1**	**13.7**	**0.00%**	**22271.1**	**21.7**	**0.00%**
T2P8_F6K2A2	1	49223.9	45.4	77.88%	27671.9	45.5	0.00%	27671.9	59.7	0.00%
	2	49241.7	49.3	75.83%	28005.3	49.3	0.00%	28005.3	60.6	0.00%
	3	49037.2	33.3	70.20%	28811.4	33.4	0.00%	28811.3	49.7	0.00%
	4	48750.3	48.3	71.19%	28477.6	48.2	0.00%	28477.6	64.4	0.00%
	5	48765.7	36.9	73.65%	28082.1	36.9	0.00%	28082.1	48.4	0.00%
	6	49989.8	31.7	73.36%	28836.1	31.7	0.00%	28836.1	46.0	0.00%
	7	48613.2	29.7	71.27%	28383.9	29.8	0.00%	28383.9	44.7	0.00%
	8	51035.7	35.1	74.05%	29323.2	35.1	0.00%	29323.2	51.6	0.00%
	9	49366.4	35.6	77.78%	27768.4	35.7	0.00%	27768.4	50.6	0.00%
	10	49287.6	35.8	80.71%	27274.3	35.9	0.00%	27274.3	52.9	0.00%
Avg.		**49331.1**	**38.1**	**74.59%**	**28263.4**	**38.2**	**0.00%**	**28263.4**	**52.9**	**0.00%**
T2P8_F6K2A2	1	51424.4	3639.7	55.19%	33137.3	3639.9	0.00%	33137.2	3662.7	0.00%
	2	51500.2	130.9	54.39%	33356.3	131.0	0.00%	33356.3	158.9	0.00%
	3	51316.4	3579.2	49.94%	34224.3	3579.3	0.00%	34224.2	3599.7	0.00%
	4	51024.3	81.5	51.36%	33711.6	81.5	0.00%	33711.5	107.4	0.00%
	5	50999.9	80.3	51.66%	33627.9	80.4	0.00%	33627.9	102.6	0.00%
	6	52222.8	39.9	52.27%	34297.1	40.1	0.00%	34297.0	64.2	0.00%
	7	50907.5	1946.1	51.01%	33712.4	1946.2	0.00%	33712.3	1968.9	0.00%
	8	53247.4	3472.5	52.00%	35031.5	3472.7	0.00%	35031.5	3493.4	0.00%
	9	51564.5	39.8	54.93%	33282.1	39.9	0.00%	33282.0	63.8	0.00%
	10	51595.9	31.9	58.15%	32623.7	31.9	0.00%	32623.7	48.8	0.00%
Avg.		**51580.4**	**1304.2**	**53.09%**	**33700.4**	**1304.3**	**0.00%**	**33700.4**	**1327.0**	**0.00%**

a Deviation compared to the best solution found by the B&Ch
b Cumulative time

Table 5: Detailed results of the 3-phase heuristic reported in Table 8 (groups T4P5_F6K2A2, T4P5_F6K3A3, T4P8_F6K2A2, and T4P8_F6K3A3)

		Phase 1			**Phase 2**			**Phase 3**		
Group	Inst.	Obj. Val.	Time(s)	Dev.[a]	Obj. Val.	Time(s)[b]	Dev.[a]	Obj. Val.	Time(s)[b]	Dev.[a]
T4P5_F6K2A2	1	60356.6	75.6	18.64%	51158.5	77.3	0.56%	50872.4	101.6	0.00%
	2	102166.1	62.4	17.98%	86995.4	62.6	0.46%	86995.4	99.3	0.46%
	3	146134.8	52.0	6.82%	139550.5	55.0	2.01%	136799.5	75.0	0.00%
	4	154163.6	77.1	9.15%	143039.3	77.4	1.27%	142566.1	129.3	0.93%
	5	108171.0	75.0	26.85%	85276.6	77.6	0.00%	85276.6	105.9	0.00%
	6	109876.6	46.1	7.18%	102518.0	46.5	0.00%	102517.9	68.3	0.00%
	7	184727.4	44.3	17.46%	157262.4	49.9	0.00%	157262.4	71.7	0.00%
	8	88778.3	35.0	16.46%	76233.9	35.8	0.00%	76233.8	53.1	0.00%
	9	126839.1	53.9	14.19%	113781.6	55.6	2.44%	113781.6	75.0	2.44%
	10	78475.4	54.7	5.68%	74255.4	56.7	0.00%	74255.4	79.1	0.00%
Avg.		**115968.9**	**57.6**	**14.04%**	**103007.2**	**59.4**	**0.67%**	**102656.1**	**85.8**	**0.38%**
T4P5_F6K2A2	1	66850.0	254.0	26.02%	56935.7	260.7	7.33%	56935.7	336.7	7.33%
	2	112521.7	290.9	18.65%	98010.8	291.7	3.35%	95386.1	1240.7	0.58%
	3	162472.4	206.9	23.29%	128948.4	209.2	-2.15%	124566.2	249.8	-5.47%
	4	151721.8	481.3	10.74%	141689.6	483.2	3.41%	140203.2	992.4	2.33%
	5	91890.0	147.9	4.81%	87670.0	152.9	0.00%	87670.0	216.1	0.00%
	6	107562.7	63.7	7.60%	99961.6	64.2	0.00%	99961.6	101.6	0.00%
	7	126812.7	109.5	17.12%	116886.2	112.7	7.95%	118167.3	172.6	9.13%
	8	83105.2	52.0	12.52%	73859.8	52.3	0.00%	73859.8	88.5	0.00%
	9	128816.0	149.5	13.96%	118672.0	150.8	4.99%	118672.0	183.7	4.99%
	10	75086.3	88.5	25.05%	60964.9	90.2	1.53%	60964.9	128.0	1.53%
Avg.		**110683.9**	**184.4**	**15.98%**	**98359.9**	**186.8**	**2.64%**	**97638.7**	**371.0**	**2.04%**
T4P8_F6K2A2	1	174027.7	229.2	2.16%	164829.6	246.4	-3.24%	164543.5	296.8	-3.41%
	2	238394.3	125.7	13.78%	203931.1	131.0	-2.67%	203931.1	188.7	-2.67%
	3	211653.4	195.4	11.99%	180926.2	219.3	-4.27%	180926.2	260.8	-4.27%
	4	216212.8	2220.9	14.50%	190458.3	2228.3	0.86%	183090.3	3268.9	-3.04%
	5	217630.2	3980.5	9.44%	211047.1	3995.9	6.13%	200091.2	4060.4	0.62%
	6	196030.5	151.7	13.60%	171037.6	183.0	-0.88%	171037.6	230.2	-0.88%
	7	156857.8	111.7	25.88%	124608.0	115.6	0.00%	124607.9	160.1	0.00%
	8	149446.1	103.3	13.72%	131411.7	109.0	0.00%	131411.7	138.6	0.00%
	9	154478.4	186.9	19.91%	132928.9	194.7	3.18%	132928.8	234.1	3.18%
	10	76791.6	99.4	9.86%	69899.9	116.7	0.00%	69899.8	177.3	0.00%
Avg.		**179152.3**	**740.5**	**13.48%**	**158107.8**	**754.0**	**-0.09%**	**156246.8**	**901.6**	**-1.05%**
T4P8_F6K2A2	1	181369.3	1189.7	4.68%	168631.2	1207.6	-2.67%	168631.2	1315.8	-2.67%
	2	191390.0	1373.6	12.70%	170631.0	1380.2	0.48%	169221.9	1472.0	-0.35%
	3	170753.3	716.4	3.41%	156984.1	726.7	-4.92%	156984.0	804.2	-4.93%
	4	176531.5	1197.1	4.75%	167916.9	1203.7	-0.36%	167916.8	1268.3	-0.36%
	5	165729.3	1722.1	-5.03%	157584.7	1736.5	-9.70%	157584.7	1979.4	-9.70%
	6	172512.9	233.9	0.67%	158394.8	291.1	-7.57%	158394.7	373.1	-7.57%
	7	138418.9	192.2	12.94%	122554.4	201.7	0.00%	122554.3	333.5	0.00%
	8	140166.2	151.5	9.40%	128122.8	152.0	0.00%	128122.7	282.5	0.00%
	9	181051.5	1605.0	12.47%	160202.3	1669.7	-0.48%	158174.6	1799.7	-1.74%
	10	79961.0	186.6	19.96%	67923.0	192.7	1.90%	67922.9	285.9	1.90%
Avg.		**159788.4**	**856.8**	**7.60%**	**145894.5**	**876.2**	-2.33%	145550.8	**991.4**	**-2.54%**

a Deviation compared to the best solution found by the B&Ch
b Cumulative time

D Additional experiments on critical parameters

We performed computational experiments for a sample of instances on different levels of demand and backlogging costs. These results provide insights about the performance of the proposed exact solution methods for different characteristics of the instances. We tested the 10 instances in group T2P5_F6K2A1 on 4 different cases described as follows:

- **Original case:** backlogging costs are set up as h^-_{it} and inventory unit costs are set as h^+_{it}. Demand parameters are defined based on U(a,b), where a=200,000 and b=250,000.
- **Case A:** backlogging costs are set as $\frac{1}{5}h^-_{it}$, $\forall i \in N; t \in T$.
- **Case B:** backlogging costs are set as $5h^-_{it}$, $\forall i \in N$; $t \in T$.
- **Case C:** demand levels are based on $U(a', b')$, where $a' = 75,000$ and $b' = 375,000$.
- **Case D:** constant demand as $d_i = 225,000$, $\forall i \in N$.

The average results of these experiments are presented in Table 6. These results show that higher backlogging costs and a higher demand variance (i.e., cases B and C) make the instances more difficult to be solved and affect the performance of both solution methods. However, the results for this sample of instances showed that the B&Ch is able to find optimal and good quality solutions whereas the linearized model presents large optimality gap after 3 hours (i.e., solutions with average gaps up to 21%). Instances with lower backlogging costs (i.e., Case A) appear to be less complex and can be effectively solved by both solution approaches. Results show that the linearized model and the B&Ch can find optimal solutions in a few seconds for these cases.

Table 6: Average results of the solution approaches on critical parameters

	Linearized Model					Branch-and-check					
	UB	LB	Gap	Time (s)	#Nodes	UB	LB	Gap	Time (s)	# Nodes	#cuts
Original Case	17336.0	14884.0	14.9%	9810.7	322230.6	17336.0	17335.8	0.0%	64.7	2938.5	293.7
Case A	5093.6	5093.6	0.0%	6.0	2.1	5093.6	5093.6	0.0%	0.5	0.0	0.0
Case B	19999.2	15858.0	20.2%	10800.0	335700.1	19999.2	19371.6	1.5%	1309.0	76594.3	370.9
Case C	18994.6	15083.2	21.1%	10800.0	541292.3	18994.6	18993.9	0.0%	183.0	9734.1	327.7
Case D	17920.2	15006.4	17.0%	10800.0	497459.5	17920.2	17919.7	0.0%	74.5	4353.3	328.1