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Abstract: The purpose of this study is to design incentives for price and advertising coordination in a bilateral monopoly. I prove that a wholesale price reduction and a cooperative advertising program allow a decentralized channel to reach the performance of a vertically integrated one. I identify the coordinating wholesale price regions where (i) the Stackelberg manufacturer is interested in their implementation (ii) they are Pareto-improving. I then compute the optimal coordination wholesale price resulting from an egalitarian sharing of the profit surplus.

Keywords: Differential games, advertising, pricing, incentive strategies, channel coordination, Stackelberg manufacturer

1 Introduction

This study uses a dynamic perspective to investigate the issue of channel coordination (i.e. cooperation) through the implementation of incentive strategies. More specifically, I examine a differential game that takes place in a bilateral monopoly where the manufacturer plays the role of a Stackelberg leader that offers incentives to a single retailer in order to push him to play the cooperative levels of price and advertising decisions.

The literature on marketing and supply chain management already demonstrated the positive impacts of coordinating channel members' decisions on individual and total channel profits. This result is not surprising since in coordinated channels members set their control variables at the levels that maximize the total channel profit, as if they were dictated by a single decision-maker (i.e., a centralized decision). Consequently, the performance of a coordinated bilateral monopoly is equal to that of a vertically integrated structure. In contrast, decentralized channels are characterized by independent members that usually seek the maximization of their individual outcomes. This selfish behavior leads to double-marginalisation and to underinvestment in non-price marketing variables, two sources of inefficiency in the channel.

Since vertical integration is often illegal and hard to achieve, an important research stream in the channels literature is devoted to finding alternative, less drastic solution, the main objective being identifying coordinating mechanisms that replicate the performance of a jointly owned channel in a structure where the members remain independent institutions.

The design of incentives to reach this objective started with the seminal paper of Jeuland and Shugan (1983). The authors demonstrated that quantity discounts that align the individual channel members' objectives with those of the whole channel allow independent members to maximize the total profit while maximizing their own profits. As a result, both channel members set the price and the non-price marketing variables at channel-coordinating levels.

A significant game-theoretic literature followed the work of Jeuland and Shugan (1983). Ingene, Taboubi, and Zaccour (2012) surveyed this literature and highlighted the fact that most of it investigates the channel-coordination issue in a static setting. Hence, these studies disregard the carryover effects of marketing decisions and the repetitive interactions among channel members. Chintagunta and Jain (1992) were among the first authors to extend this literature to a dynamic setting. They considered a channel where the members' marketing efforts contribute to the building of their brand goodwill (i.e., brand reputation or equity). The authors confirmed the efficiency of coordination but did not indicate how this level of efficiency can be reached when the channel is decentralized. Furthermore, their study considered channel members that only control their marketing efforts, while prices are taken as constant. To overcome these limitations, Zaccour (2008) examined the scenario where channel members control both price and non-price marketing variables, and proved that the pricing scheme suggested by Jeuland and Shugan (1983) no longer plays a coordinating role when the intertemporal effects of non-price marketing variables are introduced into the problem.

To the best of my knowledge, the only studies that have provided a solution to the issue of channel coordination in a dynamic setting are Jørgensen and Zaccour (2003) and De Giovanni, Reddy, and Zaccour (2016). In both papers, the authors suggested using two-sided incentive strategies. These mechanisms are designed in a way that allows the strategies of each channel member to depend on the other channel member's choices. Hence, when the incentive strategies are carried out jointly, the cooperative solution is reached as an equilibrium, and neither channel member can improve its outcome when deviating. Two-sided incentives are used in both studies to deal with situations where channel members set the level of their marketing instruments independently and simultaneously (i.e., play a Nash game). In the channels literature, this situation corresponds to the case where the channel has no leader.

When the channel has a leader, the channels literature has often attributed this role to the manufacturer who sets its optimal decisions by taking into account the retailer's (i.e., follower's) reaction functions. Since decisions are announced sequentially, there is no need to implement two-sided incentives; one-sided incentives offered by the manufacturer to the retailer are then the appropriate mechanism for the coordination of manufacturer Stackelberg channels.

In Jørgensen, Taboubi, and Zaccour (2006), the authors examined an advertising game where the incentive takes the form of a cooperative advertising allowance, offered by the manufacturer to its retailer in order to induce the latter to play the cooperative level of the local advertising effort. Such an incentive leads to channel coordination when the manufacturer commits to setting its national advertising effort at the cooperative level. Taboubi (2017) extended this study by introducing wholesale and retail prices as additional decision variables. In this case, the manufacturer offers two incentives to coordinate the price and advertising decisions. However, since the incentives are one-sided, and the manufacturer does not improve its profit when committing to play cooperatively, these incentives fail to mimic the performance of a vertically integrated channel, although they perform better than a decentralized channel without incentives.

In this paper, I contribute to the above-cited literature by designing one-sided incentives that a Stackelberg manufacturer can offer its retailer. I demonstrate that, when the manufacturer commits to playing its cooperative part of the contract, it can induce channel coordination by offering a wholesale price reduction and a cooperative advertising program where he subsidizes the retailer's advertising costs. I identify the conditions for the incentives' implementation and compute the optimal cooperative wholesale price value that the manufacturer should choose to guarantee an egalitarian sharing of the additional profits resulting from coordination.

The rest of the paper is organized as follows: In the next section, I present the main features of the model and the different scenarios, and compute the equilibria for the first two scenarios: channel decentralization and channel coordination through vertical integration. The latter is used as the desired solution in the design of incentives, while the former is used as a benchmark to verify the conditions for incentive implementation. In Section 3, I compute the one-sided incentive strategies and examine the conditions for their implementation. Section 4 concludes.

2 Model and scenarios

I consider a Stackelberg game where a monopolist manufacturer acts as the channel leader controlling the wholesale price $w(t)$ and the national advertising level $a_M(t)$ for its brand. The brand is sold via a single retailer, which controls the retail price $p(t)$ and the level of local advertising $a_R(t)$.

The brand's goodwill, denoted by $G(t)$, evolves according to the following capital accumulation function:

$$\dot{G}(t) = \alpha a_M(t) + \beta a_R(t) - \delta G(t), \tag{1}$$
$$G(0) = G_0 \geq 0$$

This function is an extension of the Nerlove-Arrow (1962) model. It captures the carryover effects of both channel members' advertising efforts in building the goodwill stock. α and β are positive parameters representing these impacts, and δ is a decay rate.

I assume that the manufacturer's production cost is constant (c) and that both channel members face quadratic cost functions given by

$$C(a_i) = \frac{(a_i(t))^2}{2}, \; i \in \{M, R\}.$$

The demand function, denoted by $D(t)$ is

$$D(t) = (\lambda - \theta p(t)) G(t). \tag{2}$$

Its expression indicates that demand decreases in the retail price and is positively affected by the goodwill stock, resulting from the channel-members' investments in advertising efforts.

Since the objective of the study is to design incentive strategies that allow for channel coordination and to investigate the conditions for their implementation, it is necessary to start the analysis by computing the channel members' strategies and profits at the equilibria under a channel decentralization scenario (denoted by the superscript D) and under a channel coordination scenario (denoted by the superscript C).

A final scenario, denoted by the superscript I, will be devoted to incentive design.

J_M and J_R stand for the objective functionals of the manufacturer (M) and the retailer (R), respectively. In all the scenarios, I consider that the channel members discount their stream of profits over an infinite horizon by using the same discount rate $\rho \geq 0$ subject to the state dynamics given by (1). Finally, I consider the information structure to be Markovian, that is, channel members' strategies depend on the current level of goodwill.

2.1 Channel decentralization

This scenario provides a benchmark to investigate whether or not the incentives can be implemented. It corresponds to the case where channel members are independent institutions maximizing their individual payoffs without providing any incentives.

The objective functionals of the retailer and the manufacturer are given by the following equations:

$$J_R = \int_0^\infty e^{-\rho t} \left[(p(t) - w(t)) (\lambda - \theta p(t)) G(t) - \frac{(a_R(t))^2}{2} \right] dt, \tag{3}$$

$$J_M = \int_0^\infty e^{-\rho t} \left[(w(t) - c) (\lambda - \theta p(t)) G(t) - \frac{(a_M(t))^2}{2} \right] dt. \tag{4}$$

Since the retailer is the follower, I start by solving its maximization problem in order to obtain its reaction functions. Then I substitute these reaction functions into the manufacturer's problem and solve it. The retailer's strategies at equilibrium are computed after substituting of manufacturer's strategies under equilibria into the retailer's reaction functions. The following proposition gives the equilibrium strategies, demand, channel members' value functions, and the goodwill level at the steady-state under the decentralization scenario.

Proposition 1 *Assuming an interior solution, the equilibrium strategies under decentralization are given by the following expressions:*

$$w^D = \frac{\lambda + c\theta}{2\theta},$$

$$a_M^D = \frac{(\lambda - c\theta)^2}{8\theta (\delta + \rho)} \alpha,$$

$$p^D = \frac{3\lambda + c\theta}{4\theta},$$

$$a_R^D = \frac{(\lambda - c\theta)^2}{16\theta (\delta + \rho)} \beta.$$

Demand is given by

$$D^D(G) = \frac{(\lambda - c\theta)}{4} G$$

and the channel members' value functions are

$$V_M^D(G) = S_1 G + S_2 \tag{5}$$

$$V_R^D(G) = T_1 G + T_2 \tag{6}$$

where $S_1, S_2, T_1,$ and T_2 are positive parameters. Their expressions are given in Appendix A.

The goodwill level at the steady state is given by

$$G_{SS}^D = \frac{(2\alpha^2 + \beta^2)(\lambda - c\theta)^2}{16\theta\delta (\delta + \rho)}.$$

Proof. See Appendix A. □

2.2   Channel coordination

This scenario corresponds to the case where the channel is coordinated via vertical integration. It makes it possible to compute the desired levels for the marketing instruments that the manufacturer wants to replicate by offering the incentive strategies. These levels are used to design the incentives in the last scenario.

Under this scenario, both channel members agree to cooperate by maximizing the sum of their individual outcomes. The objective functional is given by the following expression:

$$
\begin{aligned}
J_C = J_M + J_R \\
= \int_0^\infty e^{-\rho t} \left[(p(t) - c)(\lambda - \theta p(t)) G(t) - \frac{(a_R)^2 + (a_M)^2}{2} \right] dt,
\end{aligned}
\tag{7}
$$

subject to (1).

Optimal strategies, demand, the channel's value function, and the goodwill level at the steady state under this scenario are given in the following proposition.

Proposition 2 *Optimal strategies under coordination are given by the following expressions:*

$$
p^C = \frac{\lambda + c\theta}{2\theta},
$$

$$
a_M^C = \frac{(\lambda - c\theta)^2}{4\theta(\delta + \rho)}\alpha,
$$

$$
a_R^C = \frac{(\lambda - c\theta)^2}{4\theta(\delta + \rho)}\beta.
$$

Demand is given by

$$
D^C(G) = \frac{(\lambda - c\theta)}{2}G.
$$

The channel's value function is

$$
V^C(G) = K_1 G + K_2
\tag{8}
$$

where K_1, K_2 are positive parameters. Their expressions are given in Appendix B. The goodwill level at the steady state is given by

$$
G_{SS}^C = \frac{(\alpha^2 + \beta^2)(\lambda - c\theta)^2}{4\theta\delta(\delta + \rho)}.
$$

Proof. See Appendix B.   □

As expected, these results indicate that retail prices, national and local advertising levels, demands, total channel outcomes, and goodwill at the steady state under the coordinated and the decentralized channels compare as follows:

$$
\begin{aligned}
p^C(G) < p^D(G),\ a_M^C(G) > a_M^D(G),\ a_R^C(G) > a_R^D(G), \\
D^C(G) > D^D(G),\ V^C(G) > V_M^D(G) + V_R^D(G) \text{ and } G_{SS}^C > G_{SS}^D.
\end{aligned}
$$

Hence, the manufacturer, who acts as a leader, could be tempted to implement incentives that allow the decentralized channel to reach this level of efficiency. The next section is devoted to the design and computation of these incentives. It corresponds to the third scenario.

3   Incentives for channel coordination

I consider that the manufacturer, as he plays the role of the channel leader, offers two incentives designed to push the retailer to play the cooperative levels p^C and a_R^C of the retail price and of the local advertising efforts. These incentives are based on a wholesale price reduction and a cooperative advertising program.

The expression of both incentives are given by the following equations:

$$w^I\left(p\right) = w^C + \psi\left(p - p^C\right) \tag{9}$$

$$I\left(a_R\right) = \eta C(a_R) = \frac{\eta}{2}\left(a_R\right)^2 \tag{10}$$

where ψ and η are positive parameters that must be set by the manufacturer in order to push the retailer to choose the channel-coordinating levels $p^C\left(t\right)$ and $a_R^C\left(t\right)$ for the retail price and the retailer's advertising efforts, respectively.

Notice that the price-coordinating mechanism given by Equation (9) is designed as an incentive strategy. It indicates that the manufacturer adjusts its wholesale price by providing either a price discount, or a price increase as "punishment," depending on whether the retailer fixes the retail price over or under the channel-optimal retail price.

The second equation corresponds to a different type of incentive, since it doesn't link the channel members' advertising strategies one to the other, as is the case with the incentive strategies suggested by Jørgensen and Zaccour (2003) and De Giovanni et al. (2016). The incentive in (10) corresponds to a cooperative advertising program where the manufacturer subsidizes the retailer's local advertising costs according to a participation rate η. This participation rate is not considered a control variable, but its value is chosen by the manufacturer whose aim is to push the retailer to set $a_R^I = a_R^C$.

In order to reach the objective of channel coordination, the manufacturer commits to play its part in the coordinated solution, that is, it commits to setting the wholesale price at the level w^C and the national advertising level at its coordinating level a_M^C, as given in Proposition 2.

I follow the same steps as in the decentralization scenario and start by solving the retailer's maximization problem after substituting w^C, p^C, and a_M by their respective values. The retailer's HJB equation is given by

$$
\begin{aligned}
\rho V_R^I\left(G\right) &= \max_{p, a_R \geq 0}\left[\left(p - w^I\left(p\right)\right)\left(\lambda - \theta p\right) G + \frac{(1-\eta)}{2}\left(a_R\right)^2 + \frac{dV_R^I}{dG}\left(\alpha a_M + \beta a_R - \delta G\right)\right] \\
&= \max_{p, a_R \geq 0}\left[\left(p - \left(w^C + \psi\left(p - \frac{\lambda + c\theta}{2\theta}\right)\right)\right)\left(\lambda - \beta\theta\right) G + \frac{(1-\eta)}{2}\left(a_R\right)^2 \right. \\
&\quad \left. + \frac{dV_R^I}{dG}\left(\alpha a_M + \beta a_R - \delta G\right)\right],
\end{aligned}
\tag{11}
$$

subject to equations (9) and (10).

Note that w^C vanishes under channel coordination. Hence, its value can be determined ex-post by channel members. Hence, I consider here only two constraints on $w^C\left(t\right)$: its convergence to a nonnegative value when t goes to infinity; and $w^{VI}\left(t\right) \geq c$, a condition that guarantees a positive margin for the manufacturer.

The first-order conditions resulting from the maximization of Equation (11) with respect to the retailer's control variables give the following reaction functions:

$$
p^I = \left\{
\begin{array}{l}
\frac{2\left(\lambda + \theta w^C\right) - \psi\left(3\lambda + c\theta\right)}{4\theta(1-\psi)} \text{ if } p > 0 \\
0 \qquad \text{otherwise}
\end{array}
\right\},
$$

$$
a_R^I = \left\{
\begin{array}{l}
\frac{\beta\left(V_R^I(G)\right)'}{(1-\eta)} \text{if } > 0 \\
0 \qquad \text{otherwise}
\end{array}
\right\}.
$$

I substitute these reaction functions in Equation (11) and conjecture that the retailer's value function is linear and given by the following expression:

$$V_R^I(G) = R_1 G + R_2$$

The parameters R_1 and R_2 are then found by identification, after rearranging the terms that correspond to the coefficient of $G(t)$ and the constant. At this stage, R_1 and R_2 depend on the model's parameters and on the incentives parameters' ψ and η. Since $a_R^I = \frac{\beta(V_R^I(G))'}{(1-\eta)}$, then $a_R^I = \frac{\beta R_1}{(1-\eta)}$.

The values of ψ and η are then obtained by setting $p^I = p^C$ and $a_R^I = a_R^C$ and solving the system. The solution is given by the following expressions:

$$\left\{ \begin{array}{l} \psi = \frac{2(w^C - c)\theta}{\lambda - c\theta} \\ \eta = \frac{2(w^C - c)\theta}{\lambda - c\theta} \end{array} \right\}. \tag{12}$$

These expressions indicate that the manufacturer's incentives designed to push the retailer to play the cooperative solution are affected by the manufacturer's unit margin $(w^C - c)$ under the cooperative scenario. Indeed, the manufacturer will increase its support for the retailer by reducing the wholesale price and offering a higher support for its local advertising if the manufacturer's unit margin under the cooperative scenario increases.

Then I substitute ψ and η from (12) in the expressions of R_1 and R_2 in order to obtain the retailer's value function.

The manufacturer's HJB equation under the incentives scenario is given by the following equation:

$$\rho V_M^I(G) = (w^I - c)(\lambda - \theta p^I) G - \frac{(a_M)^2}{2} - \frac{\eta}{2}(a_R^I)^2 + \frac{dV_M^I}{dG}(\alpha a_M + \beta a_R^I - \delta G). \tag{13}$$

In order to compute $V_M^I(G)$, I substitute p^I, a_M, and a_R^I by their coordinating levels p^C, a_M^C, and a_R^C given in Proposition 2 and the parameter η by its value from (12) and conjecture that the manufacturer's value function is linear and given by:

$$V_M^I(G) = M_1 G + M_2$$

Here again, the parameters M_1 and M_2 are obtained by identification.

The following proposition gives the incentive mechanisms that the manufacturer should implement to induce the retailer to act in a cooperative manner, and the value functions of both channel members when these incentives are implemented.

Proposition 3 *When the manufacturer commits to play cooperatively and offers the retailer the incentives given by the expressions*

$$w^I(p) = w^C + \left(\frac{2(w^C - c)\theta}{\lambda - c\theta}\right)\left(p - \frac{\lambda + c\theta}{2\theta}\right), \tag{14}$$

$$I(a_R) = \frac{2(w^C - c)\theta}{\lambda - c\theta}(a_R)^2, \tag{15}$$

the decentralized channel reaches the efficiency of a vertically integrated one.

The retailer's and manufacturer's value functions are given by

$$V_R^I(G) = \frac{(\lambda - c\theta)(\lambda - \theta(2w^C - c))}{4\theta(\delta + \rho)} G + \frac{(\lambda - c\theta)^3 (2\alpha^2 + \beta^2)(\lambda - \theta(2w^C - c))}{32\theta^2 \rho(\delta + \rho)^2},$$

$$V_M^I(G) = \frac{(\lambda - c\theta)(w^C - c)}{2(\delta + \rho)} G + \frac{(c\theta - \lambda)^3 (\alpha^2 \lambda - 2w^C \theta(2\alpha^2 + \beta^2) + c\theta(3\alpha^2 + 2\beta^2))}{32\theta^2 \rho(\delta + \rho)^2}.$$

Note here that we have the following equalities:

$$K_1 = R_1 + M_1, \text{ and } K_2 = R_2 + M_2.$$

It follows that we can write

$$V^C(G) = V_M^I(G) + V_R^I(G).$$

In order to investigate whether the incentives can be implemented, it is necessary to prove that the manufacturer improves its profits when it offers these incentives to the retailer and commits to play its cooperative part of the contract. Hence, one needs to compare the manufacturer's profit when offering the incentive with its profit when the channel is decentralized and no incentives are offered (i.e., the status quo). Since the planning horizon is infinite, I focus the analysis on the steady-state payoffs and compute $V_M^I \left(G_{SS}^I \right)$ and $V_M^D \left(G_{SS}^D \right)$. The next proposition indicates the minimum level for the coordinating wholesale price under which the manufacturer is not interested in implementing the incentives.

Proposition 4 *The manufacturer is interested in implementing the incentives under the following condition on the cooperative wholesale price level:*

$$w^C \geq \frac{\lambda\left(\left(28\alpha^2\left(\delta+\rho\right)+15\beta^2\left(\delta+2\rho\right)\right)+c\theta(36\alpha^2\left(\delta+\rho\right)+17\beta^2\left(\delta+2\rho\right)\right)}{32\theta(2\alpha^2\left(\delta+\rho\right)+\beta^2\left(\delta+2\rho\right))}.$$

Proof. It suffices to compute $V_M^I \left(G_{SS}^I \right) - V_M^D \left(G_{SS}^D \right)$ and to satisfy the condition on its positivity. Notice that this lower bound is compatible with the constraint $p^C - w^C \geq 0$. □

Moving on to the retailer's problem, I its possible, even though the retailer is a follower, to identify the conditions under which theses incentives are also profitable, and consequently, Pareto-improving. Following the same logic, I compute $V_R^I \left(G_{SS}^I \right) - V_R^D \left(G_{SS}^D \right)$ and identify the following condition on the upper bound of w^C, which bound is given by

$$w^C \leq \frac{\lambda\left(\left(\alpha^2\left(5\delta+2\rho\right)+\beta^2\left(\delta+\rho\right)\right)+c\theta(\alpha^2\left(11\delta+14\rho\right)+\beta^2\left(7\delta+15\rho\right)\right)}{8\theta(2\alpha^2\left(\delta+\rho\right)+\beta^2\left(\delta+2\rho\right))}.$$

Hence, these bounds define the interval within which the incentive strategies offered by the manufacturer to the retailer are Pareto-improving.

If we consider a situation where the channel members adopt an egalitarian principle allowing them to equally share the surplus resulting from implementing the cooperative solution through these incentives, it is possible to compute the optimal value for w^C according to this principle.

This can be done by computing the function $f\left(w^C\right)$ given by

$$f\left(w^C\right) = \left(V_M^I \left(G_{SS}^I \right) - V_M^D \left(G_{SS}^D \right)\right) - \left(V_R^I \left(G_{SS}^I \right) - V_R^D \left(G_{SS}^D \right)\right)$$

and solving $f\left(w^C\right) = 0$.

The result indicates that the optimal level for the wholesale price under a cooperative scenario when the egalitarian principle is used is given by

$$w^C = \frac{\lambda\left(\left(12\alpha^2\left(4\delta+3\rho\right)+\beta^2\left(19\delta+34\rho\right)\right)+c\theta(\alpha^2\left(80\delta+92\rho\right)+\beta^2\left(45\delta+94\rho\right)\right)}{64\theta(2\alpha^2\left(\delta+\rho\right)+\beta^2\left(\delta+2\rho\right))}$$

To illustrate these results, I provide a numerical example where the model's parameters are set at the following levels:

$$\alpha = 1, \delta = 0.1, c = 2, \lambda = 10, \rho = 0.1, \beta = 1, \text{ and } \theta = 1$$

and the cooperative wholesale price w^C varies in the interval defined by $c = 2$ and $p^C = 6$. This interval is chosen in order to guarantee positive margins for both channel members.

With these values, the demand level, the strategies, the individual profits and the goodwill at the steady state are positive under all the investigated scenarios. Figure 1 illustrates the difference in the channel members profits between the scenarios I and D for various values of w^C. The dashed line corresponds to the variations in manufacturer's profits (ΔVM). It indicates that the manufacturer is interested in implementing the incentives only when w^C is higher than 3.28. The bold line, which corresponds to the variations in the retailer's profits (ΔVR) indicates that the retailer is interested by the incentives only when w^C is lower than 5.6. Hence, all the values of w^C lying in the interval $]3.28, 5.6[$ are Pareto-improving. The level of w^C that allows both channel members to share equally the profit surplus resulting form incentives implementation is given at the intersection of both lines which is obtained for $w^C = 4.45$.
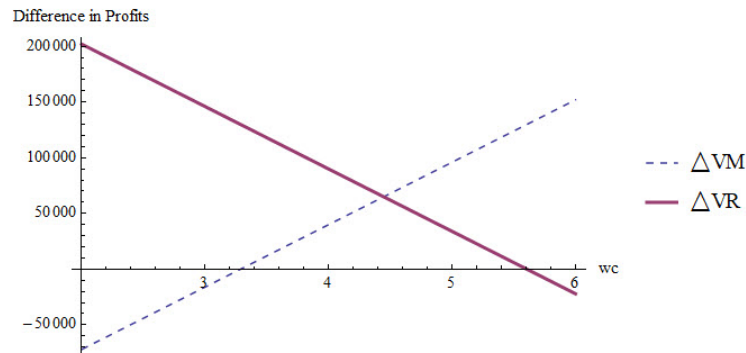

Figure 1: Incentives implementation interval

4 Conclusion

In their survey of game-theoretic models of cooperative advertising, Jørgensen and Zaccour (2014) pointed out that most of the studies in this literature "have almost completely overlooked the channel coordination problem." As a future research direction, they suggested investigating whether coordination could be reached through this mechanism. According to these authors, important questions about the existence and design of incentive contracts that push channel members to set their decision variables at the channel-optimal levels should be raised and addressed.

This study responds to this request. Not only does it demonstrate that such incentives exist and can be implemented, but it also gives conditions where they can be Pareto-improving and provides the appropriate design that leads to coordination. Another contribution of this research is that it provides mechanisms that allow the coordination of both price and non-price marketing decisions (i.e., local and national advertising) in a dynamic setting, for a channel where the manufacturer acts as a Stackelberg leader.

The main drawback of this study is that the model structure predicts degenerate strategies for prices and advertising efforts. Hence, an interesting extension could be to use different expressions to capture the goodwill dynamics and demand function in order to generate state-dependent strategies.

Appendix A

I need to establish the existence of two bounded and continuously differentiable value functions $V_R^D(G)$ and $V_M^D(G)$, which satisfy, for all $G(t) \geqslant 0$, the Hamilton-Jacobi-Bellman (HJB) equations of the retailer (R) and the manufacturer (M), given by

$$\rho V_R^D (G) = \max_{p, a_R \geq 0} \left[(p - w)(\lambda - \theta p) G + \frac{(a_R)^2}{2} + \frac{dV_R^D}{dG}(\alpha a_M + \beta a_R - \delta G) \right], \tag{16}$$

$$\rho V_M^D (G) = \max_{w, a_M \geq 0} \left[(w - c)(\lambda - \theta p) G - \frac{(a_M)^2}{2} + \frac{dV_M^D}{dG}(\alpha a_M + \beta a_R - \delta G) \right]. \tag{17}$$

Since the retailer is the follower, I start by maximizing the right-hand side of Equation (16) w.r.t. the control variables p and a_R and solve the resulting system of equations. The result gives the following pair of reaction functions:

$$p = \frac{\theta w + \lambda}{2\theta}, \tag{18}$$

$$a_R = \beta \frac{dV_R^D}{dG}. \tag{19}$$

Then I substitute p and a_R by their expressions from the reaction functions in Equation (17). Performing the maximization of this expression w.r.t. manufacturer's control variables yields

$$w^D = \frac{\theta c + \lambda}{2\theta}, \tag{20}$$

$$a_M^D = \alpha \frac{dV_M^D}{dG}. \tag{21}$$

I insert (21) and (20) on the right-hand side of the manufacturer's HJB equation and conjecture that $V_R^D(G)$ and $V_M^D(G)$ are linear value functions given by (6) and (5), respectively. Rearranging all the terms corresponding to the coefficient of G and the constant allows me to find the values of S_1 and S_2 by identification. Their expressions are given by

$$S_1 = \frac{(\lambda - c\theta)^2}{8\theta(\delta + \rho)},$$

$$S_2 = \frac{(\lambda - c\theta)^2 \left(\alpha^2 (\lambda - c\theta)^2 + 16T_1 \beta^2 \theta (\delta + \rho) \right)}{128\theta^2 \rho (\delta + \rho)^2}. \tag{22}$$

Since $S_1 = \frac{dV_M^D}{dG}$ and w^D is given by (20), I compute the expressions of a_M^D and p^D given in the proposition by substituting S_1 in (21) and w^D in (18). Then I substitute w^D, p^D, a_M^D, and the expression of a_R from Equation (19) into Equation (16) and obtain the values of T_1 and T_2 by identification, when rearranging the coefficient of G in the value function and the constant. I obtain the following values:

$$T_1 = \frac{(\lambda - c\theta)^2}{16\theta(\delta + \rho)},$$

$$T_2 = \frac{(\lambda - c\theta)^4 \left(4\alpha^2 + \beta^2 \right)}{512\theta^2 \rho (\delta + \rho)^2}.$$

The value of S_2 is computed by substituting T_1 into Equation (22), which gives the following result:

$$S_2 = \frac{(\lambda - c\theta)^4 \left(\alpha^2 + \beta^2 \right)}{128\theta^2 \rho (\delta + \rho)^2}.$$

The goodwill level at the steady state is computed by setting $\dot{G}(t) = 0$ and solving for G after replacing a_M and a_R by their respective values a_M^D and a_R^D given in proposition 1.

Appendix B

Following the same approach as in Appendix A, the values of p^C, a_R^C , and a_M^C under coordination with vertical integration are obtained from the maximization of the right-hand side of the following HJB equation:

$$\rho V^C\left(G\right) = \max_{p,a_R,a_M \geq 0}\left[\left(p-c\right)\left(\lambda - \theta p\right)G - \frac{\left(a_R\right)^2}{2} - \frac{\left(a_M\right)^2}{2} + \frac{dV^C}{dG}\left(\alpha a_M + \beta a_R - \delta G\right)\right],$$

where $V^C\left(G\right)$ denotes the value function under this scenario. Solving the system of equations resulting from this maximization gives the the following expressions of p^C, a_R^C , and a_M^C:

$$p^C = \frac{\theta c + \lambda}{2\theta},$$

$$a_R^C = \beta\frac{dV^{VI}}{dG},$$

$$a_M^C = \alpha\frac{dV^{VI}}{dG}.$$

I substitute these expressions in the HJB equation and conjecture that this equation has the linear form given in Proposition 2. I obtain the parameters of this value function by rearranging the terms in order to separate the coefficient of $G\left(t\right)$ and the constant, which correspond to the following expressions of K_1 and K_2:

$$K_1 = \frac{\left(\lambda - c\theta\right)^2}{4\theta\left(\delta + \rho\right)},$$

$$K_2 = \frac{\left(\lambda - c\theta\right)^4\left(\alpha^2 + \beta^2\right)}{32\theta^2\rho\left(\delta + \rho\right)^2}.$$

The optimal values of a_R^C and a_M^C given in the proposition are obtained after substitution of $\frac{dV^C}{dG}$ by its value, which corresponds to K_1.

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