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Depth-of-interaction bias in the achievable ultra-fast timing of detectors for TOF-PET

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Abstract: In Positron Emission Tomography (PET), deep crystals (>20 mm) must be used to enhance detection efficiency and increase overall scanner sensitivity. However, for fast time-of-flight (TOF) scanners, this may come at a cost for the achievable coincidence time resolution (CTR), since the propagation time of annihilation photons differs depending on the depth-of-interaction (DOI) location. In the literature, DOI effect in CTR computation is modeled by incorporating the attenuation probability density function (PDF) with the PDFs for the scintillation pulse emission, propagation and detection. However, the resulting PDF would not describe accurately the variation, in timestamps distribution, due to annihilation photon DOI. In this study, we propose to investigate the DOI bias effect on CTR for a typical and a near-ideal scintillation detector. We also calculated the CTR of some estimators (k-th trigger, average of first-k triggers, and Gauss-Markov estimator) based on the ordered primary detected scintillation photons. The root-mean-square error (RMSE) of the coincidence detection process is a straightforward metric to compare the quality of different estimators. The RMSE of the different estimators was calculated using DOI probability in coincidence. A small difference in the calculated CTR was found for a typical LYSO/SiPM scintillation detector when assessing RMSE; this was expected since the DOI error influence remains negligible against other parameters. However, the difference becomes crucial for near-ideal scintillation detectors: the standard method predicts 10-20 ps CTR, but RMSE remains above ∼55 ps CTR, which seems more likely since the difference in TOF introduced by the crystal length can almost be of the same magnitude. Thus, in the absence of DOI measurement, investigation of the achievable ultra-fast CTR in TOF-PET detectors should include the bias introduced by DOI.

Keywords: Coincidence timing resolution, scintillation detectors, time of flight positron emission tomography, depth of interaction

Résumé : En tomographie par émission de positron (PET), des cristaux longs (>20 mm) doivent être utilisés pour augmenter l'efficacité de détection et la sensibilité de la caméra. Cependant, pour des caméras exploitant un temps de vol (TOF) rapide, ce choix entraîne des répercussions sur la limite atteignable de la résolution temporelle en coïncidence (CTR), puisque le temps de propagation des photons d'annihilations varie selon la profondeur d'interaction (DOI). Dans la littérature, l'effet de la DOI sur le CTR est modélisé en incorporant la densité de probabilité (PDF) de l'atténuation avec les PDFs d'émission, de propagation et de détection du signal de scintillation. Cependant, la PDF résultante ne décrit pas correctement la variation, dans la distribution d'étiquette temporelle, causée par la DOI du photon d'annihilation. Dans cette étude, nous proposons d'étudier l'effet du biais induit par la DOI sur le CTR pour un détecteur typique et un idéal. Nous avons aussi calculé le CTR de quelques estimateurs (k-ième, la moyenne des k premiers et Gauss-Markov) basé sur les premiers photons de scintillations détectés. L'erreur quadratique moyenne (RMSE) du processus de détection en coïncidence est une métrique appropriée pour comparer la performance des différents estimateurs. Le RMSE des différents estimateurs a été calculé en utilisant la probabilité de la DOI en coïncidence. Une légère différence entre le CTR prédit par l'approche standard et celui du RMSE a été trouvée pour un détecteur typique de type LYSO/SiPM; ce qui était prévisible puisque le biais induit par la DOI est négligeable relativement aux autres sources d'erreur. Cependant, la différence devient significative pour des détecteurs presque idéaux: l'approche standard prédit un CTR de 10-20 ps, alors que le RMSE prédit un CTR au-dessus du 55 ps, ce qui semble plus probable puisque la différence de TOF introduit par la longueur du cristal est de cet ordre. Donc, dans l'absence de mesure de DOI, l'étude du CTR atteignable pour un détecteur TOF-PET ultra-rapide devrait inclure le biais induit par le DOI.

Mots clés : Résolution temporel en coïncidence, détecteur à scintillation, tomographie par émission de positons avec temps de vol, profondeur d'interaction

1 Introduction

In most clinical Positron Emission Tomography (PET) scanners, deep crystals (>20 mm) are used to enhance detection efficiency and increase overall scanner sensitivity. However, for fast time-of-flight (TOF) scanners, this may come at a cost for the achievable coincidence time resolution (CTR), since the propagation time of annihilation photons differs depending on the depth-of-interaction (DOI) location. For example, in coincident LYSO crystals, the TOF difference due to DOI can reach ∼30 ps/cm. In TOF-PET, such DOI bias might become significant if other CTR degrading factors (e.g. crystal emission, photodetector response) are not prominent [4].

The typical method to incorporate DOI in the scintillation signal modeling is to come up with a single probability density function (PDF) of the scintillation photons propagation time that includes all possible DOIs, and weight it with a PDF defined from the 511-keV exponential attenuation profile in the crystal [1, 3]. Hence, for any given DOI, the propagation time of a scintillation photon is drawn from a single PDF. However, scintillation photons emitted at different DOIs do not produce the same timestamps distribution. Thus, a different approach needs to be used to correctly incorporate DOI blurring into the achievable CTR calculations.

In this study, we propose to investigate the DOI bias effect on CTR for a typical and a near-ideal scintillation detector. We also calculate the CTR of some estimators based on the ordered primary detected scintillation photons and we propose a different metric to compare the quality of different estimators.

2 Methods

2.1 Modeling of DOI in coincidence timing

The achievable CTR in scintillation detectors can be studied through modeling of the complete scintillation signal with the PDF of each process (emission, transport, and detection). As mentioned above, the DOI is currently included by merging its PDF directly with the PDFs of these processes, which results in a global PDF to model the complete process. It is worth noting that the DOI is typically correctly included as a delay error when using extensive Monte Carlo simulations for extracting timestamps. It is when one comes to the calculations of the achievable CTR that there can be some underestimation.

For instance, let us consider an ideal scintillation detector with the emission, transit, and detection of the scintillation photons being defined by dirac-like PDFs. The resultant PDF in this case would have the same form as the exponential attenuation probability [4]. Let us now consider a crystal emitting a very large number of scintillation photons. In the case where these photons are drawn from a single global PDF (convolution of all processes with DOI attenuation), this would result in all the primary triggers simulated over many 511-keV interactions being positioned at the start of the global PDF whereas they should actually be distributed along the exponential attenuation function. This highlights that a single global PDF overshadows the DOI blur and we propose an alternative method to model the DOI bias for a more accurate evaluation of the achievable CTR.

2.2 Signal modeling

Let PDF_d describes the production, transport, and detection of the scintillation photons for DOI of d in a $3{\times}3{\times}20\ \mathrm{mm}^3$ crystal. The scintillation photon transport in the crystal was assessed with an analytical model using the same light collection parameters (polished surfaces, air gap between crystal and 98%-reflectivity specular reflector, coupling grease with n=1.45 between crystal and photosensor) outlined in the paper of Cates et al. [1]. We simulated two different scintillation detectors (SD) with characteristics detailed in Table 1: a typical one (Typical SD) using a $\mathrm{Lu_{1.9}Y_{0.1}SiO_5}$ (LYSO) crystal and a conventional SiPM-like photosensor, and an ideal one (Ideal SD) with near-perfect crystal and photosensor.

Table 1: The two scintillation detector (SD) modeled, the Typical SD having characteristics of LYSO crystal and conventional SiPM-like photosensor, and the Ideal SD with near-perfect crystal and photosensor.

	Typical SD	**Ideal SD**
Crystal		
Rise time (ps)	72	30
Decay time (ns)	43	8
Light yield (ph/MeV)	27,000	100,000
Light transfer efficiency (%)	39	100
Photodetector		
Photon detection efficiency (%)	55	100
Single photon time resolution (ps)	100	3

2.3 MSE expected value for CTR

The mean squared error (MSE) of an estimator $\hat{\theta}$ with respect to an unknown parameter θ is defined as the sum of the variance of the estimator and the squared bias of the estimator:

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}(\hat{\theta}) + \mathrm{Bias}(\hat{\theta}, \theta)^2 \tag{1}$$

where in our case, θ represents the time origin when a 511-keV enters the crystal. With ordered statistics on the first detected scintillation photons, it is possible to estimate the variance (span of distributions) and the DOI bias (mean position of distributions) as a function of the trigger index. These distributions were obtained by simulating the difference in trigger arrival times between two detectors in coincidence using the PDF_d calculated for each DOI. Three standard estimators were studied, the k^{th} trigger arrival time estimator, the average arrival time estimator for the first k triggers, and the Gauss-Markov estimator [5, 2]. With two detectors A and B, a MSE value is calculated for each combination in coincidence of DOI $(d_{\mathrm{A}}, d_{\mathrm{B}})$. The expected value of MSE can thereby be obtained by using a DOI probability table (based on 511-keV exponential attenuation) in coincidence, where $\mathsf{P}(d_{\mathrm{A}}, d_{\mathrm{B}}) = \mathsf{P}(d_{\mathrm{A}})\mathsf{P}(d_{\mathrm{B}})$. Finally, a root-mean-square error (RMSE) is calculated using the expected value of MSE of all possible combinations of DOI:

$$\mathrm{RMSE} = \left[\sum_{d_{\mathrm{A}}}\sum_{d_{\mathrm{B}}}\left(\mathrm{MSE}_{d_{\mathrm{A}}}^{\mathrm{single}} + \mathrm{MSE}_{d_{\mathrm{B}}}^{\mathrm{single}}\right) \times \mathsf{P}(d_{\mathrm{A}}, d_{\mathrm{B}})\right]^{1/2} \tag{2}$$

which can then be converted to Full-Width at Half Maximum (FWHM) to obtain a CTR value of $2.35 \times \mathrm{RMSE}$.

3 Results and discussion

Figure 1 displays the distributions of the 1^{st}, 10^{th}, and 50^{th} primary triggers arrival times in a single Typical SD as a function of DOI. A large DOI value corresponds to a annihilation photon interaction occurring near the photosensor.

By using the mean peak position in Figure 1, we can assess the bias of primary triggers depending on the DOI difference. Figure 2 shows the observed bias for five k^{th} arrival time triggers when a Typical SD is in coincidence with another Typical SD where interactions occur at a fixed DOI of 0.5 mm.

Figure 3 presents the CTR of three estimators as a function of the first 50 triggers for the Typical SD and Ideal SD. Two interpretations of the CTR for each estimator are shown, the standard one found in the literature, denoted Std, and defined by (2) without bias, and an RMSE version incorporating the bias (2). One can see a relatively small difference between the Std and RMSE versions for Typical SD. This is expected since the DOI error effect remains negligible in comparison to other parameters (e.g. SPTR of 100 ps). The difference becomes crucial for the Ideal SD, where the Std version reaches 10-20 ps CTR but the RMSE version remains above ∼55 ps CTR. This highlights that when aiming at ultra-fast CTR of 10-20 ps with a rapid and intense signal with an Ideal-like SD, the DOI introduces a critical bias that would not be observed with the standard calculation of achievable CTR.

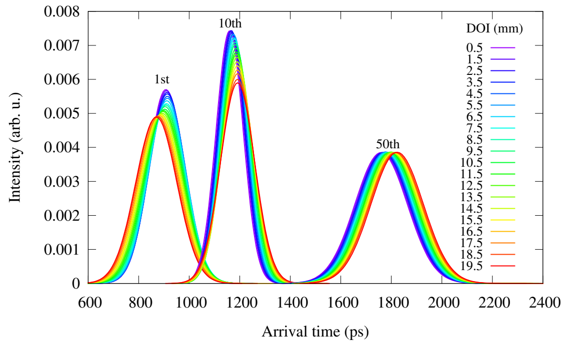

Figure 1: Distributions of three primary triggers arrival times (1^{st}, 10^{th}, 50^{th}) in a single Typical SD as a function of DOI.

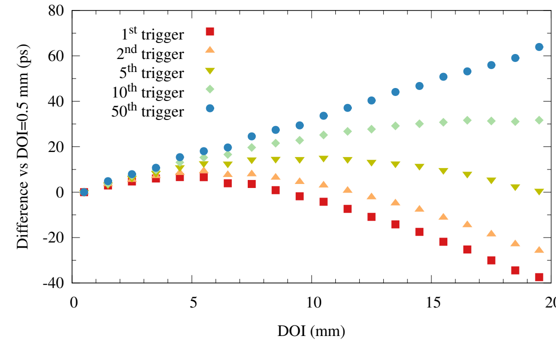

Figure 2: Bias for five k^{th} arrival time triggers when a Typical SD is in coincidence with another Typical SD where interactions occur at a fixed DOI of 0.5 mm.

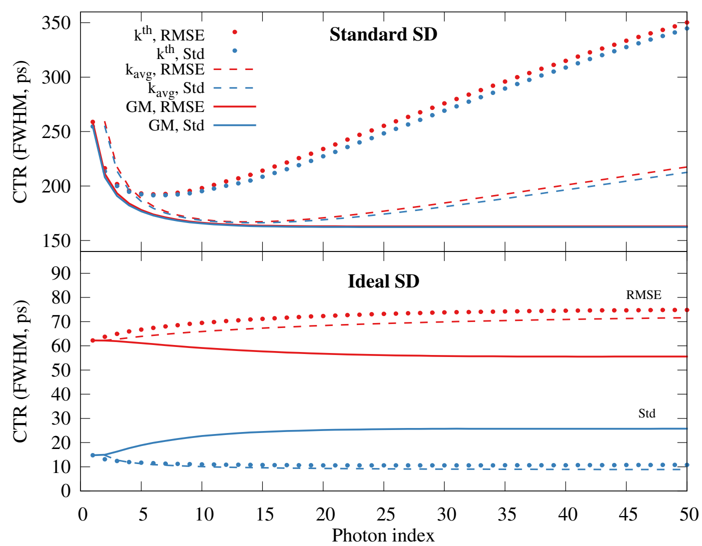

Figure 3: (Top) Coincidence time resolution of three estimators as a function of the number of primary triggers for the Typical SD. The CTR of three estimators are displayed: dots refer to the k^{th} arrival time estimator, dashed-lines refer to the average arrival time obtained for the first k detected scintillation photons, while full lines refer to the Gauss-Markov (GM) estimator [2]. (Bottom) Same as top graph, for the Ideal SD. Two interpretations of CTR are displayed, the standard one found in the literature (Std), and an RMSE version incorporating the bias (2).

4 Conclusion

The expected value of a mean squared error MSE estimator which adds the DOI bias error to the other stochastic blurs of the scintillation signal (emission, transit, and detection of scintillation photons) was calculated using DOI probability in coincidence. A small difference in the calculated CTR was found for a typical LYSO/SiPM scintillation detector when assessing the RMSE (square root of MSE), but was expected since the DOI error influence remains negligible relative to other parameters. The difference becomes significant for a near-ideal scintillation detector, where the standard method without correctly taking into account the DOI bias predicts a 10-20 ps CTR, although the RMSE version shows a CTR of $\sim$55 ps. We conclude that investigation of the achievable ultra-fast CTR in TOF-PET detectors should include the bias induced by the DOI in the absence of suitable DOI measurement. The Cramr-Rao lower bound on CTR with biased DOI information will also be addressed at the conference.

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