

**Open pit stochastic optimization
with in-pit tailings storage**

A. Rimélé, M. Gamache,
R. Dimitrakopoulos

G-2017-35

Mai 2017

Cette version est mise à votre disposition conformément à la politique de libre accès aux publications des organismes subventionnaires canadiens et québécois.

Avant de citer ce rapport, veuillez visiter notre site Web (<https://www.gerad.ca/fr/papers/G-2017-35>) afin de mettre à jour vos données de référence, s'il a été publié dans une revue scientifique.

This version is available to you under the open access policy of Canadian and Quebec funding agencies.

Before citing this report, please visit our website (<https://www.gerad.ca/en/papers/G-2017-35>) to update your reference data, if it has been published in a scientific journal.

Les textes publiés dans la série des rapports de recherche *Les Cahiers du GERAD* n'engagent que la responsabilité de leurs auteurs.

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2017
– Bibliothèque et Archives Canada, 2017

The authors are exclusively responsible for the content of their research papers published in the series *Les Cahiers du GERAD*.

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2017
– Library and Archives Canada, 2017

Open pit stochastic optimization with in-pit tailings storage

Adrien Riméle^a

Michel Gamache^b

Roussos Dimitrakopoulos^a

^a GERAD & COSMO Stochastic Mine Planning Laboratory & Department of Mining and Materials Engineering, McGill University, Montréal (Québec) Canada, H3A 0E8

^b GERAD & Department of Mathematics and Industrial Engineering, Polytechnique Montréal (Québec) Canada, H3C 3A7

adrien.rimele@mail.mcgill.ca

michel.gamache@polymtl.ca

roussos.dimitrakopoulos@mcgill.ca

Mai 2017

Les Cahiers du GERAD

G–2017–35

Copyright © 2017 GERAD

Abstract: Management and storage of waste and tailings are critical concerns in open pit mining, especially when the available space is limited but also to reduce the impact of the activities on the environment. This paper presents a new SIP model which simultaneously optimize the blocks extraction sequence and the in-pit disposal of tailings and waste material. The model, which also considers geological uncertainty with a set of simulations, is tested over a low dip iron ore deposit. The results are convincing in terms of space saving and impact on the production and thus demonstrate the potential of such an approach.

Acknowledgments: The work in this paper was funded by the FRQNT grant “Développement d’une stratégie globale d’optimisation de sites miniers avec incertitude: Amélioration de la viabilité économique et de la gestion environnementale des résidus miniers d’une mine de fer dans le nord” G240453 FRQ-NT MI-181547 and COSMO - Stochastic Mine Planning Laboratory, a global centre for leading-edge research and graduate education in “orebody modelling and strategic mine planning with uncertainty”, is supported by AngloGold Ashanti, Barrick Gold, BHP Billiton, De Beers, Kinross, Newmont, Vale, and Canada Research Chairs Program, NSERC.

1 Introduction

Long term planning is of major importance to evaluate the viability of an open pit mine project. Operations research with Mixed Integer Programming (MIP) models has been used for decades to optimize the extraction sequence of mining blocks with the objective to maximize the discounted cash flow (DCF) and to satisfy the diverse production constraints. [1], [2] However, the traditional approach relies on deterministic optimization since it does not consider any source of uncertainty. Numerous studies [3]–[5] have demonstrated the significant impact of geological uncertainty over the mine production forecasted by those models. The geological information comes from sparse drillholes and is traditionally interpolated to the all block model with an estimated method (like krigging) to generate a single but smoothed representation of the orebody which typically under represents waste and high grades. [6] In order to consider the inevitable uncertainty associated with such representation, stochastic conditional simulations, which can be seen as a Monte Carlo type technique which represents the local grade and material type variability, provide a set of equiprobable representations of the deposit. The information provided by a set of simulations about the local grade variability can be used to maximize the DCF and manage the risk associated to the yet undiscovered geology. In 2008, Dimitrakopoulos and Ramazan proposed a two stages Stochastic Integer Programming (SIP) model with fixed recourse for the open pit mine scheduling problem which successfully managed the risk of the production. [7] While the quality of the obtained schedules has improved with such SIP model, the complexity of solving them has also increased. To tackle large instances and include more elements of the value chain, metaheuristic methods have been developed such as tabu search [8], simulated annealing [9]–[12] or progressive hedging, a scenario-based decomposition technique [13]. Those techniques have successfully managed to deal with real large size deposits of hundreds of thousands of mining blocks and to give a more realistic forecast of the production.

Besides traditional constraints for the block extraction scheduling (accessibility, tonnage capacities, average grade, etc.), tailings and waste material management may be considered. Indeed, the volumes induced are significant and can be a great concern if the external space for stockpiles is limited. Moreover, this material also deeply impacts the environment and needs to be moved again during the rehabilitation phase. An option that can be taken is to store tailings and waste material directly into the free spaces inside the pit, during the operations, in a way that it does not affect a lot the production. The literature on this topic is rather scant; the only found related work is from Zuckerberg et al. [14]. The authors developed an extended version of the mine planning software Blasor of BHP Billiton that they called Blasor-InPitDumping (BlasorIPD). For confidentiality reasons, the mathematical model is not given but they explained the main ideas of their work, conducted in a deterministic framework. They aggregate blocks and assign them a period of extraction. Then processing decisions only are given to subdivisions of those aggregates. When an AGG is determined to be sent to the waste, a percentage can be sent to an external stockpile and the rest to a zone inside the pit. The material sent back to the pit is associated to a “refill AGG”, a larger aggregate than initial AGG that also respects slope constraints. Their method allows to reduce the external space for storage, however, the waste material disposal inside the pit can be sparse (can result in a not very practical plan), ore accessibility is not mentioned and the aggregation of blocks coupled to smoothing of grades can mislead forecast results (mining selectivity and more waste than expected).

The problem addressed in this paper is to simultaneously optimize the block extraction sequence and the disposal of tailings and waste material disposal inside the mined-out areas of the pit, considering the geological uncertainty of a low dip deposit. The planning will be performed at the block level without predefined cut-off policy (extraction sequence and destination policy) with the objective to maximize the discounted cash flow. For the in-pit storage, several requirements are considered to satisfy practicability issues. First, to control the locality and accessibility of the in-pit storage area, the latter must be unique (i.e., spatially continuous). Once the material is stored, it cannot be shifted, thus the area can only grow from one period to another. Of course, once an area (set of strips) is used for storage, the remaining ore blocks are sterilized since they cannot be accessed anymore. To prevent transforming such ore blocks to waste, a given minimum percentage of ore blocks within a strip is required to be mined prior to considering the strip for storage. The volume available for storage within a strip will be considered equal to the volume represented by the extracted blocks. The external available space for storage will be determined as being a maximum number of blocks which can be stored in external stockpiles.

A new SIP formulation called open pit mine planning stochastic integer program with in-pit tailings disposal will be introduced followed by a description of the heuristic solving mechanism. Then the model will be tested in a case study to demonstrate its potential for managing tailings and waste while maximizing the discounted cash flow.

2 Method

The formulation is a two-stage stochastic model with fixed recourses [15] to take into account geological uncertainty. The in-pit storage will be defined by two extreme strips following the dip for every period and the amount of stored material within each strip.

2.1 Variables

The first stage decision variables concern the blocks extraction period and destination as well as the position and volume of the in-pit storage area. The mining schedule is determined by the binary variables $x_{i,d,p}$ which are equal to 1 when a given block $i \in \mathcal{B}$ has been sent to destination $d \in \mathcal{D}$ by period $p \in \mathcal{P} = \{1, \dots, P\}$ and 0 otherwise. The delimitation of the in-pit storage area is given by the binary variables $u_{k,p}$ and $l_{k,p}$ which take the value 1 if strip $k \in \mathcal{K} = \{1, \dots, K\}$ is respectively the top and bottom strip of the storage area in period p , 0 otherwise (note that $k = 1$ will be qualified as the bottommost strip and $k = K$ as the topmost strip). The availability (1) or non-availability (0) of a strip k for storage in period p is given by the binary variable $z_{k,p}$. Finally, variable $y_{k,p} \in \mathbb{R}^+$ defines the number of tailings blocks stored in strip k in period p . The second stage variables correspond to the impact of geological uncertainty, not yet revealed during the first stage decision taking. In geological scenario $s \in \mathcal{S}$, the deviation in terms of metallurgical characteristic $c \in \mathcal{C}$ in period p is denoted by the variable $dev_{c,p,s}^\pm \in \mathbb{R}^+$ (excess +, shortage -).

2.2 Objective function

$$\max Z = \overbrace{\frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{B}} \sum_{d \in \mathcal{D}} \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} d_p \cdot v_{i,d,s} * (x_{i,d,p} - x_{i,d,p-1})}^{\text{Part 1}} - \overbrace{\frac{1}{|\mathcal{S}|} \sum_{c \in \mathcal{C}} \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} d_p * (pen_{c,p}^{dev+} * dev_{c,p,s}^+ + pen_{c,p}^{dev-} * dev_{c,p,s}^-)}^{\text{Part 2}} \quad (\text{EQ 1})$$

Where: d_p denotes the discount factor applied in period p , $v_{i,d,s}$ denotes the economic value of block i in scenario s (positive or negative) if sent to destination d , and $pen_{c,p}^{dev\pm}$ denotes the penalty cost of deviating from targets of characteristic $c \in \mathcal{C}$ in period p (excess +, shortage -).

In the objective function, Part 1 aims to maximize the expected discounted cash flow generated by the extracted blocks while Part 2 penalizes the expected deviations from the production targets.

2.3 Constraints

$$\forall i \in \mathcal{B}, \forall d \in \mathcal{D}, \forall p \in \mathcal{P} \quad x_{i,d,p} - x_{i,d,p-1} \geq 0 \quad (\text{EQ 2})$$

$$\forall i \in \mathcal{B}, \forall p \in \mathcal{P} \quad \sum_{d \in \mathcal{D}} x_{i,d,p} \leq 1 \quad (\text{EQ 3})$$

$$\forall i \in \mathcal{B}, \forall j \in \Gamma_i^-, \forall p \in \mathcal{P} \quad \sum_{d \in \mathcal{D}} x_{i,d,p} \leq \sum_{d \in \mathcal{D}} x_{j,d,p} \quad (\text{EQ 4})$$

$$\forall c_1 \in \mathcal{C}_1, \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \quad \sum_{i \in \mathcal{B}} (q_{c_1,i,s} * (x_{i,d,p} - x_{i,d,p-1})) - dev_{c_1,p,s}^\pm \leq target_{c_1,p}^\pm \quad (\text{EQ 5})$$

$$\forall c_2 \in \mathcal{C}_2, \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \quad \sum_{i \in \mathcal{B}} (q_{c_2,i,s} * t_{i,s} * (x_{i,d,p} - x_{i,d,p-1})) - dev_{c_2,p,s}^\pm \leq target_{c_2,p}^\pm * \sum_{i \in \mathcal{B}} (t_{i,s} * (x_{i,d,p} - x_{i,d,p-1})) \quad (\text{EQ 6})$$

$$\forall p \in \mathcal{P} \quad \begin{cases} \sum_{k \in \mathcal{K}} u_{k,p} \leq 1 \\ \sum_{k \in \mathcal{K}} l_{k,p} \leq 1 \end{cases} \quad (\text{EQ 7})$$

$$\forall p \in \mathcal{P} \quad \sum_{k \in \mathcal{K}} k * (u_{k,p} - l_{k,p}) \geq 0 \quad (\text{EQ 8})$$

$$\forall p \in \mathcal{P} \setminus \{1\} \quad \begin{cases} \sum_{k \in \mathcal{K}} k * u_{k,p} \geq \sum_{k \in \mathcal{K}} k * u_{k,p-1} \\ \sum_{k \in \mathcal{K}} k * l_{k,p} \leq \sum_{k \in \mathcal{K}} k * l_{k,p-1} \end{cases} \quad (\text{EQ 9})$$

$$\forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad z_{k,p} = \sum_{j=k}^{K-1} (u_{j,p} - l_{j+1,p}) + u_{K,p} \quad (\text{EQ 10})$$

$$\forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad y_{k,p} \leq N_k * z_{k,p} \quad (\text{EQ 11})$$

$$\forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad \sum_{p'=1}^p y_{k,p'} \leq \sum_{i \in \mathcal{K}^{-1}(k)} \sum_{d \in \mathcal{D}} x_{i,d,p} \quad (\text{EQ 12})$$

$$\forall p \in \mathcal{P} \quad \sum_{k \in \mathcal{K}} y_{k,p} \leq \sum_{i \in \mathcal{B}} \sum_{d \in \mathcal{D}} (x_{i,d,p} - x_{i,d,p-1}) \quad (\text{EQ 13})$$

$$\forall k \in \mathcal{K}, \forall i \in \mathcal{K}^{-1}(k), \forall p \in \mathcal{P} \quad \sum_{d \in \mathcal{D}} (x_{i,d,p} - x_{i,d,p-1}) \leq 1 - z_{k,p} \quad (\text{EQ 14})$$

$$\sum_{i \in \mathcal{B}} \sum_{d \in \mathcal{D}} x_{i,d,p} - \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} y_{k,p} \leq \pi \quad (\text{EQ 15})$$

$$\forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad \gamma * N_k * z_{k,p} \leq \sum_{i \in \mathcal{K}^{-1}(k)} \sum_{d \in \mathcal{D}} x_{i,d,p} \quad (\text{EQ 16})$$

Where: Γ_i^- denotes the set of direct predecessor blocks of block i , $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$ such as \mathcal{C}_1 denotes the linear characteristics (tonnages) and \mathcal{C}_2 the nonlinear characteristics (grades), $q_{c,i,s}$ denotes the characteristic c of block i in scenario s , $target_{c,p}^{\pm}$ denotes the upper (+) and lower (-) bounds targets of average characteristic c in scenario s , $t_{i,s}$ denotes the tonnage of block i in scenario s , N_k denotes the number of blocks in strip k , $\mathcal{K}^{-1}(k)$ denotes the set of blocks that belong to strip k , π denotes the maximum number of blocks that can be stored outside of the pit, the parameter γ denotes the percentage of the total number of blocks of a strip that need to be extracted before considering the strip for storage.

Equations 2-6 are typical constraints for long-term stochastic open-pit scheduling. They respectively refer to reserve constraints, destination policy, predecessor constraints, average quantity targets and average grade targets. More details can be found in the literature [7], [16]. Equations 7-16 are additional specific constraints for the in-pit tailings and waste material storage. Equation 7 defines the uniqueness of both the top and bottom strip delimiting the in-pit area of storage. Equation 8 enforces the good relative order of the top and bottom strip (i.e., the top strip must be located ‘‘above’’ the bottom one). Equation 9 constrains the area of storage to remain similar or to grow only, without translation (a strip once considered for storage will remain so for the rest of the life of mine). Equation 10 states that a strip is available for storage during a given period if and only if it is located between the top and bottom strip. Equation 11 allow a strip to receive stored material once this strip is available for storage. Equation 12 limits the number of tailings or waste material stored in a strip to the number of blocks that have been previously mined in the same strip (available space). Equation 13 states that the number of tailings or waste material blocks stored during one period cannot be higher than the number of extracted blocks during this period. Equation 14 claims that once a strip has been designated for storage, its remaining blocks cannot be extracted anymore. Equation 15 limits the number for tailings blocks stored in an external stockpile to π . Finally, Equation 16 ensures that a given percentage γ of the ore blocks present in a strip have been extracted before considering the strip for storage.

The open-pit mine scheduling problem is a very complex problem to solve. Geological uncertainty using geostatistical simulations in addition to the in-pit material disposal formulation makes the model impossible to solve with a traditional mathematical solver like Cplex, even with a reasonably small deposit (a few thousand blocks). The

method chosen in this paper to solve the model is a typical sliding time window heuristic method which can be found for instance in Cullenbine et al. [17]. This is an iterative method for which at each iteration all the variables of all the periods, except the ones of the “window”, are relaxed (set continuous instead of integer). The solution for the first period of the window is saved and the variables fixed before moving the window up by one period and repeat the process. Here, the window will be composed of one period only. In addition, to reduce again the complexity of the model, the latest periods (i.e., separated from the “window” by at least two periods) are gathered in groups. For instance, for the first iteration, the “window” is period 1 (binary variables), periods 2 and 3 are relaxed (continuous variables), period 4 and 5 are relaxed and grouped (represented as one period of double capacity), etc.

3 Case study

The model was tested on a low dip iron ore deposit composed of 3177, $100*100*15m^3$ mining blocks. The schedule was performed over 10 periods and considered two possible destinations for every block: the mill or the waste dump (external or in-pit). Geological uncertainty was represented by a set of 10 stochastic conditional simulations generated by Direct Block Simulation with min/max autocorrelation factor [18] which provide for every block the iron grade (Fe%), the silica grade (SiO₂%) and the density. The main production constraints concern the concentrate tonnes per period, the average Iron grade, and the average Silica grade. Following the definition of the model, the in-pit storage is considered by strips toward the dip (West-East), the maximum number of blocks that can be stored outside of the pit is set to $\pi = 500$ and the percentage of ore blocks that must be mined in every strip before storage is set to $\gamma = 75\%$. The average grade of the blocks and swelling phenomena motivate the assumption that, on average, an extracted block results in 80% of its initial volume as tailings (or waste material) to store. The proposed model contains a total of 64 680 binary variables, 680 continuous variables, and 413 000 constraints. It was solved in 6h32min with the previously introduced sliding time window with grouped periods heuristic method with the use of the solver Cplex v12.4 [19] (processor i7-2600S 2.8GHz and 8GB RAM). To evaluate the quality of the heuristic method, the objective value is compared to the one of the relaxed model (upper bound of the optimal solution). The gap ($gap = \frac{Obj^{relaxed} - Obj^{heuristic}}{Obj^{relaxed}}$) is 1.76%, which demonstrates the good performance of the method.

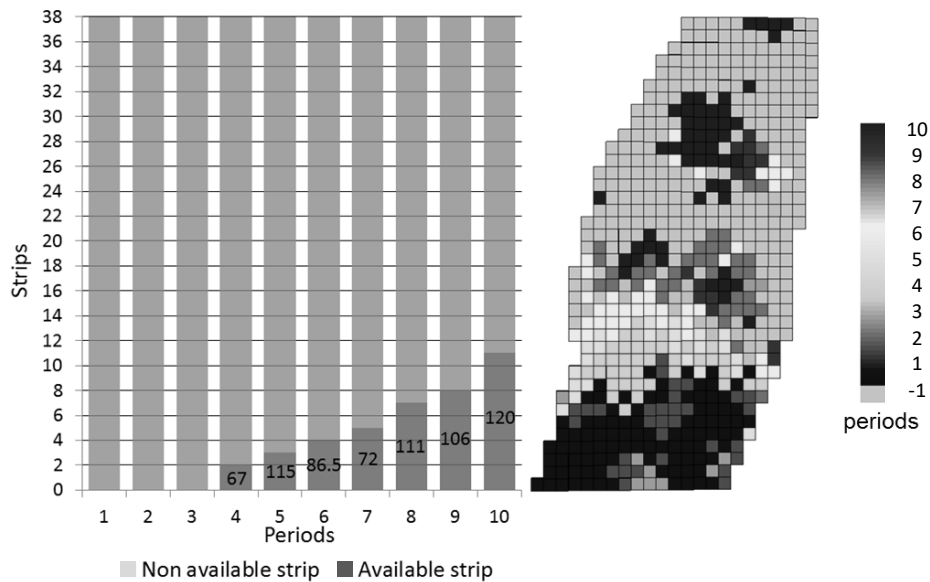


Figure 1 Strips storage availability and number of stored blocks per period (left), mine extraction schedule (right)

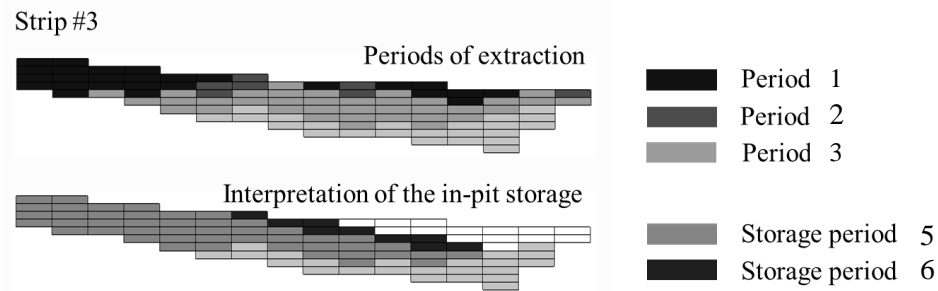


Figure 2 Dip toward cross section of the schedule (top) and the in-pit storage disposal (bottom)

Figure 1 shows a top view of the schedule (right) and the storage within the different strips (left). One can note that the in-pit storage has started in period 4 (once the external stockpile was full). The first strips used for storage are the two bottommost strips (1 and 2), followed by the above ones. As expected, those strips are also the first that have been mined, to free space for storage. From period 5, 100% of the storage is performed inside the pit. Figure 2 presents an example of a cross section (3rd strip). This strip has been extracted more than 75% within the first 3 periods. With the previously defined assumption about the volumes transformations, 57.8 extracted blocks in period 5 and 11.2 blocks in period 6 resulted in respectively 46 and 9 tailings or waste material blocks stored in strip 3.

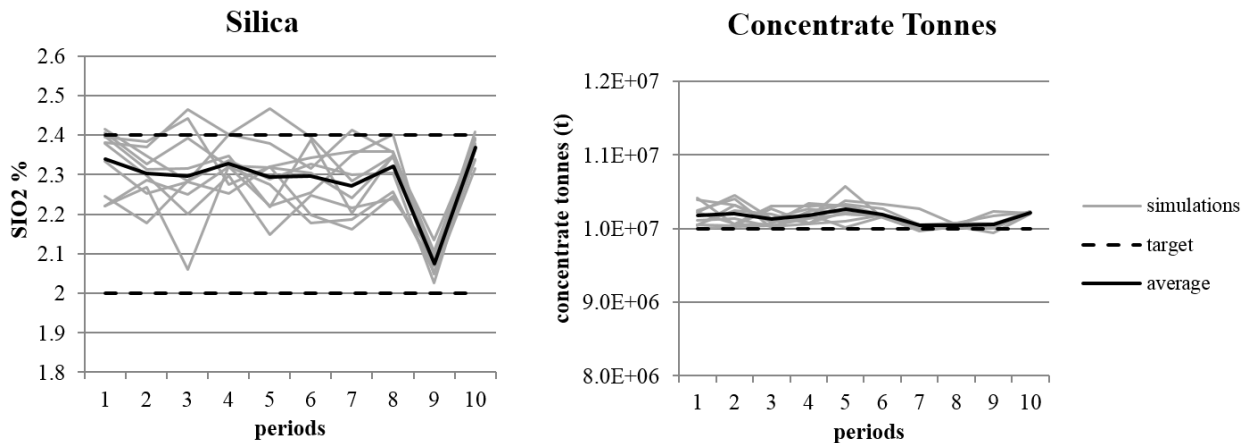


Figure 3 Risks profiles of the average Silica grade and the concentrate tonnes

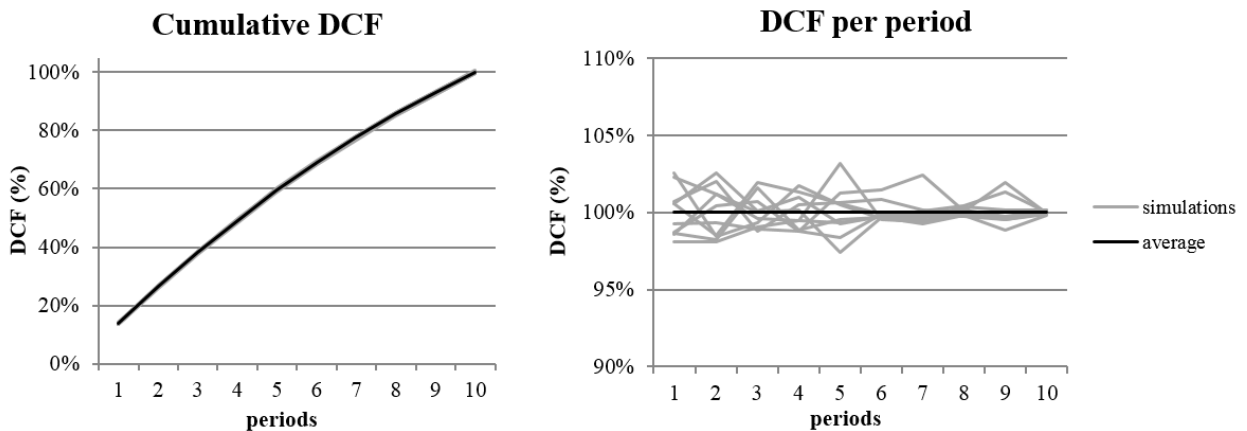


Figure 4 Risk profiles of the cumulative (left) and per period (right) discounted cash flow (relative to the average)

Figure 3 presents the risk profiles associated with the average Silica grade and the concentrate tonnes per period. One can note that the SIP model has managed to control the geological risk by respecting the constraints not only on average but also for each individual scenario. Figure 4 also presents risk profiles but in terms of discounted cash flow. The graph on the left shows the cumulative DCF on which, because of low variability, the different scenarios cannot be differentiated. The graph on the right, however, shows the DCF per period on which the fluctuations from one scenario to another are more perceptible but always contained within 3% of the average.

Finally, during the 10 periods of extraction, 1 177 blocks have been extracted from which 677 resulted in tailings or waste material stored inside the pit. This material represents a volume of 81 million m³, equivalent to a stockpile of 1200m of diameter for 100m of height. Also, the objective value the model was compared the one obtained with an infinite external storage capacity (solved with an exact method), equivalent to a simple extraction sequence schedule. The gap was 1.77%, this result outlines the low impact over the production of storing material inside the pit, especially since the re-handling costs of the rehabilitation phase were not considered.

4 Conclusions

The open pit mine planning SIP with in-pit tailings disposal model presented in this paper has formulated a practical way of storing tailings and waste material inside the pit during the operation, for a low dip deposit. It has managed to simultaneously optimize the storage and the block extraction sequence and destination policy so that the first produces a very low impact on the production. Considering the option of storing inside the pit reduces considerably the external space for the stockpile, saves re-handling costs and limits the impact on the environment and landscape during the operations.

References

- [1] S. Ramazan and R. Dimitrakopoulos, Traditional and New MIP Models for Production Scheduling With In-Situ Grade Variability, *Int. J. Surf. Mining, Reclam. Environ.*, 18(2), 85–98, 2004.
- [2] A. M. Newman, E. Rubio, R. Caro, A. Weintraub, and K. Eurek, A Review of Operations Research in Mine Planning, *Interfaces (Providence)*, 40(3), 222–245, Jun. 2010.
- [3] P. Dowd, Risk assessment in reserve estimation and open-pit planning, *Trans. Inst. Min. Metall. (Section A Min. Ind.)*, 103(January), 148–154, 1994.
- [4] P. J. Ravenscroft, Risk analysis for mine scheduling by conditional simulation, *Trans. Inst. Min. Metall. Sect. A. Min. Ind.*, 101:A104–A108, 1992.
- [5] R. Dimitrakopoulos and M. Godoy, Demand driven and risk managing strategic mine planning, *Minerales*, 60(263), 45–55, 2006.
- [6] P. Goovaerts, *Geostatistics for natural resources evaluation*. Oxford University Press, 1997.
- [7] R. Dimitrakopoulos and S. Ramazan, Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions, *Min. Technol. IMM Trans. Sect. A*, 117(4), 155–160, 2008.
- [8] A. Lamghari and R. Dimitrakopoulos, A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty, *Eur. J. Oper. Res.*, 222(3), 642–652, 2012.
- [9] L. Montiel and R. Dimitrakopoulos, Optimizing mining complexes with multiple processing and transportation alternatives: An uncertainty-based approach, *Eur. J. Oper. Res.*, 247(1), 166–178, 2015.
- [10] L. Montiel and R. Dimitrakopoulos, A heuristic approach for the stochastic optimization of mine production schedules, *J. Heuristics*, Press, 2017.
- [11] R. C. Goodfellow and R. Dimitrakopoulos, Global optimization of open pit mining complexes with uncertainty, *Appl. Soft Comput.*, 40, 292–304, 2016.
- [12] R. Goodfellow and R. Dimitrakopoulos, Stochastic Optimisation of Mineral Value Chains, *Math. Geosci.*, 2017.
- [13] A. Lamghari and R. Dimitrakopoulos, Progressive hedging applied as a metaheuristic to schedule production in open-pit mines accounting for reserve uncertainty, *Eur. J. Oper. Res.*, 253(3), 843–855, 2016.
- [14] M. Zuckerberg, P. Stone, R. Pasyar, and E. Mader, Joint Ore Extraction And In-Pit Dumping Optimization, *SME Annu. Meet.*, 1–3, 2007.
- [15] J. R. Birge and F. Louveaux, *Introduction to stochastic programming*. Springer Science & Business Media, 2011.
- [16] A. Riméle, Méthode heuristique d'optimisation stochastique de la planification minière et positionnement des résidus miniers dans la fosse, *École Polytechnique de Montréal*, 2016.
- [17] C. Cullenbine, R. K. Wood, and A. Newman, A sliding time window heuristic for open pit mine block sequencing, *Optim. Lett.*, 5(3), 365–377, Aug. 2011.
- [18] A. Boucher and R. Dimitrakopoulos, Block Simulation of Multiple Correlated Variables, *Math. Geosci.*, 41(2), 215–237, Feb. 2009.
- [19] IBM ILOG, *CPLEX User's Manual, Version 12 Release 6*, International Business Machines Corporation. 2014.