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# Interval observer approach to output stabilization of linear impulsive systems

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**Abstract:** The problem of output stabilization is studied for a class of linear hybrid systems subject to signal uncertainties: linear impulsive systems under dwell-time constraints. Two problems are considered. First, an interval observer estimating the set of admissible values for the state is designed. Next, an output stabilizing feedback design problem is studied where the stability is checked using linear matrix inequalities (LMIs). To the best of our knowledge, interval observer approach has never been proposed for the stabilization of this class of hybrid systems. Efficiency of the proposed approach is demonstrated by computer experiments for Fault Detection and Isolation (FDI) and Fault-Tolerant Control (FTC) of a power split device with clutch for heavy-duty military vehicles.

**Keywords:** Interval estimation, linear impulsive systems and control, stabilization

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## 1 Introduction

Impulsive systems are an important class of hybrid systems that include both continuous and discrete event dynamics. The continuous dynamics are generally represented by differential equations and the discrete one by switching laws, which govern discontinuous jumps of continuous states [17, 15]. The instants of these jumps can be time-dependent or state-dependent [3, 17]. Many systems can be modeled as impulsive ones [10]. Biological systems and intelligent vehicle/highway ones are some examples. Impulsive system stabilization is an important problem treated in the literature [4, 5, 7]. When all uncertainty is included in corresponding intervals or polytopes, synthesis of a conventional controller using the estimates of true values of the state may be complicated [1, 22]. In such a case the problem of pointwise estimation can be substituted by the interval one [9]. Using input-output measurements an observer has to estimate the set of admissible values (interval) for the state at each instant of time [18]. A major advantage of interval estimation is that it allows many types of uncertainties to be taken into account in the system [14].

Control of impulsive systems has been studied in previous works [4, 5, 6, 20]. In general in the existing literature, disturbances or uncertain parameters are such that the design of observers that can converge to the true values of the state is assumed to be possible. The problem of control design of impulsive systems becomes very challenging when one has to take into account the presence of a disturbance or uncertain parameters whose values should be considered to be an interval, rather than a single point measurement. The use of interval observer approach can be a solution in these cases [13, 12].

This paper sets out to make a contribution at two levels. Firstly, an extension of the results from [4] on input-to-state stability analysis for linear impulsive systems is proposed. Secondly, using the interval observer from [10], a stabilizing control design based on interval observers as in [13, 12] is presented. Since the interval estimates satisfy  $\underline{x}(t) \leq x(t) \leq \bar{x}(t)$  for all  $t \geq 0$ , then the stabilization of the bounds  $\underline{x}(t)$  and  $\bar{x}(t)$ , which are considered as outputs of an interval observer, ensures the same property for the considered linear impulsive system. To the best of our knowledge, the interval observer approach has never been proposed for the stabilization of this class of hybrid systems.

The outline of the paper is as follows. Some basic facts from the theories of interval estimation and hybrid systems are given in Section 2. In Section 3 we investigate the necessary results from hybrid systems robust stability under ranged dwell-time, which are applied in Sections 3 and 5 to design respectively the interval observer and the controller. Finally, the methodology is applied to Fault Detection and Isolation (FDI) and Fault-Tolerant Control (FTC) of a power split device with clutch for heavy-duty military vehicles in Section 6 in order to demonstrate the efficiency of the developed technique.

## 2 Preliminaries

The real and integer numbers are denoted by  $\mathbb{R}$  and  $\mathbb{Z}$  respectively,  $\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$  and  $\mathbb{Z}_+ = \mathbb{Z} \cap \mathbb{R}_+$ . Euclidean norm for a vector  $x \in \mathbb{R}^n$  will be denoted as  $|x|$ , the symbol  $|A|$  for a matrix  $A \in \mathbb{R}^{n \times n}$  corresponds to the induced matrix norm. The sequence of integers  $1, \dots, n$  is denoted as  $\overline{1, n}$ . The symbols  $I_n$  and  $E_{n \times m}$  denote the identity matrix with dimension  $n \times n$  and the matrix with all elements equal to 1 with dimensions  $n \times m$ , respectively. For a bounded input  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  the symbol  $\|u\|_{[t_0, t_1]}$  denotes its  $L_\infty$  norm

$$\|u\|_{[t_0, t_1]} = \sup_{t \in [t_0, t_1]} |u(t)|,$$

if  $t_1 = +\infty$  then we simply write  $\|u\|$ . We denote as  $\mathcal{L}_\infty$  the set of all inputs  $u$  with the property  $\|u\| < \infty$ . A matrix  $A \in \mathbb{R}^{n \times n}$  is called Metzler if all its elements outside the main diagonal are nonnegative, *i.e.*  $A_{i,j} \geq 0$  for  $1 \leq i \neq j \leq n$ . The vector of the eigenvalues of the matrix  $A$  is denoted as  $\lambda(A)$ .

## 2.1 Interval relations

For two vectors  $x_1, x_2 \in \mathbb{R}^n$  or matrices  $A_1, A_2 \in \mathbb{R}^{n \times n}$ , the relations  $x_1 \leq x_2$  and  $A_1 \leq A_2$  are understood elementwise. The relation  $P \prec 0$  ( $P \succ 0$ ) means that the matrix  $P \in \mathbb{R}^{n \times n}$  is negative (positive) definite, the class of such matrices is denoted as  $S_{>0}^n$  ( $S_{<0}^n$ ). Given a matrix  $A \in \mathbb{R}^{m \times n}$ , define  $A^+ = \max\{0, A\}$ ,  $A^- = A^+ - A$  (similarly for vectors) such that the matrix of absolute values of all elements  $|A|_{\text{abs}} = A^+ + A^-$ .

**Lemma 1** [11] *Let  $x \in \mathbb{R}^n$  be a vector variable,  $\underline{x} \leq x \leq \bar{x}$  for some  $\underline{x}, \bar{x} \in \mathbb{R}^n$ .*

(1) *If  $A \in \mathbb{R}^{m \times n}$  is a constant matrix, then*

$$A^+ \underline{x} - A^- \bar{x} \leq Ax \leq A^+ \bar{x} - A^- \underline{x}. \quad (1)$$

(2) *If  $A \in \mathbb{R}^{m \times n}$  is a matrix variable and  $\underline{A} \leq A \leq \bar{A}$  for some  $\underline{A}, \bar{A} \in \mathbb{R}^{m \times n}$ , then*

$$\begin{aligned} \underline{A}^+ \underline{x}^+ - \bar{A}^+ \underline{x}^- - \underline{A}^- \bar{x}^+ + \bar{A}^- \bar{x}^- &\leq Ax \\ &\leq \bar{A}^+ \bar{x}^+ - \underline{A}^+ \bar{x}^- - \bar{A}^- \underline{x}^+ + \underline{A}^- \underline{x}^-. \end{aligned} \quad (2)$$

Furthermore, if  $-\bar{A} = \underline{A} \leq 0 \leq \bar{A}$ , then the inequality (2) can be simplified:  $-\bar{A}(\bar{x}^+ + \underline{x}^-) \leq Ax \leq \bar{A}(\bar{x}^+ + \underline{x}^-)$ .

## 2.2 Stability of hybrid systems under ranged dwell-time

Consider an impulsive linear system with external inputs

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t) \quad \forall t \in [t_i, t_{i+1}), i \in \mathbb{Z}_+, \\ x(t_{i+1}) &= Gx(t_{i+1}^-) + g(t_{i+1}) \quad \forall i \geq 1, \end{aligned} \quad (3)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector and  $x(t_{i+1}^-)$  is the left-sided limit of  $x(t)$  for  $t \rightarrow t_{i+1}$ ;  $A, G \in \mathbb{R}^{n \times n}$ ;  $f: \mathbb{R}_+ \rightarrow \mathbb{R}^n$ ,  $f \in \mathcal{L}_\infty$  is the input for  $t \in [t_i, t_{i+1})$ ;  $g: \mathbb{R}_+ \rightarrow \mathbb{R}^n$ ,  $g \in \mathcal{L}_\infty$  is the input at time instants  $t_{i+1}$  for all  $i \geq 1$ . The sequence of impulse events  $t_i$  with  $i \in \mathbb{Z}_+$  is assumed to be positively incremental, i.e.  $T_i = t_{i+1} - t_i > 0$  and  $t_0 = 0$ .

**Theorem 1** [4] *Consider the system (3) with  $\|f\|_\infty = \|g\|_\infty = 0$  and a ranged dwell-time  $T_i \in [T_{\min}, T_{\max}]$  for all  $i \in \mathbb{Z}_+$ , where  $0 < T_{\min} \leq T_{\max} < +\infty$  are given constants. Then it is asymptotically stable provided that there exist matrices  $P \in S_{>0}^n$  and  $Q \in S_{>0}^n$  such that for all  $\theta \in [T_{\min}, T_{\max}]$*

$$G^T e^{A^T \theta} P e^{A \theta} G - P = -Q. \quad (4)$$

The proof of the above theorem is based on the fact that in this case  $W(x) = x^T P x$  is a Lyapunov function for (3) at discrete instants of time  $t_i$ .

## 3 Robust stability of hybrid systems under ranged dwell-time

Following [8, 19], robustness with respect to the inputs  $f$  and  $g$  can be proven (see the definition of the input-to-state stability (ISS) property given in those works).

**Lemma 2** *Consider system (3) with a ranged dwell-time  $T_i \in [T_{\min}, T_{\max}]$  for all  $i \in \mathbb{Z}_+$ , where  $0 < T_{\min} \leq T_{\max} < +\infty$  are given constants. Then, provided that there exist matrices  $P \in S_{>0}^n$  and  $Q \in S_{>0}^n$  such that for all  $\theta \in [T_{\min}, T_{\max}]$  the LMI (4) is satisfied, (3) is ISS and the following asymptotic gain is guaranteed*

$$\lim_{t \rightarrow +\infty} |x(t)| \leq [\rho_{P,Q,W} \|g\|_{\infty} + T_{\max}(1 + \rho_{P,Q,W} |G| \varrho(A)) \|f\|_{\infty}] \varrho(A), \quad (5)$$

$$\rho_{P,Q,W} = \frac{\sqrt{\frac{\lambda_{\max}(W)}{\lambda_{\min}(P)}}}{1 - \sqrt{\frac{\lambda_{\max}(P) - \frac{1}{2}\lambda_{\min}(Q)}{\lambda_{\min}(P)}}}, \quad (6)$$

where  $W = P + \sup_{\theta \in [T_{\min}, T_{\max}]} 2PGe^{A\theta} Qe^{A^T\theta} G^T P$  and  $\varrho(A) = \begin{cases} e^{\mu(A)T_{\max}} & \mu(A) > 0 \\ 1 & \mu(A) \leq 0, \end{cases}$

for  $\mu(A) = \max_{i=1, \dots, n} \lambda\left(\frac{A+A^T}{2}\right)$  being a logarithmic norm of the matrix  $A$ .

This result implies that (3) has bounded solutions for any bounded inputs  $f$  and  $g$  if the LMI (4) is valid. All proofs are skipped due to the space limitation.

## 4 Interval observer

Consider a hybrid (impulsive) linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + f(t) \quad \forall t \in [t_i, t_{i+1}), i \in \mathbb{Z}_+, \\ x(t_{i+1}) &= Gx(t_{i+1}^-) + Du(t_{i+1}) + g(t_{i+1}) \quad \forall i \geq 1, \\ y(t) &= Cx(t) + v(t), \end{aligned} \quad (7)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector and  $x(t_{i+1}^-)$  is the left-sided limit of  $x(t)$  for  $t \rightarrow t_{i+1}$ ;  $u(t) \in \mathbb{R}^m$  is the control;  $A, G \in \mathbb{R}^{n \times n}$ ;  $B, D \in \mathbb{R}^{n \times m}$ ;  $f : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ ,  $f \in \mathcal{L}_{\infty}$  is the input for  $t \in [t_i, t_{i+1})$ ;  $g : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ ,  $g \in \mathcal{L}_{\infty}$  is the input at time instants  $t_{i+1}$  for  $i \geq 1$ ;  $y(t) \in \mathbb{R}^p$  is the output signal available for measurements;  $v \in \mathcal{L}_{\infty}$  is the measurement noise;  $C \in \mathbb{R}^{p \times n}$ . As before, the sequence of impulse events  $t_i$  with  $i \in \mathbb{Z}_+$  is assumed to be positively incremental, *i.e.*  $T_i = t_{i+1} - t_i > 0$  and  $t_0 = 0$ .

We need the following Assumptions for system (7)

**Assumption 1** Let  $T_i = t_{i+1} - t_i \in [T_{\min}, T_{\max}]$  for all  $i \in \mathbb{Z}_+$ , where  $0 \leq T_{\min} \leq T_{\max} < +\infty$  are given constants.

**Assumption 2** There exist matrices  $L \in \mathbb{R}^{n \times p}$ ,  $M \in \mathbb{R}^{n \times p}$ ,  $P \in S_{>0}^n$  and  $Q \in S_{>0}^n$  such that

i) the LMI

$$(G - MC)^T e^{(A-LC)^T\theta} P e^{(A-LC)\theta} (G - MC) - P = -Q \quad (8)$$

holds for all  $\theta \in [T_{\min}, T_{\max}]$ ;

ii) the matrix  $(A - LC)$  is Metzler;

iii) the matrix  $(G - MC)$  is nonnegative.

When Assumption 2.i holds, the quadratic form  $W(x) = x^T P x$  is a discrete-time Lyapunov function for the LTI discrete-time system  $z_{i+1} = e^{(A-LC)\theta} (G - MC) z_i$  for all  $\theta \in [T_{\min}, T_{\max}]$  and  $i \in \mathbb{Z}_+$  by Theorem 1.

**Assumption 3** Let

i) two functions  $\underline{f}, \bar{f} : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ ,  $\underline{f}, \bar{f} \in \mathcal{L}_{\infty}$  be given such that

$$\underline{f}(t) \leq f(t) \leq \bar{f}(t) \quad \forall t \in \mathbb{R}_+;$$

ii) two functions  $\underline{g}, \bar{g} : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ ,  $\underline{g}, \bar{g} \in \mathcal{L}_{\infty}$  be given such that

$$\underline{g}(t) \leq g(t) \leq \bar{g}(t) \quad \forall t \in \mathbb{R}_+;$$

iii) a constant  $0 \leq V \leq +\infty$  be given such that  $\|v\| < V$ .

Assumption 1 is common in the existing literature concerning observer design, and it implies that the intervals of time between switching are lower and upper bounded. Assumptions 2.ii and 2.iii are essential for the approach but are rather restrictive. They can be relaxed using a transformation of coordinates [23], but such a relaxation will lead to a more complex notation and it is omitted here for brevity. Assumptions 3.i and 3.ii state that the inputs of the hybrid system (7) are known up to some interval errors  $\bar{f}(t) - \underline{f}(t)$  and  $\bar{g}(t) - \underline{g}(t)$ . Assumption 3.iii suggests an upper bound  $V$  for the amplitude of the noise  $v$ .

Under the introduced Assumptions an interval observer equation for (7) takes the form

$$\begin{aligned}
\dot{\underline{x}}(t) &= (A - LC)\underline{x}(t) + Ly(t) + Bu(t) + \underline{f}(t) \\
&\quad - \bar{L}V \quad \forall t \in [t_i, t_{i+1}), \\
\underline{x}(t_{i+1}) &= (G - MC)\underline{x}(t_{i+1}^-) + My(t_{i+1}) \\
&\quad + Du(t_{i+1}) + \underline{g}(t_{i+1}) - \bar{M}V, \\
\dot{\bar{x}}(t) &= (A - LC)\bar{x}(t) + Ly(t) + Bu(t) + \bar{f}(t) \\
&\quad + \bar{L}V \quad \forall t \in [t_i, t_{i+1}), \\
\bar{x}(t_{i+1}) &= (G - MC)\bar{x}(t_{i+1}^-) + My(t_{i+1}) \\
&\quad + Du(t_{i+1}) + \bar{g}(t_{i+1}) + \bar{M}V,
\end{aligned} \tag{9}$$

$\forall i \in \mathbb{Z}_+$ , where  $\underline{x}(t) \in \mathbb{R}^n$  and  $\bar{x}(t) \in \mathbb{R}^n$  are the lower and the upper interval estimates for the state  $x(t)$ , respectively,  $\bar{L} = (L^+ + L^-)E_{p \times 1}$  and  $\bar{M} = (M^+ + M^-)E_{p \times 1}$ .

**Theorem 2** [10] *Let Assumptions 2.ii, 2.iii and 3 be satisfied. Then in (7),(9),*

$$\underline{x}(t) \leq x(t) \leq \bar{x}(t) \quad t > 0 \tag{10}$$

*provided that  $\underline{x}(0) \leq x(0) \leq \bar{x}(0)$ . If  $x \in \mathcal{L}_\infty$  and Assumptions 1 and 2.i are valid, then  $\underline{x}, \bar{x} \in \mathcal{L}_\infty$ .*

## 5 Control design

The idea of this work consists in solving the stabilization problem for the completely known system (9) instead of (7). Under conditions of Theorem 2, if both  $\underline{x}(t)$  and  $\bar{x}(t)$  converge to zero, then the state  $x(t)$  also has to converge to zero, and boundedness of  $x(t)$  follows by the same property of  $\underline{x}(t)$  and  $\bar{x}(t)$ . In this case the signal  $y(t)$  is treated in the system (9) as a state dependent disturbance.

**Corollary 1** *Let Assumptions 2.ii, 2.iii and 3 be satisfied, then*

$$|y(t)| \leq |C|(|\underline{x}(t)| + |\bar{x}(t)|), \quad \forall t \in \mathbb{R}_+.$$

Hence, one has to stabilize the system (9) uniformly (or robustly) with respect to a Lipschitz nonlinearity  $y$ . The control is chosen as a conventional state linear feedback

$$\begin{aligned}
u(t) &= \underline{K}x(t) + \bar{K}\bar{x}(t), \quad \forall t \in [t_i, t_{i+1}), \\
u(t_{i+1}) &= \underline{J}\underline{x}(t_{i+1}^-) + \bar{J}\bar{x}(t_{i+1}^-),
\end{aligned} \tag{11}$$

where  $\underline{K}$ ,  $\bar{K}$ ,  $\underline{J}$  and  $\bar{J}$  are four feedback gains to be designed. When substituting the control (11) into (9), it follows that

$$\begin{aligned}
\dot{\underline{x}}(t) &= (A - LC + B\underline{K})\underline{x}(t) + Ly(t) + B\bar{K}\bar{x}(t) + \underline{f}(t) \\
&\quad - \bar{L}V \quad \forall t \in [t_i, t_{i+1}), \\
\underline{x}(t_{i+1}) &= (G - MC + D\underline{J})\underline{x}(t_{i+1}^-) + My(t_{i+1}) \\
&\quad + D\bar{J}\bar{x}(t_{i+1}^-) + \underline{g}(t_{i+1}) - \bar{M}V,
\end{aligned} \tag{12}$$



$$\begin{aligned}\dot{\bar{x}}(t) &= (A - LC + B\bar{K})\bar{x}(t) + Ly(t) + B\underline{K}x(t) + \bar{f}(t) \\ &\quad + \bar{L}V \quad \forall t \in [t_i, t_{i+1}), \\ \bar{x}(t_{i+1}) &= (G - MC + D\bar{J})\bar{x}(t_{i+1}^-) + My(t_{i+1}) \\ &\quad + D\underline{J}x(t_{i+1}^-) + \bar{g}(t_{i+1}) + \bar{M}V,\end{aligned}$$

and it is necessary to analyse stability of this nonlinear system. Let us introduce  $\varepsilon(t) = [\underline{x}^T(t) \bar{x}^T(t)]^T$  and the matrices

$$\begin{aligned}R &= \begin{bmatrix} A - LC + B\underline{K} & B\bar{K} \\ B\underline{K} & A - LC + B\bar{K} \end{bmatrix}, \\ S &= \begin{bmatrix} G - MC + D\underline{J} & D\bar{J} \\ D\underline{J} & G - MC + D\bar{J} \end{bmatrix}, \\ \delta(t) &= \begin{bmatrix} f(t) - \bar{L}V \\ \bar{f}(t) + \bar{L}V \end{bmatrix}, \quad \varsigma(t_{i+1}) = \begin{bmatrix} g(t_{i+1}) - \bar{M}V \\ \bar{g}(t_{i+1}) + \bar{M}V \end{bmatrix},\end{aligned}$$

then one can rewrite the system (12) as

$$\begin{aligned}\dot{\varepsilon}(t) &= R\varepsilon(t) + \delta(t) + \begin{bmatrix} Ly(t) \\ Ly(t) \end{bmatrix} \\ \varepsilon(t_{i+1}) &= S\varepsilon(t_{i+1}^-) + \varsigma(t_{i+1}) + \begin{bmatrix} My(t_{i+1}) \\ My(t_{i+1}) \end{bmatrix}.\end{aligned}\tag{13}$$

**Theorem 3** *Let Assumptions 1, 2.ii, 2.iii and 3 hold,  $\underline{x}(0) \leq x(0) \leq \bar{x}(0)$  and there exist matrices  $\underline{K} \in \mathbb{R}^{m \times n}$ ,  $\bar{K} \in \mathbb{R}^{m \times n}$ ,  $\underline{J} \in \mathbb{R}^{m \times n}$ ,  $\bar{J} \in \mathbb{R}^{m \times n}$ ,  $P \in S_{>0}^{2n}$  and  $Q \in S_{>0}^{2n}$  such that the matrix inequality*

$$e^{R^T\theta} S^T P e^{R\theta} - P + Q = 0\tag{14}$$

is satisfied for all  $\theta \in [T_{\min}, T_{\max}]$ , and

$$\begin{aligned}\rho_{P,Q,W} \sqrt{2}|M| + \sqrt{2}T_{\max}(1 + \\ \rho_{P,Q,W}|S|\varrho(R)|L|\varrho(R)|C| < 1,\end{aligned}$$

where  $W = P + \sup_{\theta \in [T_{\min}, T_{\max}]} 2PSe^{R\theta} Qe^{R^T\theta} S^T P$ . Then system (13) is ISS with respect to the inputs  $\delta$  and  $\varsigma$ .

**Remark 1** *The design of the control (11) may be affected by computational complexity problems. The feedback gains  $\underline{K}$ ,  $\bar{K}$ ,  $\underline{J}$  and  $\bar{J}$  are chosen a priori to satisfy (14). The LMI (14) can be reformulated using convexity [2].*

## 6 Application

In this section, the interval observer and the controller that have been designed in this work are applied to Fault Detection and Isolation (FDI) and Fault-Tolerant Control (FTC) of a complex uncertain system.

Hybrid electric vehicles (HEVs) can be classified with respect to their energy flow used for propulsion as either series or parallel [24]. Combining these two systems one can obtain the so-called series-parallel HEVs, which have the advantages of these two basic architectures, but have a more complicated structure. The Power Split Device (PSD) that divides the power coming from various power sources into the drivetrain (see Figure 1) plays a major role in the suitable energy management strategy of series-parallel HEVs [16]. A hybrid powertrain with a high availability for heavy-duty military vehicles is considered for our application. A series-parallel HEV architecture is considered along with the Ravigneaux geartrain as PSD [24].

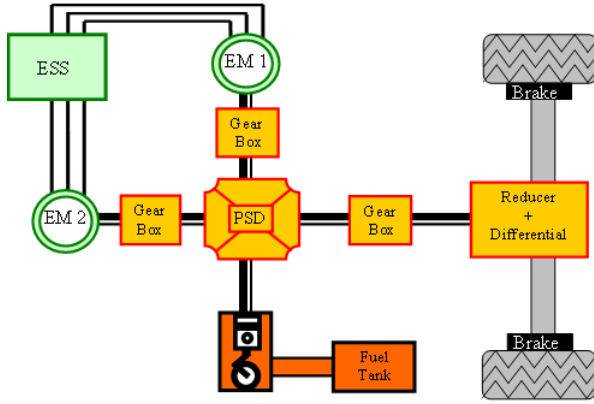


Figure 1: Series-parallel HEV configuration with PSD [24]

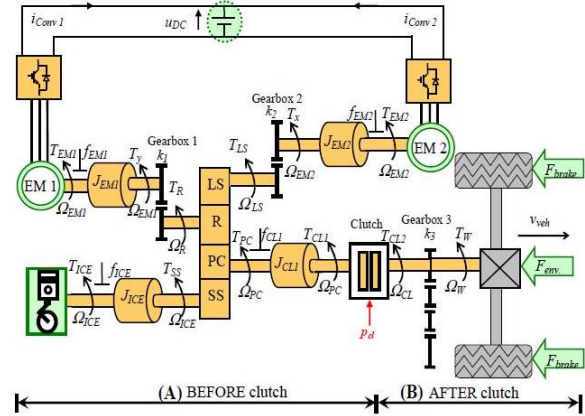


Figure 2: Considered system layout [24]

The considered architecture is comprised of a PSD mounted with shafts connected to two electric machines (EM1 and EM2) through gearboxes, an Internal Combustion Engine (ICE) and transmission with clutch through a gearbox [24] (see Figure 2). The role of a clutch is to connect the driving shaft to a driven shaft, so that the driven shaft may be started or stopped at will, without stopping the driving shaft. In conventional vehicles, the clutch allows for power to be transmitted from the ICE to the wheels in order to change the speed ratio using the gearbox [24].

The behavior of the clutch is nonlinear, and two different states are considered: the slipped (open) one and the locked (closed) one. During the locked position of the clutch, the system is considered as a single equivalent inertia: the disks are rigidly coupled with each other.

With reference to the notations in Table 1, the locked position of the clutch can be modeled by the following equations

$$\begin{aligned} J_{CL1} \frac{d}{dt} \Omega_{CL} + f_{CL1} \Omega_{CL} &= T_{CL2} - T_{PC}, \\ T_{CL1} &= T_{CL2} = T_{CL}, \\ \Omega_{PC} &= \Omega_{CL}. \end{aligned}$$

Table 1: Parameters used in the Application

| Param.        | Meaning  |
|---------------|--|
| $T_{CL1}$     | Torque provided upstream of the clutch         |
| $T_{CL2}$     | Torque provided downstream of the clutch       |
| $\Omega_{CL}$ | Speed of the primary shaft after the clutch    |
| $\Omega_{PC}$ | Speed of the secondary shaft before the clutch |
| $f_{CL1}$     | Friction coefficient of the shaft              |
| $J_{CL1}$     | Inertia of the shaft                           |

Let us consider a failure mode of the braking phase of the heavy-duty military vehicle with  $T_{CL1} = T_{CL2} = 0$ , and assume that there is a coefficient  $\kappa = 1$ , which is added to the friction coefficient  $f_{CL1}$  at all instants  $t = 3k$  with  $k \in \mathbb{Z}_+$ . The case where a decrease in normal force leads to an increase in friction is considered. This situation leads to a negative friction coefficient [25]. For the application, the case with  $f_{CL1} = -0.5$  and  $J_{CL1} = 1kg.m^2$  is considered. Dissipation losses, vibration, abrasion and temperature effects are neglected. Then this failure mode can be represented by the following system with  $x(t) = \Omega_{CL}$  and  $u(t) = T_{PC}$

$$\begin{aligned}
\dot{x}(t) &= ax(t) + bu(t) + f(t) \quad \forall t \neq 3k, k \in \mathbb{Z}_+, \\
x(t) &= hx(t^-) + du(t) + g(t) \quad \forall t = 3k, k \in \mathbb{Z}_+, \\
y(t) &= cx(t) + v(t),
\end{aligned} \tag{15}$$

where  $a$ ,  $b$ ,  $c$ ,  $d$  and  $h$  are defined as follows

$$\begin{aligned}
a &= \frac{-f_{CL1}}{J_{CL1}} = 0.5, \quad b = \frac{-1}{J_{CL1}} = -1, \\
c &= 10, \quad d = -1, \quad h = \frac{-f_{CL1} + \kappa}{J_{CL1}} = 1.5,
\end{aligned}$$

and  $x(t) \in \mathbb{R}$ ,  $y(t) \in \mathbb{R}$  are the state and the output, respectively. The external disturbances and noises  $f(t)$ ,  $g(t)$  and  $v(t)$  for simulation are selected as follows

$$\begin{aligned}
f(t) &= \beta \sin(2t), \quad g(t) = \delta \sin(t), \\
v(t) &= V \cos(t),
\end{aligned}$$

where  $\beta = 10^{-3}$ ,  $\delta = 10^{-2}$  and  $V = 2$  are given parameters. Thus,

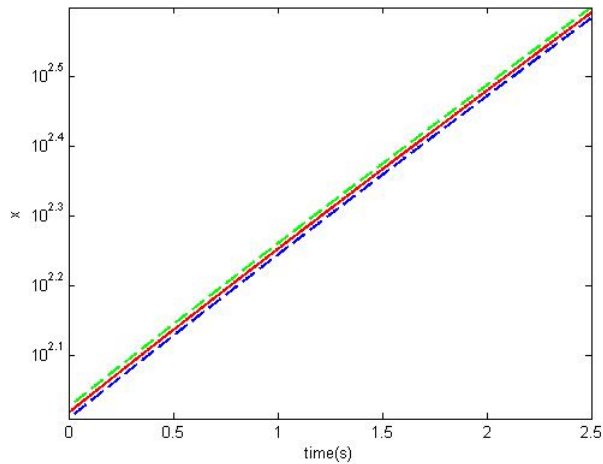
$$\begin{aligned}
\underline{f}(t) &= -\beta, \quad \bar{f}(t) = \beta, \\
\underline{g}(t) &= -\delta, \quad \bar{g}(t) = \delta.
\end{aligned}$$

Assumption 3 is then satisfied. Assumption 2.ii is verified for  $l = 0$ :  $a - lc = 0.5$  is Metzler but not Hurwitz stable. Assumption 2.iii is verified for  $m = 0.14$ :  $g - mc = 0.1$  is nonnegative. By applying Matlab YALMIP toolbox [21] with discretization to solve the LMIs we found that Assumption 2.i holds for all  $\theta \in [0, 4.6051]$ . Then the system (9) with the minimum dwell-time  $\theta \in [0, 4.6051]$  is ISS. Therefore all conditions of Theorem 2 are satisfied and the interval observer (9) solves the problem of interval state estimation for the Fault Detection and Isolation (FDI). The results of simulation are shown in Figure 3 where the solid line represents the state  $x$ , and the dash lines are used for the interval estimates  $\underline{x}$  and  $\bar{x}$  which are given in the logarithmic scale.

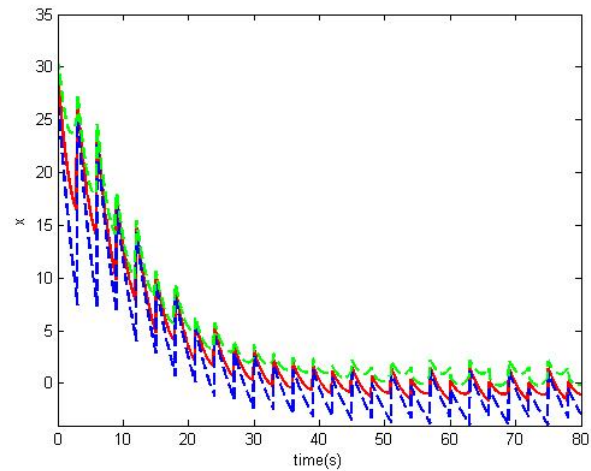
The default on the state is detectable and isolable since  $x$  appears in the Analytical Redundancy Relation (15) and the Fault signature matrix is distinguishable. Hence it is required to stabilize the state, which represents the speed of the primary and secondary shaft after and before the clutch during the braking phase of the heavy-duty military vehicle in the considered failure mode. The time response is required to be less than 60 seconds. Equations 14 and 6 are satisfied for  $\bar{k} = 0.1$ ,  $\underline{k} = 0.65$ ,  $\bar{j} = 0.1$  and  $\underline{j} = -0.15$ . The matrices

$$R = \begin{bmatrix} -0.15 & -0.1 \\ -0.65 & 0.4 \end{bmatrix}, \quad S = \begin{bmatrix} 0.25 & -0.1 \\ 0.15 & 0 \end{bmatrix}$$

satisfy all conditions of Theorem 3 for all  $\theta \in [0, 4.6051]$  and the controller (11) solves the problem of stabilization of the speed  $\Omega_{CL}$ . Then the system (7), (9), (11) is ISS with respect to the inputs  $f$  and  $g$ . The results of simulation are shown in Figure 4 where the solid line represents the state  $x$ , and the dash lines are used for the interval estimates  $\underline{x}$  and  $\bar{x}$ . From these results we can conclude that the speed  $\Omega_{CL}$  is stabilized and the time response which is  $t_R \approx 32$  seconds meet the time response requirements.



**Figure 3: Results of the simulation of the interval state estimation**



**Figure 4: Results of the simulation of the stabilization**

## 7 Conclusion

The problems of interval estimation and robust stabilization for a class of linear hybrid systems subject to signal uncertainties have been considered in this paper. The goal of the proposed approach is to take into account the presence of disturbance or uncertain parameters during the synthesis. A new approach for output feedback design is proposed for this class of systems where an interval observer is used instead of a conventional one. Knowing the estimates of the upper and the lower bounds of the state, the problem of output stabilization is reduced to a problem of robust state feedback design. The stability can be checked using linear matrix inequalities (LMIs). Efficiency of the proposed approach is shown on computer simulations for Fault Detection and Isolation (FDI) and Fault-Tolerant Control (FTC) of a power split device with clutch for heavy-duty military vehicles. Future work can focus on nonlinear hybrid systems with parameter uncertainties, and the reformulations of the conditions of the LMIs (14) and (6) using convexity.

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