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G–2016–97

November 2016
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November 2016

Les Cahiers du GERAD
G–2016–97

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**Abstract:** Mineral value chains consist of material from multiple sources, several processing streams, and transportation systems that combine to generate various saleable products. These sources of material can be mineral deposits, stockpiles or material purchased from external vendors. Typically, there is a high level of uncertainty associated with these sources of material, which propagates through the mineral value chain. Uncertainty in material types/grades can be modelled and used in the assessment of risk in mining plans. This uncertainty can also be incorporated during optimization in stochastic frameworks.

This paper presents an approach that first utilizes a set of scenarios derived from stochastic simulations of the different sources of material to perform risk analysis and then applies a simultaneous stochastic optimization framework to develop a robust mine plan for a multipit gold complex. The scenarios include stochastic simulations of initial stockpiles and incorporate the uncertainty of external material by means of Monte Carlo simulations over historical data. The analysis allows for potential risks associated with mine plans to be clearly quantified, particularly the extraction schedule’s ability to meet key project targets on a yearly basis. The implementation of a stochastic optimization framework generates a mine plan with feasible additive consumption and expected NPV 10% higher than a base case plan.

**Keywords:** Stochastic optimization, multipit gold complex, geological risk
1 Introduction

Due to the current low metal prices, mining operations are facing the challenge of lowering costs and improving operational efficiencies. This requires improving the synergies associated with the different components of the mineral value chain. Rather than optimizing individual components independently, mining companies should look at the entire operation. For example, controlling the quality of the input material to a process can lead to better throughputs, reduced costs and increased recoveries (Wharton, 2004). Orebody modelling and mine optimization methods discretize mineral deposits into mining blocks. Traditional pit design and scheduling techniques assign economic values to blocks considering economic and operating parameters. Blocks are comprised of metals and materials and their contribution to the operation depends on how material is extracted, blended and processed. Therefore, mining blocks cannot be defined in terms of economic values prior to optimization. Mineral deposits may not be the only source of material in a mining complex. In some operating mines, material extracted from the deposit(s) may also be blended with material coming from pre-existing stockpiles or external sources. As multiple sources of uncertainty are added to the mineral value chain model, the cumulative effects of risk are propagated through to the final products sold to customers, herein referred to as the global supply uncertainty.

Several methods have incorporated the geological uncertainty associated with mineral deposits into pit design and mine production scheduling (Godoy, 2003; Godoy and Dimitrakopoulos, 2004; Lamghari and Dimitrakopoulos, 2012; Bendorf and Dimitrakopoulos, 2013; Goodfellow and Dimitrakopoulos, 2013; Lamghari, Dimitrakopoulos and Ferland, 2013; Montiel and Dimitrakopoulos, 2013). These approaches, however, ignore the uncertainty associated with the material that comes from the stockpiles and the external sources as global supply uncertainty does. Similarly to mineral deposits, the uncertainty associated with a set of initial stockpiles can be modelled using stochastic simulations (Goovaerts, 1997; Mustapha et al., 2014; Straubhaar and Malinvernì, 2014; Maleki and Emery, 2015). The uncertainty of the material that comes from external sources can be modelled using Monte Carlo simulations using historical data. Incorporating global supply uncertainty into the optimization of mineral value chains allows for the quantification and minimization of the risk associated with not meeting project targets, such as production rates, metal quantities, blending requirements, etc..

The simultaneous optimization of multiple components of the mining value chain in deterministic frameworks includes the work of Hoeger et al. (1999), Stone et al. (2007), Chanda (2007), Wooller (2007, 2010). Some methods have been developed to incorporate uncertainty in the optimization of mining complexes: Groeneveld et al. (2010) incorporate uncertainty in market price, costs, utilization of equipment, plant recovery and time for building options (infrastructure) while simultaneously optimizing mining, stockpiling, processing and port policies; Bodon (2010) models the problem of supplying exports in a coal chain as a discrete event simulation model (DES). Both of these stochastic methods, however, ignore geological uncertainty—a key contributor of not meeting production targets and net present value (NPV) forecasts (Vallee, 2000; Dimitrakopoulos et al., 2002).

This paper presents an approach that incorporates global supply uncertainty in the optimization of mining complexes with multiple pits, stockpiles, external sources, and processing streams. The approach focuses on generating mining, stockpiling and blending schedules that maximize expected NPV while controlling production rates and blending requirements. The following section describes the proposed methodology to follow for incorporating global supply uncertainty into mining complex optimization. The implementation of the method at a multipit gold complex is presented in Section 3. Finally, conclusions and recommendations for future work are presented in Section 4.

2 Optimizing a multipit mining complex

The global supply uncertainty associated with a set of mineral deposits and stockpiles can be described using a set of scenarios generated by means of stochastic spatial simulation techniques (Figure 1). The uncertainty associated with material coming from external sources is modelled by means of Monte Carlo simulations using historical data. A particular scenario for the global supply uncertainty space results from combining the scenarios associated with the different raw material sources.

The material coming from the different sources are combined to provide feasible production rates and satisfy blending requirements while the expected NPV is maximized. For the blending operation, the different ore types can
be classified into different categories based on the grades of different properties (Wharton, 2005). A stockpile may be used for each category, which provides flexibility to blend, however, the complexity of the optimization problem increases drastically as the number of blending stockpiles increases. Incorporating stockpile simulations during optimization is a demanding task given the dynamic behavior of stockpiles; that is, mine operators are not only reclaiming material from stockpiles (similar to extracting material from mineral deposits) but they are also feeding the stockpiles, which implies the necessity of modelling the way material is disposed and reclaimed from the stockpiles.

![Diagram of material sources and process](image)

Figure 1: Uncertainty associated with the different sources of material.

There is an uncertainty associated with the material coming from external sources that may play an important role in the global supply uncertainty. If the uncertainty of this material is too high, a stochastic optimizer will blend the material from the mineral deposits and stockpiles to minimize the downside risk associated with the use of the external material. Ignoring this uncertainty will generate solutions that do not consider the global supply uncertainty, thus not minimizing the downside risk and maximizing the upside potential.

### 2.1 Stochastic framework formulation

The goal is to generate a mining, stockpiling, blending and processing plan that maximizes the expected NPV while generating feasible production rates that satisfy blending requirements. To achieve the goal, the optimization model uses the following objective function:

$$\text{Maximize } O = \sum_{t=1}^{T} \left( \frac{1}{2} \sum_{s=1}^{S} \text{dis CPR} \text{ofit}(s, t) - \text{penalty}(s, t) \right)$$

(1)

where $T$ is the number of years of the life-of-mine (LOM), $S$ is the set of scenarios that model the global supply uncertainty, $\text{dis CPR} \text{ofit}(s, t)$ is the discounted profit in period $t$ and scenario $s$, and $\text{penalty}(s, t)$ are the penalized deviations in period $t$ and scenario $s$. The discounted profits are evaluated using Equation (2) where $\text{revenue}(s, t)$ is the revenue obtained by selling the different products in period $t$ and scenario $s$, $\text{mine} \text{cost}(s, t)$ is the cost of mining the material at the different mineral deposits in period $t$ and scenario $s$, $\text{proc} \text{ost}(s, t)$ is the cost of processing the material at the different processing destinations in period $t$ and scenario $s$, $\text{stock} \text{cost}(s, t)$ is the cost of stockpiling material in the different blending stockpiles in period $t$ and scenario $s$, $\text{rehand} \text{le} \text{cost}(s, t)$ is the cost of rehandling material from the different blending stockpiles and $\text{drate}$ is the economic discount rate.

$$\text{dis CPR} \text{ofit}(s, t) = \frac{\text{revenue}(s, t) - \text{mine} \text{cost}(s, t) - \text{proc} \text{ost}(s, t)}{\text{stock} \text{cost}(s, t) - \text{rehand} \text{le} \text{cost}(s, t)} \left(1 + \text{drate} \right)^t$$

(2)

To calculate the revenue, non-linear recovery curves that account for deleterious elements can be considered. The total extraction cost accounts for different per-unit mining costs associated with depth and type of material by
considering mining cost adjustment factors. The penalties, \( \text{penalty}(s, t) \), are calculated by considering mining and processing capacities and operational ranges for blending properties. The penalty term in the objective function depends on the magnitude of the deviations and associated per-unit penalty values. The per-unit penalty values must be chosen to balance the two terms of the objective function; a high per-unit penalty cost will generate solutions with poor improvement in expected NPV, however, using low per-unit penalty costs will generate impractical solutions that deviate from operational ranges. As a result, the penalties control the level of risk that is obtained with the stochastic solution and allows prioritizing operational targets that are more relevant; e.g., in an autoclave, it may be more important to control the sulfide sulfur than the carbonate, which can be expressed by assigning larger penalty values to the sulfide sulfur.

The revenues, costs, productions and deviations can be evaluated as in Montiel and Dimitrakopoulos (2015). The flexibility associated with the different components of the mining value chain and the ways the different mineral deposits are discretized generate large non-linear optimization models that are unable to be solved using commercially available optimization tools. To solve the problem, a stochastic approach based on simulated annealing has been developed and implemented (Metropolis et al., 1953; Kirkpatrick et al., 1983). The next sections describe the method and its implementation at a multipit gold complex.

2.2 Optimization methodology

The stochastic optimization of a mineral value chain demands the simultaneous optimization of mining, stockpiling, blending and processing schedules under global supply uncertainty. The solution approach explores the solution domain (the set of all feasible solutions to the problem) by perturbing an initial solution in order to obtain a solution as close as possible to the global optimum. As optimal extracting, stockpiling, blending and processing policies must be defined simultaneously, different perturbation mechanisms are performed iteratively. Five different types of perturbation are implemented to extensively explore the solution domain: (i) block extraction period perturbations; (ii) block destination perturbations; (iii) multipit perturbations; (iv) single reclaim perturbations; and (v) double reclaim perturbations. Any perturbation will be accepted or rejected based on the decision rule of simulated annealing (Metropolis et al., 1953; Kirkpatrick et al., 1983).

2.2.1 Block extraction period perturbation

This perturbation mechanism randomly selects a block and modifies its extraction period. There are two different cases for a given block: (i) the candidate period is earlier than the current period; (ii) the candidate period is later than the current period.

By assigning a different extraction period to a block, some slope constraint violations may occur. The mechanism to correct slope constraint violations varies depending on the cases described previously. To check slope constraint violations the method uses the predecessor and successor envelopes (Figure 2). The predecessor envelope of a given block represents the minimum set of predecessor blocks that needs to be checked to ensure that slope constraints for that block are respected. The predecessor envelope accounts for different geotechnical zones with different slope angles at different azimuths (any slope configuration can be considered). Similarly, the successor envelope helps in checking slope constraint violations of successor blocks.

Figure 2: Predecessor envelope of a given block.

In case (i), the slope correction procedure will check the set of blocks in the predecessor envelope. If there is a block in the predecessor envelope with an extraction period later than the candidate period, the current extraction period of that block is replaced with the candidate one. The opposite occurs for case (ii) where the procedure will check
the set of blocks in the successor envelope. Any block in the successor envelope with extraction period earlier than the candidate period gets assigned the candidate period.

The perturbation mechanism modifies the extraction period of a certain block as well as the periods of the blocks violating slope constraints in the predecessor or successor envelope. This implies bigger swaps of material when compared with other swapping mechanisms (Godoy and Dimitrakopoulos, 2004) which allows a more extensive exploration of the solution domain. Furthermore, by swapping periods of multiple blocks at once, the final stochastic schedule will be smoother and easier to transform into a practical one.

2.2.2 Block destination perturbation

For each block and every destination, the algorithm accumulates the economic value of the block in the destination along the different scenarios. The ‘best’ destination of a mining block may be seen as the destination with the highest accumulated economic value, however, given the blending requirements and the time value of money, the optimal destination of a mining block may not coincide with the best destination; the destination of a mining block should not be defined prior to optimization as it depends on the properties of the block and the mining and blending schedule.

This perturbation mechanism randomly selects a block and swaps its destination from the set of available processes. It seeks to minimize the misclassification of material, i.e., sending waste to the mill or a particular ore type to a destination that does not process this type of material. Given the uncertainty associated with the raw material, a mining block that is classified as a certain ore type in one scenario may be classified as a different one in other scenario; e.g., a block classified as sulfide ore in scenario 1 can be classified as oxide ore in scenario 2. A block sent to the wrong destination represents a cost with none or marginal revenue.

The perturbation of block destinations also controls the way material is stockpiled. If the accumulated economic value of a given block in its best destination is positive, stockpiling the block is also considered as one of the options. Alternatively, if the value is negative, the perturbation mechanism will consider only the set of available processing destinations, from which one or several waste dumps may exist (processing destinations with zero recoveries).

2.2.3 Multipit perturbation

When the mining complex is comprised of multiple pits, the method performs multipit perturbations. This perturbation mechanism randomly selects two blocks, each one from different pits and swaps their extraction periods. The extraction periods of the blocks belonging to the predecessor or successor envelopes must also be modified accordingly. This perturbation mechanism allows larger movements in the solution domain as it acts as performing twice a block extraction period perturbation (Section 2.2.1). This perturbation mechanism modifies the production rates of the different pits, prioritizing the extraction of one pit that has more profitable material than another.

2.2.4 Single reclaim perturbation

This perturbation mechanism evaluates improvements in the objective function by reclaiming material from the blending stockpiles. It randomly selects a processing destination $d$ and a given period $t$ and evaluates the average shortfall in production in destination $d$ in period $t$. Then, it chooses a blending stockpile $b$ and assigns a proportion of the shortfall of material from $b$ to destination $d$. This allows modifying the reclaiming of material from the blending stockpiles while reducing the shortfall in production at the different processing destinations.

2.2.5 Double reclaim perturbation

This perturbation mechanism swaps units of material reclaimed from two different blending stockpiles in different periods at a given processing destination. If a given mill is fed with blending stockpile A in year 2 and blending stockpile B in year 4, this perturbation mechanism will send some units of blending stockpile A to year 4 and some units of blending stockpile B to year 2. This mechanism does not impact much on the production rates of the processing destinations but has a large influence on how the material is blended to meet the specific blending requirements. It helps in allocating the blend type materials in the periods where they are required.
2.2.6 The methodology

The method uses an initial solution that can be generated using standard industry practices and performs the different set of perturbations iteratively to generate a final risk-based solution (Figure 3). A cycle is defined as one round of perturbations of every type. The main input parameters of the method are the number of perturbations of each type in every cycle. If the mining complex has a large number of blending stockpiles and stringent blending requirements, the number of double reclaim perturbations should be considerable large. If the main goal is to increase the expected NPV, block based perturbations should be prioritized. Double swap perturbations are significant when there is large flexibility in mining rates. Other important parameters to evaluate are the number of cycles, the initial temperature for annealing and the per-unit penalty costs.

![Figure 3: Methodology.](image)

3 Case study: A multipit gold complex

The method is implemented in a gold complex comprised of 2 pits and 4 processing destinations (Figure 4). The material extracted from Pit A provides sulphide ore that can be sent to an autoclave or a set of sulphide stockpiles. The sulphide ore is classified into 51 different blend types based on the gold grades, sulphide sulfur (SS), carbonate (CO\(_3\)) and organic carbon (OC). This implies that 51 different sulphide stockpiles can be considered for blending, from which 26 already have material. The 51 different blend types can be grouped in 11 different categories based on the SS, CO\(_3\) and OC grades (Table 1). Pit B provides oxide ore that can be sent to an oxide mill plant, a leaching plant or an oxide stockpile. The waste material from both pits is sent to a waste dump.

![Figure 4: Multipit gold complex.](image)
The autoclave processes sulphide ore coming from Pit A, the sulphide stockpiles and three different external sources. The sulphide ore must be blended to meet very stringent blending requirements in the autoclave in terms of SS, CO$_3^-$, OC and SS/CO$_3^-$ ratio. To decrease the CO$_3^-$ in the autoclave to meet the blending requirements, an acid can be added; however, the amount of acid in a certain year cannot exceed a specified maximum amount. The metallurgical recovery in the autoclave is a non-linear function that depends on gold and OC. The recovery functions in the oxide mill and oxide leach are non-linear functions that depend only on gold grades.

### 3.1 Base case solution

A base case solution for the multipit mining complex has been generated using standard industry practices in a deterministic framework. The sequence of extraction of both pits can be observed in Figure 5 and the mine production rates in Figure 6. The mining complex will operate at full mine capacity only in year three, after which the mine production will gradually decrease until year 11 when there is no more production from both pits. After year 7 there is no production from Pit A. Despite the lack of production from the mine after year 11, the autoclave will be fed until year 14 using the material from the stockpiles and the external sources (Figure 7 – left). The autoclave will be fed to full capacity, except in the last year due to the depletion of the sulphide ore. The oxide mill will be fed until year 13 when the oxide ore is consumed (Figure 7 – right). There are big shortfalls in the production of the oxide mill between years five and seven; however, since the autoclave is the more relevant process, the solution prioritizes feeding the autoclave to its full capacity.

<table>
<thead>
<tr>
<th>Sulphide Blend Types</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blend type A</td>
<td>Low SS and medium-low CO$_3^-$</td>
</tr>
<tr>
<td>Blend type B</td>
<td>High SS and medium CO$_3^-$</td>
</tr>
<tr>
<td>Blend type C</td>
<td>Medium-high SS and low CO$_3^-$</td>
</tr>
<tr>
<td>Blend type D</td>
<td>Low SS and low CO$_3^-$</td>
</tr>
<tr>
<td>Blend type E</td>
<td>High OC and medium-low CO$_3^-$</td>
</tr>
<tr>
<td>Blend type F</td>
<td>High CO$_3^-$</td>
</tr>
<tr>
<td>Blend type G</td>
<td>Low SS and medium-high CO$_3^-$</td>
</tr>
<tr>
<td>Blend type H</td>
<td>High SS and low CO$_3^-$</td>
</tr>
<tr>
<td>Blend type I</td>
<td>Very high CO$_3^-$</td>
</tr>
<tr>
<td>Blend type K</td>
<td>High OC and High CO$_3^-$</td>
</tr>
<tr>
<td>Blend type O</td>
<td>Very High OC</td>
</tr>
</tbody>
</table>

Figure 5: Extraction sequences of Pit A (left) and Pit B (right).

Figure 6: Mine production.
The largest amount of in-situ gold going to the autoclave occurs in year four (Figure 8). The blending requirements are satisfied in the different periods with a very low shortfall for SS and CO$_3$ in the last two years. The CO$_3$ and the SS/CO$_3$ ratio are very close to the lower limit of the operational ranges. To keep the quality of the material inside the operational ranges, acid is added along the different years to reduce the concentration of CO$_3$. The amount of acid consumed is smaller than the maximum amount in all the different years (Figure 9). The largest amount of gold recovered from the autoclave will occur in year 4 when the amount of in-situ gold sent to the autoclave is largest.

Under the deterministic assumption, the base case solution has feasible production rates while keeping the blending properties inside the operational ranges. However, there is large uncertainty associated with the raw material that needs to be considered to evaluate the solution. This analysis will be done in the following subsections.

### 3.2 Modelling the global supply uncertainty

Twenty orebody simulations are used for each deposit to model geological uncertainty. For Pit A, which provides sulphide ore, the multi-element simulations were generated using DBMAFSIM (Boucher and Dimitrakopoulos, 2009). The simulated properties are Au, SS, CO$_3$ and OC (Figure 10). The simulations show more variability than the
estimated model because, unlike estimation methods, simulations do not smooth out the grades. For Pit B, which provides oxide ore, only the gold grades are simulated as there are no blending requirements to process oxide ore.

Several existing stockpiles have been sampled to assess their quality and variability, which is used to generate a set of stochastic simulations (Figure 11). The stockpiles where no drilling campaign was performed have been simulated using production grade control data. For the material coming from external sources, the uncertainty is described using a set of Monte Carlo simulations using historical data.
3.3 Risk analysis of the base case solution

A risk analysis for the base case solution is performed using a set of scenarios that quantify/describe the global supply uncertainty. The production of the autoclave in all scenarios will be very close to the forecasted in the deterministic assumption (Figure 12). Although the amount of material sent to the autoclave is very similar in stochastic and deterministic frameworks, the amount of gold sent to the autoclave decreases on average 5% in the first 6 years along the scenarios when compared to the forecasted in the deterministic assumption. This will have a significant impact in the NPV as less gold is processed in the autoclave in the first years. Furthermore, larger fluctuations are observed for SS and SS/CO$_3$ ratio (before adding the acid). The amount of acid required to accommodate the SS/CO$_3$ ratio inside the operational ranges largely exceed the maximum amount in some periods which implies that the solution is not practical (Figure 13).

Figure 14 shows the OC, Au recovered in the autoclave and the risk profile of the NPV. The OC that affects the metallurgical recovery can be seen as a deleterious element. The lower quantity of gold sent to the autoclave and the highest concentration of OC in the scenarios imply a lower amount of gold recovered in the autoclave which translates into a 6% less expected NPV than forecasted in the deterministic framework.

Figure 12: Risk analysis - Production, In-situ Au, SS and SS/CO$_3$ ratio of the autoclave.
Figure 13: Risk analysis - Acid consumption in the autoclave.

Figure 14: Risk analysis - OC, Au recovered in Autoclave and NPV.
3.4 The risk-based solution

The multipit stochastic optimization method is implemented after setting up the optimization parameters. The sequence of extraction for both pits is shown in Figure 15. Pit A produces up to year 10 whereas production ceases in year 7 for the base case solution (Figure 16).

![Figure 15: Risk-based extraction sequences of Pit A (left) and Pit B (right).](image1)

The autoclave operates at full capacity for the risk-based solution. The fluctuations in SS and SS/CO\(_2\) ratios are smaller than for the base case solution. As a result, the probability for increased acid demands exceeding the maximum amount is very small for the different periods, which implies that the blending strategy implemented makes the solution practical.

![Figure 16: Mine production with the risk-based solution.](image2)

The risk-based solution blends material in a way that allows the acid demand to remain feasible. Furthermore, it recovers more gold in the initial years than the base case solution, which generates an expected NPV 10% higher than expected for the base case. The stochastic methodology described in this paper shows its ability to control production rates and blending requirements while improving the expected NPV, thus minimizing the downside risk and maximizing the upside potential of the mining complex.

It is worth mentioning that the risk-based solution considers the same ultimate pits as the deterministic base case solution (Figure 19). Stochastic optimal pit limits do not coincide with deterministic ones (Albor and Dimitrakopoulos, 2009). Ongoing work includes the re-evaluation of the optimal pit limits in stochastic frameworks and the generation of mineable extraction sequences by considering mining phases derived from the risk-based solution.
Figure 17: Production, SS, SS/CO$_3$ ratio and acid consumption with the risk-based solution.

Figure 18: OC, total Au recovered and NPV with the risk-based solution.
This paper describes a method for incorporating the global supply uncertainty in the optimization of mining complexes comprised of multiple pits, ore types, stockpiles, processing destinations and blending requirements. The method generates a risk-based solution by implementing a simulated annealing algorithm with five different perturbation mechanisms: (i) block extraction perturbations; (ii) block destination perturbations; (iii) multipit perturbations; (iv) single reclaim perturbations; and (v) double reclaim perturbations.

The case study outlines the ability of the method to improve the blending of material from multiple sources in a stochastic framework. A risk analysis over a deterministic base case solution is performed. The analysis shows that the demand of acid required to maintain the blending properties in an autoclave within operational ranges will exceed the maximum allowable amount, which emphasizes the importance of accounting for the uncertainty of the raw material. The method described in this paper generates a solution that keeps the demand of acid within the allowable amount. Furthermore, the risk-based solution has an expected NPV 10% higher than the one expected for the base case solution due to the production of more gold, mainly in the initial years of mining.

Ongoing and future work will focus on evaluating the optimal pit and the mineability of the risk-based extraction sequences.

References


