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Abstract: It is an established result in the literature that if the knowledge spillover between firms is sufficiently high, then R&D investments are higher when firms cooperate than when they compete. We show that this result does not necessarily hold true when non-innovating firms are present in the industry.

Keywords: Investments in R&D, cooperation; competition; knowledge spillover

Résumé : La littérature en R&D a établi que si le paramètre mesurant le débordement en connaissance est suffisamment élevé, alors les firmes investissent plus en recherche et développement quand elles coopèrent que quand elles ne coopèrent pas. Nous montrons que ce résultat ne tient pas nécessairement quand l'industrie comporte des firmes non actives en R&D.

Mots clés : Investissements en R&D, coopération, concurrence, débordement de connaissances

1 Introduction

Initiated by the seminal paper in d'Aspremont and Jacquemin (1988) (AJ), a huge literature on investments in research and development (R&D) in oligopoly has developed during the last three decades. The focal question is whether firms' cooperation in R&D leads to higher investments and payoffs than under noncooperation (see Silipo (2008) for a general discussion). The setup/assumptions in this literature can be summarized as follows: (i) The firms are symmetric, produce a homogenous product and are all active in R&D. (ii) The game is played in two stages, with firms deciding, cooperatively or not, their R&D expenditures in the first stage and competing à la Cournot (i.e., choosing their output levels) in the second stage. (iii) R&D investments are process-oriented, i.e., they aim at reducing the production cost. (iv) Each firm leaks part of its knowledge to competitors (the spillover effect) and, similarly, benefits (generally) gratuitously from its competitors' R&D efforts.

In AJ, cooperation meant choosing R&D levels that maximize the firms' joint profits, given the spillover parameter $\beta \in [0, 1]$. In another influential paper, Kamien et al. (1992) considered different R&D arrangements in an industry with N firms defined in terms of coordination (cartelization/joint optimization or competition), information sharing (which refers to the value of β), or both. Kamien et al. (1992) called research joint venture (RJV) the case where the firms maximize knowledge spillover, that is, they set $\beta = 1$. Consequently, the following two-stage games are defined: (i) Cartelization and RJV (Ca, RJV); (ii) Cartelization and no information sharing (Ca, NIS); (iii) Competition and no information sharing (Co, NIS); and finally (iv) Competition and RJV (Co, RJV).

An important result in AJ, Kamien et al. and in the literature that followed, is that for sufficiently high degree of spillover, i.e., $\beta \geq 1/2$, cartelization leads to higher investments in R&D than competition. In this paper, we show that this result does not necessarily hold true when the industry includes firms that are not active in R&D. Ceccagnoli (2005), Ben Abdelaziz et al. (2008) and Ben Brahim et al. (2016) are the only studies to consider such a heterogenous industry. Ceccagnoli (2005) analyzed the impact of the knowledge spillover to non-innovating firms on the incentives of innovating firms to continue their cost-reducing R&D effort. Ben Abdelaziz et al. (2008) obtained the intuitive result that the presence of non-innovating firms (called surfers) in an industry leads to lower individual investments in R&D, to a lower collective level of knowledge and to a higher product price. These differences did not alter the established result that cooperation leads to higher R&D investments than competition. Surprisingly, Ben Brahim et al. (2016) showed that surfers' presence yields higher welfare for some parameter values.

The rest of the paper is structured as follows: In Section 2, we introduce the model. In Section 3, we characterize equilibria in the two scenarios of cooperation and noncooperation in R&D. In Section 4, we compare these equilibria and establish our main result. We briefly conclude in Section 5.

2 The model

As in Ben Abdelaziz (2008) and Ben Brahim et al. (2016), the set of firms $\mathcal{N} = \{1, \ldots, N\}$ is divided into two subsets $\mathcal{I} = \{1, \ldots, I\}$ and $\mathcal{S} = \{1, \ldots, S\}$, with $\mathcal{I} \cap \mathcal{S} = \emptyset$. Subset $\mathcal{I}$ consists of firms that had built in the past a research infrastructure, e.g., laboratory, 3d-printing capacity, database, etc. and are active in R&D, i.e., they have the technical personnel to carry out process R&D. Subset $\mathcal{S}$ is made up of firms that do not possess research facilities and are inactive in R&D. We shall refer to subset $\mathcal{I}$ as the $\mathcal{I}$nvestors (or $\mathcal{I}$nnovators) and to subset $\mathcal{S}$ as the $\mathcal{S}$urfers.

Denote by q_j the output of firm $j \in \mathcal{N}$ and by $Q = \sum_{j=1}^{N} q_j$ the total production. The inverse demand is affine and given by $P(Q) = a - Q$, where $a > 0$. Following the literature, we assume that knowledge is not perfectly appropriable, i.e., a firm that is active in R&D helps (involuntarily) its competitors to also reduce their cost. An investor, as well as a surfer, benefits from all investors' R&D efforts at zero cost. The assumption of zero cost finds support in Arrow (1962) where it is argued that inventing something new is costly, but copying it is costless. However, the two types of firms differ in their capacity to capture others' R&D results. Indeed, we assume, not unrealistically, that by having a research facility, an investor is better placed than a surfer to absorb the knowledge produced by the innovators. Denote by β the spillover rate between innovators, and by γ the rate of knowledge that spills over to a surfer, with $0 \leq \gamma < \beta \leq 1$.

Denote by x_i the investment in R&D by firm $i, i \in \mathcal{I}$, and by X_k the total level of knowledge available to firm $k \in \mathcal{N}$, that is,

$$\begin{array}{rcl} X_i = & x_i + \sum_{j \neq i, j \in I} \beta x_j, & i \in \mathcal{I}, \\ X_s = & \gamma \sum_{i \in I} x_i, & s \in \mathcal{S}. \end{array} \quad (1)$$

The unit production cost function is given by

$$F_k\left(X_k\right) = c - f(X_k), \qquad k \in \mathcal{N},$$

where c is the initial cost satisfying $0 < c < a$. We assume that $f\left(\cdot\right)$ is a (strictly) concave increasing function, satisfying $f\left(0\right) = 0$, and adopt the following functional form:

$$\begin{array}{rcl} F_i\left(\bar{x}\right) = & c - \left(\sqrt{\frac{x_i + \sum_{j \neq i, j \in I} \beta x_j,}{\delta}}\right), & i \in \mathcal{I}, \\ F_s\left(\bar{x}\right) = & c - \left(\sqrt{\frac{\gamma \sum_{i \in I} x_i}{\delta}}\right), & s \in \mathcal{S}, \end{array} \quad (2)$$

where $\bar{x} = \left(x_i\right)_{i \in \mathcal{I}}$ and δ is a positive scaling parameter.

Remark 1 *The strict concavity assumption of $f\left(\cdot\right)$ was also made in, e.g., Kamien et al. (1992) and Amir (2000), and our functional form is from Amir. In d'Aspremont and Jacquemin (1988) as well as in Ben Abdelaziz et al. (2008) and Ben Brahim et al. (2016), which are the closest papers to ours, $f\left(\cdot\right)$ is linear.*

The profit function of firm k reads as follows:

$$\begin{array}{rcl} \pi_i = & \left(a - \sum_{j \in \mathcal{N}} q_j - F_i\left(\bar{x}\right)\right) q_i - x_i, & i \in \mathcal{I}, \\ \pi_s = & \left(a - \sum_{j \in \mathcal{N}} q_j - F_s\left(\bar{x}\right)\right) q_s, & s \in \mathcal{S}. \end{array} \quad (3)$$

As in Kamien et al. (1992), our cost of R&D effort is linear, while in d'Aspremont and Jacquemin (1988), it was quadratic. Qualitatively speaking, a linear cost and concave knowledge efficiency is equivalent to quadratic (convex) cost and linear efficiency.

3 Equilibria

We characterize the equilibria in the two scenarios, where the innovators compete or cooperate in the R&D stage. In both cases, all firms in the industry compete in the product market. To derive a subgame-perfect Nash equilibrium, we solve the two-stage game backward.

Introduce the following notation:

$$\begin{array}{rcl} K & = & \dfrac{N - \beta(I-1)}{\sqrt{1 + \beta(I-1)}} - \dfrac{\gamma S}{\sqrt{\gamma I}}, \\ L & = & (S+1)\sqrt{1 + \beta(I-1)} - S\sqrt{\gamma I}, \\ M & = & \dfrac{\sqrt{\gamma I}\left(N\beta - I + 1\right) - \gamma\left(N - I\right)\sqrt{1 + \beta(I-1)}}{\sqrt{1 + \beta(I-1)}\sqrt{\gamma I}}, \\ R & = & K + (I-1)M. \end{array}$$

The following proposition recalls the result from Kamien et al. (1992) that gives the second-stage output as function of knowledge acquired in the first stage of the game.

Proposition 1 *(Kamien et al. (1992)). The equilibrium output of firm $k \in \mathcal{N}$ is given by*

$$q_k^{nc}(X) = \frac{a - c + f(X_k) - \left(\sum_{j \neq k} f(X_j) - f(X_i)\right)}{N+1},$$

where $X = (X_k)_{k \in \mathcal{N}}$.

The next two propositions characterize the unique equilibria in the two scenarios.

Proposition 2 *Assuming an interior solution, the unique symmetric subgame-perfect equilibrium strategies when innovators do not cooperate in R&D are given by*

$$\sqrt{x_i^{nc}} = \frac{\sqrt{\delta}(a-c)K}{\delta(N+1)^2 - KL}, \; i \in \mathcal{I}, \tag{4}$$

$$q_i^{nc} = \frac{(a-c)\left((\delta(N+1)^2 - KL) + K((S+1)\sqrt{1+\beta(I-1)} - S\sqrt{\gamma I})\right)}{(\delta(N+1)^2 - KL)(N+1)}, \quad i \in \mathcal{I}, \tag{5}$$

$$q_s^{nc} = \frac{(a-c)\left((\delta(N+1)^2 - KL) + K\left(N\sqrt{\gamma I} - (I-1)\sqrt{1+\beta(I-1)}\right)\right)}{(\delta(N+1)^2 - KL)(N+1)}, \quad s \in \mathcal{S}. \tag{6}$$

Proof. See Appendix. □

Proposition 3 *Assuming an interior solution, the unique symmetric subgame-perfect equilibrium strategies when the innovators cooperate in R&D are given by*

$$\sqrt{x^c} = \frac{\sqrt{\delta}(a-c)R}{\delta(N+1)^2 - RL}, \quad i \in \mathcal{I}, \tag{7}$$

$$q_i^c = \frac{(a-c)\left((\delta(N+1)^2 - RL) + R\left((S+1)\sqrt{1+\beta(I-1)} - S\sqrt{\gamma I}\right)\right)}{(\delta(N+1)^2 - RL)(N+1)}, \quad i \in \mathcal{I}, \tag{8}$$

$$q_s^c = \frac{(a-c)\left((\delta(N+1)^2 - RL) + R\left(N\sqrt{\gamma I} - (I-1)\sqrt{1+\beta(I-1)}\right)\right)}{(\delta(N+1)^2 - RL)(N+1)}, \quad s \in \mathcal{S}. \tag{9}$$

Proof. See Appendix. □

4 Main result

The difference between the interior solution investments in R&D under the two scenarios is given by

$$\Delta = \sqrt{x_i^{nc}} - \sqrt{x^c} = -\left(\frac{\sqrt{\delta}(a-c)\left(\delta(N+1)^2\right)(I-1)}{(\delta(N+1)^2 - KL)(\delta(N+1)^2 - RL)}\right) f(\beta),$$

where

$$f(\beta) = M = \frac{\sqrt{\gamma I}\,(N\beta - I + 1) - \gamma(N - I)\sqrt{1+\beta(I-1)}}{\sqrt{1+\beta(I-1)}\sqrt{\gamma I}}.$$

Our objective is to prove the following:

Proposition 4 *Under the assumption of interior cooperative and noncooperative equilibria, there exists at least one $\beta > 0.5$ such that $\Delta > 0$.*

To show our main result, we need the following:

Lemma 1 *The assumption of interior solutions implies*

$$\delta(N+1)^2 - KL > 0 \text{ and } \delta(N+1)^2 - RL > 0.$$

Proof. As $a > c$, to prove the result it suffices to show that K and R are positive, which is the case. Indeed,

$$\begin{aligned}
K &= \frac{N-\beta(I-1)}{\sqrt{1+\beta(I-1)}} - \frac{\gamma S}{\sqrt{\gamma I}} = \frac{\sqrt{I}\left(N-\beta(I-1)\right) - \sqrt{\gamma}S\sqrt{1+\beta(I-1)}}{\sqrt{1+\beta(I-1)}\sqrt{I}}, \\
&> \frac{\sqrt{I}\left(N-\beta I)\right) - \sqrt{\gamma}S\sqrt{1+\beta(I-1)}}{\sqrt{1+\beta(I-1)}\sqrt{I}} > \frac{\sqrt{I}\left(N-\beta I)\right) - \sqrt{\gamma}S\sqrt{1+\beta I}}{\sqrt{1+\beta(I-1)}\sqrt{I}}, \\
&> \frac{\sqrt{I}\left(N-I)\right) - \sqrt{\gamma}S\sqrt{1+\beta I}}{\sqrt{1+\beta(I-1)}\sqrt{I}} = \frac{\sqrt{I}S - \sqrt{\gamma}S\sqrt{1+\beta I}}{\sqrt{1+\beta(I-1)}\sqrt{I}} > \frac{S\sqrt{I}\left(1-\sqrt{\gamma}\sqrt{\beta}\right)}{\sqrt{1+\beta(I-1)}\sqrt{I}} > 0.
\end{aligned}$$

$$\begin{aligned}
R &= K + (I-1)M \\
&= \frac{(1-\beta)\left(N+I-1\right) + \beta I\left(N-1\right) + \beta)}{\sqrt{1+\beta(I-1)}} > 0.
\end{aligned}$$

□

Consequently, under the assumption of interior solutions, for Δ to be positive, we need to have $f(\beta)$ is negative. We have the following:

Lemma 2 *For* $\beta \in [0,1]$*,* $f(\beta)$ *is strictly convex increasing, and therefore achieves its minimum at* $\beta = 0$*, and its maximum at* $\beta = 1$*.*

Proof. It suffices to compute the first two derivatives

$$\begin{aligned}
f'(\beta) &= \frac{\left(2N\left(1-\beta\right) + \beta NI + I\left(I-1\right)\right)}{2\sqrt{1+\beta(I-1)}\left(1+\beta(I-1)\right)\gamma I} > 0, \quad \forall \beta \in [0,1], \\
f''(\beta) &= -\frac{\sqrt{1+\beta(I-1)}}{\gamma I}\frac{\left(4N\left(1-\beta\right) + 4NI + 3I^2\left(I-1\right) + N\beta I^2\right)}{4\left(1+\beta(I-1)\right)\left(1+\beta(I-1)\right)^2} < 0, \quad \forall \beta \in [0,1],
\end{aligned}$$

to get the result. □

The minimum and maximum values are given by

$$\begin{aligned}
f(0) &= \frac{\sqrt{I}\left(-I+1\right) - \sqrt{\gamma}\left(N-I\right)}{\sqrt{I}} < 0, \\
f(1) &= \frac{\left(N-I\right)\left(1-\sqrt{\gamma}\right) + 1}{\sqrt{I}} > 0.
\end{aligned}$$

By continuity of $f(\beta)$, there must exist a value $\tilde{\beta} \in [0,1]$ satisfying $f\left(\tilde{\beta}\right) = 0$, i.e.,

$$f\left(\tilde{\beta}\right) = \frac{\sqrt{\gamma I}\left(N\tilde{\beta} - I + 1\right) - \gamma\left(N-I\right)\sqrt{1+\tilde{\beta}(I-1)}}{\sqrt{1+\tilde{\beta}(I-1)}\sqrt{\gamma I}} = 0, \tag{10}$$

which is equivalent to have

$$\sqrt{\gamma I}\left(N\tilde{\beta} - I + 1\right) = \gamma\left(N-I\right)\sqrt{1+\tilde{\beta}(I-1)}. \tag{11}$$

Table 1: Values of $\tilde{\beta}^{+}$ for different combinations of γ and I

γ	Number of innovators								
	2	3	4	5	6	7	8	9	10
0.05	0.24	0.32	0.40	0.49	0.57	0.66	0.74	0.82	0.9
0.10	0.30	0.37	0.44	0.52	0.60	0.68	0.76	0.83	0.9
0.15	0.36	0.41	0.48	0.56	0.63	0.70	0.77	0.84	0.9
0.20	0.40	0.45	0.51	0.58	0.65	0.72	0.78	0.84	0.9
0.25	0.44	0.48	0.54	0.60	0.67	0.73	0.79	0.85	0.9
0.3	0.48	0.52	0.57	0.63	0.69	0.75	0.80	0.85	0.9

Squaring both sides of the above equation, we obtain

$$N^2 I\tilde{\beta}^2 - \tilde{\beta}\,(I-1)\left(2IN + \gamma\,(N-I)^2\right) + I\,(I-1)^2 - \gamma\,(N-I)^2 = 0.$$

The roots of the above second-degree polynomial are given by

$$\tilde{\beta} = \frac{(I-1)\left(2IN + \gamma\,(N-I)^2\right) \pm (N-I)\sqrt{\gamma}\sqrt{(I-1)^2\left(\gamma\,(N-I)^2 + 4IN\right) + 4N^2 I}}{2N^2 I}.$$

The right-hand side of (11) is positive, and therefore we must select the root that yields a positive left-hand side of (11). It is easy to verify that $\tilde{\beta}^{+}$ is the root that satisfies this requirement, i.e.,

$$\tilde{\beta}^{+} = \frac{(I-1)\left(2IN + \gamma\,(N-I)^2\right) + (N-I)\sqrt{\gamma}\sqrt{(I-1)^2\left(\gamma\,(N-I)^2 + 4IN\right) + 4N^2 I}}{2N^2 I}. \tag{12}$$

To prove Proposition 4, it suffices to show that there exists one value of $\tilde{\beta}^{+}$ larger than 0.5 such that $\Delta > 0$. Table 1 gives some examples of such values for different γ and I, for $N = 10$. The results in Table 1 can be summarized in one sentence: the larger the spillover to surfers, or the number of innovators, the larger the interval $\left[\frac{1}{2}, \tilde{\beta}^{+}\right]$ of values β for which non-cooperation in R&D yields larger investments than cooperation.

The value $\tilde{\beta}^{+}$ depends on three parameters, namely, γ, N and I, with $\gamma < \beta$ and $I \leq N$. It is immediate to see from (12) that $\tilde{\beta}^{+}$ is increasing in γ and hence its minimum, with respect to this parameter, is achieved at $\gamma = 0$, with

$$\tilde{\beta}^{+}_{\gamma=0} = \frac{I-1}{N},$$

which implies

$$\tilde{\beta}^{+}_{\gamma=0} \geq \frac{1}{2} \Leftrightarrow I \geq \frac{N+2}{2}. \tag{13}$$

The above condition, and the result that $\tilde{\beta}^{+}$ is increasing in γ, leads to the following statement: If the number of innovators in the industry is sufficiently high, then non-cooperative investments in R&D exceed their cooperative counterparts. Further, the higher the spillover to surfers, the lower the threshold required on I for the result to hold. In d'Aspremont and Jacquemin (1988), where $N = 2$, the condition in (13) is satisfied only when both firms are active.

5 Conclusion

Considering a heterogeneous industry with both innovating and non innovating firms, we showed that the statement in the R&D literature that cooperation leads to higher investments for $\beta > \frac{1}{2}$ does not necessarily hold in our setting. The composition of the industry, that is, the number of innovators and surfers, as well as the spillover γ to non-innovating firms play a role in ranking cooperative and non-cooperative investments. A high spillover among innovators is an incentive to cooperate, but the presence of surfers changes in fact the game and reduces this incentive.

Appendix 1: Proof of Proposition 2

The payoff function of firm i is given by

$$\pi_i = q_i^2 - x_i,$$

and the first-order equilibrium condition is given by:

$$\frac{\partial \pi_i}{\partial x_i} = 0, \tag{14}$$

$$2q_i \frac{\partial q_i}{\partial x_i} - 1 = 0. \tag{15}$$

Using the result in Kamien et al. (1992), the optimal output expression is given by

$$q_i = \frac{a - c + Nf(X_i) - \sum_{i \in I} f(X_j) - \sum_{i \in S} f(X_s)}{N+1},$$

and we then have

$$\frac{\partial q_i}{\partial x_i} = \frac{1}{N+1}\left(Nf(X_i) - \sum_{i\in I} f(X_j) - \sum_{i \in S} f(X_s)\right), \tag{16}$$

where

$$f'(X_i) = \frac{1}{2\sqrt{\delta X_i}}, \tag{17}$$

$$f'(X_j) = \frac{\beta}{2\sqrt{\delta X_j}}, \tag{18}$$

$$f'(X_s) = \frac{\gamma}{2\sqrt{\delta X_s}} \tag{19}$$

Under symmetry, we get

$$\frac{\partial q_i}{\partial x_i} = \frac{1}{2\sqrt{\delta}(N+1)}\left(\frac{N}{\sqrt{X_i}} - \frac{\beta(I-1)}{\sqrt{X_j}} - \frac{\gamma S}{\sqrt{X_s}}\right), \tag{20}$$

and consequently the optimal output level is given by

$$q_i = \frac{a - c + (S+1)f(X) - Sf(X_s)}{N+1}, \tag{21}$$

$$q_i = \frac{\sqrt{\delta}(a-c) + (S+1)\sqrt{1+\beta(I-1)x} - S\sqrt{\gamma I x}}{\sqrt{\delta}(N+1)}. \tag{22}$$

To have the optimal R&D investment level, we can write the first-order equilibrium condition as follows:

$$2\left(\frac{\sqrt{\delta}(a-c) + (S+1)\sqrt{1+\beta(I-1)x} - S\sqrt{\gamma I x}}{\sqrt{\delta}(N+1)}\right) \times \tag{23}$$

$$\frac{1}{2\sqrt{\delta}(N+1)}\left(\frac{N}{\sqrt{X_i}} - \frac{\beta(I-1)}{\sqrt{X_j}} - \frac{\gamma S}{\sqrt{X_s}}\right) = 1 \tag{24}$$

$$2\left(\frac{\sqrt{\delta}(a-c) + (S+1)\sqrt{1+\beta(I-1)x} - S\sqrt{\gamma I x}}{\sqrt{\delta}(N+1)}\right) \times \tag{25}$$

$$\frac{1}{2\sqrt{\delta x}(N+1)}\left(\frac{N - \beta(I-1)}{\sqrt{1+\beta(I-1}} - \frac{S\gamma}{\sqrt{\gamma I}}\right) = 1 \tag{26}$$

Let us denote by:

$$\frac{N - \beta(I-1)}{\sqrt{1+\beta(I-1)}} - \frac{S\gamma}{\sqrt{\gamma I}} = K$$

and

$$(S+1)\sqrt{1+\beta(I-1)} - S\sqrt{\gamma I} = L$$

Finally, the optimal non cooperative R&D investment level is given by:

$$\sqrt{x^n} = \frac{\sqrt{\delta}(a-c)K}{\delta(N+1)^2 - KL},$$

Appendix 2: Proof of Proposition 3

In R&D cooperation, innovators coordinate their R&D investment decisions maximizing their joint payoffs as follows:

$$\frac{\partial \sum_{i\in I}\pi_i}{\partial x_i} = 0 \tag{27}$$

$$\frac{\partial \pi_i}{\partial x_i} + \frac{\partial \sum_{j\neq i, j\in I}\pi_j}{\partial x_i} = 0 \tag{28}$$

The first order condition is given by:

$$\frac{2q_i(\bar{x})}{N+1}\left(-N\frac{\partial F_i(\bar{x})}{\partial x_i} + \sum_{j\in\mathcal{I}, j\neq i,}\frac{\partial F_j(\bar{x})}{\partial x_i} + \sum_{s\in\mathcal{S}}\frac{\partial F_s(\bar{x})}{\partial x_i}\right)$$
$$+\sum_{j\in\mathcal{I}, j\neq i}\frac{2q_j(\bar{x})}{N+1}\left(-N\frac{\partial F_j(\bar{x})}{\partial x_i} + \sum_{j\in\mathcal{I}, j\neq i,}\frac{\partial F_j(\bar{x})}{\partial x_i} + \sum_{s\in\mathcal{S}}\frac{\partial F_s(\bar{x})}{\partial x_i}\right) - 1 = 0,\ i\in I.$$

In the symmetric case it becomes:

$$2q^*\left(\frac{\partial q_i}{\partial x_i} + (I-1)\frac{\partial q_j}{\partial x_i}\right) = 1 \tag{29}$$

$$\tag{30}$$

The first order condition is given in this case by:

$$\frac{1}{\delta(N+1)^2}[\sqrt{\delta}(a-c) + L\sqrt{x}][\frac{K}{\sqrt{x}} + (I-1)\frac{M}{\sqrt{x}}] = 1$$

where M is given by:

$$M = \frac{N\beta - (I-1)}{\sqrt{1+\beta(I-1)}} - \frac{S\gamma}{\sqrt{\gamma I}}$$

Assuming an interior solution, the unique symmetric subgame-perfect equilibrium when the innovators cooperate in the first stage is given by

$$\sqrt{x^c} = \frac{\sqrt{\delta}(a-c)R}{\delta(N+1)^2 - RL},$$

where

$$R = K + (I-1)M$$

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