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# Design of a REDD mechanism with application to Madagascar

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**Abstract:** In this paper, we design a REDD mechanism (Reduction of Emissions from Deforestation and forests Degradation) to help a region in Madagascar manage its tropical forest in a sustainable way. To do so, we use viability theory and devise an algorithm to compute viable solutions. Our results highlight some conditions which are required in order to reach a sustainable policy. These conditions are stated in terms of population growth rate as well as economic and environmental development.

**Key Words:** REDD mechanism, viability theory, forest exploitation, sustainability, Madagascar.

**Résumé:** Dans cet article, nous montrons les conditions d'efficacité d'un mécanisme financier de type REDD+ (initiative internationale pour la réduction des émissions liées à la déforestation et à la dégradation des forêts dans les pays en développement) pour aider une région de Madagascar à gérer sa forêt tropicale de manière durable. Pour cela, notre méthodologie repose sur la théorie de la viabilité et sur un algorithme permettant de trouver des solutions viables. Nos résultats mettent en évidence certaines conditions nécessaires pour une gestion durable de cette forêt. Ces conditions sont énoncées en fonction du taux de croissance de la population et de développement économique et environnemental.

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# 1 Introduction

According to Metz (2007) and van der Werf et al. (2009), 17% of greenhouse gas (GHG) emissions are due to deforestation and forest degradation. Avoiding deforestation, especially in developing countries, is therefore an issue that has retained a considerable attention of scientists and decision makers. In 2005, the United Nations Framework Convention on Climate Change (UNFCCC) set up a program of financial compensation for the Reduction of Emissions from Deforestation and forests Degradation (REDD), whose objective is to give a financial compensation for avoided deforestation. Some of the current issues surrounding this mechanism are the definition of a baseline deforestation rate to measure “avoided deforestation” (Busch et al., 2009), as well as the design of a transfer that is efficient (van Soest and Lensink, 2000) and ensures equity between heterogeneous countries (Cattaneo et al., 2010).

In this paper, we design a financial transfer intended to help a region in Madagascar to manage its tropical forest in a sustainable fashion. These financial transfers can be conditional or unconditional. In the latter, the support is a lump sum given by a donor community to a forestry country to reduce its deforestation rate. Among many other authors, Fredj et al. (2006) argue that an unconditional transfer is inefficient as it does not provide the right incentive to the recipient country to reduce its deforestation rate. A variety of conditional mechanisms have been proposed in the literature with the transfer being dependent on alternative use of deforested areas. For instance, Bellassen and Gitz (2008) use carbon credits for compensating farmers in Cameroon for conservation of primary land forest instead of cultivating this land, whereas Lu and Liu (2013) use a spatial model to compare what farmers can get from a REDD+ mechanism to revenues from palm oil in a region in Indonesia. Following Ollivier (2012), we propose a transfer based on current forest area and on the difference between the deforestation rate and a historical baseline. To implement this proposal, we need to evaluate the price per preserved forest unit, which we will do using viability theory (see Aubin, 1991).

Viability theory is particularly well-suited to study sustainable exploitation of renewable resources such as forests, fisheries and environment; see, e.g., Aubin et al. (2011), Aubin et al. (2005), Béné and Doyen (2008), Doyen et al. (2007), Martinet and Blanchard (2009), Martinet and Doyen (2007). An interesting feature of this theory is that it quite naturally allows for the integration of economic, ecological, and social dimensions of sustainability in the form of a series of constraints. To summarize, the main ingredients of a viability problem are a dynamical system describing the evolution over time of some state variables of interest (e.g., forest land, population) as function of some control variables (e.g., deforestation rate, monetary transfers) and a constraint set describing what is sustainable. Solving a viability problem amounts to determining the trajectories of the state variables that always satisfy the constraints. In the specific context of forests, examples of applications of viability theory include Rapaport et al. (2006), who studied sustainability of an aged-structure forest, Domenech et al. (2011), who considered the sustainability of the world’s forest, Bernard and Martin (2013), and Andrès-Domenech et al. (2014), who focused on some areas in Madagascar.

In this paper, we use viability theory to determine sustainable exploitation policies for the forest of Haute-Matsiatra, a region in the south-east of Madagascar. This region is known for its endemic flora and fauna, and, as in many other regions of Madagascar, aid programs are needed to help conserve its rainforest.

In a preliminary study,<sup>1</sup> an evaluation of carbon sequestered by forest area has been carried out in view of the implementation of a REDD mechanism. Here, we focus on the design of such a mechanism and its associated policies to reduce deforestation while ensuring the economic development of the region. We take stock of the model proposed in Bernard and Martin (2013), where monetary transfers were unconditional, with the implication that there is no guarantee that these transfers will indeed lead to a reduction in the deforestation rate. In this paper, we propose to link the payment to the preserved forest area and to the deforestation rate. As REDD does not have the vocation to exist forever, but is seen as transitional tool, we also explore alternative policies based on wages increase and productivity enhancement associated to REDD mechanism.

<sup>1</sup>[https://madagascar.helvetas.org/fr/projects\\_a\\_madagascar/ancien\\_projects222/projet\\_foreca/](https://madagascar.helvetas.org/fr/projects_a_madagascar/ancien_projects222/projet_foreca/)

The rest of the paper is organized as follows: Section 2 introduces the model, and Section 3 describes the algorithm that is used to obtain viable solutions. In Section 4, we consider different plausible scenarios and discuss the results. Section 5 briefly concludes.

## 2 Model

Bernard and Martin (2013) proposed an unconditional monetary transfer in the aim of ensuring the coexistence of a preserved forest area and the fulfilment of the population needs in a region in the south east of Madagascar. More specifically, the area of interest lies in the forest corridor of Fianarantsoa, an area that is well-known for its rain-forest and its endemic fauna and flora. The subsistence of the population depends mainly upon the local rice-growing industry, and given the high rate of population growth, there is a continuous pressure to deforest and cultivate new fields. Depending on the season, the workforce moves from one side to the other of the corridor of Fianarantsoa.

We reconsider the same model in Bernard and Martin (2013), with two important changes, namely: (i) that the monetary transfer will be conditional to the deforestation rate as well as to the forest area, and designed to satisfy some ecological and economic constraints to be defined below; and (ii) that the population is not a given parameter but evolves according to a logistic curve. One of the outputs of our model is the minimal monetary transfer in a REDD program that allows the satisfaction of the ecological and economic constraints at each period of time. As stated above, we will also consider that some important parameters could change over time, particularly salaries and land yields, which may change as a result of enhanced crop productivity.

The corridor of Fianarantsoa (CF) economic, environmental and demographic system is described by three state variables, namely, the agriculture land  $S$ , the population living in the area  $P$ , and the physical capital  $K$ . Time is discrete and denoted by  $t \in \mathbb{N}$ .

**Agricultural land dynamics:** The total land surface is assumed to be shared between a forest area and an agricultural area. Denote by  $\rho(t)$  the proportion of active population working outside the agricultural sector, and by  $1 - \rho(t)$  the proportion of farmers. We assume that  $\rho(t) \in [\rho_{min}, \rho_{max}]$ , where  $\rho_{min}$  and  $\rho_{max}$  are the minimum and maximum proportion of the population that does not work in agriculture. We denote by  $\delta(t) \in [0, \delta_{max}]$ , the per-capita deforested land, measured in hectare (ha), at time  $t$ . The evolution of the agricultural area is given by the following difference equation:

$$S(t+1) = S(t) + \delta(t)(1 - \rho(t))P(t), \quad S(0) \text{ given.} \quad (1)$$

The proportion of outside-agriculture workers  $\rho(t)$  and the effort for developing new agricultural area  $\delta(t)$  are the control (decision) variables. At each time, the decision maker can select any feasible values and check if they are part of a sustainable solution. As there is no reforestation effort in our model, the only way to sustain a given size of the forest at any time  $\tau$  onward, is to impose  $\delta(t) = 0$  for all  $t \geq \tau$ .

**Population dynamics:** We suppose that the population  $P$  grows according to the logistic curve

$$P(t+1) = P(t) + r \cdot P(t) \cdot \left(1 - \frac{P(t)}{P_{car}}\right), \quad P(0) \text{ given} \quad (2)$$

where  $P_{car}$  is the carrying capacity<sup>2</sup> and  $r$  the yearly growth rate. We shall analyze two scenarios for population growth, namely, a low growth rate of 0.009, and a business-as-usual scenario in which the (rural Malagasy) population grows at the current rate  $r = 0.031$ .<sup>3</sup>

<sup>2</sup>Following data from the INSTAT, we fix the carrying capacity  $P_{car} = 1,260,000$ , which is the maximal population observed in the corridor of Fianarantsoa during the last twenty years.

<sup>3</sup>Data from the last census of the Malagasy population in 1993.

**Capital dynamics:** The total capital available evolves according to the following dynamics:

$$\begin{aligned} K(t+1) = & K(t) - cP(t) - \beta\delta(t)(1 - \rho(t))P(t) \\ & + \omega\rho(t)P(t) \\ & + pe \cdot \min(S(t), \gamma(1 - \rho(t))P(t)), \quad K(0) \text{ given.} \end{aligned}$$

The constant  $c$  corresponds to the per-capita consumption expressed in US\$. Hence, the term  $cP(t)$  is the total consumption expenditures. The cost of adding a hectare to agricultural land is given by  $\beta$  (US\$/ha). This cost includes eventually chopping trees, clearing land, developing an irrigation system and labor cost. The corresponding yearly total cost is given by  $\beta\delta(t)(1 - \rho(t))P(t)$ . On the revenue side, we have two inputs. The first one is the revenue earned by the active population working outside the agriculture sector, that is,  $\omega\rho(t)P(t)$ , where  $\omega$  is the average earned wage (US\$/capita/year). The total revenue from agriculture activities depend on three items, namely, the price of the crop  $p$  (rice in US\$/kg), the productivity of the land denoted  $e$  and expressed in kg/ha/year, and finally the cultivated surface. Recall that the total available surface for farming is  $S$ . Now, it may be the case that not all this surface is exploited. Denote by  $\gamma$  the per-capita exploited area, which implies a total cultivated area of  $\gamma(1 - \rho(t))P(t)$ . Consequently, the revenues from agriculture activities in US\$ per year are given by the last term in the above equations, namely,  $pe \cdot \min(S(t), \gamma(1 - \rho(t))P(t))$ .

When managing this system, that is, choosing the values of the control variables, the policy maker (regulator, government, etc.) may impose a series of constraints to satisfy some social, economic and environmental objectives. The set of these constraints define what is then meant by a sustainable system. In our context, we follow Bernard and Martin (2013) and retain the following four constraints:

**Ecological constraint:** As forests act as carbon sinks and play an important role in biodiversity conservation, we require the forest surface to never be below the level  $F_{min}$ . In terms of cultivated area, this translates into the constraint

$$S(t) \leq F_0 - F_{min}, \quad (3)$$

where  $F_0$  is a (historical) reference forest area measured in ha.

**Economic constraints:** We impose two constraints on per-capita available capital  $k(t)$ , namely: (i) it must be larger or equal to a lower bound  $k_{min}$ , that is,

$$k_{min} \leq k(t) = \frac{K(t)}{P(t)}, \quad (4)$$

and (ii) it does not decrease over time, i.e.,

$$k(t+1) \geq k(t), \quad (5)$$

where  $k_{min}$  is given in US\$/capita/year.

**Population constraint:** We suppose that the system is viable (or sustainable) when the active population satisfy the constraint

$$P_{min} \leq P(t) \leq P_{max}, \quad (6)$$

where  $P_{min} = 500,000$  and  $P_{max} = 1,255,300$ . Note that this constraint is independent of the control variables. Exceeding the upper bound could be interpreted as a level where food safety would become a serious issue. The lower bound is simply a historical statistical figure.

**Transfer mechanism:** Recall that a REDD mechanism is an incentive payment from developed countries to developing countries to help them to curb tropical deforestation rate and to reduce CO<sub>2</sub> emissions from deforestation and forest degradation. Usually, the current deforestation rate is compared to a baseline (or reference level) deforestation rate, which is determined by the recipient country. Although this approach is intuitive, basing a transfer only on the progress of the deforestation rate may not provide sufficient incentive to countries having historically a low deforestation level. Indeed, in this case the difference between the

historical and actual levels is too small, and it may be too costly to reduce the deforestation rate. To correct for this potential bias, we follow Ollivier (2012) and propose to also include the forest area in the transfer payment. We believe that a payment that accounts for both the forest area and the deforestation rate will be attractive to countries that did not deforest much during the last years and deter countries from deforesting a lot before joining a REDD project.

Denote by  $d_{bas}$  the baseline deforestation rate, by  $R$  the price of carbon sequestered by a hectare of tropical forest, and by  $\theta(t)$  the transfer payment per hectare of preserved forested land at time  $t$ . We assume that the actual size of the forest is measured by the difference between the historical reference value  $F_0$  and the current agriculture surface  $S(t)$ . Recalling that the current deforestation rate is given by  $\delta(t)(1 - \rho(t))P(t)$ , then the relevant difference in deforestation is measured by  $d_{bas} - \delta(t)(1 - \rho(t))P(t)$ . Consequently, the monetary transfer at time  $t$  is given by

$$\tau(t) = \max \left( 0, \theta(t) \cdot (F_0 - S(t)) + R \cdot (d_{bas} - \delta(t)(1 - \rho(t))P(t)) \right). \quad (7)$$

In the above formula, the price of carbon  $R$  is a given parameter by, e.g., carbon trading, while  $\theta$  is a control variable.

## 2.1 Sustainable management

To wrap up, our three-dimensional controlled dynamic system can be represented as follows:

$$(\mathcal{X}) \quad \begin{cases} S(t+1) &= S(t) + \delta(t)(1 - \rho(t))P(t), \\ P(t+1) &= P(t) + r \cdot P(t) \cdot \left(1 - \frac{P(t)}{P_{car}}\right), \\ K(t+1) &= K(t) - cP(t) - \beta\delta(t)(1 - \rho(t))P(t) \\ &\quad + \omega\rho(t)P(t) + pe \cdot \min(S(t), \gamma(1 - \rho(t))P(t)), \\ \text{with } u &\triangleq [\delta; \rho; \theta] \in U, \end{cases} \quad (8)$$

where  $U$  is the feasible set of controls defined by

$$U = [0, \delta_{max}] \times [\rho_{min}, \rho_{max}] \times [\theta_{min}, \theta_{max}]. \quad (9)$$

The *constraint set*  $\mathcal{K}$  is given by

$$\mathcal{K} = \left\{ [S(t); P(t); K(t)] \text{ such that } \begin{array}{l} S(t) \leq F_0 - F_{min}, \\ k_{min} \leq \frac{K(t)}{P(t)}, \\ k(t) \leq k(t+1), \\ P_{min} \leq P(t) \leq P_{max} \end{array} \right\}. \quad (10)$$

Put differently, a sustainable (or viable) trajectory is a triplet  $(S(t), P(t), K(t))$  that satisfies the ecological, economic and demographic constraints. Starting from any given initial values of the state variables, there may be many, one or no evolutions that satisfy these constraints during the planning horizon, which can be finite or infinite. Those trajectories that satisfy the constraints are called *viable evolutions*, and the set of all viable evolutions is defined by

$$\mathcal{S} = \left\{ [S(\cdot); P(\cdot); K(\cdot)] \mid \forall t > 0, \exists [\delta(t); \rho(t); \theta(t)] \text{ such that } [S(t); P(t); K(t)] \in \mathcal{K} \right\}. \quad (11)$$

In words, the set  $\mathcal{S}$  contains all possible (starting) values of the state variables for which we can find at least one path of the control variables such that the resulting trajectories remain in the set  $\mathcal{K}$ , that is, satisfy the constraints during the whole planning horizon.

If there is more than one viable evolution, then the management has to select one, or a subset of them, based on some additional criteria or objectives. In our illustrative numerical simulations, we shall retain two criteria, namely:



- **Criteria 1:** Largest forest area at each time.
- **Criteria 2:** Lowest deforestation effort.

In case of several viable trajectories, we will apply first Criteria 1 and next Criteria 2. We note that in all simulations, these criteria were sufficient to end up with only one viable trajectory. For all the simulations performed in Section 4, the parameter values, which are taken from Bernard and Martin (2013), are given in Table 1.

Table 1: Parameter values

Definition	Notation	Unit	Value
Consumption	$\underline{c}$	US\$/capita/year	200
Cost of developing agriculture land	$\beta$	US\$/ha	25
Rice price	$p$	US\$/kg	0.30
Rice productivity	$e$	kg/ha/year	1,000
Cultivated area per capita	$\gamma$	ha/capita	0.46
Agricultural wage	$\omega$	US\$/capita/year	200
Maximal effort of development	$\delta_{max}$	ha/capita/year	0.008
Reference forest area	$F_0$	ha	110,000
Lower bound on forest area	$F_{min}$	ha	65,000
Lower bound on capital	$k_{min}$	US\$/capita	800
Lower bound on population	$P_{min}$	capita	500,000
Upper bound on population	$P_{max}$	capita	1,255,300

### 3 Computation

Viable trajectories are found and selected by using an optimal control algorithm. The dynamic system can be expressed as  $x_{t+1} = f(x_t, u_t)$ , where  $x_t$  is the state vector and  $u_t$  is control vector at time  $t$ . For algorithmic purposes, it is practical to normalize the controls in  $\mathcal{U} = \{u \in \mathbb{R}^3, \|u\|_\infty \leq 1\}$ . The original control variables  $[\delta_t; \rho_t; \theta_t]$  can be computed from  $u_t = [u_t^\delta; u_t^\rho; u_t^\theta]$  by setting:

$$\delta_t = \frac{\delta_{max} + \delta_{min}}{2} + \frac{\delta_{max} - \delta_{min}}{2} u_t^\delta, \quad (12)$$

and similarly for  $\rho_t$  and  $\theta_t$ . A trajectory is defined as a succession of  $t_{max}$  commands in  $\mathcal{U}$  and is noted  $\mathbf{U} = [u_0; u_1; \dots; u_{t_{max}-1}] \in \mathcal{U}^{t_{max}}$ . As in most optimal control algorithms, a Cartesian sampling  $\mathcal{U}^S \subset \mathcal{U}$  is defined such that  $[0; 0; 0] \in \mathcal{U}^S$ . This sampling is used to perturb the selected trajectory  $\mathbf{U}$ .

The state space is partitioned into cells. At each time step, only one particle will be selected in each cell, which ensures that the number of particles remaining at each time step is bounded by the number of cells. Thus, the number of model calls at each time step is bounded by  $\text{card}(\mathcal{U}^S) \times \text{number of cells}$ .

The algorithm is divided into the following steps:

1. The trajectory  $\mathbf{U}$  is initialized at 0, which means that  $u_t = [u_t^\delta; u_t^\rho; u_t^\theta] = [0; 0; 0], \forall t$ . The parameter  $\lambda$  that controls the magnitude of the perturbations applied to  $\mathbf{U}$  is initialized at 1.
2. The set of particles is initialized with a single particle at the starting point  $x_0$ .
3. At each time step  $t$ , each particle is propagated by evaluating all the controls in  $\{u, u = u_t + \lambda v, v \in \mathcal{U}^S, u \in \mathcal{U}\}$ . All the particles that are not inside the acceptable state space are disregarded as well as the particles that do not satisfy the constraint set defined in Eq.(10).
4. In each cell of the state space, a single particle is selected according to the previously defined criteria.
5.  $t \leftarrow t + 1$ . If  $t < t_{max}$ , go to step 3 and continue the propagation.
6. If  $t = t_{max}$ , a single particle is selected amongst all the particles in the whole state space. The new selected trajectory  $\mathbf{U}$  is computed by backward programming from this particle.
7. If the number of refinements of the trajectory is not reached, go to step 2 and refine the trajectory  $\mathbf{U}$  by re-propagating it from the start with  $\lambda \leftarrow \lambda/2$ .

An illustration of this algorithm is given in Figure 1 for  $x \in \mathbb{R}$ ,  $u \in \mathbb{R}$ . In this illustration, the state space is divided into 3 cells and  $\mathcal{U}^S = \{-1; 0; +1\}$ . In the REED mechanism problem, the state space is divided in  $75^3 = 421,875$  cells, and  $\text{card}(\mathcal{U}^S) = 15^3 = 3,375$ . The trajectory is propagated 3 times.

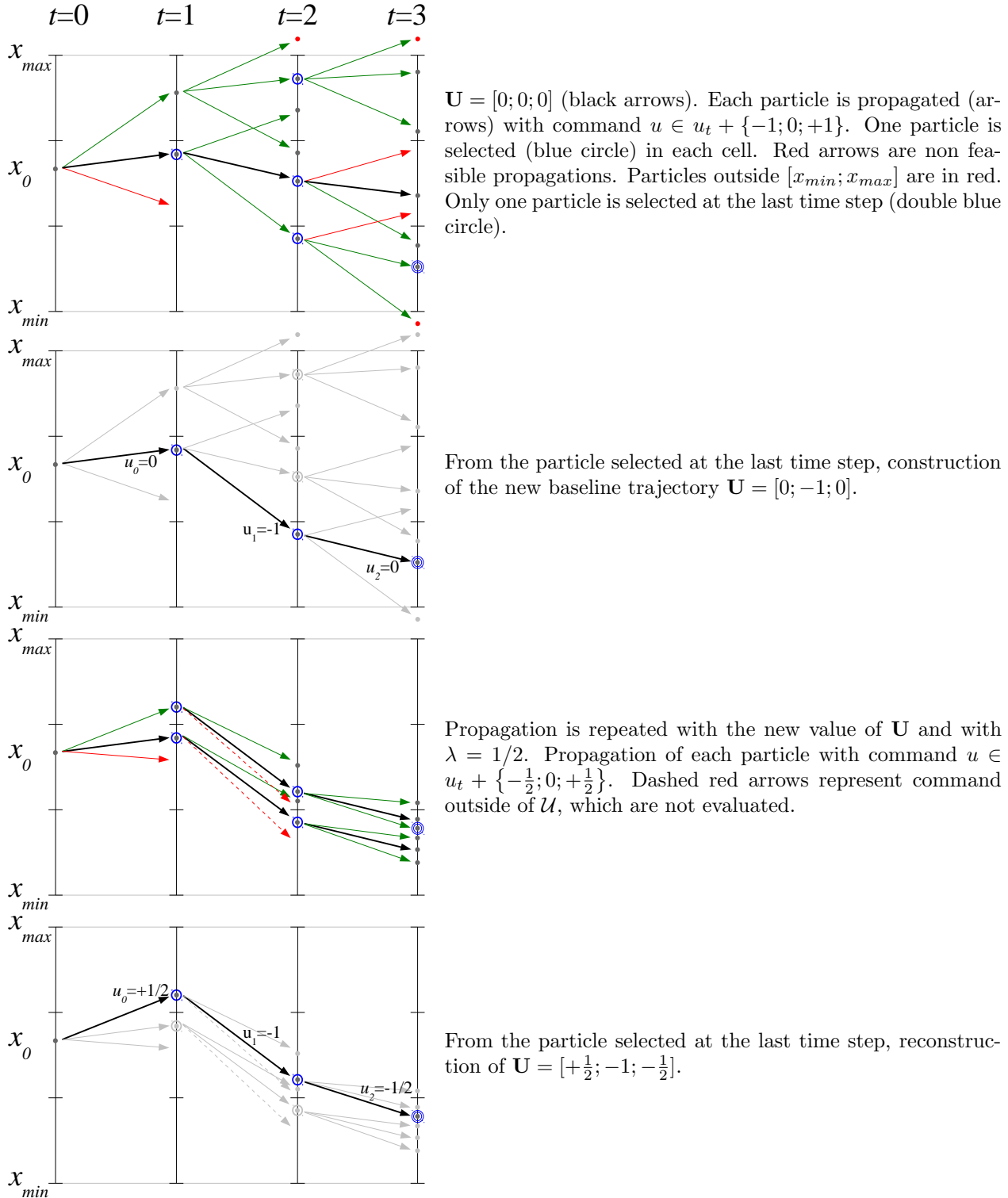


Figure 1: Illustration of the propagation algorithm

## 4 Scenarios

We focus on possible evolutions starting from the current state of the system. Further, it is more interesting to present the results per capita rather than for total population. For this, we denote by  $k$  the *per capita* capital and by  $s$  the *per capita* cultivated area. The current state is given by the triplet  $[s; P; k] = [0.033; 10^6; 800]$ .

We shall consider five different scenarios, and in each case run simulations with and without monetary transfers (or REDD mechanism). The trajectories of the state variables start from current state and are computed for a time horizon of 105 years.

### 4.1 Laissez-faire scenario

In this *laissez-faire* or benchmark scenario, we do not impose any constraints, i.e., the population can deforest at will, and there is no minimal capital requirement. Further, we suppose that the population will continue growing at its current rate of 0.031 during the whole time horizon. The results are presented in Figure 2. The *per capita* capital is negative during the whole period, and all the forest land is deforested, with the cultivated area stabilizing after 30 years. The development of new agriculture land is at its maximal during the first half of the horizon, and becomes zero afterwards as there is no more forest to clear. The proportion of outside-agriculture workers is stable.

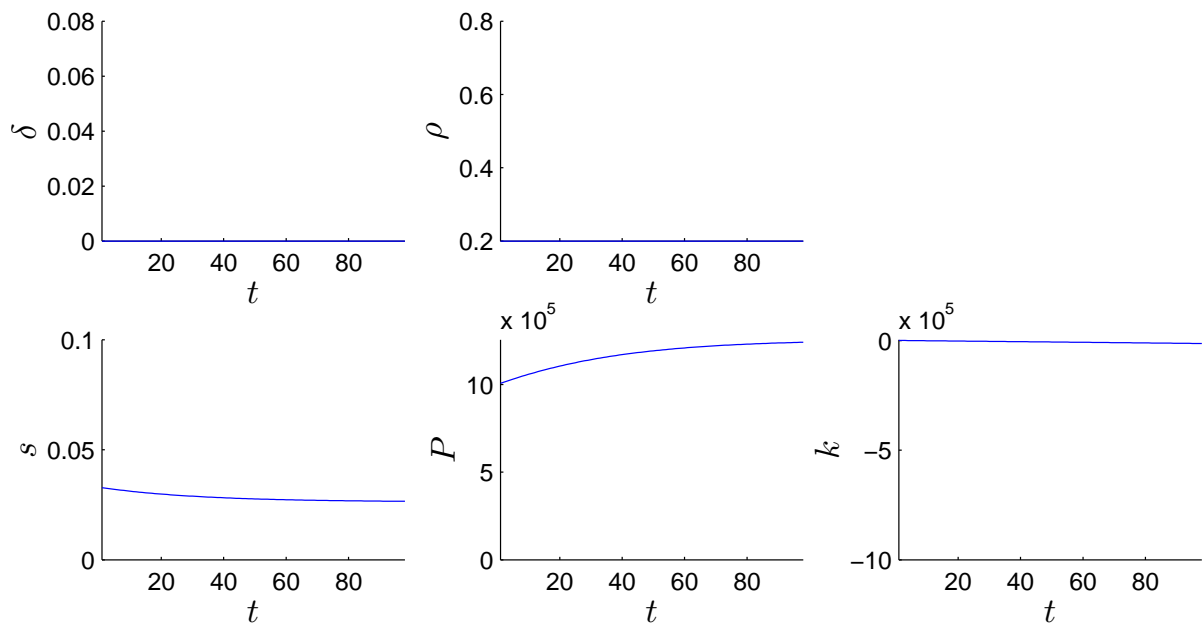


Figure 2: Evolution of the current situation in the "laissez-faire" scenario.

Given the outcome, a natural question is whether monetary transfers can slow deforestation. To check for this, we considered a REDD mechanism in the form discussed in Section 2. Recall that one part of the payment is proportional to the difference between the baseline deforestation rate and the current one, and the other part depends on the forest area scaled by a coefficient  $\theta$ , which is considered as a control variable taking values between  $\theta_{min} = 100$  and  $\theta_{max} = 350$ :

$$\tau(t) = \max\left(0, \theta \cdot (F_0 - S(t)) + R \cdot (d_{bas} - \delta(t)(1 - \rho(t))P(t))\right). \quad (13)$$

The results, which are summarized in Figure 3, show that adding no monetary transfer within the acceptable range can lead to a positive *per capita* capital.

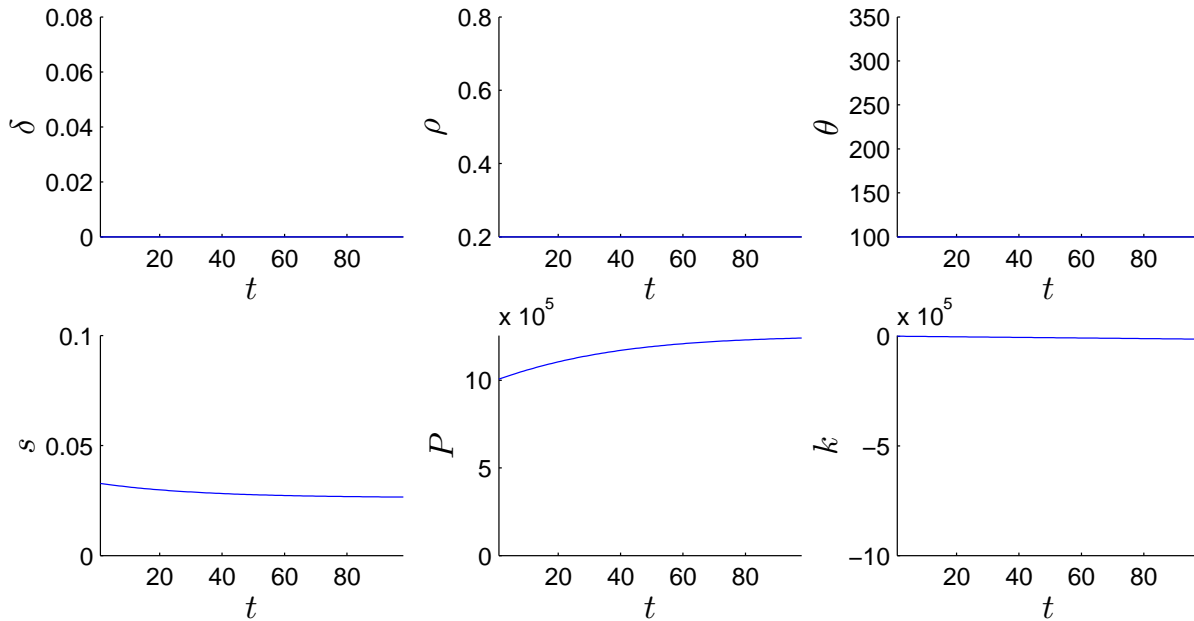


Figure 3: Evolution of the current situation in the "laissez-faire" scenario with the addition of controlled REDD payment.

## 4.2 Ecological and economic constraints

In this scenario, we check if sustainable development can be achieved if we add the constraints imposing that the agricultural area *per capita* cannot exceed  $s_{max} = 0.08$  and that the *per capita* capital cannot be lower than  $k_{min} = 800$ .

First, we run a series of simulations assuming that the current growth population rate ( $r = 0.031$ ) will be in force during the whole planning horizon. If monetary transfers are not on the menu, then we obtain the same grim result as above, namely, we do not find a viable trajectory that starts from the current situation. In particular, the *per capita* capital is less than  $k_{min}$ . Adding a monetary transfer per hectare of preserved forest, that is, a  $\theta \in [110, 350]$ , does not help.

Second, we did the same exercise, assuming however a lower population growth rate, namely,  $r = 0.009$ , which corresponds to the rate observed in developed countries. With and without monetary transfers, we reached the same result as before, that is, the set of viable evolutions is empty.

Given these results, we explore in the next scenarios the impact on the sustainability of adding a wage increase and/or a crop productivity enhancement.

## 4.3 Higher wages

Up to now, we have assumed that the wage (also named defray) in agriculture is \$200. Does an increase of this figure to \$220 lead to a viable solution? Our findings are as follows:

1. With and without REDD payments, there is no viable solution if the population continues growing at its current rate of 0.031 during the retained planning horizon.
2. Without REDD transfers, having a low population growth rate of 0.009 is not sufficient to reach a viable solution.
3. If the wage is \$220 and the population growth is low ( $r = 0.009$ ), then there exists at least one viable solution provided monetary transfers are offered. As these transfers depend on the control variable  $\theta \in [110, 350]$ , we can determine the lowest value that is necessary to induce viability at each period.

By doing so, we would have in fact designed a REDD mechanism. The results in Figure 4 show that a viable solution requires an increasing  $\theta$  over time, starting from \$200 per hectare of preserved forest and ending at \$230.

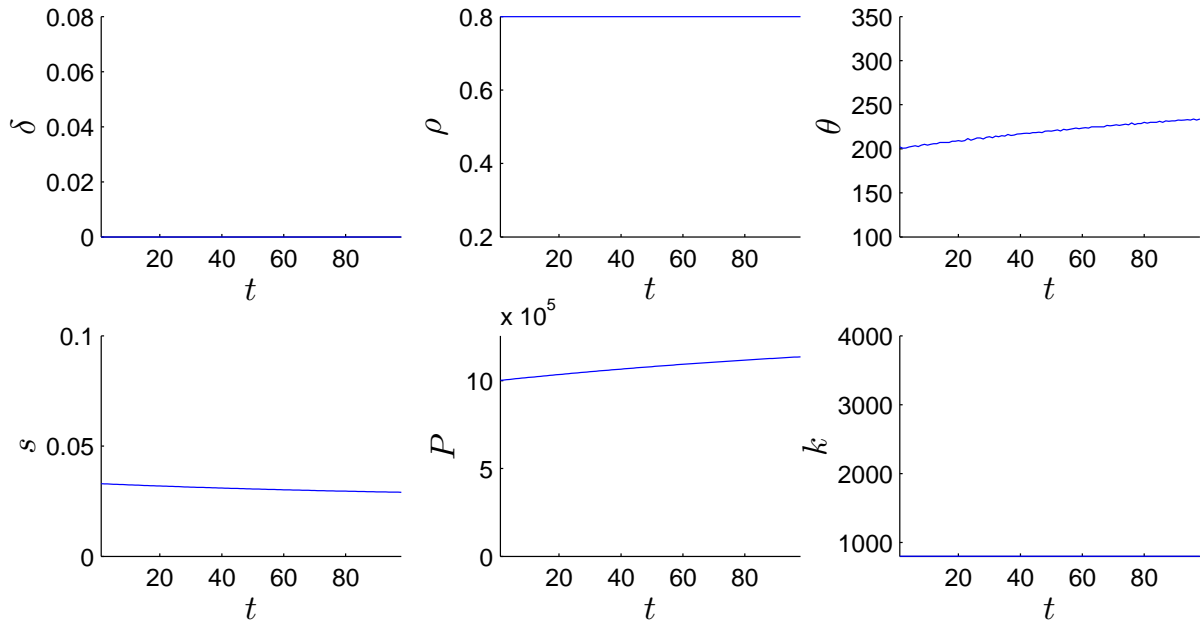


Figure 4: Evolution from current situation with monetary transfers, low population growth rate ( $r = 0.009$ ) and higher wage ( $\omega = 220\$$ ).

#### 4.4 Higher crop productivity

We reconsider the scenario with ecological and economic constraints, that is, the scenario with  $s_{max} = 0.08$  and  $k_{min} = 800$ , with an increase, however, in crop productivity. In our benchmark (see Appendix) the rice productivity  $e$  is 1,000 kg/ha/year. We tested different values, but present only the results with  $e = 1500$ , which corresponds to an increase of productivity by 50%.<sup>4</sup> Our results are as follows:

1. If the population continues growing at current rate of 0.031, then there is no viable solution even with monetary transfers.
2. If the population grows at the same rate as in developed countries, that is,  $r = 0.009$ , and there is no REDD mechanism, then again starting from the current situation, there is no viable trajectory.
3. If the population growth rate is low ( $r = 0.009$ ) and a REDD mechanism is implemented, then there exists at least one viable solution. The results are exhibited in Figure 5. Note that the value of  $\theta$  is around \$300, which is significantly higher than in the previous scenario with higher wages.

#### 4.5 Higher wages and higher crop productivity

In this last scenario, we couple the two previous ones by considering a wage increase to \$220 and crop productivity enhancement to 1,500 kg/ha/year. We obtain that at the current population growth rate, there is no viable evolution unless a REDD mechanism is implemented. Figure 6 summarizes the results. Note that in this scenario, the value of  $\theta$  is around 190.

<sup>4</sup>In the studied region of Madagascar, productivity can be improved. The different value tested for rice productivity are inspired from Tsujimoto et al. (2009)

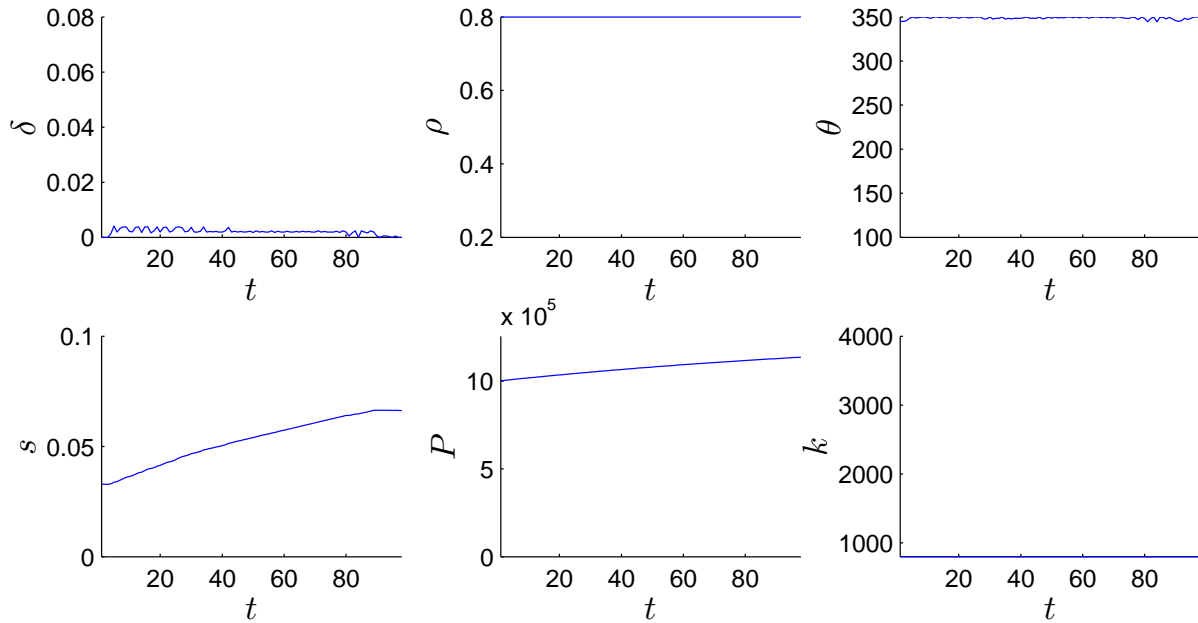


Figure 5: Evolution of the current situation in scenario 4 with monetary transfer and growth rate  $r = 0.09$  in the case of a crop productivity enhancement equal to  $e = 1500$ .

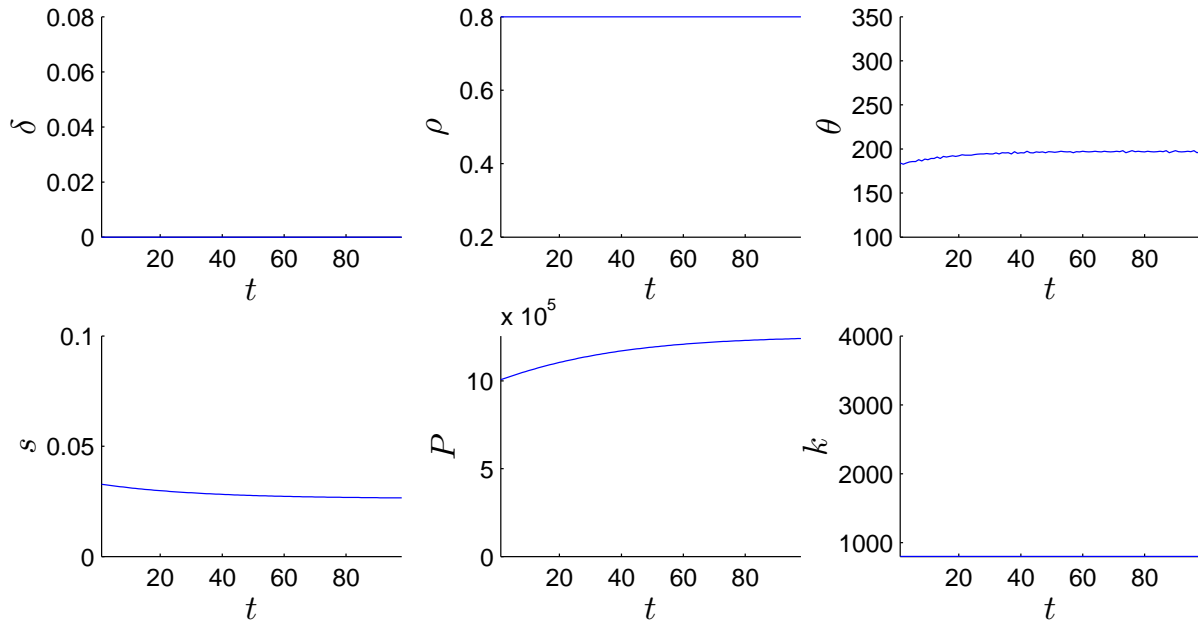


Figure 6: Evolution of the current situation in scenario 5 with monetary transfer and growth rate  $r = 0.09$  in the case of a wage increase equal to  $\omega = 200$  and a crop productivity enhancement equal to  $e = 1500$ .

## 5 Conclusion

In this paper, we explored different scenarios for a sustainable exploitation of the forest of Haute-Matsiatra in Madagascar. A first conclusion is that a laissez-faire scenario is not viable. A second is that a REDD mechanism and either an increase in wage and/or better crop productivity is required to reach a viable solution. The solution also depends on the rate of population growth.

The main contribution of this study is twofold. First, it gives decision makers in Madagascar (government and international agencies) the tools to assess different policies in their search for a sustainable development

of the forest in this region, and possibly other regions in the country that share similar characteristics. Second, using viability theory, we provide a new approach for designing a REDD mechanism, which does not need to be fully defined a priori. Put differently, the control or design parameter values will depend on certain features and policies involved in the process. To illustrate, the transfers may vary with the expected population growth rate (at least indirectly), with economic conditions (e.g., wages) and other elements such as crop productivity. Of course, the payments are also related to the forest's size, and not given as a lump sum.

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