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Abstract: Commercial piracy and counterfeiting are widespread phenomena in different businesses, ranging from software and video games to luxury fashion products. The International Chamber of Commerce estimates that \$650 billion in counterfeit goods were sold in 2008 and that the cost of lost tax revenues due to counterfeit goods was \$125 billion in developed countries alone.

Starting from the point of view that piracy cannot be deterred, due to, e.g., the absence of a concrete action by strong institutions, we model the problem as a dynamic game involving a legal producer and a pirate. To allow for the fact that pirate's entry onto the market occurs with a given delay after the launch of the product by the firm, our game involves two subperiods, namely, before and after entry. Each player controls his own retail price and advertising budget. We characterize the equilibrium pricing and advertising strategies of the two players, and assess the impact of the pirate's entry date on these strategies. Further, by contrasting the results of the scenarios with and without counterfeiting, we determine the conditions under which the presence of the illegal producer is beneficial to consumers and (even) to the legal firm.

Key Words: Counterfeiting; Pricing; Advertising; Dynamic Games.

Résumé : Le piratage commercial et la contrefaçon sont des phénomènes répandus dans différents secteurs d'activités, allant de logiciels et jeux vidéo à des produits de mode de luxe. La Chambre de commerce internationale estime à 650 milliards de dollars les ventes de marchandises de contrefaçon en 2008 et le coût des recettes fiscales perdues à 125 milliards de dollars seulement dans les pays développés.

Partant du point de vue que la piraterie ne peut être dissuadée, en raison par exemple d'absence d'une action concrète par des institutions fortes, nous modélisons le problème comme un jeu dynamique impliquant une entreprise (légale) et un pirate. Pour tenir compte du fait que l'entrée de pirate sur le marché s'effectue avec un certain retard donné après le lancement du produit par l'entreprise, notre jeu comporte deux sous-périodes, à savoir, avant et après l'entrée. Chaque joueur contrôle son propre prix de détail et le budget de publicité. Nous caractérisons le prix d'équilibre et les stratégies publicitaires des deux joueurs, et évaluons l'impact de la date d'entrée du pirate sur ces stratégies. En outre, en comparant les résultats des scénarios avec et sans contrefaçon, nous déterminons les conditions dans lesquelles la présence du producteur illégal est bénéfique pour les consommateurs et (même) à l'entreprise légale.

Mots clés : Contrefaçon; prix ; publicité; jeux dynamiques.

1 Introduction

This paper is about competition between the producer of a good or service and a counterfeiter with the technical and commercial ability to provide the same product or service at a lower price. Counterfeiting is defined as the act of producing or selling a product under a sham brand that is an intentional and calculated reproduction of a genuine brand, while piracy is described as the exact, unauthorized and illegal reproduction on a commercial scale of a copyrighted work or trademarked product. Moreover, Cordell, Wongtada and Kieschnick in [3] give this definition: "Any unauthorized manufacturing of goods whose special characteristics are protected as intellectual property rights (trademarks, patents and copyrights) constitutes product counterfeiting." In the context of fashion design, the so-called "fast-fashion" producers, which are agents that are able to replicate original designs at a high speed, on a large scale and at a low cost, represent a sizeable threat for fashion houses ([6]). The international trade in counterfeit products is estimated to exceed six per cent of global trade. It is not only damaging to business and to investment opportunities but it also negatively impacts on society and on the global economy. The International Chamber of Commerce[1] estimates that $650 billion in counterfeit goods were sold in 2008 and that the cost of lost tax revenues due to counterfeit goods was $125 billion in developed countries alone. Furthermore, 2.5 million jobs have been lost as a result of fake products. By 2015, the ICC expects the value of counterfeit goods worldwide to exceed $1.7 trillion, which is more than 2% of the world's total current economic output. Jeffrey Hardy, advisory group coordinator for the anti-counterfeiting program at ICC observed[2] that, "The whole business has just exploded. And it goes way beyond music and Gucci bags."

Owners of intellectual property must take action to protect their rights against such forms of piracy. A number of anti-counterfeiting strategies are recommended by numerous researchers, such as aggressively cutting prices, providing financial incentives to distributors so they will reject counterfeits, and advertising to consumers the harmful effects of fake goods [2]. Shultz and Saporito in [14] offer ten anti-counterfeiting strategies, among them, advertising as a tool to differentiate real products from phony ones, pricing to influence demand and finally, involvement in coalitions with organizations that have similar intellectual property rights (IPR) interests. Few papers have used game-theoretic models to analyze competition and the possibility of cooperation between players in the context of counterfeiting. Wong (see [17]) and Posner (see [12]) proposed static games to evaluate the impact of greater legal protection on the incentives to fashion designers of filing lawsuits to protect their designers, and those to fast-fashion firms of making replicas of these designs.

In this paper, we study the strategic interactions between a firm and a pirate in a dynamic framework. The dynamic feature is crucial to capture important phenomena such as brand goodwill (reputation or brand equity) and the counterfeiter's date of entry onto the market. Our starting point is that although the pirate is stealing part of the market from the legal producer, it may actually be helping to enlarge the market through its advertising. In that sense, some similarities can be found in the literature dealing with software piracy, where network externality effects are present. The interested reader may refer to Belleflamme and Peitz in [1], for a comprehensive survey of digital piracy.

Our differential-game model accounts for pricing and advertising interactions between the two players, namely, the legal producer and the counterfeiter. We assume that the counterfeiter's date of entry onto the market is known and we assess its impact on the firm's advertising and pricing strategies. The goodwill dynamics are à la Nerlove-Arrow (see Huang et al. [8] for a recent survey of differential games in advertising). Whereas the legal producer may use different means to advertise the product, it is most likely that the illegal one will use the Internet. As pointed out by Warschauer [16], e-commerce is the easiest way for a pirate to advertise a fake product. We assume that the legal demand negatively depends on price, whereas the illegal demand increases with the legal price. We focus on one selling season and retain a finite-time horizon divided into two sub-periods, namely, before and after counterfeiter entry onto the market. We characterize the optimal solutions in two settings, i.e., a case where there is no illegal demand and a case that involves a pirate. By contrasting the two scenarios, we will be able to quantify the impact of counterfeiting on the

[1] http://www.iccwbo.org/Advocacy-Codes-and-Rules/BASCAP/Library/BASCAP-publications-A-Z/

[2] http://money.cnn.com/2012/09/27/news/economy/counterfeit-goods/index.html

firm's prices and advertising strategies, as well as its effect on profits and consumer. In a nutshell, our paper aims to answer the following research questions:

1. How does the pirate entry's date affect the first-period equilibrium strategies?
2. Under which conditions is the presence of an illegal producer beneficial to consumers?
3. Under which conditions, if any, does piracy benefit the legal producer?

The rest of the paper is organized as follows: In Section 2, we introduce the model; in Section 3, we characterize the optimal pricing and advertising strategies for the legal producer in the absence of piracy. In Section 4, we determine the advertising and price-equilibrium strategies in the scenario where a counterfeiter enters the market at a given date. In Section 5, we compare the two scenarios, and in Section 6, we conclude.

2 Model

We refer by ℓ to the firm that legally markets the product and by i to the illegal producer (pirate). Time t is continuous and the planning horizon is T. The firm produces at unit cost c, which we set equal to zero for simplicity. A positive unit cost would most likely only have a purely quantitative impact on our results.[3] The pirate is a "fast-fashion" producer, that is, an agent with the capacity to copy the product at a very low cost, also set equal to zero, who then markets it after a certain delay $\mathcal{D}$ at a price[4] $m(t)$. The delay may depend on the product's complexity, the availability of production capacity, etc. The assumption that the delay is exogenous is essentially made for tractability. Note however that we will conduct a sensitivity analysis to assess the impact of $\mathcal{D}$ on the equilibrium strategies and payoffs.

Denote by $p(t)$ the price of the legal producer at time t, and by $G(t)$ the goodwill of the product. (Think of goodwill as the brand equity or reputation of the firm.) The demand system reads as follows:

$$\begin{aligned} d_{\ell 1} &= d_{\ell 1}(p, G), & t &\in [0, \mathcal{D}], \\ d_{\ell 2} &= d_{\ell 2}(p, m, G), & t &\in (\mathcal{D}, T], \\ d_i &= d_i(p, m, G), & t &\in (\mathcal{D}, T], \end{aligned}$$

with

$$\begin{aligned} &\frac{\partial d_{\ell 1}}{\partial p} < 0, && \frac{\partial d_{\ell 1}}{\partial G} > 0, && \\ &\frac{\partial d_{\ell 2}}{\partial p} < 0, && \frac{\partial d_{\ell 2}}{\partial m} > 0, && \frac{\partial d_{\ell 2}}{\partial G} > 0, \\ &\frac{\partial d_i}{\partial p} > 0, && \frac{\partial d_i}{\partial m} < 0, && \frac{\partial d_i}{\partial G} > 0, \end{aligned}$$

that is, both legal and illegal demands are increasing in the product's goodwill, and each player's demand is decreasing in its own price and increasing in its rival's.

To keep the model tractable, we assume the following forms for the demands, which clearly satisfy the above properties:

$$\begin{aligned} d_{\ell 1}(t) &= (\alpha - \beta p(t)) G(t), & t &\in [0, \mathcal{D}], \\ d_{\ell 2}(t) &= (\alpha - \beta p(t) + \varphi(m(t) - p(t))) G(t), & t &\in (\mathcal{D}, T], \\ d_i(t) &= \varphi(p(t) - m(t)) G(t), & t &\in (\mathcal{D}, T], \end{aligned}$$

where α, β and φ are non-negative parameters, with $\beta \geq \varphi$, that is, the own-price effect is larger than the cross-price effect. Note that the second-period demands include a price-differential term (as it does in, e.g.,

[3] The assumption of zero cost is of course much more justified in the case of software production, where the cost is negligible, than in the case of fashion counterfeiting.

[4] Under the assumption of zero production costs for the two players, their retail prices could also be interpreted as margins.

Eliashberg and Jeuland[5][4]) and Martín-Herrán et al. [10]), and that prices interact multiplicatively with the goodwill value. It is as if the temptation to buy the illegal product is proportional to the price gap and to the extent to which the product is famous (measured here by the goodwill). In particular, if $m(t) \geq p(t)$, then we assume that the illegal demand vanishes, meaning that (some) consumers are willing to buy the fake product only if it is cheaper than the legal product; otherwise, they prefer the legal product. This is reminiscent to consumers' intrinsic preference for the legal product.

Remark 1 *Although the total demand during the second period is given by*

$$d_{\ell 2}(t) + d_i(t) = d_2(t) = (\alpha - \beta p(t))\, G(t), \qquad t \in (\mathcal{D}, T],$$

one should not conclude that the parameter φ does not play any role here. As we will see, the equilibrium prices will be a function of φ, and consequently, the total demand will also depend on this parameter. Also, clearly, the legal firm's price $p(t)$ will take different values during the two periods.

We suppose that the goodwill increases with the information provided about the product in different media by the firm and the pirate. To keep the terminology compact, we shall generically refer to this information as advertising, and denote by $a_j(t)$ the advertising effort of player $j \in \{\ell, i\}$, at time $t \in [0, T]$. The evolution of the goodwill (continuous state variable) is governed by the following linear-differential equation:

$$\begin{aligned} \dot{G}(t) &= \begin{cases} \gamma_\ell a_\ell(t), & t \in [0, \mathcal{D}], \\ \gamma_\ell a_\ell(t) + \gamma_i a_i(t), & t \in (\mathcal{D}, T], \end{cases} \\ G(0) &= G_0, \end{aligned} \tag{1}$$

where $\gamma_j, j \in \{\ell, i\}$, are non-negative parameters measuring the efficiency of advertising, G_0 is the initial goodwill of the product, and $G(\mathcal{D})$ is the goodwill value at entry time $\mathcal{D}$. We assume that the advertising costs are convex increasing, and given by the following quadratic functions:

$$C_j(a_j) = \frac{\kappa_j}{2} a_j^2, \quad j \in \{\ell, i\}.$$

As the pirate's advertising effort is (possibly) mainly done by e-mailing, we expect κ_i to be much lower than κ_ℓ, but we do not make a precise assumption for the moment.

Assuming profit-maximization behavior, the players' optimization problems are given by

$$\begin{aligned} J_\ell &= \max_{p(t), a_\ell(t)} \left[\int_0^{\mathcal{D}} \left(p(t)(\alpha - \beta p(t))\, G(t) - \frac{\kappa_\ell}{2} a_\ell^2(t) \right) dt \, + \right. \\ &\qquad \left. \int_{\mathcal{D}}^T \left(p(t)(\alpha - \beta p(t) + \varphi(m(t) - p(t)))\, G(t) - \frac{\kappa_\ell}{2} a_\ell^2(t) \right) dt \right], \end{aligned} \tag{2}$$

$$J_i = \max_{m(t), a_i(t)} \int_{\mathcal{D}}^T \left(m(t)\, \varphi(p(t) - m(t))\, G(t) - \frac{\kappa_i}{2} a_i^2(t) \right) dt, \tag{3}$$

subject to the dynamics in (1).

We shall characterize and compare the solutions in the following two scenarios:

No piracy: In this scenario, the only demand is legal, and the firm acts as a monopolist and optimizes its profit given by

$$J = \max_{p(t), a(t)} \int_0^T \left(p(t)(\alpha - \beta p(t))\, G(t) - \frac{\kappa_\ell}{2} a^2(t) \right) dt,$$

with the goodwill dynamics and demand given by

[5]The setting in Eliashberg and Jeuland (1986) is similar to ours in the sense that the time horizon is divided into two subperiods; in the first, there is a monopolist selling a new (durable) product, and in the second, another firm enters the market.

$$\begin{aligned}\dot{G}(t) &= \gamma_\ell a(t),\\ G(0) &= G_0,\\ d_\ell(t) &= (\alpha_\ell - \beta_\ell p(t))\, G(t).\end{aligned}$$

The optimal solution will be superscripted with $\mathcal{N}$ (for $\mathcal{N}$o piracy). This is our benchmark scenario, which could correspond to, e.g., situations where the product life cycle is very short and $\mathcal{D} \geq T$, or where the law is efficient in deterring piracy.

Piracy: In this case, the pirate enters the market at time $\mathcal{D}$. The pirate and the firm play the finite-horizon differential game described by (2)-(3) and (1). An open-loop Nash equilibrium will be sought, and the optimal state and strategy will be superscripted with $\mathcal{C}$ (for $\mathcal{C}$ounterfeiting).

By comparing the outcomes of the two scenarios, we will be able to measure the impact of piracy on the firm's profit and on consumer. We delete from now on the time argument when no ambiguity may arise.

3 No Piracy

We characterize in this section the optimal pricing and advertising policies in the absence of piracy. The following proposition summarizes our findings.

Proposition 2 *Assuming an interior solution, the optimal price and advertising are given by*

$$p^{\mathcal{N}}(t) = \frac{\alpha}{2\beta}, \tag{4}$$

$$a^{\mathcal{N}}(t) = \frac{\alpha^2\gamma_\ell}{4\beta\kappa_\ell}(T-t). \tag{5}$$

The goodwill trajectory is given by

$$G^{\mathcal{N}}(t) = G_0 + \frac{\alpha^2\gamma_\ell^2}{8\beta\kappa_\ell}(2T-t)\,t, \tag{6}$$

and the total outcome by

$$J^{\mathcal{N}} = \frac{\alpha^2}{4\beta}T\left(G_0 + \frac{\alpha^2\gamma_\ell^2 T^2}{24\beta\kappa_\ell}\right). \tag{7}$$

Proof. See Appendix. □

We first note that the optimal solution is indeed interior. Next, we make the following observations: (i) Advertising is monotonically decreasing over time and equal to zero at T. This is expected in view of the finite-horizon assumption and absence of a salvage function valuing the goodwill stock at T. The optimal advertising policy is obtained by equating the marginal cost of advertising ($\kappa_\ell a$) to the marginal revenue given by the shadow price of goodwill multiplied by the marginal effectiveness of advertising, that is, $\lambda\gamma_\ell$. (ii) The price is constant over time, which is due to its absence in the state dynamics. Further, it is increasing in the market potential α and decreasing in the consumer sensitivity parameter β. (iii) The total profit is increasing in α, γ_ℓ, G_0 and T, and decreasing in the cost parameter κ_ℓ and β. In summary, the results in this benchmark scenario are fully intuitive.

4 Game with Piracy

In this section, we assume that a counterfeiter has the technical ability to produce and to market a fake product after a certain delay $\mathcal{D} \in (0, T]$. The problem can therefore be divided into two parts: a first period, where the firm is a monopolist; and a second period, where it competes with the pirate. To solve the game, we proceed backward and start by computing the Nash equilibrium in the second period, assuming that $G_{\mathcal{D}}$

is the value of state variable at date $\mathcal{D}$. This allows us to characterize the legal producer's optimal payoff as a function of the state value $G_{\mathcal{D}}$; we denote this quantity by $J_{\ell 2}^{\mathcal{C}}(G_{\mathcal{D}})$. Next, we solve the overall optimization problem

$$\max_{p_1(t),a_{\ell 1}(t)} \left[\int_0^{\mathcal{D}} \left(p_1(t)(\alpha - \beta p_1(t)) G_1(t) - \frac{\kappa_\ell}{2} a_{\ell 1}^2(t) \right) dt + J_{\ell 2}^{\mathcal{C}}(G_1(\mathcal{D})) \right]. \tag{8}$$

where the subscript 1 stands for first period, while the subscript 2 stands for second period. The following proposition characterizes the equilibrium strategies during the duopoly period, and describes the analytical form of the term $J_{\ell 2}^{\mathcal{C}}(G_{\mathcal{D}})$ for all admissible values $G_{\mathcal{D}}$ of the state variable at time $\mathcal{D}$.

Proposition 3 *Assuming an interior solution, the second-period price and advertising Nash-equilibrium strategies are given by*

$$p_2^{\mathcal{C}}(t) = \frac{2\alpha}{4\beta + 3\varphi}, \tag{9}$$

$$m^{\mathcal{C}}(t) = \frac{\alpha}{4\beta + 3\varphi}, \tag{10}$$

$$a_{\ell 2}^{\mathcal{C}}(t) = \frac{4\alpha^2(\beta+\varphi)}{(4\beta+3\varphi)^2} \frac{\gamma_\ell}{\kappa_\ell}(T-t), \tag{11}$$

$$a_i^{\mathcal{C}}(t) = \frac{\alpha^2\varphi}{(4\beta+3\varphi)^2} \frac{\gamma_i}{\kappa_i}(T-t). \tag{12}$$

The goodwill trajectory and total second-period payoffs are as follows:

$$\begin{aligned}
G_2^{\mathcal{C}}(t) &= G_{\mathcal{D}} + \frac{\alpha^2}{2(4\beta+3\varphi)^2}\left(\frac{4(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)(t-\mathcal{D})(2T-t-\mathcal{D}),\\
J_{\ell 2}^{\mathcal{C}}(G_{\mathcal{D}}) &= \frac{4\alpha^2(\beta+\varphi)(T-\mathcal{D})}{(4\beta+3\varphi)^2}\left(G_{\mathcal{D}} + \frac{\alpha^2(T-\mathcal{D})^2}{3(4\beta+3\varphi)^2}\left(\frac{2(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)\right),\\
J_i^{\mathcal{C}} &= \frac{\varphi\alpha^2(T-\mathcal{D})}{(4\beta+3\varphi)^2}\left(G_{\mathcal{D}} + \frac{\alpha^2(T-\mathcal{D})^2}{6(4\beta+3\varphi)^2}\left(\frac{8(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)\right).
\end{aligned}$$

Proof. See Appendix. □

The results call for the following comments: (i) As assumed, the equilibrium solution is interior. (ii) The prices are constant and strategic complements, that is, an increase in the price by one player triggers an increase in the price by the rival. This can be seen by looking at the reaction functions (see Appendix) given by

$$\begin{aligned}
p_2^{\mathcal{C}} &= \frac{\alpha + \varphi m^{\mathcal{C}}}{2(\beta+\varphi)}, & \frac{\partial p_2^{\mathcal{C}}}{\partial m^{\mathcal{C}}} &= \frac{\varphi}{2(\beta+\varphi)} > 0,\\
m^{\mathcal{C}} &= \frac{p_2^{\mathcal{C}}}{2}, & \frac{\partial m^{\mathcal{C}}}{\partial p_2^{\mathcal{C}}} &= \frac{1}{2} > 0.
\end{aligned}$$

We note that the fake product is offered to consumers at half the price of the legal product. (iii) As in the previous scenario, the advertising strategies are decreasing over time and equal to zero at T. Note that the advertising policies are strategically independent, which is due to the absence of direct interaction between them in the payoff functions and in the dynamics. (iv) The two players' second-period payoffs are increasing in the initial goodwill value $G_{\mathcal{D}}$, and decreasing in $\mathcal{D}$. As goodwill shifts both demands upward, the first observation, that payoffs are increasing functions in the initial goodwill is not surprising. Neither is the fact that it is in the counterfeiter's best interest to enter the market as soon as he possibly can. The result that the legal firm's second-period profit is decreasing in $\mathcal{D}$ is a priori surprising, but may be explained by the fact that the counterfeiter contributes positively to the common public good (the goodwill). In fact, the relevant comparison is between the total payoffs achieved by the legal firm under the two scenarios. We postpone this discussion to later.

Now, we look at the overall optimization problem of the firm (8), that is, we find the optimal pricing and advertising expenditures before the illegal producer's entry onto the market.

Proposition 4 *Assuming an interior solution, the price and advertising strategies during the time interval* $[0,\mathcal{D}]$ *are given by*

$$p_1^{\mathcal{C}}(t) = \frac{\alpha}{2\beta}, \tag{13}$$

$$a_{\ell 1}^{\mathcal{C}}(t) = \frac{\alpha^2\gamma_\ell}{\kappa_\ell}\left(\frac{(\mathcal{D}-t)}{4\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})}{(4\beta+3\varphi)^2}\right). \tag{14}$$

The goodwill trajectory and the total profit are given by

$$G_1^{\mathcal{C}}(t) = G_0 + \frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}\left(\frac{(2\mathcal{D}-t)\,t}{8\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})\,t}{(4\beta+3\varphi)^2}\right),$$

$$J_\ell^{\mathcal{C}} = J_{\ell 2}^{\mathcal{C}}\left(G_1^{\mathcal{C}}(\mathcal{D})\right) + \frac{\alpha^2 G_0}{4\beta}\mathcal{D} + \frac{\alpha^4\gamma_\ell^2}{2\kappa_\ell}\left\{\frac{\mathcal{D}^3}{48\beta^2} - \frac{16(\beta+\varphi)^2}{(4\beta+3\varphi)^4}(T-\mathcal{D})^2\mathcal{D}\right\}.$$

Proof. See Appendix. □

We observe that the optimal solution is interior and that the pricing and advertising strategies can be interpreted as previously; therefore, there is no need to repeat the same arguments here. The above proposition answers our first research question, which is about how the pirate's entry date affects the first-period equilibrium strategies. We obtain that the price is independent of $\mathcal{D}$, whereas advertising is increasing in that date. Indeed, we have

$$\frac{\partial a_{\ell 1}^{\mathcal{C}}}{\partial \mathcal{D}} = \frac{\alpha^2\gamma_\ell}{\kappa_\ell}\left(\frac{9\varphi^2+8\beta\varphi}{4\beta(4\beta+3\varphi)^2}\right) > 0,$$

$$\frac{\partial G_1^{\mathcal{C}}}{\partial \mathcal{D}} = \frac{\alpha^2\gamma_\ell}{\kappa_\ell}\left(\frac{9\varphi^2+8\beta\varphi}{4\beta(4\beta+3\varphi)^2}\right) t > 0.$$

Given that

$$d_{\ell 1}^{\mathcal{C}}(t) = \left(\alpha - \beta p_1^{\mathcal{C}}(t)\right) G_1^{\mathcal{C}}(t) = \frac{\alpha G_1^{\mathcal{C}}(t)}{2}, \qquad t \in [0,\mathcal{D}],$$

we conclude that the larger is $\mathcal{D}$, the higher the demand during the monopoly period, for all $t \in [0,\mathcal{D}]$.

Before moving on to comparing the two scenarios, we state the following lemma, which will help visualize the goodwill trajectories.

Lemma 1 *The goodwill trajectories in both scenarios, that is,*

$$G^{\mathcal{N}}(t) = G_0 + \frac{\alpha^2\gamma_\ell^2}{8\beta\kappa_\ell}(2T-t)\,t,$$

$$G_1^{\mathcal{C}}(t) = G_0 + \frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}\left(\frac{(2\mathcal{D}-t)\,t}{8\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})\,t}{(4\beta+3\varphi)^2}\right),$$

$$G_2^{\mathcal{C}}(t) = G_1^{\mathcal{C}}(\mathcal{D}) + \frac{\alpha^2}{(4\beta+3\varphi)^2}\left(\frac{4(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)\frac{(t-\mathcal{D})(2T-t-\mathcal{D})}{2},$$

are strictly concave and increasing in $t \in [0,T)$.

Proof. See Appendix. □

5 Comparison

In this section, we compare the equilibrium strategies and outcomes under the two scenarios and answer our second and third research questions. The next two propositions compare the pricing and advertising strategies of the legal firm.

Proposition 5 *The legal product's prices compare as follows:*

$$p^{\mathcal{N}} = p_1^{\mathcal{C}} > p_2^{\mathcal{C}}.$$

Proof. The first equality is obvious. To show that $p_1^{\mathcal{C}} > p_2^{\mathcal{C}}$, it suffices to compute

$$p_2^{\mathcal{C}} - p_1^{\mathcal{C}} = -\frac{3\varphi\alpha}{2\beta\left(4\beta+3\varphi\right)} < 0.$$

□

In the piracy game, the legal producer cuts its price by $r = 3\varphi\alpha/2\beta\left(4\beta+3\varphi\right)$ during the duopoly period, with respect to the first period. The magnitude of the rebate r depends in particular on the cross-price parameter φ; if consumers are highly sensitive to the price gap between the two products, then the legal producer has no choice but to heavily discount its price. Indeed, we have $\partial r/\partial\varphi = 6\alpha/\left(4\beta+3\varphi\right)^2 > 0$. For fashion and luxury products, we would expect φ to be rather low, whereas for invisible products (e.g., software, video games), the conjecture is that φ would be high. The result that $p^{\mathcal{N}} = p_1^{\mathcal{C}}$ means that the legal firm does not make any attempt in the piracy game to deter the entry of the illegal producer by, e.g., selling at a lower price than the monopoly one. This is due to our assumption about the absence of the price in the goodwill dynamics.

Proposition 6 *The advertising strategies compare as follows: $a_\ell^{\mathcal{N}}(t) > a_\ell^{\mathcal{C}}(t)$ for all $t \in [0,T]$.*

Proof. See Appendix □

The main message from this result is that the classical free-riding problem in a public-good provision (here, goodwill) is present. Indeed, in the duopoly game, the legal firm cannot alone appropriate the benefit of its goodwill, and consequently, it reduces its advertising in the piracy game with respect to its level in the no-piracy scenario. It is important to stress here that this reduction occurs during the whole planning interval, and not only during the period $(\mathcal{D},T]$. In this sense, the legal producer is forward-looking when setting its advertising strategy for the before-entry period. Further, the result is independent of the counterfeiter's actual date of entry onto the market. A corollary to the above proposition is that the goodwill in the no-piracy game is larger than the goodwill in the piracy game during the time interval $[0,\mathcal{D}]$. Indeed, for all $t \in [0,\mathcal{D}]$, we obtain that

$$G^{\mathcal{N}}(t) = G_0 + \int_0^t a_\ell^{\mathcal{N}}(s)\,ds > G_0 + \int_0^t a_\ell^{\mathcal{C}}(s)\,ds = G_1^{\mathcal{C}}(t). \tag{15}$$

The next proposition compares the goodwill trajectories.

Proposition 7 *Defining*

$$\begin{aligned} A &= \frac{\varphi\left(9\varphi+8\beta\right)}{4\beta}\frac{\gamma_\ell^2}{\kappa_\ell}, \\ B &= \frac{\varphi\gamma_i^2}{\kappa_i}, \\ \tilde{\mathcal{D}} &= T\left(\frac{B-A}{A+B}\right) \in (0,T), \end{aligned}$$

the goodwill trajectories compare as follows:

Case 1: *if* $A \geq B$, *then* $G^{\mathcal{N}}(t) > G^{\mathcal{C}}(t)$ *for all* $t \in [0, T]$;
Case 2: *if* $A < B$ *and* $\mathcal{D} \geq \tilde{\mathcal{D}}$, *then* $G^{\mathcal{N}}(t) > G^{\mathcal{C}}(t)$ *for all* $t \in [0, T]$;
Case 3: *if* $A < B$ *and* $\mathcal{D} < \tilde{\mathcal{D}}$, *then there exists a unique* $\tilde{t} > \mathcal{D}$ *such that* $G^{\mathcal{N}}(t) \geq G^{\mathcal{C}}(t)$ *for all* $t \in [0, \tilde{t}]$, *while* $G^{\mathcal{N}}(t) < G^{\mathcal{C}}(t)$ *for all* $t \in (\tilde{t}, T]$, *where*

$$\tilde{t} = T - \sqrt{(T - \mathcal{D})\left(T - \frac{(B+A)}{(B-A)}\mathcal{D}\right)}.$$

Proof. See Appendix. □

The above proposition shows that the difference in the goodwill trajectories involves three quantities, namely, A, B and $\tilde{\mathcal{D}}$, where A (resp. B) depends on the legal (resp. illegal) producer's advertising parameters, and $\tilde{\mathcal{D}}$ is a function of the two previous quantities and the planning horizon T. To interpret the results in the proposition, we need to look at $A - B$, which is given by

$$A - B = \varphi \left[\frac{\gamma_\ell^2}{\kappa_\ell} \underbrace{\frac{(9\varphi + 8\beta)}{4\beta}}_{>2} - \frac{\gamma_i^2}{\kappa_i} \right],$$

Observe first that if the game were fully symmetric, that is, $\gamma_\ell = \gamma_i$ and $\kappa_\ell = \kappa_i$, then $A > B$ for all parameter values, and $G^{\mathcal{N}}(t)$ would be larger than $G^{\mathcal{C}}(t)$ for all $t \in [0, T]$. This means that if the illegal producer is as efficient as the firm in terms of impact and cost of advertising, then counterfeiting is unambiguously damaging the firm's reputation. Second, clearly $A - B$ can be positive for many combinations of parameter values. For instance, if the pirate's (resp. legal producer's) efficiency in raising the goodwill is sufficiently low (resp. sufficiently high), then $A - B$ would be positive, and the goodwill in the no-piracy game would dominate its no-piracy counterpart for all $t \in [0, T]$. An instance where $A - B$ is negative is when the pirate's marginal advertising cost is sufficiently low. This is plausible, as pirates typically advertise their fake products through the Internet, at a very low cost. However, this is not sufficient to fully characterize the sign of $G^{\mathcal{N}}(t) - G^{\mathcal{C}}(t)$; if the entry occurs sufficiently late during the planning horizon, that is, $\mathcal{D} \geq \tilde{\mathcal{D}}$, we get $G^{\mathcal{N}}(t) > G^{\mathcal{C}}(t)$ for all $t \in [0, T]$. Otherwise, that is, $\mathcal{D} < \tilde{\mathcal{D}}$, then we obtain that the two goodwill trajectories intersect at date $\tilde{t}$, with the no-piracy goodwill being larger on $[0, \tilde{t}]$. This invites an assessment of the impact of the parameters β and φ on the threshold $\tilde{\mathcal{D}}$. The derivatives are given by

$$\begin{aligned} \frac{\partial \tilde{\mathcal{D}}}{\partial \beta} &= \frac{TB}{(A+B)^2} \frac{9\varphi^2 \gamma_\ell^2}{2\kappa_\ell \beta^2} > 0, \\ \frac{\partial \tilde{\mathcal{D}}}{\partial \varphi} &= -\frac{TB}{(A+B)^2} \frac{\gamma_\ell^2 (18\varphi + 8\beta)}{2\beta\kappa_\ell} < 0. \end{aligned}$$

Therefore, the larger is φ (consumer sensitivity to the difference in the prices of the two products), the lower is the threshold $\tilde{\mathcal{D}}$, which implies that for Case 3 to hold, entry must occur early in the game. The reverse can be stated for β (consumer sensitivity to the legal product's price).

Now, we compare the demands in the two scenarios. A first clear-cut result is that demand for the legal producer is higher during the monopoly period in the no-piracy game than in the piracy game. To show this, recall that the demands are given by

$$d^{\mathcal{N}}(t) = \frac{\alpha}{2} \cdot G^{\mathcal{N}}(t), \quad d_{\ell 1}^{\mathcal{C}}(t) = \frac{\alpha}{2} \cdot G_1^{\mathcal{C}}(t), \quad \text{for all } t \in [0, \mathcal{D}].$$

Therefore,

$$\text{sign}\left(d^{\mathcal{N}}(t) - d_{\ell 1}^{\mathcal{C}}(t)\right) = \text{sign}\left(G^{\mathcal{N}}(t) - G_1^{\mathcal{C}}(t)\right),$$

which is positive by (15). Consequently, we state the following:

Proposition 8 *During the time interval* $[0, \mathcal{D}]$, *we have*

$$d^{\mathcal{N}}(t) > d_{\ell 1}^{\mathcal{C}}(t).$$

Given that $p^{\mathcal{N}} = p_1^{\mathcal{C}}$, the conclusion here is that, irrespective of the parameter values, including the illegal producer's entry date, the consumer's surplus is lower under piracy than in the no-piracy game, for all $t \in [0, \mathcal{D}]$. The difference in total demands during the after-entry period is characterized below.

Proposition 9 *If $\gamma_i^2/\kappa_i > \gamma_\ell^2/\kappa_\ell$ then:*

- *either $d^{\mathcal{N}}(t) \geq d_2^{\mathcal{C}}(t)$ for all $t \in (\mathcal{D}, T]$,*
- *or, in a left neighborhood of T, we have that $d^{\mathcal{N}}(t) < d_2^{\mathcal{C}}(t)$.*

Proof. See Appendix. □

Under the conditions in the proposition, we conclude that piracy is detrimental to the consumer. Unfortunately, we cannot analytically characterize the difference in total demands when the condition in the previous proposition's assumption is not satisfied. In particular, this occurs when the pirate's advertising cost is sufficiently low. As alluded to before, this possibility can by no means be ruled out, and is actually plausible. The next result concerns the demands to the legal producer.

Proposition 10 *If $A < B$ and $\mathcal{D} < \tilde{\mathcal{D}}$ then, in a left neighborhood of T, we have that*

$$d^{\mathcal{N}}(t) < d_{\ell 2}^{\mathcal{C}}(t).$$

Proof. We notice that for all $t \in [\mathcal{D}, T]$

$$d^{\mathcal{N}}(t) - d_{\ell 2}^{\mathcal{C}}(t) = \frac{\alpha}{2}\left(G^{\mathcal{N}}(t) - \underbrace{\frac{4\beta + 6\varphi}{4\beta + 3\varphi}}_{>1} G^{\mathcal{C}}(t)\right) < \frac{\alpha}{2}\left(G^{\mathcal{N}}(t) - G^{\mathcal{C}}(t)\right).$$

From Proposition 6, we get that, in a left neighborhood of T, the following inequality holds:

$$d^{\mathcal{N}}(t) < d_{\ell 2}^{\mathcal{C}}(t).$$

□

The difference between the legal firm's total profit under the two scenarios, i.e., $J_\ell^{\mathcal{C}} - J^{\mathcal{N}}$, is a huge expression that involves all the parameters and cannot be assessed analytically. Therefore, to get some insight into this difference, we shall resort to some numerical simulations. Our model has 10 parameters, namely:

$$\begin{aligned} \text{Demand parameters} &: \alpha, \beta, \varphi, \\ \text{Advertising parameters} &: \gamma_\ell, \kappa_\ell, \gamma_i, \kappa_i, \\ \text{Dates of entry and planning horizon} &: \mathcal{D}, T, \\ \text{Initial goodwill} &: G_0. \end{aligned}$$

With 10 parameters, we can obviously run a huge number of simulations, but this will not necessarily lead to a clear answer to our research question. Instead, we shall focus on the most meaningful parameters in our context. Based on the discussion so far, we know that α and G_0 do not play a crucial role in the comparative results. Further, by assumptions, the parameters φ and $\mathcal{D}$ are bounded as follows: $0 < \varphi \leq \beta$, and $0 < \mathcal{D} \leq T$. What really matters is how close these parameters are to their upper bounds. Consequently, α, β, T and G_0 can be fixed once and for all without much qualitative loss. Further, we observe that, in all the expressions to be compared, the advertising parameters always appear in the form of one of the following pairs of ratios:

$$\left(\frac{\gamma_\ell^2}{\kappa_\ell}, \frac{\gamma_i^2}{\kappa_i}\right) \text{ or } \left(\frac{\gamma_\ell}{\kappa_\ell}, \frac{\gamma_i}{\kappa_i}\right).$$

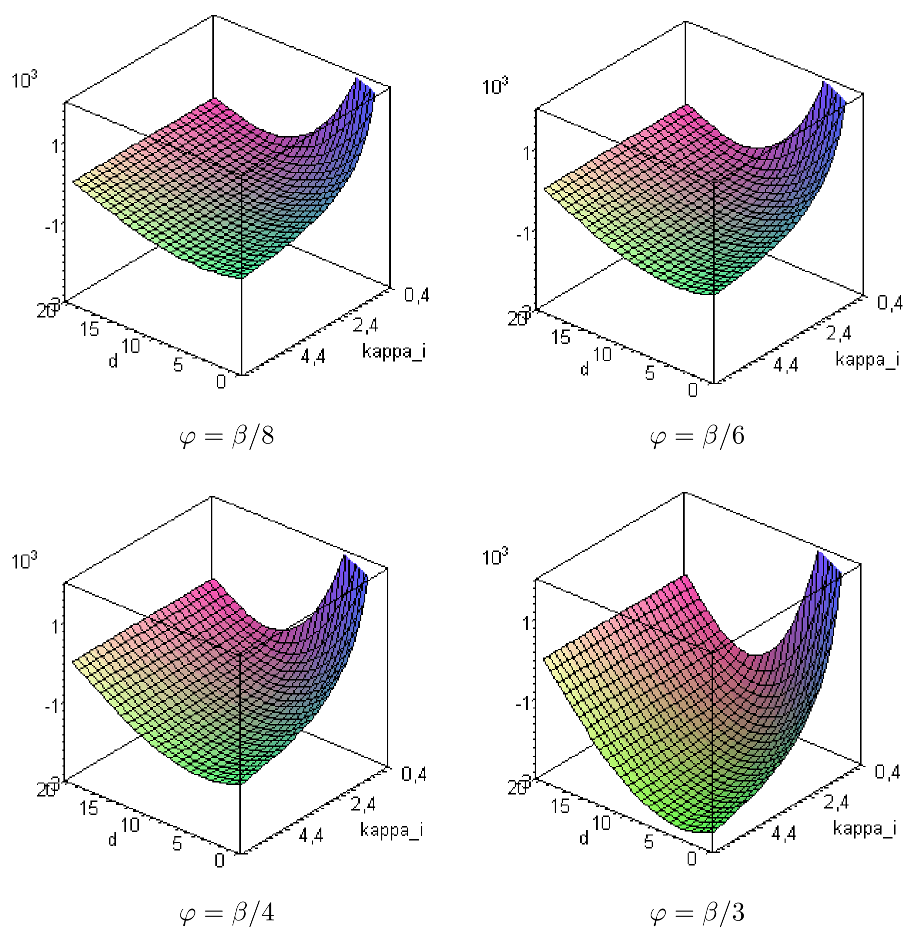
Consequently, we can normalize either the kappas or the gammas, and still fully capture the effect of differential efficiency in advertising on profits and demand. Efficiency can be assessed by the impact of the advertising dollar on goodwill, or by the cost of the advertising effort. As costs are probably easier to visualize than the marginal impact of advertising on goodwill, we normalize γ_ℓ and γ_i to one, and let $\kappa_i \in (0, \kappa_\ell]$, where κ_ℓ will be assigned a given, once-and-for-all value. This means that our experiments are run under the (plausible) assumption that the illegal producer's advertising cost is lower than the legal producer's cost.

To summarize, our numerical simulations are conducted in settings characterized by

$$\begin{aligned} \alpha &= 10, \beta = 4, \gamma_\ell = \gamma_i = 1, \kappa_\ell = 6, T = 20, G_0 = 10, \\ \mathcal{D} &\in (0, T], \quad \kappa_i \in (0, \kappa_\ell], \ \varphi \in (0, \beta]. \end{aligned}$$

Denote by Δ the difference between the firm's total profit under piracy and under the benchmark, that is, $\Delta = J_\ell^{\mathcal{C}} - J^{\mathcal{N}}$. In the next figures, we plot the value of Δ in the $(\kappa_i, \mathcal{D})$-, $(\varphi, \mathcal{D})$- and (κ_i, φ)-space, respectively. To assess the impact of the left-out parameter on Δ, and by the same token, the robustness (in a qualitative sense) of the results, we let this parameter take four different values.

From Figure 1, we observe that for a given φ, the higher the pirate's advertising cost (higher κ_i), or the earlier the entry date $\mathcal{D}$, then the higher the legal producer's profit difference. The intuition behind this result is that the larger the pirate' advertising cost, the lower his contribution to the goodwill. The consequence is a lower demand for the firm's product (recall that the goodwill shifts the demand up) and a lower profit in the counterfeiting scenario than in the benchmark scenario. Similarly, the earlier the pirate enters the market, the less the firm invests in advertising, with the same chain of consequences as for the entry date

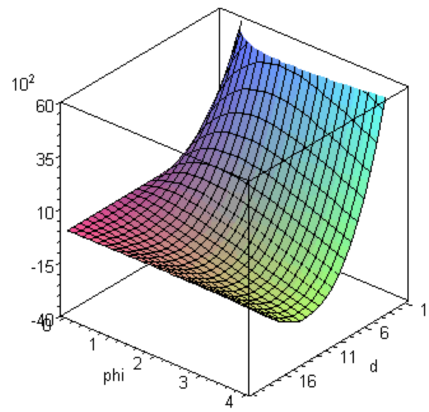


$\varphi = \beta/8$ $\varphi = \beta/6$

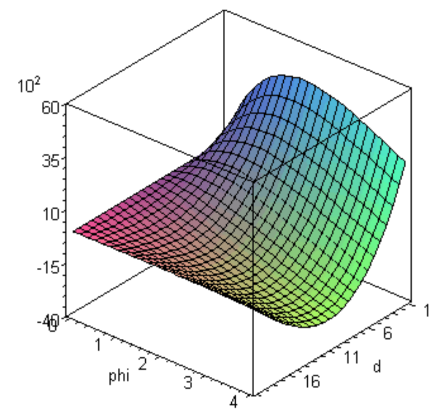
$\varphi = \beta/4$ $\varphi = \beta/3$

Figure 1

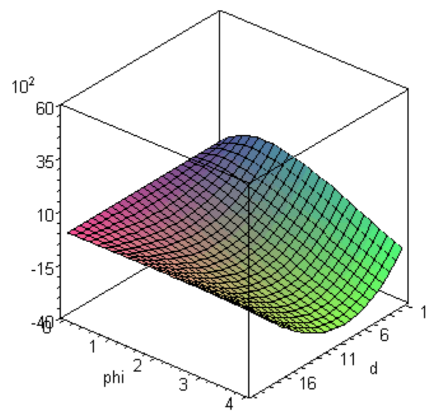
$\mathcal{D}$. Now, increasing φ, i.e., increasing consumer sensitivity to the price differential, deteriorates the firm's profit even more under piracy $J_\ell^{\mathcal{C}}$, and consequently, Δ is lower. Again, if we associate a low value of φ to a scenario involving a visible product (e.g., fashion accessories) and a high value to an invisible product (e.g., software), then, from Figure 1, we conclude that piracy is more damaging to firms marketing invisible products. Figures 2 and 3 reveal exactly the same information as Figure 1, and therefore, there is no need to repeat the interpretation.



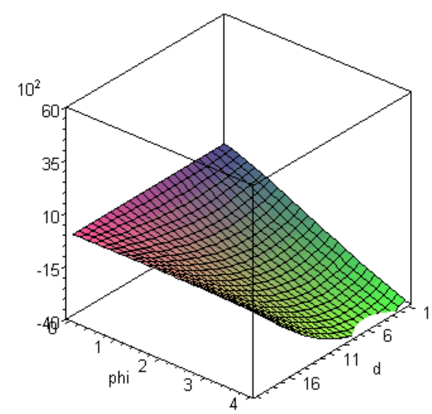
$\kappa_i = \kappa_\ell/20$



$\kappa_i = \kappa_\ell/10$



$\kappa_i = \kappa_\ell/5$



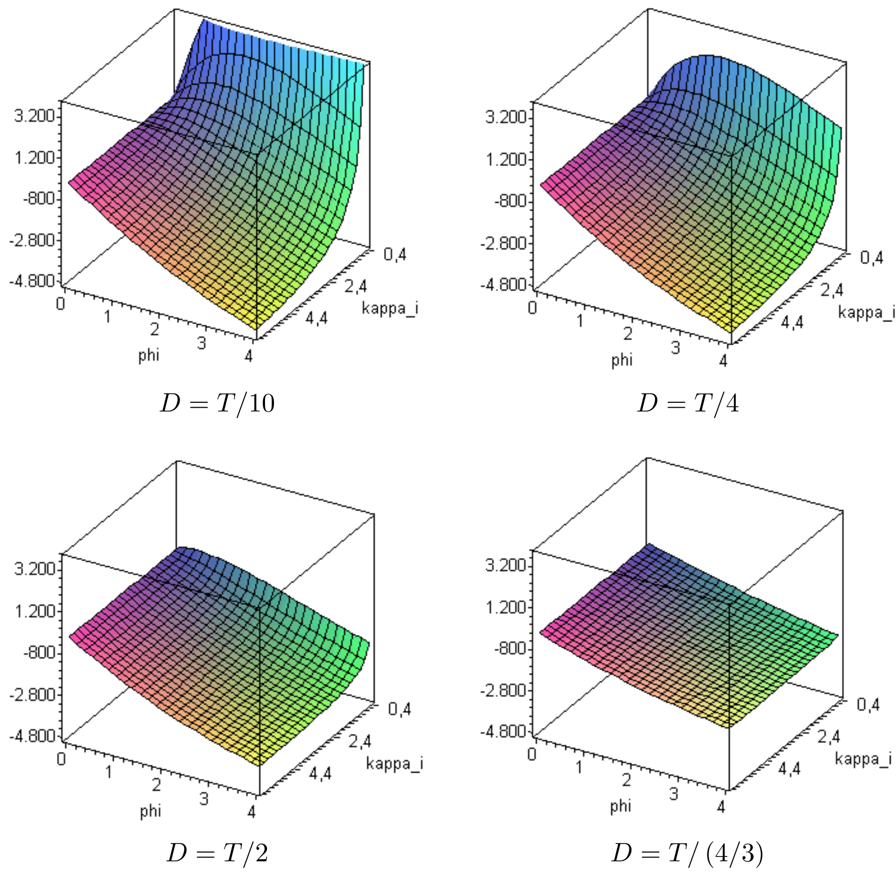
$\kappa_i = \kappa_\ell/2$

Figure 2

6 Implications and Concluding Remarks

The main takeaways of this work lie in the answers to our three research questions. First, regarding the impact of piracy on the firm's strategy, we obtained that counterfeiting leads to reduced advertising throughout the planning horizon, and to a lower price after the pirate's entry. The decrease in advertising is due to the fact that the pirate's entry changes the nature of the goodwill from being for a private good to being for a public good. We recover here the classically established result of the under-provision of a public good, meaning that agents under-invest in an endeavour when they cannot fully appropriate the benefits of their investments. The decrease in the price is a consequence of the competitive pressure induced by the arrival of a rival in the market.

Second, by comparing the demand under the two scenarios, we showed that, independently of the parameter values, piracy is detrimental to consumers during the before-entry period. Given that the firm sets its price to the same value in both scenarios during the interval $[0, \mathcal{D}]$, the clear-cut conclusion is that the consumer surplus is lower under piracy during this time interval. After entry, no definitive result can be



$D = T/10$ $D = T/4$

$D = T/2$ $D = T/(4/3)$

Figure 3

established, as the difference between the demands of the two scenarios depends on the parameter values, and more specifically, on the pirate's advertising efficiency.

Finally, the impact of piracy on a firm's profit is ambiguous, which is interesting in of itself because it challenges the intuition that counterfeiting is necessarily bad. (Our focus here is on profit and not on ethical or moral issues.) Our results show that in some region of the parameter space, it is in the best interest of a legal producer that there be a pirate offering a fake product. Indeed, based on Figures 1–3, we see that a legal producer benefits if a low-cost advertising pirate shows up, provided that this pirate does not enter the market too early and that consumers are not too sensitive to the price difference between the legal and fake products. As alluded to before, this scenario probably corresponds to the market for fashion and luxury products, where we can expect that it takes some time to produce the fake product, rather than to market for, e.g., software or video game products, where security protection is broken relatively easily and rapidly.

As in any modelling effort, the results depend on the assumptions made, and some of these are restrictive. We see our work as a first exploration of the area of dynamic strategic interactions between legal and illegal producers. Our starting point is that the illegal producer's will take place anyway, and that the firm can do nothing to deter it. It can of course adapt its strategy to counterfeiter's presence. In view of what is observed in practice (see the Introduction), we believe that this assumption of an exogenous entry is not that severe. In any event, our context can be seen as one of cheap counterfeiting (recall the zero cost assumption) where deterrent institutions are not strong enough. In this sense, our research design is an if-then experiment: if a counterfeiter can enter the market and produce at a very low cost, then how does this affect the legal producer and consumers? From a conceptual point of view, it is clearly of interest to adopt a framework where it is feasible to deter entry, and then answer the question of whether it is in the firm's best interest to do so, and when.

A second important assumption in our work is the positive impact of the counterfeiter's advertising on goodwill. This was seen as a proxy of the network effect. It is clearly of interest to analyze the case where the externality is negative. In such a context, entry deterrence obviously becomes much more important than here. Finally, one could think of different specifications of the demand functions, including having an à la Bass diffusion aspect.

7 Appendix

Proof of Proposition 2

The optimization problem is given by

$$\begin{aligned} J &= \max_{p(t),a(t)} \int_0^T \left(p(t)(\alpha - \beta p(t)) G(t) - \tfrac{\kappa_\ell}{2} a^2(t)\right) dt, \\ &\quad \text{s.t.} \quad \dot{G}(t) = \gamma_\ell a_\ell(t), \\ &\quad\quad\quad G(0) = G_0. \end{aligned}$$

The firm's Hamiltonian reads

$$H = p(\alpha - \beta p) G - \frac{\kappa_\ell}{2} a^2 + \lambda \gamma_\ell a,$$

where λ is the adjoint variable associated with the dynamics of goodwill.
Assuming an interior solution, the first-order optimality conditions include

$$\begin{aligned} \frac{\partial H}{\partial p} &= (\alpha - 2\beta p) G = 0, \\ \frac{\partial H}{\partial a} &= -\kappa_\ell a + \lambda \gamma_\ell = 0, \\ \dot{\lambda}(t) &= -p(t)(\alpha - \beta p(t)), \\ \lambda(T) &= 0. \end{aligned}$$

Solving the above equations, we get

$$\begin{aligned} p^{\mathcal{N}}(t) &= \frac{\alpha}{2\beta}, \\ \lambda(t) &= \frac{\alpha^2}{4\beta}(T - t), \\ a^{\mathcal{N}}(t) &= \frac{\alpha^2 \gamma_\ell}{4\beta\kappa_\ell}(T - t). \end{aligned}$$

Substituting for $a_\ell(t)$ in the dynamics and solving the resulting differential equation gives

$$G^{\mathcal{N}}(t) = G_0 + \frac{\alpha^2 \gamma_\ell^2}{8\beta\kappa_\ell}\left(2Tt - t^2\right).$$

The demand is given by

$$d_\ell(t) = \frac{\alpha}{2}\left(G_0 + \frac{\alpha^2 \gamma_\ell^2}{8\beta\kappa_\ell}\left(2Tt - t^2\right)\right).$$

Substituting for $a_\ell^{\mathcal{N}}(t), p^{\mathcal{N}}(t)$ and $G^{\mathcal{N}}(t)$ in the objective function and integrating, we get

$$J^{\mathcal{N}} = \frac{\alpha^2}{4\beta} T \left(G_0 + \frac{\alpha^2 \gamma_\ell^2 T^2}{24\beta\kappa_\ell}\right).$$

Proof of Proposition 3

The players' profit-maximization problems are given by

$$\begin{aligned}
J_{\ell 2} &= \max_{p_2(t),a_{\ell 2}(t)} \int_{\mathcal{D}}^{T} \left(p_2(t)\left(\alpha-(\beta+\varphi)p_2(t)+\varphi m(t)\right)G_2(t)-\frac{\kappa_\ell}{2}a_{\ell 2}^2(t)\right)dt,\\
J_i &= \max_{m(t),a_i(t)} \int_{\mathcal{D}}^{T} \left(m(t)\varphi\left(p_2(t)-m(t)\right)G_2(t)-\frac{\kappa_i}{2}a_i^2(t)\right)dt,
\end{aligned}$$

subject to

$$\begin{aligned}
\dot{G}_2(t) &= \gamma_\ell a_{\ell 2}(t)+\gamma_i a_i(t),\\
G_2(\mathcal{D}) &= G_{\mathcal{D}}.
\end{aligned}$$

Introduce the players' Hamiltonians

$$\begin{aligned}
H_{\ell 2} &= p_2\left(\alpha-(\beta+\varphi)p_2+\varphi m\right)G_2-\frac{\kappa_\ell}{2}a_{\ell 2}^2+\mu_{\ell 2}\left(\gamma_\ell a_{\ell 2}+\gamma_i a_i\right),\\
H_i &= m\varphi\left(p_2-m\right)G-\frac{\kappa_i}{2}a_i^2+\mu_i\left(\gamma_\ell a_{\ell 2}+\gamma_i a_i\right),
\end{aligned}$$

where $\mu_{\ell 2}$ and μ_i are the adjoint variables appended to the state equation by player ℓ and i. Assuming an interior solution, the first-order equilibrium conditions include

$$\begin{aligned}
\frac{\partial H_{\ell 2}}{\partial p_2} &= \left(\alpha-2(\beta+\varphi)p_2+\varphi m\right)G_2=0,\\
\frac{\partial H_{\ell 2}}{\partial a_{\ell 2}} &= -\kappa_\ell a_{\ell 2}+\mu_{\ell 2}\gamma_\ell=0,\\
\frac{\partial H_i}{\partial m} &= \varphi\left(p_2-2m\right)G_2=0,\\
\frac{\partial H_i}{\partial a_i} &= -\kappa_i a_i+\mu_i\gamma_i=0,\\
\dot{\mu}_{\ell 2}(t) &= -p_2(t)\left(\alpha-(\beta+\varphi)p_2(t)+\varphi m(t)\right),\\
\mu_\ell(T) &= 0,\\
\dot{\mu}_i(t) &= -m(t)\varphi\left(p_2(t)-m(t)\right),\\
\mu_i(T) &= 0.
\end{aligned}$$

From the first and third equations, we obtain the reaction functions

$$p_2^{\mathcal{C}}(t)=\frac{\alpha+\varphi m^{\mathcal{C}}(t)}{2(\beta+\varphi)}, \qquad m^{\mathcal{C}}(t)=\frac{p_2^{\mathcal{C}}(t)}{2}.$$

Hence, solving for the prices, we get

$$p_2^{\mathcal{C}}(t)=\frac{2\alpha}{4\beta+3\varphi}, \qquad m^{\mathcal{C}}(t)=\frac{\alpha}{4\beta+3\varphi}.$$

Inserting the results in the adjoint equations and integrating yields

$$\mu_{\ell 2}(t)=\left(\frac{4\alpha^2(\beta+\varphi)}{(4\beta+3\varphi)^2}\right)(T-t), \qquad \mu_i(t)=\left(\frac{\alpha^2\varphi}{(4\beta+3\varphi)^2}\right)(T-t).$$

Consequently, we get the following advertising functions:

$$a_{\ell 2}^{\mathcal{C}}(t)=\frac{4\alpha^2(\beta+\varphi)}{(4\beta+3\varphi)^2}\frac{\gamma_\ell}{\kappa_\ell}(T-t), \qquad a_i(t)=\frac{\alpha^2\varphi}{(4\beta+3\varphi)^2}\frac{\gamma_i}{\kappa_i}(T-t).$$

Inserting in the goodwill dynamics and solving leads to

$$G_2^{\mathcal{C}}(t) = G_{\mathcal{D}} + \frac{\alpha^2}{2\left(4\beta+3\varphi\right)^2}\left(\frac{4\left(\beta+\varphi\right)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)\left(t-\mathcal{D}\right)\left(2T-t-\mathcal{D}\right).$$

The demands are given by

$$\begin{aligned} d_{\ell 2}^{\mathcal{C}}(t) &= \left(\frac{2\alpha\left(\beta+\varphi\right)}{4\beta+3\varphi}\right)G_2^{\mathcal{C}}(t), \\ d_i^{\mathcal{C}}(t) &= \left(\frac{\alpha\varphi}{4\beta+3\varphi}\right)G_2^{\mathcal{C}}(t). \end{aligned}$$

The legal firm's total outcome on $(\mathcal{D},T]$ is given by

$$J_{\ell 2}^{\mathcal{C}}\left(G_{\mathcal{D}}\right) = \frac{4\alpha^2\left(\beta+\varphi\right)}{\left(4\beta+3\varphi\right)^2}\left(T-\mathcal{D}\right)\left(G_{\mathcal{D}} + \frac{\alpha^2\left(T-\mathcal{D}\right)^2}{3\left(4\beta+3\varphi\right)^2}\left[\frac{2\left(\beta+\varphi\right)\gamma_l^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right]\right).$$

The counterfeiter's total payoff is given by

$$J_i^{\mathcal{C}} = \frac{\varphi\alpha_\ell^2}{\left(4\beta+3\varphi\right)^2}\left(T-\mathcal{D}\right)\left(G_{\mathcal{D}} + \frac{\alpha^2\left(T-\mathcal{D}\right)^2}{6\left(4\beta+3\varphi\right)^2}\left(\frac{8\left(\beta+\varphi\right)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)\right).$$

Proof of Proposition 4

The overall payoff function of the firm is given by

$$J_\ell = \max_{p_1(t),a_{\ell 1}(t)}\left[\int_0^{\mathcal{D}}\left(p_1(t)\left(\alpha-\beta p_1(t)\right)G_1(t) - \frac{\kappa_\ell}{2}a_{\ell 1}^2(t)\right)dt + J_{\ell 2}^{\mathcal{C}}\left(G_1(\mathcal{D})\right)\right],$$

where

$$J_{\ell 2}^{\mathcal{C}}\left(G_{\mathcal{D}}\right) = \frac{4\alpha^2\left(\beta+\varphi\right)}{\left(4\beta+3\varphi\right)^2}\left(T-\mathcal{D}\right)\left(G_{\mathcal{D}} + \frac{\alpha^2\left(T-\mathcal{D}\right)^2}{3\left(4\beta+3\varphi\right)^2}\left[\frac{2\left(\beta+\varphi\right)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right]\right).$$

Introducing the Hamiltonian

$$H_{\ell 1} = p_1\left(\alpha-\beta p_1\right)G_1 - \frac{\kappa_\ell}{2}a_{\ell 1}^2 + \mu_\ell\gamma_\ell a_{\ell 1},$$

where μ_ℓ is the adjoint variable appended to the goodwill dynamics. Assuming an interior solution, the first-order optimality conditions include

$$\begin{aligned} \frac{\partial H_{\ell 1}}{\partial p_1} &= \left(\alpha-2\beta p_1\right)G_1 = 0, \\ \frac{\partial H_{\ell 1}}{\partial a_{\ell 1}} &= -\kappa_\ell a_{\ell 1} + \mu_\ell\gamma_\ell = 0, \\ \dot{\mu}_{\ell 1}(t) &= -p_1(t)\left(\alpha-\beta p_1(t)\right), \\ \mu_{\ell 1}(\mathcal{D}) &= \frac{\partial J_{\ell 2}^{\mathcal{C}}\left(G_{\mathcal{D}}\right)}{\partial G_{\mathcal{D}}} = \frac{4\alpha^2\left(\beta+\varphi\right)}{\left(4\beta+3\varphi\right)^2}\left(T-\mathcal{D}\right). \end{aligned}$$

From the first equation, we obtain

$$p_1^{\mathcal{C}}(t) = \frac{\alpha}{2\beta}.$$

Solving for the adjoint equation, we get

$$\dot{\mu}_{\ell 1}(t) = -\frac{\alpha^2}{4\beta},$$

$$\mu_{\ell 1}(t) = \frac{\alpha^2}{4\beta}\left(\mathcal{D}-t\right) + \frac{4\alpha^2\left(\beta+\varphi\right)}{\left(4\beta+3\varphi\right)^2}\left(T-\mathcal{D}\right).$$

Consequently, the optimal advertising level is given by

$$a_{\ell 1}^{\mathcal{C}}(t) = \frac{\alpha^2\gamma_\ell}{\kappa_\ell}\left(\frac{(\mathcal{D}-t)}{4\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})}{(4\beta+3\varphi)^2}\right).$$

Inserting in the state dynamics and solving, we get

$$G_1^{\mathcal{C}}(t) = G_0 + \frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}\left(\frac{(2\mathcal{D}-t)\,t}{8\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})\,t}{(4\beta+3\varphi)^2}\right).$$

At $t=\mathcal{D}$, we have

$$G_1^{\mathcal{C}}(\mathcal{D}) = G_0 + \frac{\alpha^2\gamma_\ell^2\mathcal{D}}{\kappa_\ell}\left(\frac{\mathcal{D}}{8\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})}{(4\beta+3\varphi)^2}\right).$$

The total payoff is given by

$$\begin{aligned}
J_\ell^{\mathcal{C}} &= \int_0^{\mathcal{D}}\left(p_1^{\mathcal{C}}(t)\left(\alpha-\beta p_1^{\mathcal{C}}(t)\right)G_1^{\mathcal{C}}(t) - \frac{\kappa_\ell}{2}\left(a_\ell^{\mathcal{C}}(t)\right)^2\right)dt + J_{\ell 2}^{\mathcal{C}}\left(G_1^{\mathcal{C}}(\mathcal{D})\right),\\
&= \frac{\alpha^2}{4\beta}G_0\mathcal{D} + \frac{\alpha^4\gamma_\ell^2}{2\kappa_\ell}\left\{\frac{\mathcal{D}^3}{48\beta^2} - \frac{16(\beta+\varphi)^2}{(4\beta+3\varphi)^4}(T-\mathcal{D})^2\mathcal{D}\right\} + J_{\ell 2}^{\mathcal{C}}\left(G_1^{\mathcal{C}}(\mathcal{D})\right).
\end{aligned}$$

Proof of Lemma 1

It suffices to compute the following derivatives to get the results:

$$\begin{aligned}
\dot{G}^{\mathcal{N}}(t) &= \frac{\alpha^2\gamma_\ell^2}{4\beta\kappa_\ell}(T-t) > 0,\\
\ddot{G}^{\mathcal{N}}(t) &= -\frac{\alpha^2\gamma_\ell^2}{4\beta\kappa_\ell} < 0;\\
\dot{G}_1^{\mathcal{C}}(t) &= \frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}\left(\frac{(\mathcal{D}-t)}{4\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})}{(4\beta+3\varphi)^2}\right) > 0,\\
\ddot{G}_1^{\mathcal{C}}(t) &= -\frac{\alpha^2\gamma_\ell^2}{\beta\kappa_\ell} < 0;\\
\dot{G}_2^{\mathcal{C}}(t) &= \frac{\alpha^2}{(4\beta+3\varphi)^2}\left(\frac{4(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)(T-t) > 0,\\
\ddot{G}_2^{\mathcal{C}}(t) &= -\frac{\alpha^2}{(4\beta+3\varphi)^2}\left(\frac{4(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right) < 0.
\end{aligned}$$

Proof of Proposition 6

First of all, we notice that $a_\ell^{\mathcal{C}}(t)$ is a continuous control. Moreover, we notice that both the advertising controls are decreasing over time and that both vanish at time T. Comparing the derivatives of the advertising strategies in the interval $[\mathcal{D},T]$ we observe that

$$\begin{aligned}
\dot{a}_\ell^{\mathcal{N}}(t) &< \dot{a}_{\ell 2}^{\mathcal{C}}(t) \quad\Leftrightarrow\\
-\frac{\alpha^2\gamma_\ell}{4\beta\kappa_\ell} &< -\frac{4\alpha^2(\beta+\varphi)}{(4\beta+3\varphi)^2}\frac{\gamma_\ell}{\kappa_\ell} \quad\Leftrightarrow\\
20\beta+9\varphi &> 0.
\end{aligned}$$

Hence in $[\mathcal{D},T]$ we have that $a_\ell^{\mathcal{N}}(t) > a_\ell^{\mathcal{C}}(t)$.

Something different happens in the interval $[0,\mathcal{D}]$ because, in this interval, it is straightforward to prove that $\dot{a}_{\ell 1}^{\mathcal{C}}(t) < \dot{a}_{\ell}^{\mathcal{N}}(t)$. Hence, there are no intersections between these two linear affine functions inside the interval $[0,\mathcal{D})$ if and only if

$$\begin{array}{rcl} a_{\ell}^{\mathcal{C}}(0) & \leq & a_{\ell 1}^{\mathcal{N}}(0) \qquad \Leftrightarrow \\ 9\varphi + 8\beta & \geq & 0. \end{array}$$

Proof of Proposition 7

Let us compute the difference between the two trajectories in $[0,\mathcal{D}]$:

$$\begin{aligned} G^{\mathcal{N}}(t) - G_1^{\mathcal{C}}(t) &= \frac{\alpha^2\gamma_\ell^2}{8\beta\kappa_\ell}(2T-t)\,t + \\ & -\frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}\left(\frac{(2\mathcal{D}-t)\,t}{8\beta} + \frac{4(\beta+\varphi)(T-\mathcal{D})\,t}{(4\beta+3\varphi)^2}\right) \\ &= \frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}(T-\mathcal{D})\,t\left[\frac{9\varphi^2+8\beta\varphi}{4\beta(4\beta+3\varphi)^2}\right] > 0. \end{aligned}$$

This implies that the gap between the two goodwill trajectories at time $\mathcal{D}$ is

$$G^{\mathcal{N}}(\mathcal{D}) - G_1^{\mathcal{C}}(\mathcal{D}) = \frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}(T-\mathcal{D})\,\mathcal{D}\left[\frac{9\varphi^2+8\beta\varphi}{4\beta(4\beta+3\varphi)^2}\right] > 0.$$

In $[\mathcal{D},T]$ the comparison can be written in the following way:

$$\begin{gathered} G^{\mathcal{N}}(t) - G_2^{\mathcal{C}}(t) = \\ = \underbrace{G^{\mathcal{N}}(\mathcal{D}) - G_1^{\mathcal{C}}(\mathcal{D})}_{>0} + \underbrace{\int_{\mathcal{D}}^{t}\left(\gamma_\ell a_\ell^{\mathcal{N}}(s) - \gamma_\ell a_{2\ell}^{\mathcal{C}}(s)\right)ds}_{>0} - \int_{\mathcal{D}}^{t}\gamma_i a_i^{\mathcal{C}}(s)\,ds, \end{gathered}$$

where

$$\begin{gathered} \int_{\mathcal{D}}^{t}\left(\gamma_\ell a_\ell^{\mathcal{N}}(s) - \gamma_\ell a_{2\ell}^{\mathcal{C}}(s)\right)ds = \\ = \int_{\mathcal{D}}^{t}\left(\frac{\alpha^2\gamma_\ell^2}{4\beta\kappa_\ell}(T-s) - \frac{4\alpha^2(\beta+\varphi)}{(4\beta+3\varphi)^2}\frac{\gamma_\ell^2}{\kappa_\ell}(T-s)\right)ds, \\ = \frac{9\varphi^2+8\beta\varphi}{4\beta(4\beta+3\varphi)^2}\frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}\frac{(t-\mathcal{D})(2T-\mathcal{D}-t)}{2}, \end{gathered}$$

while

$$\begin{aligned} \int_{\mathcal{D}}^{t}\gamma_i a_i^{\mathcal{C}}(s)\,ds &= \int_{\mathcal{D}}^{t}\frac{\alpha^2\varphi}{(4\beta+3\varphi)^2}\frac{\gamma_i^2}{\kappa_i}(T-s)\,ds, \\ &= \frac{\alpha^2\varphi}{(4\beta+3\varphi)^2}\frac{\gamma_i^2}{\kappa_i}\frac{(t-\mathcal{D})(2T-\mathcal{D}-t)}{2}. \end{aligned}$$

Therefore,

$$\begin{gathered} G^{\mathcal{N}}(t) - G_2^{\mathcal{C}}(t) = G^{\mathcal{N}}(\mathcal{D}) - G_1^{\mathcal{C}}(\mathcal{D}) + \\ + \frac{\alpha^2}{(4\beta+3\varphi)^2}\left[\frac{(9\varphi^2+8\beta\varphi)}{4\beta}\frac{\gamma_\ell^2}{\kappa_\ell} - \frac{\varphi\gamma_i^2}{\kappa_i}\right]\frac{(t-\mathcal{D})(2T-\mathcal{D}-t)}{2}. \end{gathered} \tag{16}$$

Let us define

$$A = \frac{\left(9\varphi^2 + 8\beta\varphi\right)}{4\beta}\frac{\gamma_\ell^2}{\kappa_\ell},$$
$$B = \frac{\varphi\gamma_i^2}{\kappa_i},$$

then we have to consider three different scenarios:

- if $A \geq B$ then $G^{\mathcal{N}}(t) \geq G_2^{\mathcal{C}}(t)$ for all $t \in [\mathcal{D}, T]$;
- if $A < B$ then
  - either $G^{\mathcal{N}}(t) \geq G_2^{\mathcal{C}}(t)$ for all $t \in [\mathcal{D}, T]$ when $G^{\mathcal{N}}(T) \geq G_2^{\mathcal{C}}(T)$;
  - or there exists a unique $\tilde{t} \in [\mathcal{D}, T]$ such that $G^{\mathcal{N}}(t) \geq G_2^{\mathcal{C}}(t)$ for all $t \in \left[\mathcal{D}, \tilde{t}\right]$, but $G^{\mathcal{N}}(t) < G^{\mathcal{C}}(t)$ for all $t \in \left(\tilde{t}, T\right]$.

We want to better explain the conditions that imply the last two situations. Using (16), the inequality $G^{\mathcal{N}}(T) \geq G_2^{\mathcal{C}}(T)$ becomes

$$A\mathcal{D} + [A - B]\frac{(T - \mathcal{D})}{2} \geq 0,$$
$$\mathcal{D} \geq T\left(1 - \frac{2A}{A + B}\right).$$

We notice that $2A/(A + B) \in (0, 1)$; hence, if we define

$$\tilde{\mathcal{D}} = T\left(1 - \frac{2A}{A + B}\right),$$

we get that $\tilde{\mathcal{D}} \in (0, T)$. This allows us to characterize the first item: if $\mathcal{D} \geq \tilde{\mathcal{D}}$, then $G^{\mathcal{N}}(t) \geq G_2^{\mathcal{C}}(t)$ for all $t \in [\mathcal{D}, T]$.

Finally, let us assume that $\mathcal{D} < \tilde{\mathcal{D}}$. We want to find the intersection between the two goodwill trajectories:

$$\begin{aligned} G^{\mathcal{N}}(t) - G_2^{\mathcal{C}}(t) &= 0 \\ 2(T - \mathcal{D})\mathcal{D}A + (A - B)(t - \mathcal{D})(2T - \mathcal{D} - t) &= 0 \\ t^2 - 2Tt + \frac{2BT\mathcal{D} - (B + A)\mathcal{D}^2}{(B - A)} &= 0 \end{aligned} \tag{17}$$

We note that the discriminant of (17) is

$$(T - \mathcal{D})\frac{B(T - \mathcal{D}) - A(T + \mathcal{D})}{(B - A)}.$$

This quantity is strictly positive because, using $\mathcal{D} < \tilde{\mathcal{D}}$, it is simple to show that $B(T - \mathcal{D}) > A(\mathcal{D} + T)$. Hence, the unique solution of (17) smaller than T is

$$\tilde{t} = T - \sqrt{(T - \mathcal{D})\left(T - \frac{(B + A)}{(B - A)}\mathcal{D}\right)}.$$

Finally, we have to prove that $\tilde{t} > \mathcal{D}$; writing the explicit formulas for these two quantities, we obtain

$$\begin{aligned} T - \sqrt{(T - \mathcal{D})\left(T - \frac{(B + A)}{(B - A)}\mathcal{D}\right)} &> \mathcal{D} \quad \Leftrightarrow \\ \left(T - \frac{(B + A)}{(B - A)}\mathcal{D}\right) &< (T - \mathcal{D}) \Leftrightarrow \\ \frac{(B + A)}{(B - A)} &> 1 \quad \Leftrightarrow \\ A &> 0. \end{aligned}$$

Proof of Proposition 9

By the continuity of the global demand, we have that

$$\begin{aligned} d^{\mathcal{N}}(\mathcal{D}) - d_2^{\mathcal{C}}(\mathcal{D}) &= d^{\mathcal{N}}(\mathcal{D}) - d_{\ell 1}^{\mathcal{C}}(\mathcal{D}), \\ &= \frac{\alpha}{2}\left(G^{\mathcal{N}}(\mathcal{D}) - G_1^{\mathcal{C}}(\mathcal{D})\right), \\ &= \frac{\alpha}{2}\frac{\alpha^2\gamma_\ell^2}{\kappa_\ell}(T-\mathcal{D})\,\mathcal{D}\left[\frac{9\varphi^2+8\beta\varphi}{4\beta(4\beta+3\varphi)^2}\right] > 0. \end{aligned}$$

For $t \in (\mathcal{D}, T]$, the difference in total demand is given by

$$\begin{aligned} d^{\mathcal{N}}(t) - d_2^{\mathcal{C}}(t) &= \left(\alpha - \beta p^{\mathcal{N}}\right) G^{\mathcal{N}}(t) - \left(\alpha - \beta p_2^{\mathcal{C}}\right) G^{\mathcal{C}}(t) \\ &= \frac{\alpha}{2} G^{\mathcal{N}}(t) - \alpha\left(\frac{2\beta+3\varphi}{4\beta+3\varphi}\right) G^{\mathcal{C}}(t); \end{aligned}$$

hence,

$$\begin{aligned} \dot{d}^{\mathcal{N}}(t) - \dot{d}_2^{\mathcal{C}}(t) &= \frac{\alpha}{2}\dot{G}^{\mathcal{N}}(t) - \alpha\left(\frac{2\beta+3\varphi}{4\beta+3\varphi}\right)\dot{G}^{\mathcal{C}}(t), \\ &= \alpha^3\left[\frac{\gamma_\ell^2}{8\beta\kappa_\ell} - \frac{2\beta+3\varphi}{(4\beta+3\varphi)^3}\left(\frac{4(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right)\right](T-t). \end{aligned}$$

If we can prove that the quantity inside the square bracket is always negative, then the difference between the demands is strictly decreasing; hence, either $d^{\mathcal{N}}(t) \geq d_2^{\mathcal{C}}(t)$ for all $t \in (\mathcal{D}, T]$, or $d^{\mathcal{N}}(t) < d_2^{\mathcal{C}}(t)$ in a left neighborhood of T (it depends on the sign of the quantity $\frac{\alpha}{2}G^{\mathcal{N}}(T) - \alpha\left(\frac{2\beta+3\varphi}{4\beta+3\varphi}\right)G^{\mathcal{C}}(T)$). Therefore, to close the proof, we have to prove that

$$\begin{aligned} \frac{\gamma_\ell^2}{8\beta\kappa_\ell} - \frac{2\beta+3\varphi}{(4\beta+3\varphi)^3}\left(\frac{4(\beta+\varphi)\gamma_\ell^2}{\kappa_\ell} + \frac{\varphi\gamma_i^2}{\kappa_i}\right) &< 0 \qquad \Leftrightarrow \\ \frac{36\beta\varphi+27\varphi^2}{16\beta^2+24\beta\varphi} - 1 &< \frac{\gamma_i^2}{\kappa_i}\Big/\frac{\gamma_\ell^2}{\kappa_\ell}. \end{aligned}$$

Now, under our assumptions

$$\frac{\gamma_i^2}{\kappa_i}\Big/\frac{\gamma_\ell^2}{\kappa_\ell} = 1 + (\text{something positive})$$

the previous inequality is satisfied if and only if

$$\begin{aligned} \frac{36\beta\varphi+27\varphi^2}{16\beta^2+24\beta\varphi} - 2 &< 0, \\ 27\varphi^2 - 12\beta\varphi - 32\beta^2 &< 0. \end{aligned}$$

This is a parabola in φ and it is negative if and only if

$$\varphi \in \left(-\frac{8}{9}\beta, \frac{4}{3}\beta\right).$$

However, $\varphi \in (0, \beta]$; hence, it is always negative.

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