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Abstract: In this paper, we present a mixed integer programming model for solving the long-term planning problem of an underground mine. This model establishes the sequence of mining for a period of 20 years. This involves determining which lens of the geological model will be extracted and in what order, while respecting the production capacity of equipment. To reduce the computation time, several different strategies are proposed.

Key Words: Long-term planning, scheduling, optimization, underground mining, mixed integer programming model, cut-off grade.

Résumé : Dans cet article, nous présentons un programme linéaire mixte pour résoudre le problème de planification à long terme dans une mine souterraine. Le modèle permet d'établir une séquence de minage pour une période de planification de 20 ans. La séquence indique quelles lentilles du gisement seront exploitées et dans quel ordre, tout en respectant les contraintes de capacité sur les équipements. Afin de réduire le temps de résolution, plusieurs stratégies sont proposées.

1 Strategic Planning in Underground Mines

Raglan Mine, owned by Glencore, is located in the Nunavik region in the north of the province of Quebec (62^{nd} parallel) about 1800 km from Montreal. The ore resources of the mine are grouped into several small deposits of elongated and finite shapes called lenses. The lenses are distributed over the entire property (which stretches about 70 km from east to west) and have different tonnages and grades. Because of the spreading of the lenses, they are grouped into subsets according to their proximity. Each subset represents an underground mine. The lenses that belong to a mine are interconnected by a main ramp and a network of drifts (horizontal tunnels) and share the same infrastructure (shelters, ventilation system, etc.).

The long-term planning in mining operations, often called strategic planning, consists of scheduling the mining activities to maximize the net present value of profits over a planning horizon, which is estimated based on the geological reserves. The term used by mining engineers to describe this level of planning is "*Life Of Mine*" or "*LOM*". At Raglan Mine, the long-term planning horizon extends over 15 to 20 years. The long-term planning involves several strategic decisions including, among others, the opening and closing of a mine, the choice of cut-off grade (to be explained later), the decision whether or not to exploit a lens (and, if so, when it should be exploited), the determination of the rate of production of lenses, the start of ramp and drift excavation, etc.

The cut-off grade is a concept used by mining engineers to determine if a block of material should be considered ore (so exploited and sent to the ore processing plant) or waste (rock without economic value that preferably should remain in place). To achieve this ranking, geologists divide the deposit surrounding each lens in cubic blocks of 125 m^3 (i.e. $5m \times 5m \times 5m$). Based on the data of exploration drilling, a geostatistical approach is used to estimate the various geomechanical and geochemical characteristics of each block, among them the ore grade. Thereafter, all blocks with a grade greater than or equal to the cut-off grade will be considered as a block of ore, while other blocks will be waste blocks. For the Raglan Mine deposit, given that the blocks of higher grade are at the center and at the bottom of lenses and those of lower grade are at the periphery, the choice of the cut-off grade will impact the morphology of lenses. When the cut-off grade increases, the average grade of the lens also increases, but the tonnage of the latter decreases.

Usually, the selection of the cut-off grade and strategic planning are performed sequentially. The value of the cut-off grade is first selected and then the design of the mine and the long-term planning are carried out according to this cut-off grade. In this paper, we propose a mixed-integer programming (MIP) model for solving the strategic planning at Raglan Mine. This model takes into account all the operational constraints of the mine deemed essential for long-term planning. In addition, the MIP model let the optimizer determine the best cut-off grade for each lens to maximize the net present value of profit.

This paper is organized as follows. In Section 2, the details of the mining methods and the development of the operations in Raglan Mine are explained. Section 3 describes all the elements of the mathematical model devised to build the long-term planning. In Section 4, some strategies which are used to accelerate the resolution of the mathematical model are presented. The computational experiments done for each strategy are shown in Section 5.

2 Mining Methods and Development

Operations of underground mines can be divided into two groups of activities: development activities and production activities. The details of the activities in each group vary greatly from one underground mine to another. In this paper, we describe only the activities encountered at the Raglan Mine. The explanation of these activities is essential to better understand the description of the mathematical formulation which will follow in the next section.

To access the lenses, engineers must excavate ramps, also called inclines, and drifts, which are horizontal cuts that allow access to the lenses from the ramp. These excavation activities are carried out in areas of waste rock, yielding no income, and they generate considerable expenses. However, these development activities are essential to exploit the lenses. Development activities are divided into two categories: the development

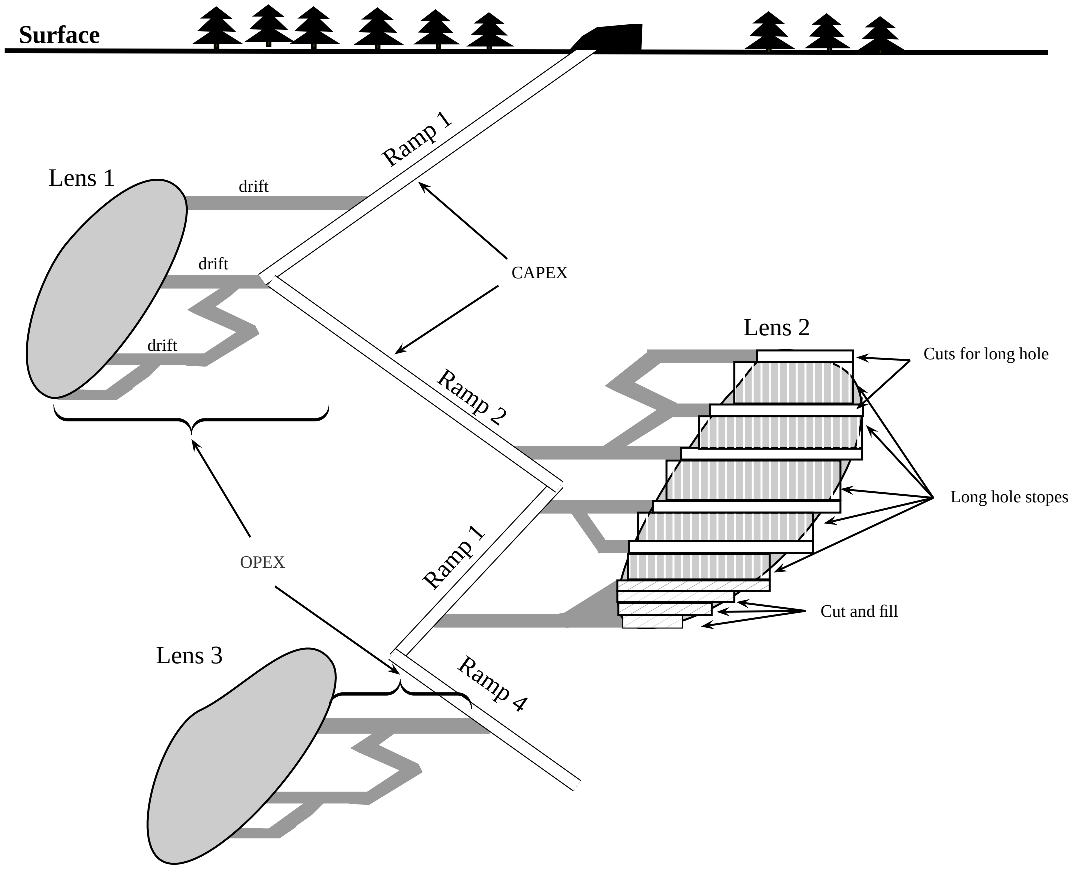

Figure 1: Mining operations

of common infrastructure (CAPEX, which stands for **CAP**ital **EX**penditure) and operational development (OPEX, which stands for **OP**erationnal **EX**penditure). CAPEX activities consist of infrastructure that will be used throughout the life of the mine while OPEX activities consist of the development of drifts giving access to lenses from the main ramp. If the ore of a lens is not extracted, the OPEX development associated with this lens will not be made. The number of meters of OPEX development done for a lens depends on the size of the lens. Indeed, a lens with a high tonnage requires more OPEX development. Both types of development are shown in Figure 1.

Several mining methods can be encountered in underground mines. The choice of the mining method depends mainly on the mechanical properties of the bedrock. At Raglan Mine, two mining methods are used to extract the ore from lenses: the cut and fill method and the long hole method. Cut and fill mining is a selective method of mining in which horizontal cuts (slices of ore) are removed advancing upwards. Upon completion, the cut is filled back to the access ramp with backfill material, which is often composed of waste material excavated from ramps and drifts. Another drift is driven on top of the filled cut. This process continues until the top of the stope is reached. This method is shown at the bottom of lens 2 in Figure 1.

The long hole method consists first of drilling horizontal cuts through the ore (see cuts for long hole in Figure 1). These cuts separate the orebody into sublevels and each sublevel is divided into stopes. These cuts, which are used to enable the miner to blast off vertical slabs of ore, are similar to those done by the cut and fill method and, for this reason, they are considered in the MIP model as being part of cut and fill activities. Using these cuts, the miner drills a set of parallel vertical holes, with a height equal to one sublevel, and fills them with explosives. Blasted slices of ore run into an open void within the open stope and are mucked out with loading equipment to the surface using ramps. Stopes are normally backfilled after they are mined out. Between stopes, ore sections are set aside for pillars to support the hanging wall. These pillars are normally shaped as vertical beams across the orebody. This method of mining is very cheap compare to cut and fill mining because it facilitates the extraction of high volumes of ore in a shorter period of time and with less equipments.

At Raglan Mine, the cut and fill method is used at the bottom of the lens, when the dip of the lens is low and prevents the use of the long hole method. A small proportion of each lens is operated by this technique, the rest of the lens is operated by long hole method. The proportion between the two methods varies from one lens to another. It depends on the characteristics of the lens and also on the selected cut-off grade. It is important to know the proportion of each lens to be operated by each mining method given that their production rates differ greatly: the long hole mining method has a much higher production rate than the cut and fill mining method.

Mining seldom recovers all resources present in an ore deposit. The amount of ore actually extracted from a deposit is referred to as the mining recovery rate and is expressed as a percentage. In addition, a certain amount of waste is usually mixed in with the ore during mining. Waste mixed in with ore is called dilution and is usually expressed as the dilution factor. Finally, the amount of nickel in the ore cannot be recovered at 100% at the processing plant. A recovery factor at the processing plant should also be considered to more accurately estimate the amount of extracted nickel. The mining recovery rate, the percentage of dilution, and the processing plant recovery rate that we used in our MIP model were those provided by the engineering team at Raglan Mine.

As mentioned before, the stopes must be backfilled. This is because when a stope is excavated, the void left by the excavation creates pressure on the rock that surrounds it and this may cause subsidence. To do the backfilling, the engineers use waste rock from the excavation of ramps and drifts as backfill material. During a year, if more waste is extracted than ore, the surplus that is not used for backfilling stopes must be transported to the surface. Conversely, if more ore was extracted in a year, waste material stored on surface will have to be descended into the mine to compensate for the lack of backfill material. Both operations raise operational costs, and these costs must be considered in the MIP model for each year of the planning.

Mining operations involve some precedence constraints between activities. For example, in Figure 1, ramp 1 must be done before ramp 2, and ramp 2 before ramp 3, etc. The OPEX development of lens 1 can be done only if ramp 1 is constructed. Moreover, the ore extraction by the cut and fill mining method can begin only if a proportion of OPEX was performed. As shown in Figure 1, it is not necessary for all OPEX development to be performed before starting the extraction of ore by the cut and fill method. Similarly, the extraction of ore by the long hole mining method may begin if the development of cuts for long hole was made. To allow the overlapping of certain mining activities with certain development activities, in the MIP model, OPEX development, cut and fill and long hole mining activities of each lens were split into two parts. This separation reflects the percentage of OPEX development needed before starting mining by the cut and fill method, and similarly the percentage of the cut and fill method (which, as stated before, includes the the long hole cuts) that must be done before beginning long hole mining activities. However, there is no precedence constraints between the lens. For example in Figure 1, lens 2 can be extracted before lens 1.

3 Mixed Integer Programming Model

Few authors have dealt with the scheduling problem in underground mines. As mentioned in Newman et al. (2010), the main reason for this is the wide variety of operating underground methods. However in recent years, several papers dealing with long-term planning in underground mines have been presented in the literature. Among them Carlyle and Eaves (2001), Menabde et al. (2004), Nehring and Topal (2007), Riff et al. (2009), and Epstein et al. (2012) propose models that maximize the net present value. Due to the complexity of solving this problem optimally, these models are either based on an integer linear programming model in which tasks are often aggregated or grouped together to reduce the number of variables and constraints and to facilitate the resolution, or are based on metaheuristics. In these papers, the cut-off grade is fixed to a specific value for the entire deposit. Some authors, Horsley (2005), and Wang et al. (2008), have sought to optimize the cut-off grade using a pre-established mining sequence. In this paper, we try to deal with both aspects simultaneously: looking for an optimal mining sequence and determining the optimal cut-off grade for each lens.

In this section, we describe the mixed-integer program (MIP) used to solve the problem of long-term planning. First, we define the sets, parameters and variables used. Then, the objective function and the constraints of the model are detailed.

3.1 Sets, Parameters and Variables

Based on the description given in the previous section, we can define sets for the elements of the operation, as shown in Table 1. It is important to notice that the set for the OPEX developments is not defined, given that each OPEX is strictly associated with a lens. Furthermore, for the definition of the cut-off grades, each lens can be defined by two curves: a first one that consists of the average grade vs the cut-off grade and a second one that consists of the tonnage of the lens vs the cut-off grade. These non-linear functions could be approximated by a set of tangent lines. However, the available information provided by these curves does not allow for such precision. For this reason, the functions have been discretized and only certain cut-off grades are considered in the set $\mathcal{G}$. Lastly, we also define some subsets, which are used to facilitate the definition of the model.

Table 1: List of Sets

Set	*Definition*
$\mathcal{M}$	Mines
$\mathcal{R}$	CAPEX Development Segments
$\mathcal{L}$	Lenses
$\mathcal{T}$	Periods
$\mathcal{G}$	Cut-off Grades
Γ_j	All lenses which the access requires the development of CAPEX $j \in \mathcal{R}$.
Ω_j	All CAPEX developments which the access requires the development of CAPEX $j \in \mathcal{R}$.
Θ_i	All CAPEX developments on the path from the surface to lens $i \in \mathcal{L}$.
Δ_m	All CAPEX developments belonging to mine $m \in \mathcal{M}$.
Π_m	All lenses belonging to mine $m \in \mathcal{M}$.

For each activity in the operation, we define a superscript to distinguish the parameters and the variables in the MIP model, which may have the same definition, but belong to different activities. These superscripts are presented in Table 2.

Table 2: List of Superscripts

Superscript	*Activity*
R	Capex
O	Opex
C	Cut and Fill
L	Long Hole

Given the large number of parameters in the MIP model, they are presented in Table 3 grouped by type of activity.

Table 3: List of Parameters

Name	*Definition*
L_j^R	Length (meters) of CAPEX segment $j \in \mathcal{R}$.
V_j^R	Volume (m^3) of waste rock contained in CAPEX segment $j \in \mathcal{R}$.
C_j^R	Cost (\$) to excavate CAPEX segment $j \in \mathcal{R}$.
L_{ikg}^O	Length (meters) of the k^{th} part of operation development associated with lens $i \in \mathcal{L}$ based on a cut-off grade $g \in \mathcal{G}$.
V_{ikg}^O	Volume (m^3) of the k^{th} part of operation development associated with lens $i \in \mathcal{L}$ based on a cut-off grade $g \in \mathcal{G}$.
C_{ikg}^O	Cost (\$/tons) of the k^{th} part of operation development associated with lens $i \in \mathcal{L}$ based on a cut-off grade $g \in \mathcal{G}$.
T_{ikg}^C	Mass (tons) of ore contained in the k^{th} part of lens $i \in \mathcal{L}$ mined by cut and fill, based on a cut-off grade $g \in \mathcal{G}$.
N_{ikg}^C	Mass (tons) of nickel contained in the k^{th} part of lens i mined by cut and fill, based on a cut-off grade $g \in \mathcal{G}$.
V_{ikg}^C	Volume (m^3) of ore contained in the k^{th} part of lens $i \in \mathcal{L}$ mined by cut and fill, based on a cut-off grade $g \in \mathcal{G}$.
L_{ikg}^C	Equivalent length (m) of ore contained in the k^{th} part of lens $i \in \mathcal{L}$ mined by cut and fill, based on a cut-off grade $g \in \mathcal{G}$.
C_{ikg}^C	Extraction cost (\$/tons) in the k^{th} part of lens $i \in \mathcal{L}$ mined by cut and fill, based on a cut-off grade $g \in \mathcal{G}$.
T_{ikg}^L	Mass (tons) of ore contained in the k^{th} part of lens $i \in \mathcal{L}$ mined by long hole, based on a cut-off grade $g \in \mathcal{G}$.
N_{ikg}^L	Mass (tons) of nickel contained in the k^{th} part of lens $i \in \mathcal{L}$ mined by long hole, based on a cut-off grade $g \in \mathcal{G}$.
V_{ikg}^L	Volume (m^3) of ore contained in the k^{th} part of lens $i \in \mathcal{L}$ mined by long hole, based on a cut-off grade $g \in \mathcal{G}$.
C_{ikg}^L	Extraction cost (\$/tons) in the k^{th} part of lens $i \in \mathcal{L}$ mined by long hole, based on a cut-off grade $g \in \mathcal{G}$.
Q_{ig}^L	Maximum annual mass of ore (tons/year) that can be extracted by long hole mining method in lens $i \in \mathcal{L}$, based on a cut-off grade $g \in \mathcal{G}$.
I_m	Initial investment (\$) needed to start production in mine $m \in \mathcal{M}$.
K_m	Investment (\$) needed to close mine $m \in \mathcal{M}$.
C_m	Maintenance cost (\$/year) to keep mine $m \in \mathcal{M}$ opened.
T_t	Maximal mass of ore (tons) that can be extracted at period $t \in \mathcal{T}$.
N_t	Maximal mass of nickel (tons) that can be treated at period $t \in \mathcal{T}$.
L_t	Maximal length (meters) of development that be done during period $t \in \mathcal{T}$.
LM	Maximal length (m) of development that can be done in a year.
f_t	Discount factor at time $t \in \mathcal{T}$.
S_t	Selling price (\$/tons) of nickel at time $t \in \mathcal{T}$.
CM	Treatment cost at the processing plant (\$/tons).
U	Recovery factor at the processing plant (%).
F	Swelling coefficient of waste rock.
R	Loading factor of excavated lens.
CS	Transportation costs (\$/m^3) of surplus waste material.

It is important to better describe some of the parameters used. Firstly, the factor f^t is used to discount the cash flows from the net present value. This value can be calculated as $f^t = (1+d)^t$, where d is the discount rate. Furthermore, the selling price of nickel S_t, the treatment cost CM and the recovery factor U at the processing plant can be used to calculate the revenue per ton of metal as $R_t = (S_t - CM)U$. Notice that the mining recovery rate and the dilution factor are not specified because they are already included into the parameters T_{ikg}^C, T_{ikg}^L, N_{ikg}^C, and N_{ikg}^L. Finally, the swelling coefficient of waste rock F and the loading factor of an excavated lens R must be considered for backfilling and the cost of transporting the surplus or the deficit of material is given by the parameter CS.

The variables can be separated in two groups, the binary variables and the continuous variables. Most binary variables in the MIP model are used to define the beginning of each activity, with the only exception being the binary variables COG used to select the cut-off grade for each lens.

$y^R_{jt} =$ 1 if the development of CAPEX segment $j \in \mathcal{R}$ starts at time $t \in \mathcal{T}$, 0 otherwise.
$y^O_{it} =$ 1 if the OPEX development for lens $i \in \mathcal{L}$ starts at time $t \in \mathcal{T}$, 0 otherwise.
$y^C_{it} =$ 1 if the cut and fill mining method of lens $i \in \mathcal{L}$ starts at time $t \in \mathcal{T}$, 0 otherwise.
$y^L_{ikt} =$ 1 if the k^{th} part of long hole mining method of lens $i \in \mathcal{L}$ starts at time $t \in \mathcal{T}$, 0 otherwise.
$FM_{mt} =$ 1 if mine $m \in \mathcal{M}$ is closed at time $t \in \mathcal{T}$, 0 otherwise.
$COG_{ig} =$ 1 if cut-off grade $g \in \mathcal{G}$ is used for lens $i \in \mathcal{L}$, 0 otherwise.

It is important to note that there is no need to define variables that represent the opening of mines. This is because a mine $m \in \mathcal{M}$ is always started by a single CAPEX development, which is represented by the parameter ψ_m. For this reason, the opening of a mine can be expressed by the variable y^R_{jt} associated with the CAPEX development $j = \psi_m$.

Besides the binary variables, the MIP model also includes continuous variables x to identify the proportion of activities carried out at time $t \in \mathcal{T}$ and variables MS and DS to represent the volume of waste material transported in and out of the mines, respectively. These variables are defined as follows.

$x^R_{jt} =$ Fraction (%) of CAPEX segment $j \in \mathcal{R}$ completed at time $t \in \mathcal{T}$.
$x^O_{iktg} =$ Fraction (%) of the k^{th} part of the operational development associated with lens $i \in \mathcal{L}$ completed at time $t \in \mathcal{T}$, if the the cut-off grade is $g \in \mathcal{G}$.
$x^C_{iktg} =$ Fraction (%) of the k^{th} part of the cut and fill method in lens $i \in \mathcal{L}$ completed at time $t \in \mathcal{T}$, if the the cut-off grade is $g \in \mathcal{G}$.
$x^L_{iktg} =$ Fraction (%) of the k^{th} part of the long hole method in lens $i \in \mathcal{L}$ completed at time $t \in \mathcal{T}$, if the the cut-off grade is $g \in \mathcal{G}$.
$MS_{mt} =$ Volume (m^3) of waste material from mine $m \in \mathcal{M}$ to be stored at the surface at time $t \in \mathcal{T}$.
$DS_{mt} =$ Volume (m^3) of waste to be descended from the surface to backfill a stope of mine $m \in \mathcal{M}$ at time $t \in \mathcal{T}$.

3.2 Objective Function

The objective is to maximize the net present value of Raglan Mine over a fixed planning horizon. To do this, the objective function is divided into several parts. Part 1 shows the income obtained from the sale of nickel. The amount of nickel reflects the percentages of dilution and recovery. Parts 2 and 3 show the total cost of both developments and the sum of the production costs of the two operating methods. The costs of opening and closing a mine are included in Part 4, while fixed costs to maintain a mine open are in Part 5. Finally, the costs of additional handling for backfill material (waste material) are represented by Part 6.

$$\max Z = \sum_{\forall t \in \mathcal{T}} f^t \Bigg\{ \overbrace{R_t \sum_{\forall i \in \mathcal{L}} \sum_{\forall g \in \mathcal{G}} \sum_{k=1}^{2} (N^C_{ikg} x^C_{iktg} + N^L_{ikg} x^L_{iktg})}^{\text{Part 1}}$$
$$\overbrace{- \sum_{\forall j \in \mathcal{R}} C^R_j x^R_{jt} - \sum_{\forall i \in \mathcal{L}} \sum_{\forall g \in \mathcal{G}} \sum_{k=1}^{2} C^O_{ikg} x^O_{iktg}}^{\text{Part 2}}$$
$$\overbrace{- \sum_{\forall i \in \mathcal{L}} \sum_{\forall g \in \mathcal{G}} \sum_{k=1}^{2} (C^C_{ikg} x^C_{iktg} + C^L_{ikg} x^L_{iktg})}^{\text{Part 3}}$$

$$\overbrace{-\sum_{\forall m\in\mathcal{M}}\sum_{\forall j\in\Psi_m} I_m y^R_{jt} - \sum_{\forall m\in\mathcal{M}} K_m FM_{mt}}^{\text{Part 4}}$$
$$\overbrace{-\sum_{\forall m\in\mathcal{M}} C_m(\sum_{u=1}^{t} y^R_{\psi_m u} - \sum_{u=1}^{t} FM_{mu}))}^{\text{Part 5}}$$
$$\overbrace{-\sum_{\forall m\in\mathcal{M}} CS(MS_{mt}+DS_{mt})}^{\text{Part 6}}\Bigg\} \quad (1)$$

3.3 Constraints

This subsection describes the constraints of the MIP.

3.3.1 Global Constraints

Three global constraints apply to all mines in production or in development. Constraints (2) limit the amount of ore extracted annually. This limitation is due to a provision of the *Raglan Agreement* – signed in 1995 by Société Minière Raglan du Québec (now Raglan Mine) and five Inuit partners – Makivik Corporation and Salluit and Iqaluit supported by their respective landholding corporation – indicating that the maximum annual quantity of ore handled at the processing plant must not exceed 1.32 million tons. Constraint (3) limit the amount of metal extracted per year due to the capacity of the ore processing plant. Finally, constraints (4) limit the amount of development in meters per year. These constraints are due to the limited drilling equipment and the number of working teams available on the site.

$$\sum_{\forall i\in\mathcal{L}}\sum_{\forall g\in\mathcal{G}}\sum_{k=1}^{2}(T^C_{ikg}x^C_{iktg}+T^L_{ikg}x^L_{iktg}) \le T_t \qquad \forall t\in\mathcal{T} \quad (2)$$
$$\sum_{\forall i\in\mathcal{L}}\sum_{\forall g\in\mathcal{G}}\sum_{k=1}^{2}(N^C_{ikg}x^C_{iktg}+N^L_{ikg}x^L_{iktg}) \le N_t \qquad \forall t\in\mathcal{T} \quad (3)$$
$$\sum_{\forall j\in\mathcal{R}} L^R_j x^R_{jt} + \sum_{\forall i\in\mathcal{L}}\sum_{\forall g\in\mathcal{G}}\sum_{k=1}^{2}(L^O_{ikg}x^O_{iktg}+L^C_{ikg}x^C_{iktg}) \le L_t \qquad \forall t\in\mathcal{T} \quad (4)$$

3.3.2 Beginning of Activities

Constraints (5) to (8) limit the tasks to start at most once. They are inequalities because if a lens is not economically exploitable, it will be left behind. However, this additional freedom increases the difficulty of the problem. Indeed, the number of combinations is greater when tasks are able to be performed or not.

$$\sum_{\forall t\in\mathcal{T}} y^R_{jt} \le 1 \qquad \forall j\in\mathcal{R} \quad (5)$$
$$\sum_{\forall t\in\mathcal{T}} y^O_{it} \le 1 \qquad \forall i\in\mathcal{L} \quad (6)$$
$$\sum_{\forall t\in\mathcal{T}} y^C_{it} \le 1 \qquad \forall i\in\mathcal{L} \quad (7)$$
$$\sum_{\forall t\in\mathcal{T}} y^L_{ikt} \le 1 \qquad \forall i\in\mathcal{L}, k=1,2 \quad (8)$$

3.3.3 Links between Production and Binary Variables

Constraints (9) to (14) link production variables to binary variables. If the decision is made to start an activity, the binary variable associated with this activity will have a value of 1 at time t and the continuous production variables associated with this activity can be greater than 0 during all periods $u \geq t$.

$$\sum_{u=1}^{t} x_{ju}^{R} \leq \sum_{u=1}^{t} y_{ju}^{R} \qquad \forall j \in \mathcal{R}, \forall t \in \mathcal{T} \tag{9}$$

$$\sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i1ug}^{O} \leq \sum_{u=1}^{t} y_{iu}^{O} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{10}$$

$$\sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i2ug}^{O} \leq \sum_{u=1}^{t} y_{iu}^{C} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{11}$$

$$\sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i1ug}^{C} \leq \sum_{u=1}^{t} y_{iu}^{C} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{12}$$

$$\sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i2ug}^{C} \leq \sum_{u=1}^{t} y_{i1u}^{L} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{13}$$

$$\sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{ikug}^{L} \leq \sum_{u=1}^{t} y_{iku}^{L} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T}, k = 1, 2 \tag{14}$$

3.3.4 Precedence Constraints

Constraints (15) to (21) represent the precedence between different activities. Constraints (15) ensure that a mine can only be closed if it was opened before. Constraints (16) ensure that a CAPEX development $j \in \mathcal{R}$ can only begin if all preceding CAPEX developments are completely built. Analogously, constraints (17) ensure that the OPEX development of a lens $i \in \mathcal{L}$ can only begin if all preceding CAPEX developments are completely built. Constraints (18) ensure that the cut and fill production for a lens $i \in \mathcal{L}$ can only begin after the conclusion of the first part of the associated OPEX development. Constraints (19) and (20) ensure that the first part of the long hole production for a lens $i \in \mathcal{L}$ can only begin after the conclusion of the first part of the cut and fill production and the second part of the OPEX development. Finally, constraints (21) ensure that the second part of the long hole production for a lens $i \in \mathcal{L}$ can only begin after the conclusion of the second part of the cut and fill production.

$$\sum_{u=1}^{t} FM_{mu} \leq \sum_{u=1}^{t} y_{\psi_m u}^{R} \qquad \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{15}$$

$$\sum_{u=1}^{t} y_{su}^{R} \leq \sum_{u=1}^{t} x_{ju}^{R} \qquad \forall j \in \mathcal{R}, \forall t \in \mathcal{T}, \forall s \in \Omega_j \tag{16}$$

$$\sum_{u=1}^{t} y_{su}^{O} \leq \sum_{u=1}^{t} x_{ju}^{R} \qquad \forall j \in \mathcal{R}, \forall t \in \mathcal{T}, \forall s \in \Gamma_i \tag{17}$$

$$\sum_{u=1}^{t} y_{iu}^{C} \leq \sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i1ug}^{O} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{18}$$

$$\sum_{u=1}^{t} y_{i1u}^{L} \leq \sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i1ug}^{C} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{19}$$

$$\sum_{u=1}^{t} y_{i1u}^{L} \leq \sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i2ug}^{O} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{20}$$

$$\sum_{u=1}^{t} y_{i2u}^{L} \leq \sum_{\forall g \in \mathcal{G}} \sum_{u=1}^{t} x_{i2ug}^{C} \qquad \forall i \in \mathcal{L}, \forall t \in \mathcal{T} \tag{21}$$

3.3.5 Constraints on Successive Tasks

Constraints (22) to (24) check for each year that the time required to perform successive tasks do not exceed one year. Since there is an unique path from the surface to any activity in the mine, all paths can be represented as a tree where the root node is the first ramp ψ_m, the nodes are the activities and the leaf nodes are the second part of the activities associated with the lenses (OPEX, cut and fill and long hole). Thus, constraints (22) are the restrictions for the paths ending in an OPEX development, constraints (23) are the restrictions for the paths ending in a cut and fill extraction and constraints (24) are the restrictions for the paths ending in a long hole extraction.

$$\sum_{j\in\Theta_i}(L_j^R/LM)x_{jt}^R + \sum_{\forall g\in\mathcal{G}}\sum_{k=1}^{2}(L_{ikg}^O/LM)x_{iktg}^O \le 1 \qquad \forall i\in\mathcal{L},\forall t\in\mathcal{T} \tag{22}$$

$$\sum_{j\in\Theta_i}(L_j^R/LM)x_{jt}^R + \sum_{\forall g\in\mathcal{G}}(L_{i1g}^O/LM)x_{i1tg}^O + \sum_{\forall g\in\mathcal{G}}\sum_{k=1}^{2}L_{ikg}^C/LM)x_{iktg}^C \le 1 \qquad \forall i\in\mathcal{L},\forall t\in\mathcal{T} \tag{23}$$

$$\sum_{j\in\Theta_i}(L_j^R/LM)x_{jt}^R + \sum_{\forall g\in\mathcal{G}}(L_{i1g}^O/LM)x_{i1tg}^O + \sum_{\forall g\in\mathcal{G}}(L_{i1g}^C/LM)x_{i1tg}^C + \sum_{\forall g\in\mathcal{G}}\sum_{k=1}^{2}(T_{ikg}^L/Q_{ig}^L)x_{iktg}^L \le 1 \qquad \forall i\in\mathcal{L},\forall t\in\mathcal{T} \tag{24}$$

3.3.6 Closure of a Mine

Constraints (25) ensure that each mine closes once. If these constraints were not present, the model could temporarily cease operations at a mine and return to it later. This option is possible in practice, but it is not part of the mandate. In this situation, other costs would be required. Indeed, temporarily closing a mine does not eliminate all fixed costs. A cost of reopening should also be added. In this paper, if the decision to close a mine is taken, then all the activities of this mine must stop. This behavior is obtained using by adding Constraints (26) to (29).

$$\sum_{\forall t\in\mathcal{T}} FM_{mt} \le 1 \qquad \forall m\in\mathcal{M} \tag{25}$$

$$x_{jt}^R \le 1 - \sum_{u=1}^{t} FM_{mu} \qquad \forall m\in\mathcal{M},\forall j\in\Pi_m,\forall t\in\mathcal{T} \tag{26}$$

$$\sum_{\forall g\in\mathcal{G}} x_{iktg}^O \le 1 - \sum_{u=1}^{t} FM_{mu} \qquad \forall m\in\mathcal{M},\forall i\in\Pi_m,\forall t\in\mathcal{T},k=1,2 \tag{27}$$

$$\sum_{\forall g\in\mathcal{G}} x_{iktg}^C \le 1 - \sum_{u=1}^{t} FM_{mu} \qquad \forall m\in\mathcal{M},\forall i\in\Pi_m,\forall t\in\mathcal{T},k=1,2 \tag{28}$$

$$\sum_{\forall g\in\mathcal{G}} x_{iktg}^L \le 1 - \sum_{u=1}^{t} FM_{mu} \qquad \forall m\in\mathcal{M},\forall i\in\Pi_m,\forall t\in\mathcal{T},k=1,2 \tag{29}$$

It is important to notice that there is no need to add to the MIP model constraints regarding the opening of a mine. This is because all the production variables x are bound to the first CAPEX development of each mine by the constraints described in Sections 3.3.3 and 3.3.4.

3.3.7 Cut-off Grades

Constraints (30) ensure that only one cut-off grade is selected per lens. Unlike previous constraints, it is an equality constraint. This restriction does not change the solution since the lens can be assigned a cut-off without being exploited. However, it slightly facilitates resolution of MIP. Furthermore, constraints (31) to (33) bind production variables with the cut-off grade. In a first version of the mathematical formulation, instead using the COG variables, there were binary variables to choose when to start an activity with a given cut-off grade. With the inclusion of the COG variables and by using the binding constraints, the number of binary variables was reduced by about 30% for an instance of 20 lenses, 7 cut-off grades and a period of 15 years. This proportion is larger with the increase in the number of lenses and years. However, the number of constraints increases in the order of 10% for an instance of the mentioned size.

$$\sum_{\forall g \in \mathcal{G}} COG_{ig} = 1 \qquad \forall i \in \mathcal{L} \tag{30}$$

$$\sum_{\forall t \in \mathcal{T}} x^O_{iktg} \leq COG_{ig} \qquad \forall i \in \mathcal{L}, \forall g \in \mathcal{G}, k = 1,2 \tag{31}$$

$$\sum_{\forall t \in \mathcal{T}} x^C_{iktg} \leq COG_{ig} \qquad \forall i \in \mathcal{L}, \forall g \in \mathcal{G}, k = 1,2 \tag{32}$$

$$\sum_{\forall t \in \mathcal{T}} x^L_{iktg} \leq COG_{ig} \qquad \forall i \in \mathcal{L}, \forall g \in \mathcal{G}, k = 1,2 \tag{33}$$

3.3.8 Flow Conservation for Backfill Material

Equation (34) is a flow conservation constraint for the amount of waste material produced and used. To meet this requirement, it is necessary, for each year, that the sum of the amount of waste rock extracted from development activities and the amount of waste material transported from the surface is equal to the sum of the amount of waste material used as backfill in the stopes and the amount of waste material transported to the surface. Given that the additional quantities of waste material descended from the surface or transported to the surface generates a cost, it is impossible to obtain a positive value for these two quantities in the same year.

$$\begin{aligned} & F\{\sum_{\forall j \in \Pi_m} V^R_j x^R_{jt} + \sum_{\forall i \in \Pi_m} \sum_{\forall g \in \mathcal{G}} \sum_{k=1}^{2} V^O_{ikg} x^O_{iktg}\} + DS_{mt} \\ & -R\{\sum_{\forall i \in \Pi_m} \sum_{\forall g \in \mathcal{G}} \sum_{k=1}^{2} (V^C_{ikg} x^C_{iktg} + V^L_{ikg} x^L_{iktg})\} - MS_{mt} = 0 \qquad \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \end{aligned} \tag{34}$$

4 Acceleration Strategies

Despite the performance of solvers available on the market, the MIP model described in the previous section is still difficult to solve. As a result of this, different strategies were devised to reduce the computation time. This section describes some of the most successful we have found.

4.1 Preprocessing

The first preprocessing strategy, denoted $PP1$, uses two pre-treatment procedures to calculate the earliest and latest starts of a task. This strategy is not new and has been used in the past for the long-term planning problem in open-pit mines (see Caccetta and Hill (2003) , L'Heureux et al. (2013)). In the procedure that calculates the earliest start, only the precedence constraints between activities are considered. This procedure considers each lens to be unique and determines the earliest moment when each lens can be mined. All variables associated with the lens whose index t is lower than the earliest moment found can be set *a priori* to zero. The procedure that calculates the latest start is different from the previous procedure. Indeed, it is impossible to calculate, *a priori*, the end of the mine since the beginning is variable. However, the minimum time for a mine to be profitable (payback period) can be calculated. To identify the payback period, we must solve the MIP problem for each mine independently. The payback period is then used to set to zero some variables of the linear program. For example, if a mine becomes profitable after 3 years, the binary variable associated with that mine can be set at 0 at time $T-3$ to T, where T is the planning horizon. This is logical since the addition of a new mine must make a profit to the overall problem in the simulation time. In addition, evaluating each mine, one by one, it is possible to say whether it will be opened or not. Indeed, if one mine is not profitable, it may be removed entirely from the global model. Computation time for solving a MIP for one mine is fast because there is usually few combinations of lenses of the same mine. The same logic and the same algorithm can be applied for lens. Thus, some variables associated with lenses can also be set to zero.

The second preprocessing strategy, denoted $PP2$, proposes adding links between binary variables associated with the same lens. In the exploitation a lens, there is an advantage that the sequence of activities related to it (the development of drifts, the beginning of the cut and fill method, the beginning of the long hole method) be carried out without interruption. By adding a time limit on the execution of successive tasks, several binary variables become bound. This then promotes the production of a faster integer solution since decisions in the branching tree indirectly affect other variables. Unlike the first strategy, the latter strategy eliminates feasible solutions, i.e. solutions allowing delay between activities of the same lens. However, in practice, the feasible solutions that have been removed are of less interest because it is advantageous to complete the operation of a lens as soon as possible when the extraction of ore has began. Indeed, allowing a half-excavated lens to continue to be extracted later presents risks for the safety of miners since open stopes decrease the strength of the bedrock.

Looking at the solution of the linear relaxation of the problem, it has been identified that the variables associated with the opening of a mine were very fractional and the opening of mines was spread over several years, since this activity requires important infrastructure costs. To avoid, in part, this spread, we add a constraint of imposing the opening of at least one mine in the first two years of the planning horizon. This preprocessing strategy, denoted $PP3$, will reduce the gap between the value of the linear relaxation and the value of the integer solution by setting some variables to 1 at the beginning of the planning horizon.

4.2 Local Search in the MIP

The local search implemented in this work is different from most heuristic approaches, since it is done directly in the MIP formulation. Firstly, the original formulation is solved until a given stopping criterion is reached. Afterwards, an iteratively procedure is started which, at each iteration restrict the binary variables based in the current integer solution. This restriction is included directly in the MIP formulation as follows. Given the current integer solution and an offset parameter o, if in the solution an activity starts in year t^*, all binary variables associated with this activity which have $t < t^* - o$ or $t > t^* + o$ are set to zero. This is done for all activities and it leads to a constraint in the formulation, implying that the activities in subsequent solutions may only change o years to the past or to the future. These restrictions are also added to the cutoff grade binary variables COG_{ig}, but in this case the index g is considered instead of t and the offset parameter is always 1. The MIP formulation is then solved again and, as soon as a new improving integer solution is found, the resolution stops. The old restrictions are all dropped and a complete new set of restrictions are derived from the new current integer solution. The heuristic repeats this process until no new improving integer solution is found.

This heuristic tries to obtain good quality solutions by speeding up the resolution of the mathematical model by restricting its solution space and also by guiding the solutions found using the restrictions mentioned previously. The drawback of this approach is that after some iterations, the heuristic may fall into a local optima. In this case, the heuristic would finish with a bad quality solution.

4.3 A Constructive Heuristic

The constructive heuristic is a simple heuristic which aims to quickly find a good, feasible solution. Afterwards, the solution produced by this heuristic can be used as a hot-start for the formulation or for the local search heuristic. The idea in this heuristic is that there is a natural ordering for the activities associated with a lens which attempts to maximize the profit of the lens. Based on the precedence constraints, this ordering would be to start with the capex(es) development(s) needed to reach the lens, then build the first part of the opex development. Notice that in this moment, the first part of the cut and fill production can be started.

Then, the second part of the opex development must be done to allow the start of the first part of the long hole process. Finally, the second part of the cut and fill must be done, which will allow the second part of the long hole to start.

The heuristic starts building one scenario for each available lens and possible cutoff grade. On each scenario, the activities of the selected lens are built as soon as possible in the planing horizon using the natural order, as described. Next, the heuristic chooses the scenario that is able to obtain the best profit. This scenario is then considered to be the current solution and the lens used is no longer available for the subsequent iterations. This procedure is repeated until there is no more lens available.

Furthermore, at the end of the building procedure, the heuristic tries to move the non-profitable activities to any period in the future. This can lead to a better solution because the discount factor f^t makes the costs smaller in future periods. In an attempt to maximize the reduction of the costs, the non-profitable activities of higher periods are considered the first to be moved.

4.4 Sliding Time Windows Heuristic

The idea behind the sliding time windows heuristic is to consider a window with a small amount of years. All of the variables that are not part of the time window are fixed to zero. Then, the formulation is solved iteratively. For example, if a time window of three years is being considered, the first iteration of the heuristic will consider only variables for which $1 \leq t \leq 3$. For the next iteration, the time window will move one year and the heuristic will consider the variables for which $2 \leq t \leq 4$. Moreover, all binary variables for which $t = 1$ will be fixed to the last solution. This operation is repeated until the time window reaches the last time period in $\mathcal{T}$.

This heuristic relies on the hypothesis that the production done for a given year is not strongly dependent of the production done for several years in the future. When this is true, good quality solutions can be found using a small time window. In this case, the resolution of each iteration of the heuristic is fast and the overall computational time is much lower than solving the complete formulation. However, for large instances, a small-sized time window is not enough to reflect the behavior of the future productions. To deal with this issue, instead of enlarging the time window to a size that would threaten the efficiency of the heuristic, another type of time window is introduced. The second time window is put just after the original one and considers all the variables as being continuous variables, i.e., the binary variables are relaxed. This time window is called the *continuous window*, in contrast with the original time window, which could be called the *integer window*. An illustration of the heuristic is shown in Figure 2, which shows the third iteration of the heuristic considering an integer window of size 3 and a continuous window of size 4 for an instance with 12 periods.

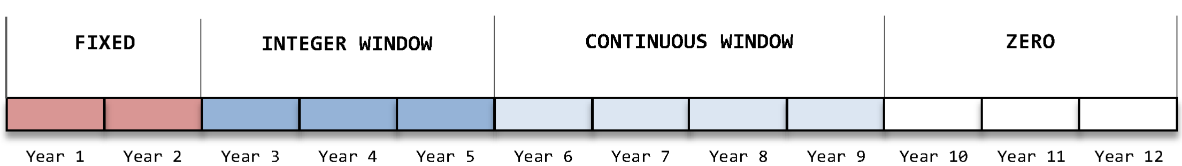

Figure 2: Third iteration of the sliding time windows heuristic with integer window = 3 and continuous window = 4 for an instance with 12 periods

4.5 Mine Sequencing

The last acceleration strategy, called mine sequencing, is simple and intuitive. Indeed, it consists of adding mines sequentially to the MIP model. The model starts with a single mine. In this case, the solver finds the best sequence quickly because there are few combinations of lenses of the same mine. When the optimal sequence is obtained, the binary variables of the first k years are then fixed and the other ones stay relaxed. The variables of a new mine are then added to the model. The formulation is once again resolved. These steps are repeated until there is no longer a mine. The choice of the processing order of mines can be done in different ways. Since the maximum net present value for each mine can be easily calculated, we opted for a sequence where the mines are sorted in decreasing order of their net present value.

5 Computational Experiments

In this section, we present the computational results when solving the problem with different strategies. The tests are performed on instances of three and five mines. The instance with 3 mines contains 20 lenses and 15 time periods. The one with 5 mines contains 47 lenses and 20 time periods. Moreover, CPLEX 12.5 is used to solve the integer formulation. For all approaches, a time limit of 2 hours (7200 seconds) has also been added.

For reasons of confidentiality of company revenues, the value of the solution of the basic MIP model for the problem with 3 mines serves as milestone. The net present value for this instance has been rescaled to 1000. The net present value for all other results were adjusted on this scale. The gaps shown represent the difference between the value of the known upper bound and the solution found by each approach. The optimal solution for the instance with 3 mines is known and can be obtained just running CPLEX, as we will further demonstrate. In contrast, solving the 5 mines instance is a difficult task and the best solution we have comes from a long run of the MIP model on CPLEX using the preprocessing strategy $PP1$ from Section 4.1. During this run, the best integer solution was found after more than 1.6 days holding a value of 2075.6. Furthermore, we allowed the run to take more than 7 days, on which CPLEX was not able to find any other integer solution, but obtaining a final upper bound of 2095.4, with 0.95% of gap.

First, we show the results for the base MIP model and for the preprocessing strategies in Table 4. The results for the instance with three mines show that the computation time is reduced by the addition of strategies $PP1$ and $PP3$. Furthermore, the second strategy reduces the computation time by a factor of about 5 when compared to strategy $PP3$, but the net present value decreases by about 0.4%. However, this decrease is negligible. The results for the instance with the five mines show that the preprocessing strategies have almost no effect on the computational time. The third strategy reduces the gap but this reduction is not sufficient enough to reduce the computational time.

Table 4: Results for the preprocessing

	3 mines			5 mines		
Model	Solution	Gap	Time	Solution	Gap	Time
Base	1000.0	0.00	5864	1917.8	8.47	7200
PP1	1000.0	0.00	3981	1932.0	7.79	7200
PP2	996.1	0.39	375	1940.0	7.41	7200
PP3	1000.0	0.00	1817	1975.5	5.72	7200

Table 5 illustrates the results for the local search heuristic for the instances with three and five mines. The results are shown for the first iteration and for all iterations of the heuristic. Furthermore, the preprocessing strategy $PP1$ is used and a time limit of 7200 seconds is set for the heuristic. Note that this heuristic was able to improve the results for both instances, especially for the five mines instance, where the improvement obtained is noticeable. The best values found for each part of the test are underlined.

Table 5: Results for the Local Search

	Offset	1		2		3		4		5	
Mines	Iter	Gap	Time	Gap	Time	Gap	Time	Gap	Time	Gap	Time
3	first	2.31	44	2.31	44	2.31	44	2.31	44	2.31	44
	all	0.03	63	0.00	251	0.00	341	0.00	413	0.00	398
5	first	18.67	601	11.25	572	16.77	490	24.90	674	15.42	525
	all	9.73	7200	2.79	7200	5.49	7200	14.42	7200	5.96	7200

Table 6 illustrates the results obtained when using the constructive heuristic as a hot-start for the local search heuristic (with an offset of 1) and for the MIP formulation (with $PP1$). For the local search test, we were expecting a good improvement on the results found. However, no improvement was found at all. On the other hand, the results found when running the MIP formulation are significantly better.

Table 6: Results for the Constructive Heuristic

	Offset	1		2		3		4		5	
Mines	Iter	Gap	Time	Gap	Time	Gap	Time	Gap	Time	Gap	Time
3	heur	0.36	5	1.20	27	<u>0.04</u>	16	3.97	19	4.57	18
	ls	0.00	1740	0.00	2765	0.00	1072	<u>0.00</u>	420	0.00	1360
	cplex	0.00	620	0.00	2670	<u>0.00</u>	323	0.00	679	0.00	530
5	heur	8.24	17	<u>5.65</u>	34	8.92	50	7.13	62	10.20	82
	ls	7.49	7200	5.47	7200	5.44	7200	7.13	7200	<u>4.90</u>	7200
	cplex	3.68	7200	1.51	7200	<u>1.16</u>	7200	1.78	7200	1.17	7200

Tables 7 and 8 illustrate the results for the sliding time windows heuristic with 3 and 5 mines. After the heuristic is done, again we use the best solution as a hot-start for the local search strategy (with an offset of 1) and for the base MIP formulation (with $PP1$). Note that this heuristic could improve all the results compared to the ones found thus far. For the instance of 5 mines, the configuration using an integer window of 3 periods of the planning horizon followed by a continuous window of 4 periods was the one that obtained the best results. Notice that during the 7 days run, a value better than this one was only found after more than 10 hours. Nevertheless, based on the tests that were carried out, one cannot identify a trend in the right configuration to use.

Table 7: Results for the Sliding Time Windows heuristic with 3 mines

	cont	1		2		3		4	
int		Gap	Time	Gap	Time	Gap	Time	Gap	Time
1	heur	—	—	29.45	44	24.76	9	22.45	11
	ls	—	—	0.03	33	1.56	36	0.03	70
	cplex	—	—	0.00	230	0.00	227	0.00	244
2	heur	24.29	9	8.96	8	3.43	7	4.46	13
	ls	0.03	29	0.00	29	0.03	27	0.00	26
	cplex	0.00	226	<u>0.00</u>	206	0.00	285	0.00	319
3	heur	2.39	8	3.13	15	<u>0.25</u>	24	4.33	32
	ls	0.03	27	0.03	38	0.00	22	0.02	18
	cplex	0.00	273	0.00	260	0.00	266	0.00	304
4	heur	1.81	55	1.81	32	0.57	66	0.56	137
	ls	0.03	8	0.03	43	<u>0.00</u>	7	0.00	16
	cplex	0.00	251	0.00	251	0.00	284	0.00	411

Finally, Table 9 shows the results for the mine sequencing heuristic. Analogous to the previous heuristics, the solution obtained by this one was used as a hot-start for the local search heuristic (with an offset of 1) and for the MIP formulation (with $PP1$). However, for both instances, this strategy is not attractive compared to the previous ones.

Finally, a final test was conducted to see the gain obtained when using a variable cut-off grade. We solved the problem by using a fixed cut-off grade for all lenses. This cut-off grade was set to the value used by the Raglan Mine. The results showed a gain of approximately 4% of the NPV when the cut-off grade is variable. This gain is significant given that the long-term revenues are in the hundreds of millions of dollars; however, one would have expected a larger gain. It should be noted, however, that the cut-off grade used by the Raglan Mine was the one that is the most often attributed to lenses in the solutions obtained by our model. This indirectly confirms that the choice of cut-off grade made by the engineers of the Raglan Mine was wise.

Table 8: Results for the Sliding Time Windows heuristic with 5 mines

int	cont	1 Gap	1 Time	2 Gap	2 Time	3 Gap	3 Time	4 Gap	4 Time
1	heur	—	—	—	—	—	—	—	—
	ls	—	—	—	—	—	—	—	—
	cplex	—	—	—	—	—	—	—	—
2	heur	86.66	26	37.94	65	26.86	99	25.29	171
	ls	85.96	7200	24.09	7200	19.94	7200	9.39	7200
	cplex	1.56	7200	1.49	7200	1.60	7200	1.51	7200
3	heur	24.42	102	18.75	143	13.89	222	15.63	496
	ls	11.70	7200	6.42	7200	6.09	7200	8.59	7200
	cplex	1.44	7200	1.43	7200	1.39	7200	1.21	7200
4	heur	17.68	206	16.14	554	12.63	1367	12.56	1163
	ls	4.74	7200	4.57	7200	6.65	7200	5.75	7200
	cplex	1.59	7200	1.49	7200	1.44	7200	1.56	7200

Table 9: Results for the Mine Sequencing Heuristic

Mines		Gap	Time
3	seq	4.81	434
	ls	4.36	513
	cplex	0.00	7200
5	seq	4.14	2251
	ls	2.54	7200
	cplex	3.76	7200

6 Conclusion

We presented a MIP model for the long term planning of an underground mine with variable cutoff grade. We showed that the resolution of this model is prohibitively expensive when using a commercial library for MIP models. For this reason, some heuristic approaches were proposed to attempt to reduce the solution times. The experimental results showed that the heuristics can provide a good solution within less than one minute. Moreover, when using these solutions as a hot-start for the MIP formulation, which is solved using CPLEX, significantt improvements are obtained.

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