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American-style options in jump-diffusion models: Estimation and evaluation

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Abstract: We propose a quasi-analytical approach for valuing American-style options under Gaussian and double exponential jumps la Merton (1976) and Kou (2002). Our approach is based on dynamic programming coupled with finite elements. Finally, we perform a numerical experiments that show convergence and efficiency. We also address the estimation problem and report an empirical investigation based on Home Depot. Jump-diffusion models outperform pure-diffusion models.

Key Words: American options, jump-diffusion process, dynamic programming, calibration, maximum likelihood.

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1 Introduction

Pure diffusion models, like that of Black and Scholes (1973), fail to capture stylized facts of the financial market, such as over/under-reactions, fat tails, and discontinuities in the underlying-asset returns. One way of solving this problem is to introduce a hybrid process made up of a Gaussian continuous component and a composite-Poisson-type discontinuous component.

While Gaussian jumps go back to Merton (1976), double-exponential jumps go back to Kou (2002). Both authors use Lévy processes, extend the pure diffusion model of Black and Scholes (1973), and provide closed-form solutions for European vanilla options. We consider their extended versions in the following.

Valuing American-style options under Lévy models is challenging due to their associated early exercise strategies. Several methodologies are proposed in the literature: the quasi-analytical approach (Bates, 1991, Gukhal, 2001, Kou and Wang, 2004), the binomial tree (Amin, 1993, Hilliard and Schwartz, 2005), partial integro-differential equations (Zhang, 1997, Andersen and Andreasen, 2000, Matache et al., 2003, 2005, Cont and Voltchkova, 2005, Almendral and Oosterlee, 2005, Toivanen, 2008, Mayo, 2008, Chiarella and Ziogas, 2009, Chockalingam and Muthuraman, 2010), the Markov chain approximation (Simonato, 2011), and Monte-Carlo simulation (DiCesare and Mcleish, 2008, Levendorskii, 2004).

We propose a quasi-analytical approach for valuing American-style options in jump-diffusion models. Our approach is based on dynamic programming coupled with finite elements. The value function under consideration is approximated by a piecewise polynomial at each decision date. We experiment with piecewise-constant, linear, and quadratic approximations. Higher-order polynomials are more accurate, but are also more time consuming. A compromise between accuracy and computing time must be found. Dynamic programming, coupled with piecewise quadratic interpolations, shows the highest degree of efficiency and competes well against alternative methodologies reported in the literature.

The estimation of the parameters of the model is also challenging problem. Under jump diffusions, the distribution of the underlying-asset return is a mixture, which results in a likelihood function with several modes. Press (1967), Bates (1991), and Hanson and Zhu (2004) use a parametric approach (historical); while Bates (1991), Andersen and Andreasen (2000), He et al. (2006), and Cont and Tankov (2004b) use a non-parametric approach (calibration). Calibration consists of matching theoretical values to the market prices of liquid contracts. We consider two historical methodologies, that is, the method of cumulants and the maximum-likelihood approach. The latter outperforms the former. All in all, we use maximum likelihood to estimate Merton's (1976) and Kou's (2002) models and maximum likelihood coupled with calibration to estimate their extended versions.

The rest of this paper is organized as follows. Section 2 presents a general jump-diffusion model and outlines Merton's (1976) and Kou's (2002) settings as special cases. While Section 3 describes the dynamic program, Section 4 presents a numerical investigation. Section 5 discusses the model estimation and Section 6 reports an empirical investigation. Section 7 concludes.

2 The jump-diffusion model

Let $\mathcal{M}$ be a frictionless market with a risk-free asset and a risky asset, a stock whose price S experiences jumps at random times, and let $(\Omega, A, \mathbf{P})$ be a probability space. Define a standard Brownian motion $(W_t)_{t\geq 0}$, a Poisson process $(N_t)_{t\geq 0}$ with a constant intensity λ, and a series of independent random variables $(\xi_n)_{n\geq 1}$ with distribution ν and density $f(\cdot)$, such that $\kappa = \mathbb{E}_{\mathbb{P}}\left[e^{\xi_n} - 1\right]$ is finite. The random variables $U_n = e^{\xi_n} - 1 = (S_{\tau_n} - S_{\tau_n -})/S_{\tau_n -} \in \,]-1, \infty[$ represent the jumps' relative amplitudes at random jump times τ_n, for $n \geq 1$.

The processes W, N, and ξ are assumed to be independent. They are used to model the continuous part of the stock-price trajectory, the jump times, and the jump amplitudes, respectively. Further, denote by $\mathcal{F}_t$ the sigma-algebra generated by $\{W_u, N_u, \xi_1, \ldots, \xi_{N_t},\ \text{for } 0 \leq u \leq t\}$.

The stock-price process S is defined by

$$S_t = S_0 e^{X_t}, \quad \text{for } t \geq 0,$$

where the stock log-return X is a Lévy process (with stationary independent increments) defined by

$$X_t = \left(\mu - \bar{d} - \frac{\sigma^2}{2} - \lambda\kappa\right) t + \sigma W_t + \sum_{n=1}^{N_t} \xi_n, \quad \text{for } t \geq 0, \tag{1}$$

where μ is the instantaneous stock return, $\bar{d}$ the proportional dividend rate, σ the stock-return volatility conditional on no jumps, and N_t the number of jumps till time $t \in [0, T]$. Set the convention $\sum_{n=1}^{0} = 0$. The process X is discontinuous at jump times.

The jump-diffusion model in eq. (1) is arbitrage free but incomplete. There exist several equivalent martingale probabilities measures that provide multiple rational values for a given option contract, all of which are consistent with the no-arbitrage principle. We provide herein a simple change of probability measure.

Let $h(\cdot)$ be a positive function such that $a = \mathbb{E}_{\mathbb{P}}[h(\xi_n)] < \infty$ and define

$$\Lambda_t^{b,h} = e^{bW_t - tb^2/2 - \lambda t(a-1)} \prod_{n=1}^{N_t} h(\xi_n), \quad \text{for } t \geq 0. \tag{2}$$

The process $\Lambda^{b,h}$ is a positive $\mathbb{P}$-martingale and has an expectation of one. The risk-neutral probability measure $\mathbb{Q}^{b,h}$, associated to the constant b and the function $h(\cdot)$, is defined by the Radon-Nikodym derivative

$$\frac{d\mathbb{Q}^{b,h}}{d\mathbb{P}}|_{\mathcal{F}_t} = \Lambda_t^{b,h}.$$

From Cont and Tankov (2004a, Proposition 9.8), $\tilde{W}_t = W_t - bt$ is a Brownian motion, $\tilde{N}_t = N_t$ is a Poisson process of intensity $\tilde{\lambda} = \lambda a$, and $\tilde{\xi}_n = \xi_n$ has the law $\tilde{\nu}$ with density $\tilde{f}(\cdot) = f(\cdot)\, h(\cdot)/a$ under $\mathbb{Q}^{b,h}$. Let $\tilde{\kappa} = \mathbb{E}_{\mathbb{Q}^{b,h}}\left[e^{\tilde{\xi}_n} - 1\right] \in \mathbb{R}$. One has

$$e^{-rt} S_t + \bar{d} \int_0^t e^{-ru} S_u du$$

is a $\mathbb{Q}^{b,h}$-martingale if, and only if,

$$b = \frac{r - \mu - (\tilde{\lambda}\tilde{\kappa} - \lambda\kappa)}{\sigma},$$

where r is the risk-free rate. Given eq. (2), the dynamics of S under the martingale probability measure $\mathbb{Q} \equiv \mathbb{Q}^{b,h}$ is

$$S_t = S_0 e^{\tilde{X}_t},$$

where $\tilde{X}$ is still a Lévy process represented by

$$\tilde{X}_t = \left(r - \bar{d} - \frac{\sigma^2}{2} - \tilde{\lambda}\tilde{\kappa}\right) t + \sigma \tilde{W}_t + \sum_{n=1}^{\tilde{N}_t} \tilde{\xi}_n, \quad \text{for } t \geq 0. \tag{3}$$

It's worth noting that the process $\{e^{-rt} S_t\}$ is a $\mathbb{Q}$-martingale.

The function $h(\cdot)$ is often chosen so that the law of jumps remains in the same family, both under the objective and martingale probabilities measures. to this end, we set

$$h(x) = \alpha e^{\beta x}, \quad \text{for } x \in \mathbb{R}, \tag{4}$$

where $\alpha \in \mathbb{R}_+^*$ and $\beta \in \mathbb{R}$. See Rémillard (2013). Appendix A provides the density function $\tilde{f}(\cdot)$.

For extended Gaussian jumps, the random variables ξ_n, for $n \geq 1$, follow $\mathcal{N}(\gamma, \delta^2)$ under $\mathbb{P}$, and $\tilde{\xi}_n$ follow $\mathcal{N}(\tilde{\gamma}, \tilde{\delta}^2)$ under $\mathbb{Q}$, where $a = \alpha e^{\beta\gamma+\beta^2\delta^2/2}$, $\tilde{\lambda} = a\lambda$, $\tilde{\gamma} = \gamma + \beta\delta^2$, $\tilde{\delta} = \delta$, and $\tilde{\kappa} = e^{\tilde{\gamma}+\tilde{\delta}^2/2} - 1$. Merton's (1976) model is obtained with $\alpha = 1$ and $\beta = 0$, which results in $b = (r - \mu)/\sigma$, $a = 1$, $\tilde{\lambda} = \lambda$, $\tilde{\gamma} = \gamma$, $\tilde{\delta} = \delta$, and $\tilde{\kappa} = e^{\gamma+\delta^2/2} - 1$. To justify this particular choice of parameters, the author claims that jump risk is not systemic and can be offset through portfolio diversification.

For extended double-exponential jumps, the parameter $\beta \in \,]-\eta_2, \eta_1[$. The random variables ξ_n, for $n \geq 1$, follow an asymmetric double-exponential distribution with a density function under $\mathbb{P}$:

$$f(\xi) = p_1\ \eta_1 e^{-\eta_1 \xi}\ \mathbb{I}_{\{\xi \geq 0\}} + p_2\ \eta_2 e^{\eta_2 \xi}\ \mathbb{I}_{\{\xi < 0\}}, \quad \text{for } \eta_1 > 1 \text{ and } \eta_2 > 0,$$

where $p_1 \geq 0$ and $p_2 \geq 0$ represent the probability of an upward and a downward jump and verify $p_1 + p_2 = 1$. The parameters η_1 and η_2 represent the mean sizes of an upward and a downward jump, respectively. The distribution of the random variables $\tilde{\xi}_n$, for $n \geq 1$, remains in the same family under $\mathbb{Q}$, where $a = \alpha p_1 \eta_1 / (\eta_1 - \beta) + \alpha p_2 \eta_2/(\eta_2 + \beta)$, $\tilde{\lambda} = a\lambda$, $\tilde{\eta}_1 = \eta_1 - \beta$, $\tilde{\eta}_2 = \eta_2 + \beta$, $\tilde{p}_1 = p_1\eta_1\tilde{\eta}_2/(p_1\eta_1\tilde{\eta}_2 + p_2\tilde{\eta}_1\eta_2)$, $\tilde{p}_2 = 1 - \tilde{p}_1$, and $\tilde{\kappa} = \tilde{p}_1\tilde{\eta}_1/(\tilde{\eta}_1 - 1) + \tilde{p}_2\tilde{\eta}_2/(\tilde{\eta}_2 + 1) - 1$. Kou's (2002) model is obtained with $\alpha = 1$ and $\beta = 1$, which results in $b = (r - \mu)/\sigma$, $a = p_1\eta_1/(\eta_1 - 1) + p_2\eta_2/(\eta_2 + 1)$, $\tilde{\lambda} = \lambda a$, $\tilde{\eta}_1 = \eta_1 - 1$, $\tilde{\eta}_2 = \eta_2 + 1$, $\tilde{p}_1 = p_1\eta_1(\eta_2 + 1)/ (p_1\eta_1(\eta_2 + 1) + \eta_2 p_2(\eta_1 - 1))$, $\tilde{p}_2 = 1 - \tilde{p}_1$, and $\tilde{\kappa} = a - 1$. In support of its choice, the authors relies on the expected utility theory and shows that a rational price can be obtained via this particular change of parameters.

While Merton (1976) derives his formula directly from an arbitrage argument, Kou (2002) considers an equilibrium model, which provides option values, that are (also) consistent with the no-arbitrage principle.

3 Valuing American-style options

3.1 The option contract

An American option on a stock with a maturity T is characterized by its known exercise value $v_t^e(s)$, where $s = S_t$ is the stock price at time $t \in [0, T]$. For example, an American vanilla option is defined by

$$v_t^e(s) = \begin{cases} (s - K)^+ & \text{for a call option} \\ (K - s)^+ & \text{for a put option} \end{cases},$$

where K is the option's exercise price and $(x)^+ \equiv \max(0, x)$. We consider its Bermudan version, which admits a finite number of exercise opportunities, given by $t_0 = 0, \ldots, t_m, \ldots, t_M = T$. For simplicity, assume that $t_{m+1} - t_m = \Delta t$ is a fixed constant.

By the no-arbitrage evaluation principle, the option's holding value at t_m is

$$\begin{aligned} v_m^h(s) &= \mathbb{E}_{\mathbb{Q}}\left[e^{-r\Delta t} v_{m+1}(S_{t_{m+1}}) \,\middle|\, S_{t_m} = s\right], \\ &\quad \text{for } m = 0, \ldots, M, \end{aligned} \tag{5}$$

where $\mathbb{E}_{\mathbb{Q}}[\,\cdot\,|S_{t_m} = s]$ is the conditional expectation under $\mathbb{Q}$. Set the convention that $v_M^h(\cdot) = 0$ to say that the option must be exercised at maturity $t_M = T$. The option's overall value at time t_m is

$$v_m(s) = \max\left(v_m^e(s), v_m^h(s)\right), \quad \text{for } s > 0. \tag{6}$$

The associated European version is characterized by $v_m^e(\cdot) = 0$, for $m = 0, \ldots, M - 1$.

3.2 The dynamic program

Let $\mathcal{G} = \{a_1, \ldots, a_p\}$ be a mesh of grid points, where $0 = a_0 < a_1 < \ldots < a_p < a_{p+1} = +\infty$. The grid $\mathcal{G}$ must be selected so that $\max_{1\leq i\leq p-1} \Delta a_i$, $\mathbb{Q}(S_t < a_1)$ and $\mathbb{Q}(S_t > a_p)$ all converge to 0 as $p \to \infty$, for $t \in \{t_1, \ldots, t_M\}$. We select the grid points to be the quantiles of S_T under $\mathbb{Q}$.

Suppose that an approximation $\tilde{v}_{m+1}(\cdot)$ of the value function $v_{m+1}(\cdot)$ is available at a given future date t_{m+1} on the grid $\mathcal{G}$. This is not a strong assumption since the value function $v_M(\cdot) = v_M^e(\cdot)$ is known in closed form.

We use a piecewise polynomial and extend the approximation $\tilde{v}_{m+1}(\cdot)$ from $\mathcal{G}$ to the overall state space $\mathbb{R}_+^*$, that is,

$$\hat{v}_{m+1}(s) = \sum_{i=0}^{p} \left(\beta_i^0 + \beta_i^1\, s + \cdots + \beta_i^d\, s^d\right) \mathbb{I}(a_i \le s < a_{i+1}), \quad \text{for } s > 0, \tag{7}$$

where d is the degree of the piecewise polynomial, whose local coefficients depend on the time step $m+1$. Eq. (5) and eq. (7) result in

$$\begin{aligned} \tilde{v}_m^h(a_k) &= \mathbb{E}_{\mathbb{Q}}\left[e^{-r\Delta t}\hat{v}_{m+1}\left(S_{t_{m+1}}\right)|S_{t_m} = a_k\right] \\ &= e^{-r\Delta t}\sum_{i=0}^{p}\left(\beta_i^0\, T_{ki}^0 + \cdots + \beta_i^d\, T_{ki}^d\right), \end{aligned} \tag{8}$$

where

$$T_{ki}^{\nu} = \mathbb{E}_{\mathbb{Q}}\left[S_{t_{m+1}}^{\nu}\mathbb{I}\left(a_i \le S_{t_{m+1}} < a_{i+1}\right)|S_{t_m} = a_k\right], \quad \text{for } \nu = 0, \ldots, d.$$

For example, T_{ki}^0 is the probability that the stock price moves from a_k at t_m and visits the interval $[a_i, a_{i+1})$ at t_{m+1}. Key ingredients for the dynamic program (DP) to run are the transition tables T^{ν}, for $\nu = 0, \ldots, d$. We derive T^{ν}, for $\nu \in \{0, 1, 2\}$, in closed form under Merton's (1976) setting (see ppendix B) and Kou's (2002) setting (see Appendix C). We find that the piecewise-quadratic interpolation ($d = 2$) is the most efficient. To reach the pure American option, set Δt as small as possible. Ben-Ameur et al. (2002) use DP for valuing Asian options in the Black and Scholes' (1973) model.

The formula in eq. (8) separates the option holding value in two parts. The first is related to the dynamics of the state process (the transition parameters) and the second to the option contract (the interpolation coefficients). The transition parameters are a fixed cost for the DP procedure as long as the time step and the model's parameters remain constant. For a given experiment and a set of the model's parameters, about 90% of CPU time is used to compute the transition parameters. Thus, the effective CPU time to run a DP experiment is about 10% of what is reported in our tables. Unlike finite differences-based methods, DP needs not a time discretization since the transition parameters do respect the true dynamics of the state process. All components of eq. (8) are computed in closed form.

From eq. (6), the approximate value function at t_m is

$$\tilde{v}_m(a_k) = \max\left(v_m^e(a_k), \tilde{v}_m^h(a_k)\right), \quad \text{for } a_k \in \mathcal{G}, \tag{9}$$

and the approximate exercise policy at t_m is characterized by

$$\tilde{v}_m^h(a_k) < v_m^e(a_k), \quad \text{for } a_k \in \mathcal{G}.$$

The local coefficients of the piecewise-quadratic interpolation $\hat{v}_{m+1}(\cdot)$ in eq. (7) verify $\hat{v}_{m+1}(\cdot) = \tilde{v}_{m+1}(\cdot)$ on $\mathcal{G}$.

The dynamic program works as follows :

1. For $m = M-1$, set $\tilde{v}_{m+1}(\cdot) = v_{m+1}(\cdot) = v_{m+1}^e(.)$ on $\mathcal{G}$;
2. Use a piecewise-quadratic interpolation as in eq. (7), and extend $\tilde{v}_{m+1}(\cdot)$, defined on $\mathcal{G}$, to $\hat{v}_{m+1}(\cdot)$, defined on the overall state space $\mathbb{R}_+^*$;

3. By eq. (8), compute $\tilde{v}_m^h(\cdot)$, defined on $\mathcal{G}$;
4. By eq. (9), compute $\tilde{v}_m(\cdot)$, defined on $\mathcal{G}$;
5. Exercise the option at (t_m, a_k) if $\tilde{v}_m^h(a_k) < v_m^e(a_k)$;
6. If $m = 0$, stop; else set $m = m - 1$, and go to step 2.

4 Numerical investigation

The code lines are written in C, compiled with GCC, and executed under a standard laptop computer running Windows 7. For each set of results, we report the highest total CPU time in seconds (fixed and linear). Our results are based on DP coupled with piecewise quadratic interpolations.

4.1 Gaussian jumps

To start with, Table 1 compares DP to the binomial tree of Amin (1993) for European put options, both with 200 time steps. It also reports Merton's (1976) true values. Set $S_0 = \$\ 40$, $\bar{d} = 0$, $\sigma^2 = 0.05$, $T = 0.25$ (years), $r = 0.08$, $\tilde{\lambda} = 5$ (jumps per year), $\tilde{\gamma} = -0.025$, and $\tilde{\delta}^2 = 0.05$. DP shows convergence and efficiency. It ensures accuracy to the fourth digit within a few seconds. DP agrees almost perfectly with Merton's (1976) true values. Amin (1993) does not report his CPU times and provides only three digits of accuracy.

Table 1: European put options

	DP with a grid size p					
K	50	100	200	400	Merton (1976)	Amin (1993)
30	0.6560	0.6711	0.6695	0.6697	0.6697	0.669
35	1.6875	1.6756	1.6728	1.6727	1.6727	1.674
40	3.5544	3.5857	3.5916	3.5920	3.5920	3.594
45	6.6251	6.6505	6.6550	6.6547	6.6547	6.656
50	10.5273	10.5413	10.5447	10.5445	10.5445	10.545
CPU	(0.00)	(0.00)	(0.03)	(0.08)		

Next, Table 2 compares DP to Bates (1991), Amin (1993), and Gukhal (2004) for Bermudan put options. Their methodologies are based on quadratic approximations, binomial trees, and compound options, respectively. Both DP and Amin (1993) run with 200 time steps. Set $S_0 = \$\ 40$, $\bar{d} = 0$, $\sigma^2 = 0.05$, $r = 0.08$, $\tilde{\lambda} = 5$ (jumps per year), $\tilde{\gamma} = -0.025$, and $\tilde{\delta}^2 = 0.05$.

Table 2: Bermudan put options

		DP with a grid size p						
T	K	50	100	200	400	Bates (1991)	Amin (1993)	Gukhal (2004)
.25	30	0.6608	0.6759	0.6743	0.6744	0.685	0.674	0.672
.25	35	1.7030	1.6902	1.6874	1.6873	1.708	1.688	1.680
.25	40	3.5811	3.6221	3.6281	3.6283	3.663	3.630	3.610
.25	45	6.7028	6.7277	6.7320	6.7318	6.787	6.734	6.695
.25	50	10.6797	10.6926	10.6957	10.6955	10.776	10.696	10.634
1	30	2.6826	2.7124	2.7175	2.7176	2.790	2.720	2.661
1	35	4.5434	4.5930	4.6000	4.6001	4.692	4.603	4.501
1	40	7.0471	7.0308	7.0249	7.0244	7.131	7.030	6.867
1	45	9.9715	9.9411	9.9477	9.9482	10.063	9.954	9.714
1	50	13.3252	13.3062	13.3123	13.3119	13.430	13.318	12.981
CPU		(0.00)	(0.01)	(0.03)	(0.11)			

DP shows convergence and efficiency. DP values are always between the approximations of Bates (1991), Amin (1993), and Gukhal (2004). They are perfectly in line with the literature. These authors report only three digits of accuracy and do not report their CPU times.

Finally, Table 3 and Table 4 compare DP to Chiarella and Ziogas (2009) and Simonato (2011). Their methodologies are based on finite differences and on the Markov chain approximation, respectively. Chiarella and Ziogas (2009) use 10,000 time steps and 5,000 space steps. Both DP and Simonato (2011) run with a daily time step. While Simonato (2011) uses 10,001 space steps, DP uses at most $p = 400$ space steps.

The parameters of Table 3 are S_0 = \$ 50, $\bar{d} = 0$, K = \$ 50, $\sigma^2 = 0.04$, $r = 0.05$, $\tilde{\lambda} = 5$ (jumps per year), $\tilde{\gamma} = -0.105$, and $\tilde{\delta}^2 = 0.01$. Those of Table 4 are $\bar{d} = 0.05$, K = \$ 100, $\sigma^2 = 0.0136$, $T = 0.5$ (years), $r = 0.03$, $\tilde{\lambda} = 1$ (jump per year), $\tilde{\gamma} = 0.0192$, and $\tilde{\delta}^2 = 0.04$.

Again, DP is in line with the literature. More importantly, DP reaches the same level of accuracy with a limited number of space steps and, consequently, exhibits very competitive CPU times.

Table 3: European call options

	DP with a grid size p					
T	50	100	200	400	Merton (1976)	Simonato (2011)
10/365	1.0214	1.0222	1.0224	1.0224	1.0224	1.0224
30/365	2.0421	2.0475	2.0473	2.0474	2.0474	2.0474
60/365	3.0780	3.0899	3.0895	3.0895	3.0895	3.0896
90/365	3.8725	3.8849	3.8847	3.8847	3.8847	3.8847
270/365	7.0897	7.1263	7.1299	7.1299	7.1299	7.1302
CPU	(0.00)	(0.00)	(0.02)	(0.06)		

Table 4: Bermudan call options

	DP with a grid size p					
S_0	50	100	200	400	Chiarella and Ziogas (2009)	Simonato (2011)
80	0.9717	0.9664	0.9650	0.9649	0.9648	0.9647
90	2.3178	2.3041	2.3065	2.3065	2.3063	2.3062
100	5.4076	5.3560	5.3603	5.3603	5.3603	5.3602
110	11.5481	11.5113	11.5073	11.5074	11.5079	11.5073
120	20.1237	20.1304	20.1323	20.1324	20.1333	20.1322
CPU	(0.00)	(0.00)	(0.02)	(0.09)		(25.169)

4.2 Double-exponential jumps

Table 5 compares DP to Kou's (2002) formula for European call options. Set S_0 = \$ 100, $\bar{d} = 0$, $\sigma^2 = 0.0256$, $T = 0.5$ (years), $r = 0.05$, $\tilde{\lambda} = 1$ (jump per year), $\tilde{\eta}_1 = 10$, $\tilde{\eta}_2 = 5$, and $\tilde{p}_1 = 0.4$. Again, DP shows convergence and efficiency. It ensures accuracy to the fourth digit within few seconds. Kou and Wang (2004) adapt the binomial tree of Amin (1993) and evaluate American-style options. DP and the binomial tree run with 1600 time steps. They also use the approximation of Barone-Adesi and Whaley (1987). Table 6 compares DP to Kou and Wang (2004) for Bermudan put options. Set S_0 = \$ 100, $\bar{d} = 0$, $\sigma^2 = 0.04$, $T = 0.25$ (years), $r = 0.05$, and $\tilde{p}_1 = 0.6$. While DP remains very accurate, the results provided by Kou and Wang (2004) are not, especially for out-of-the-money options. Some European-option values exceed their Bermudan counterparts.

Table 7 compares DP to Feng and Linetsky (2008), who use partial integro-differential equations and an extrapolation scheme based on sparse space steps for options valuation. Both procedures run with 252 time

Table 5: European call options – Kou (2002)

	DP with a grid size p				
K	50	100	200	400	Kou (2002)
90	14.7530	14.8060	14.8116	14.8119	14.8119
95	11.1737	11.1041	11.1131	11.1133	11.1133
98	9.0386	9.1574	9.1469	9.1473	9.1473
100	8.0465	7.9694	7.9598	7.9594	7.9594
105	5.3778	5.4606	5.4521	5.4518	5.4518
110	3.5276	3.5915	3.5998	3.5996	3.5996
CPU	(0.02)	(0.08)	(0.33)	(1.26)	

Table 6: Bermudan put options – Kou and Wang (2004)

				DP with a grid size p				KW (2004)		
K	$\tilde{\lambda}$	$\tilde{\eta}_1$	$\tilde{\eta}_2$	50	100	200	400	BAW	Tree	Europ.
110	3	25	25	10.5517	10.5698	10.5733	10.5738	10.43	10.48	10.1785
110	3	25	50	10.5451	10.5244	10.5193	10.5185	10.38	10.42	10.1146
110	3	50	25	10.4312	10.4517	10.4461	10.4465	10.31	10.36	9.9808
110	3	50	50	10.3771	10.3903	10.3943	10.3937	10.26	10.31	9.9151
110	7	25	25	10.9001	10.9366	10.9278	10.9287	10.79	10.81	10.6222
110	7	25	50	10.8255	10.7848	10.7912	10.7904	10.64	10.68	10.4758
110	7	50	25	10.6464	10.6203	10.6144	10.6136	10.47	10.51	10.1892
100	7	50	50	10.4634	10.4777	10.4807	10.4813	10.34	10.39	10.0337
90	3	25	25	0.7301	0.7571	0.7639	0.7746	0.76	0.75	0.7633
90	3	25	50	0.6599	0.6779	0.6823	0.6828	0.66	0.65	0.6739
90	3	50	25	0.7397	0.7044	0.7105	0.7098	0.69	0.68	0.6960
90	3	50	50	0.6468	0.6126	0.6185	0.6179	0.60	0.59	0.6067
90	7	25	25	1.0249	1.0668	1.0594	1.0603	1.04	1.03	1.0487
90	7	25	50	0.8228	0.8488	0.8552	0.8546	0.83	0.82	0.8474
90	7	50	25	0.8636	0.8951	0.8993	0.9000	0.88	0.87	0.8826
90	7	50	50	0.6589	0.6971	0.6911	0.6918	0.67	0.66	0.6589
CPU				(0.03)	(0.09)	(0.37)	(1.50)	(0.03)	(4029)	

Table 7: Put options – Feng and Linetsky (2008)

Europ.	DP with a grid size p				
S_0	50	100	200	400	Kou (2002)
85	13.6377	13.6349	13.6455	13.6462	13.6462
90	10.4557	10.4636	10.4527	10.4518	10.4518
95	7.9318	7.9051	7.9201	7.9223	7.9223
100	5.9855	5.9573	5.9788	5.9800	5.9801
105	4.5057	4.4927	4.5115	4.5132	4.5133
110	3.3841	3.3979	3.4119	3.4137	3.4137
115	2.5635	2.5993	2.5921	2.5909	2.5909
CPU	(0.01)	(0.06)	(0.25)	(1.12)	
Berm.	DP with a grid size p				
S_0	50	100	200	400	FL (2008)
85	15.0843	15.0638	15.0663	15.0693	15.0695
90	11.3726	11.3656	11.3626	11.3661	11.3662
95	8.5604	8.5469	8.5476	8.5476	8.5479
100	6.4335	6.4160	6.4168	6.4169	6.4171
105	4.8314	4.8215	4.8223	4.8223	4.8225
110	3.6153	3.6338	3.6345	3.6348	3.6347
115	2.7374	2.7505	2.7504	2.7504	2.7505
CPU	(0.02)	(0.08)	(0.28)	(1.17)	

steps. Set $\bar{d} = 0.02$, $K = \$ \ 100$, $\sigma^2 = 0.01$, $T = 1$ (year), $r = 0.05$, $\tilde{\lambda} = 3$ (jumps per year), $\tilde{\eta}_1 = 40$, $\tilde{\eta}_2 = 12$, and $\tilde{p}_1 = 0.6$. They report a CPU time of 28 seconds for their second-order IMEX midpoint method, and 1.5 seconds for their extrapolation scheme. DP must be compared to the second-order IMEX midpoint method.

5 Model estimation

We propose two historical estimation methodologies, that is, the method of cumulants and the maximum-likelihood approach, used under Merton's (1976), then under Kou's (2002) setting, respectively.

5.1 Historical approach

Let $R_t = \log\left(X_{th}/X_{(t-1)h}\right)$, for $t = 1, \ldots, M$, be the daily stock log-return, where $T = Mh$ is the last observation date. By eq. (1), one has

$$R_t = \bar{\mu}h + \sigma\left(W_{th} - W_{(t-1)h}\right) + \sum_{n=1}^{N_t} \xi_n,$$

where $\bar{\mu} = \mu - \bar{d} - \sigma^2/2 - \lambda\kappa$. From the properties of Lévy processes, the sample $R_1, \ldots, R_M$ is independent and identically distributed. The parameters to be estimated in Merton's (1976) model are $\bar{\mu}$, σ, λ, γ, and δ. The parameters to be estimated under Kou's (2002) model are $\bar{\mu}$, σ, λ, η_1, η_2, and p_1.

5.1.1 Method of cumulants

This approach matches the theoretical cumulants to their empirical counterparts. This results in a system of non-linear equations, which is solved numerically. The cumulants k_j, for $j \geq 1$, are defined by

$$\begin{aligned}\log \mathbb{E}_{\mathbb{P}}\left[\mathrm{e}^{\theta R_t}\right] &= \sum_{j=1}^{\infty} k_j \frac{\theta^j}{j!}, \\ &= \theta\bar{\mu}h + \frac{\theta^2}{2}\sigma^2 h + \lambda h\left(\phi\left(\theta\right) - 1\right),\end{aligned}$$

where θ is a real number and $\phi(\cdot)$ is the moment-generating function of ξ_n, for $n \geq 1$. The first five cumulants under Merton's (1976) setting are given by $k_1 = \mathbb{E}_{\mathbb{P}}[R_t] = \bar{\mu}h + \lambda h\gamma$, $k_2 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^2] = \sigma^2 h + \lambda h(\gamma^2 + \delta^2)$, $k_3 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^3] = \lambda h(3\gamma\delta^2 + \gamma^3)$, $k_4 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^4] - 3k_2^2 = \lambda h(3\delta^4 + 6\gamma^2\delta^2 + \gamma^4)$, and $k_5 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^5] - 10k_2k_3 = \lambda h(15\gamma\delta^4 + 10\gamma^3\delta^2 + \gamma^5)$. See Rémillard (2013) for more detail.

The first six cumulants under Kou's (2002) setting are given by $k_1 = \mathbb{E}_{\mathbb{P}}[R_t] = \bar{\mu}h + \lambda h(p_1/\eta_1 - p_2/\eta_2)$, $k_2 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^2] = \sigma^2 h + 2\lambda h(p_1/\eta_1^2 - p_2/\eta_2^2)$, $k_3 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^3] = 6\lambda h(p_1/\eta_1^3 - p_2/\eta_2^3)$, $k_4 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^4] - 3k_2^2 = 24\lambda h(p_1/\eta_1^4 - p_2/\eta_2^4)$, $k_5 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^5] - 10k_2k_3 = 120\lambda h(p_1/\eta_1^5 - p_2/\eta_2^5)$, and $k_6 = \mathbb{E}_{\mathbb{P}}[(R_t - k_1)^6] - 15k_2k_4 - 10k_3^2 - 15k_2^3 = 720\lambda h(p_1/\eta_1^5 - p_2/\eta_2^5)$.

5.1.2 Maximum likelihood

The log-likelihood of the random sample $R_1, \ldots, R_M$ under Merton's (1976) model is

$$\mathcal{L} = \sum_{t=1}^{M} \log\left[\sum_{n=0}^{\infty} \pi_n \varphi\left(R_t; \bar{\mu}h + n\gamma, \sigma^2 h + n\delta^2\right)\right],$$

where $\pi_n = P(N_h = n) = e^{-\lambda h}(\lambda h)^n/n!$, $\varphi(x; a, b)$ is the probability density function of a normal distribution with mean a and variance b, evaluated at x.

The log-likelihood of the random sample $R_1, \ldots, R_M$ under Kou's (2002) model is

$$\mathcal{L} = \sum_{t=1}^{M} \log\left(f\left(R_t | \bar{\mu}, \sigma, \lambda, \eta_1, \eta_2, p_1\right)\right),$$

where

$$\begin{aligned} f\left(R_t|\bar{\mu},\sigma,\lambda,\eta_1,\eta_2,p_1\right) &= \frac{e^{(\sigma\eta_1)^2h/2}}{\sigma\sqrt{2\pi h}}\sum_{n=1}^{\infty}\pi_n\sum_{k=1}^{n}P_{n,k}\left(\sigma\sqrt{h}\eta_1\right)^k \\ &\times\ e^{-\eta_1(R_t-\bar{\mu}h)}Hh_{k-1}\left(-\frac{R_t-\bar{\mu}h}{\sigma\sqrt{h}}+\sigma\sqrt{h}\eta_1\right) \\ &+\ \frac{e^{(\sigma\eta_2)^2h/2}}{\sigma\sqrt{2\pi h}}\sum_{n=1}^{\infty}\pi_n\sum_{k=1}^{n}Q_{n,k}\left(\sigma\sqrt{h}\eta_2\right)^k \\ &\times\ e^{\eta_2(R_t-\bar{\mu}h)}Hh_{k-1}\left(\frac{R_t-\bar{\mu}h}{\sigma\sqrt{h}}+\sigma\sqrt{h}\eta_2\right) \\ &+\ \frac{\pi_0}{\sigma\sqrt{h}}\varphi\left(-\frac{R_t-\bar{\mu}h}{\sigma\sqrt{h}}\right), \end{aligned}$$

where the function $\varphi(\cdot)$ is the normal density function. The functions $P_{n,k}(\cdot)$ and $Q_{n,k}(\cdot)$ are reported in Appendix C, as defined in Kou (2002). The function $Hh(\cdot)$, which can be viewed as a generalization of the cumulative normal distribution function, is defined in (Abramowitz and Stegun (1972)).

5.2 Relative accuracy

We simulate 2000 samples of daily log-returns from the selected jump-diffusion models. For both models, set $S_0 = \$\ 100$, $\mu = 0.08$, $\sigma = 0.12$, $T = 10$ (years), and $\lambda = 10$ (jumps per year). The jump parameters under Merton's (1976) model are $\gamma = 0.02$ and $\delta = 0.01$. The jump parameters under Kou's (2002) model are $\eta_1 = 10$, $\eta_2 = 5$, and $p_1 = 0.6$. Each sample of 2520 log-returns is used to estimate the parameters by the method of cumulants and the maximum-likelihood approach. Consistent with Rémillard (2013), the method of cumulants shows instability, due mainly to approximation errors in high cumulants.

Table 8 and Table 9 provide the root mean square error (RMSE) for each parameter and show that maximum likelihood outperforms the method of cumulants.

Table 8: RMSE for estimation methods – Merton (1976)

Method	μ	σ	λ	γ	δ
Method of cumulants	0.0544	0.0672	11.7862	0.5275	1.5621
Maximum likelihood	0.0423	0.0026	4.8153	0.0060	0.0052

Table 9: RMSE for estimation methods – Kou (2002)

Method	μ	σ	λ	η_1	η_2	p_1
Method of cumulants	0.4731	0.1211	9.2139	2.2827	5.4213	0.5411
Maximum likelihood	0.0291	0.0014	3.8898	2.7799	1.0974	0.2140

6 Empirical investigation

We consider a set of American-put options on Home Depot, issued on 01-21-2011 (evaluation date) and expiring on 05-21-2011. The initial stock level is $S_0 = \$\ 36.51$ and the risk-free rate is the one-month Libor rate, $r = 0.48\%$ (per year). We use daily stock returns from 01-13-2009 to 01-14-2011 as an estimation time window. We use Genest and Rémillard (2004) test randomness with four consecutive values and cannot reject the i.i.d structure of the random sample $R_1, \ldots, R_M$ at the level of 5%.

Figure 1 reports the time series of daily stock returns. It shows frequent peaks, which suggest a jump-diffusion dynamics for the underlying asset. We use maximum likelihood to estimate pure diffusion and

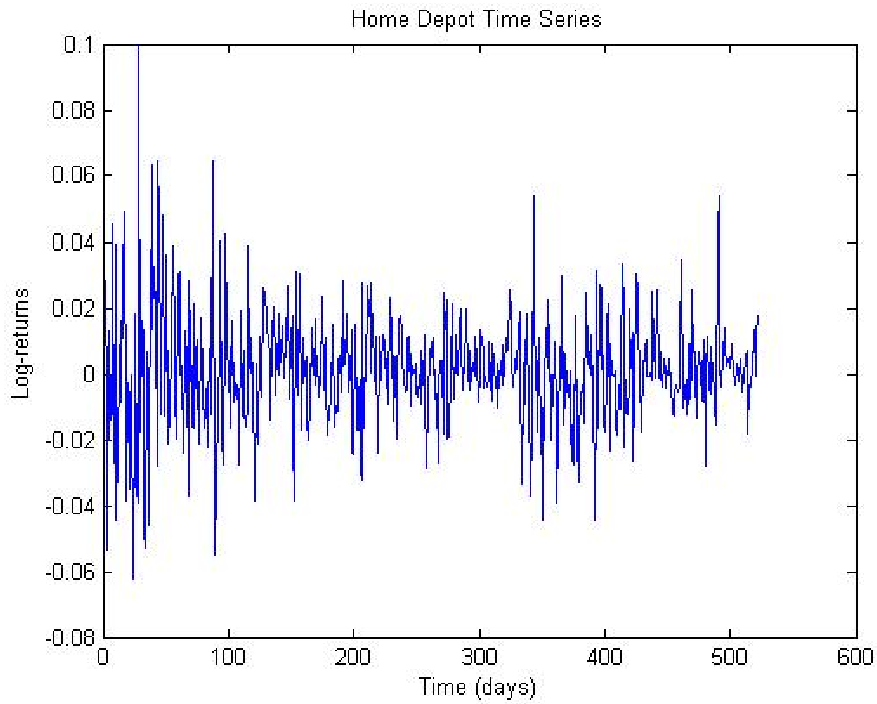

Figure 1: Home Depot log-returns

conventional jump-diffusion models, while we use maximum likelihood coupled with calibration to estimate their extended versions. We start from with maximum likelihood estimates in the conventional jump-diffusion setting, select some liquid option contracts, use calibration to approximate α and β, and revise maximum likelihood estimates as explained in Section 2. Table 10 provides maximum-likelihood estimates. Calibration results in $(\alpha, \beta) = (1, 0.228)$ for Merton and $(\alpha, \beta) = (0.9074, -0.8234)$ for Kou.

Table 10: Maximum likelihood estimates

Model	μ	σ	λ	γ	δ	η_1	η_2	p_1
Merton	0.2662	0.2559	7.5783	0.0474	$5.733.10^{-6}$	–	–	–
Kou	-0.4901	0.0434	644.7199	–	–	133.0107	117.6068	0.5999

The estimated models present a key difference: a low intensity and a high volatility for Merton's (1976) model and a high intensity and a low volatility for Kou's (2002) model. This illustrates the complexity to discern in between the pure component and the jump component of a jump-diffusion dynamics. Kou's model shows almost symmetrical relative jump amplitude $(1/\eta_1 \simeq 1/\eta_2)$.

Our empirical investigation shows that conventional jump-diffusion models outperform their pure diffusion counterparts. Moreover, extended jump-diffusion models improve further the final results.

7 Conclusion

We build a tractable framework for options pricing, where the under-lying asset price follows a jump-diffusion process la Merton (1976) and Kou (2002). Our construction is based on dynamic programming combined with finite elements. Our numerical investigation shows convergence and efficiency, and competes well against alternative methodologies. For the model estimation, we experiment with the maximum-likelihood approach alone or maximum likelihood combined with calibration. The latter outperforms the former.

Table 11: American-put options – BS (1973)

	DP with a grid size p				Market	Relative
K	50	100	200	400	price	error
32	1.0182	1.0270	1.0273	1.0273	0.630	0.6306
33	1.3477	1.3393	1.3392	1.3391	0.830	0.6134
34	1.7187	1.7063	1.7065	1.7065	1.075	0.5874
35	2.1218	2.1300	2.1304	2.1304	1.395	0.5272
36	2.5987	2.6113	2.6109	2.6109	1.790	0.4586
37	3.1605	3.1465	3.1466	3.1468	2.265	0.3893

Table 12: American-put options – Merton (1976)

	DP with a grid size p				Market	Relative
K	50	100	200	400	price	error
Merton (1976)						
32	0.6487	0.6536	0.6536	0.6554	0.630	0.0403
33	0.9132	0.9187	0.9186	0.9206	0.830	0.1092
34	1.2420	1.2474	1.2477	1.2500	1.075	0.1628
35	1.6485	1.6440	1.6441	1.6467	1.395	0.1804
36	2.1151	2.1089	2.1090	2.1167	1.790	0.1825
37	2.6480	2.6411	2.6411	2.6439	2.265	0.1673
Extended Merton						
32	0.6411	0.6461	0.6462	0.6462	0.630	0.0257
33	0.9044	0.9099	0.9097	0.9097	0.830	0.0960
34	1.2320	1.2375	1.2377	1.2377	1.075	0.1513
35	1.6378	1.6331	1.6333	1.6333	1.395	0.1708
36	2.1037	2.0973	2.0975	2.0974	1.790	0.1717
37	2.6363	2.6292	2.6293	2.6293	2.265	0.1608

Table 13: American-put options – Kou (2002)

	DP with a grid size p				Market	Relative
K	50	100	200	400	price	error
Kou (2002)						
32	0.6582	0.6652	0.6657	0.6657	0.630	0.0566
33	0.9182	0.9284	0.9290	0.9290	0.830	0.1193
34	1.2663	1.2562	1.2555	1.2555	1.075	0.1679
35	1.6676	1.6478	1.6488	1.6488	1.395	0.1819
36	2.0892	2.1115	2.1101	2.1101	1.790	0.1788
37	2.6624	2.6362	2.6387	2.6387	2.265	0.1650
Extended Kou						
32	0.5862	0.5926	0.5925	0.5925	0.630	-0.0595
33	0.8347	0.8435	0.8432	0.8432	0.830	0.0159
34	1.1655	1.1594	1.1589	1.1589	1.075	0.0780
35	1.5571	1.5433	1.5440	1.5448	1.395	0.1074
36	2.0134	1.9988	2.0001	2.0001	1.790	0.1174
37	2.5079	2.5276	2.5265	2.5266	2.265	0.1155

Appendix A. Change of measure

Recall that $h(x) = \alpha e^{\beta x}$, where $\alpha \in \mathbb{R}_+^*$ and $\beta \in \mathbb{R}$. For Merton's (1976), $a = \mathbb{E}[h(x)] = \alpha e^{\beta\gamma+\beta^2\delta^2/2}$, and the density $\tilde{f}(x)$ of $\tilde{\nu}$, under $\mathbb{Q}$, is

$$\begin{aligned}
\tilde{f}(x) &= h(x)f(x)/a \\
&= a\alpha e^{\beta x}\lambda e^{-\frac{(x-\gamma)^2}{2\delta^2}}/(\sqrt{2\pi}\delta), \\
&= a\alpha\lambda e^{-\frac{(x-\gamma)^2-2\beta\delta^2 x}{2\delta^2}}/(\sqrt{2\pi}\delta), \\
&= a\alpha\lambda e^{\beta\gamma+\beta^2\delta^2/2}e^{-\frac{(x-\gamma-\beta\delta^2)^2}{2\delta^2}}, \\
&= a\lambda e^{-\frac{(x-\gamma-\beta\delta^2)^2}{2\delta^2}}/(\sqrt{2\pi}\delta) \quad \text{for } x \in \mathbb{R},
\end{aligned}$$

where, under $\mathbb{Q}$, $\tilde{\lambda} = a\lambda$, $\tilde{\gamma} = \gamma + \beta\delta^2$, $\tilde{\delta} = \delta$, and $\tilde{\kappa} = e^{\tilde{\gamma}+\tilde{\delta}^2/2} - 1$.

For Kou's (1976) model, $\beta \in]-\eta_2, \eta_1[$, $a = \mathbb{E}[h(x)] = \alpha p_1\eta_1/(\eta_1 - \beta) + \alpha p_2\eta_2/(\eta_2 + \beta)$, and the density $\tilde{f}(x)$ of $\tilde{\nu}$, under $\mathbb{Q}$, is

$$\begin{aligned}
\tilde{f}(x) &= h(x)f(x)/a \\
&= \frac{\alpha\lambda(\eta_1-\beta)p_1\,\eta_1}{a(\eta_1-\beta)}e^{-(\eta_1-\beta)\,x}\,\mathbb{I}_{\{x\geq 0\}} + \frac{\alpha\lambda(\eta_2+\beta)p_2\,\eta_2}{a(\eta_2+\beta)}e^{(\eta_2+\beta)\,x}\,\mathbb{I}_{\{x<0\}}, \\
&= \frac{\lambda p_1\eta_1(\eta_1-\beta)}{(\eta_1-\beta)(\frac{p_1\eta_1}{\eta_1-\beta}+\frac{p_2\eta_2}{\eta_2+\beta})}e^{-(\eta_1-\beta)\,x}\,\mathbb{I}_{\{x\geq 0\}} + \\
&\qquad \frac{\lambda p_2\eta_2(\eta_2+\beta)}{(\eta_2+\beta)(\frac{p_1\eta_1}{\eta_1-\beta}+\frac{p_2\eta_2}{\eta_2+\beta})}e^{-(\eta_2+\beta)}\,\mathbb{I}_{\{x<0\}}, \\
&= \frac{p_1\eta_1(\eta_2+\beta)}{p_1\eta_1(\eta_2+\beta)+p_2\eta_2(\eta_1-\beta)}(\eta_1-\beta)e^{-(\eta_1-\beta)\,x}\,\mathbb{I}_{\{x\geq 0\}} + \\
&\qquad \frac{p_1\eta_2(\eta_1-\beta)}{p_2\eta_2(\eta_1-\beta)+p_1\eta_1(\eta_2+\beta)}(\eta_2+\beta)e^{-(\eta_2+\beta)}\,\mathbb{I}_{\{x<0\}}, \\
&= \tilde{p}_1\tilde{\eta}_1 e^{-\tilde{\eta}_1 x}\mathbb{I}_{\{x\geq 0\}} + \tilde{p}_2\tilde{\eta}_2 e^{\tilde{\eta}_2\,x}\mathbb{I}_{\{x<0\}}, \quad \text{for } x \in \mathbb{R},
\end{aligned}$$

where, under $\mathbb{Q}$, $\tilde{\lambda} = a\lambda$, $\tilde{\eta}_1 = \eta_1 - \beta$, $\tilde{\eta}_2 = \eta_2 + \beta$, $\tilde{p}_1 = p_1\eta_1\tilde{\eta}_2/(p_1\eta_1\tilde{\eta}_2 + p_2\tilde{\eta}_1\eta_2)$, $\tilde{p}_2 = 1 - \tilde{p}_1$, and $\tilde{\kappa} = \tilde{p}_1\tilde{\eta}_1/(\tilde{\eta}_1 - 1) + \tilde{p}_2\tilde{\eta}_2/(\tilde{\eta}_2 + 1) - 1$.

Appendix B. Transition tables – Merton (1976)

The transition parameters $T^{\nu}_{k,i}$, for $\nu \in \{0, 1, 2\}$, $k \in \{1, \ldots, p\}$, and $i \in \{0, \ldots, p\}$ are

$$T^{\nu}_{k,i} = \sum_{n=0}^{\infty} \mathbb{Q}\left(N_{\Delta t} = n\right)\ \eta^{\nu}_k(n)e^{c(n)^2/2}\left[\Phi\left(c_{k,i+1}(n) - c\left(n\right)\right) - \Phi\left(c_{k,i}(n) - c\left(n\right)\right)\right],$$

where $N_{\Delta t}$ is the number of jumps over $[t_m, t_{m+1}]$, $c\left(n\right) = \nu\sigma_n\sqrt{\Delta t}$, and

$$\begin{aligned}
&\mathbb{Q}\left(N_{\Delta t} = n\right) = e^{-\lambda\Delta t}\frac{\left(\lambda\Delta t\right)^n}{n!}, \\
&\sigma_n^2 = \sigma^2 + \frac{n}{\Delta t}\delta^2, \\
&\eta_k(n) = a_k e^{\left(r-\bar{d}-\lambda\kappa-\sigma_n^2/2\right)\Delta t + n(\gamma+\delta^2/2)}, \\
&c_{k,i}(n) = \frac{\log\left(a_i/a_k\right) - \left(r - \bar{d} - \lambda\kappa - \sigma_n^2/2\right)\Delta t - n(\gamma + \delta^2/2)}{\sigma_n},
\end{aligned}$$

and $\Phi\left(\cdot\right)$ is the standard normal distribution function.

Appendix C. Transition tables – Kou (2002)

The transition parameters $T^{\nu}_{k,i}$, for $\nu \in \{0,1,2\}$, $k \in \{1,\dots,p\}$, and $i \in \{0,\dots,p\}$ are

$$T^0_{k,i} = \Upsilon(\mu_0,\sigma,\lambda,p_1,\eta_1,\eta_2,x_{i+1},\Delta t) - \Upsilon(\mu_0,\sigma,\lambda,p_1,\eta_1,\eta_2,x_i,\Delta t),$$

$$T^1_{k,i} = \rho^{-1}a_k[\Upsilon(\mu_1,\sigma,\tilde{\lambda},\tilde{p}_1,\tilde{\eta}_1,\tilde{\eta}_2,x_{i+1},\Delta t) - \Upsilon(\mu_1,\sigma,\tilde{\lambda},\tilde{p}_1,\tilde{\eta}_1,\tilde{\eta}_2,x_i,\Delta t)],$$

$$T^2_{k,i} = b\rho^{-2}a_k^2[\Upsilon(\mu_2,\bar{\sigma},\bar{\lambda},\bar{p}_1,\bar{\eta}_1,\bar{\eta}_2,\bar{x}_{i+1},\Delta t) - \Upsilon(\mu_2,2\sigma,\bar{\lambda},\bar{p}_1,\bar{\eta}_1,\bar{\eta}_2,\bar{x}_i,\Delta t)],$$

where $\mu_0 = r - \frac{1}{2}\sigma^2 - \lambda\kappa, x_i = \log(a_i/a_k), \rho = \exp(-(r-\bar{d})\Delta t), \mu_1 = r + \frac{1}{2}\sigma^2 - \lambda\kappa, \tilde{\lambda} = \lambda(1+\kappa), \tilde{p}_1 = p\eta_1/(1+\kappa)(\eta_1-1), \tilde{\eta}_1 = \eta_1 - 1, \tilde{\eta}_2 = \eta_2 + 1, \bar{\sigma} = 2\sigma, \bar{\kappa} = p_1(\eta_1/2\bar{\eta}_1) + (1-p_1)(\eta_2/2\bar{\eta}_2) - 1, \mu_2 = 2r + \frac{1}{2}\bar{\sigma}^2 - \lambda\bar{\kappa}, \bar{\lambda} = \lambda(1+\bar{\kappa}), \bar{\eta}_1 = \eta_1/2 - 1, \bar{\eta}_2 = \eta_2/2 + 1, b = \exp(\sigma^2 + \lambda(\bar{\kappa} - 2\kappa)\Delta t),$ and $\bar{x}_i = x_i - \log(b)$. The function $\Upsilon(.)$ is defined by

$$\begin{aligned}
\Upsilon(\mu,\sigma,\lambda,\eta_1,\eta_2,p_1,x_i,\Delta t) \quad = \quad & \frac{e^{(\sigma\eta_1)^2\Delta t/2}}{\sigma\sqrt{2\pi\Delta t}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k}\left(\sigma\sqrt{\Delta t}\eta_1\right)^k \\
\times \quad & I_{k-1}\left(x_i - \mu\Delta t; -\eta_1, -\frac{1}{\sigma\sqrt{\Delta t}}, -\sigma\eta_1\sqrt{\Delta t}\right) \\
+ \quad & \frac{e^{(\sigma\eta_2)^2\Delta t/2}}{\sigma\sqrt{2\pi\Delta t}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k}\left(\sigma\sqrt{\Delta t}\eta_2\right)^k \\
\times \quad & I_{k-1}\left(x_i - \mu\Delta t; \eta_2, \frac{1}{\sigma\sqrt{\Delta t}}, -\sigma\eta_2\sqrt{\Delta t}\right) \\
+ \quad & \pi_0\Phi\left(-\frac{x_i - \mu\Delta t}{\sigma\sqrt{\Delta t}}\right),
\end{aligned}$$

and

$$P_{n,k} = \sum_{i=k}^{n-1} \begin{pmatrix} n-k-1 \\ i-k \end{pmatrix} \begin{pmatrix} n \\ i \end{pmatrix} \cdot \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{i-k} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{n-i} p_1^i p_2^{n-i},$$

$$Q_{n,k} = \sum_{i=k}^{n-1} \begin{pmatrix} n-k-1 \\ i-k \end{pmatrix} \begin{pmatrix} n \\ i \end{pmatrix} \cdot \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{n-i} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{i-k} p_1^{n-i} p_2^{i},$$

$$I_n(c;\alpha,\beta,\delta) = \int_c^{\infty} e^{\alpha x} Hh_n(\beta x - \delta)\,dx,$$

for arbitrary constants α, c, $\beta \in \mathbb{R}$, and $n \in \mathbb{N}$.

References

Abramowitz, M. and Stegun, I.A. (1972). Handbook of mathematical functions, 10th Printing. U.S. National Bureau of Standards, Washington, D.C.

Almendral, A. and Oosterlee, C.W. (2005). Numerical valuation of options with jumps in the underlying. Applied Numerical Mathematics, 53(1):1–18.

Amin, K. (1993). Jump diffusion option valuation in discrete time. Journal of Finance, 48:1833–1863.

Andersen, L. and Andreasen, J. (2000). Jump-diffusion processes: Volatility smile fitting and numerical methods for option pricing. Review of Derivatives Research, 4:231–262.

Barone-Adesi, G. and Whaley, R.E. (1987). Efficient analytic approximation of American option values. Journal of Finance, 42:301–320.

Bates, D.S. (1991). The crash of ’87: Was it expected? The evidence from options markets. Journal of Finance, 46:1009–1044.

Ben-Ameur, H., Breton, M., and L’Ecuyer, P. (2002). A dynamic programming procedure for pricing american-style asian options. Management Science, 48(5):625–643.

Black, F. and Scholes, M.S. (1973). The pricing of options and corporate liabilities. Journal of Political Economy, 81:637–54.

Chiarella, C. and Ziogas, A. (2009). American call options under jump-diffusion processes:A Fourier transform approach. Applied Mathematical Finance, 16:37–79.

Chockalingam, A. and Muthuraman, K. (2010). Pricing american options when asset prices jump. Operations Research Letters, 38(2):82–86.

Cont, R. and Tankov, P. (2004a). Financial Modelling with Jump Processes. Chapman & Hall/CRC, Boca Raton, FL.

Cont, R. and Tankov, P. (2004b). Non-parametric calibration of jump-diffusion option pricing models. Journal of Computational Finance, 7:1–49.

Cont, R. and Voltchkova, E. (2005). Integro-differential equations for option prices in exponential lévy models. Finance and Stochastics, 9:299–325.

DiCesare, J. and Mcleish, D. (2008). Simulation of jump diffusions and the pricing of options. Insurance: Mathematics and Economics, 43(3):316–326.

Feng, L. and Linetsky, V. (2008). Pricing options in jump-diffusion models: an extrapolation approach. Operations Research, 56:304–325.

Genest, C. and Rémillard, B. (2004). Test of independence and randomness based on the empirical copula process. Test, 13(2):335–369.

Gukhal, C.R. (2001). Analytical valuation of American options on jump-diffusion processes. Mathematical Finance, 11:97–115.

Gukhal, C.R. (2004). The compound option approach to American options on jump-diffusions. Journal of Economic Dynamics and Control, 28:2055–2074.

Hanson, F. and Zhu, Z. (2004). Comparison of market parameters for jump-diffusion distributions using multinomial maximum likelihood estimation. Decision and Control, 43rd IEEE Conference, 4:3919–3924.

He, C., Kennedy, J., Coleman, T., Forsyth, P., Li, Y., and Vetzal, K. (2006). Calibration and hedging under jump diffusion. Review of Derivatives Research, 9:1–35.

Hilliard, J.E. and Schwartz, A. (2005). Pricing European and American derivatives under a jump-diffusion process: A bivariate tree approach. Journal of Financial & Quantitative Analysis, 40:671–691.

Kou, S.G. (2002). A jump-diffusion model for option pricing. Management Science, 48:1086–1101.

Kou, S.G. and Wang, H. (2004). Option pricing under a double exponential jump diffusion model. Manage. Sci., 50:1178–1192.

Levendorskii, S.Z. (2004). Pricing of the american put under levy processes. International Journal of Theoretical and Applied Finance, 7(3):303–335.

Matache, A., Schwab, C., and Wihler, T. (2005). Fast numerical solution of parabolic integro-differential equations with applications in finance. SIAM Journal on Scientific Computing, 27:369–393.

Matache, A.-M., Nitsche, P.-A., and Schwab, C. (2003). Wavelet galerkin pricing of american options on Lévy driven assets. Quant. Finance, 5:403–424.

Mayo, A. (2008). Methods for the rapid solution of the pricing {PIDEs} in exponential and merton models. Journal of Computational and Applied Mathematics, 222(1):128–143.

Merton, R.C. (1976). Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics, 3:125–144.

Press, S.J. (1967). A compound events model for security prices. Journal of Business, 40:317–335.

Rémillard, B. (2013). Statistical Methods for Financial Engineering. Chapman & Hall/CRC, Boca Raton, FL.

Simonato, J.-G. (2011). Computing American option prices in the lognormal jump diffusion framework with a Markov chain. Finance Research Letters, 8:220–226.

Toivanen, J. (2008). Numerical valuation of european and american options under kou's jump-diffusion model. SIAM J. Sci. Comput., 30:1949–1970.

Zhang, X.L. (1997). Numerical analysis of American option pricing in a jump-diffusion model. Mathematics of Operations Research, 22:668–690.