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Abstract

It is observed that mutations in the formulations of test problems over time are not infrequent. Ensuing problems are illustrated with examples from geometric programming. Ways to avoid them are suggested.

Key Words: test problems, geometric programming, mutations.

Résumé

Il n'est pas rare d'observer au fil du temps des mutations dans les formulations de problèmes test. Les problèmes qui en découlent sont illustrés par des exemples provenant de la programmation géométrique. Des façons d'éviter ces problèmes sont suggérées.

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1 Introduction

It is well known that, despite the attention of their authors, mathematical formulae in scientific papers sometimes contain typographical errors. This is usually not considered to be a serious problem. Nevertheless, while reading a series of papers on algorithms for signomial geometric programming we were surprised to find that errors in formulations, and more precisely mutations from one version of a test problem to the next, are far from rare. This leads to wrong conclusions and unfair statements in comparative studies. Indeed, wrong coefficients or indices, rounded or truncated coefficients, missing terms, and incorrect bounds on variables lead, in some cases, to: (i) solving a different problems than that one described in the paper; (ii) referring to a problem from the literature while solving a different one; (iii) implying that one or several previous algorithms are not correct as a better (but incorrect or irrelevant) solution has been found for some test problem; (iv) reporting optimal solutions to infeasible problems; (v) reporting optimal objective function values which do not agree with those obtained by substitution of reported values of the variables.

To illustrate, we consider five test problems from a recent paper published in EJOR (Qu et al., 2008). Each time, we study the genealogy of these test problems, the advent of mutations and their consequences. We also show how these test problems can be reformulated as nonconvex quadratic programs with non convex quadratic constraints (Hansen and Jaumard, 1992). We then use the branch-and-cut QP code of Audet et al. (2000, 2008) to check the optimality of the solutions proposed for the five test problems and their mutated variants.

We stress that the aim of this paper is not to criticize colleagues (at least one of us made, on occasion, similar errors) but to study difficulties in the practice of mathematical programming, and suggest ways to alleviate them.

2 Mutations in some Geometric Programming Test Problems

We now consider five test problems from Qu et al. (2008), their genealogy, and their mutations over time.

Problem 1

This problem is a posynomial geometric programming problem and hence can be reduced to a convex program. It comes from inventory control and has three variables, one constraint, and, in some versions, lower and upper bounds on the variables. Its genealogy can be traced as follows: Qu et al. (2008) cite Shen and Zhang (2004) which cite Rijckaert and Martens (1978) which cites Kochenberger et al. (1973) which cite Smith (1970). Moreover, Qu et al. (2007b) cite directly Rijckaert and Martens (1978), and Shen and Jiao (2006) as well as Shen et al. (2008) cite Shen and Zhang (2004).

In Smith (1970), Kochenberger et al. (1973), and Rijckaert and Martens (1978), this problem is written as:

$$(\mathbf{P1}) \begin{cases} \min & G_0(x) = 5x_1 + 50000x_1^{-1} + 20x_2 + 72000x_2^{-1} + 144000x_3^{-1} + 10x_3 \\ \text{s.t.:} & \\ & G_1(x) = 4x_1^{-1} + 32x_2^{-1} + 120x_3^{-1} \leq 1. \end{cases}$$

In Shen and Zhang (2004), Shen and Jiao (2006), and Shen et al. (2008), the last term of $G_0(x)$, i.e., $10x_3$ is omitted. Moreover, lower bounds of 1 and upper bounds of 100 are added. It is easy to see that the problem is then infeasible, as $G_1(x)$ can not be lower than 1.56. Nevertheless, solutions, which of course violate these bounds, are reported in all three papers. In Qu et al. (2007b), the problem statement is similar except for a typo: the first term in the constraint is written tt_1 instead of $4x_1^{-1}$. Again, a solution violating the bounds is proposed. In Qu et al. (2008), the upper bounds are corrected to 220. $G_0(x)$, however, is very different: the term $10x_3$ is still missing, the variable index of the fourth term has changed from 2 to 1, and the coefficient of the third term has increased from 20 to 46.2. All these modifications lead to the following expression:

$$G_0(x) = 5x_1 + 50000x_1^{-1} + 46.2x_2 + 72000x_1^{-1} + 144000x_3^{-1}. \quad (1)$$

We refer to $(P1^1)$ as $(P1)$ with the correct bounds of Qu et al. (2008), $(P1^2)$ as $(P1^1)$ with the term $10x_3$ omitted in $G_0(x)$, $(P1^3)$ as $(P1^2)$ with upper bounds of 100, and $(P1^4)$ as $(P1^2)$ with (1).

Introducing new variables $y_i = x_i^{-1}$, for $i = 1, 2, 3$, problems $(P1^1)$, $(P1^2)$, $(P1^3)$, and $(P1^4)$ can be reformulated as nonconvex quadratic programs which can be solved with the QP code. For $(P1^1)$, the quadratic program is

$$
\begin{array}{ll}
\min & 5x_1 + 50000y_1 + 20x_2 + 72000y_2 + 144000y_3 + 10x_3 \\
\text{s.t. :} & \\
& 4y_1 + 32y_2 + 120y_3 \leq 1 \\
& x_i y_i = 1 \quad \text{for } i = 1, 2, 3 \\
& 1 \leq x_i \leq 220 \quad \text{for } i = 1, 2, 3.
\end{array}
$$

Table 1 presents the results reported in the cited papers together with the solutions obtained by QP for the quadratic programming formulations of $(P1^1)$, $(P1^2)$, and $(P1^4)$. In all tables of this paper, there are two sets of rows: the first set refers to the solutions and formulations presented in the cited papers; the second set refers to the solutions obtained by QP. For the results of the cited papers, the first column gives the reference, the second column gives the formulation presented in the paper, the next set of columns gives the solution reported in the paper, and the last set of columns gives the computed value of several equations of the problem using the variables values reported in the paper. For the solutions obtained by QP, the first column indicates that it is a QP solution, the second column gives the problem for which the quadratic programming formulation is solved, the next set of columns gives the solution obtained, and the last set of columns gives the computed value of several equations of the problem using the variables values obtained by QP. In order to compare fairly the results of the cited papers with those obtained by QP, the number of decimals used for the QP solutions is limited to the same value as for the reported solutions in the cited papers.

Table 1: Results for problem 1

Ref.	Reported formulation	Reported optimal solution x_1	x_2	x_3	$G_0(x)$	Computed values $G_0(x)$ of $(P1)$	Reported $G_0(x)$	$G_1(x)$
1	$(P1)$	109	85	205	6303.19	6303.213444250	-	0.998533690
2	$(P1)$	-	-	-	6297	-	-	-
3	$(P1)$	107.4	84.9	204.5	6300	6297.762364551	-	1.000955030
4	$(P1^3)$	108.734706796	85.126214158	204.324594290	6299.842427922	6299.842427919	4256.596485019	1.000000000
5	$(P1^3)$	-	-	-	6299.842427922	-	-	-
6	$(P1^3)$	107.9543	85.4785	204.4784	4259.0484	6303.832384224	4259.048384224	0.998274912
7	$(P1^4)$	109.325467810	84.048214540	214.324594290	6217.46548921	6356.716796721	6217.466884898	0.977220256
QP	$(P1^1)$	107.354281797	85.587334167	203.759898569	-	6299.824789720	6299.824789720	1.000075351
QP	$(P1^2)$	105.262705824	76.821480654	220	-	6329.528495029	4129.528495029	1.000004868
QP	$(P1^4)$	107.354281797	76.686078988	220	-	6329.678754920	5870.637877307	1.000000000

1. Smith (1970)
2. Kochenberger et al. (1973)
3. Rijckaert and Martens (1978)
4. Shen and Zhang (2004)
5. Shen and Jiao (2006); Shen et al. (2008)
6. Qu et al. (2007b)
7. Qu et al. (2008)

From Table 1, it appears that:

(i) Results in Rijckaert and Martens (1978) are given with few decimals and substitution in $G_0(x)$ does not give precisely the optimal value reported. However, the solution is close to the optimal solution of $(P1^1)$.

(ii) Despite omitting the term $10x_3$ and adding bounds which make the problem infeasible, the results reported in Shen and Zhang (2004), Shen and Jiao (2006), and Shen et al. (2008) appear to correspond to the optimal solution of $(P1^1)$.

(iii) It is not obvious which version of the problem is solved in Qu et al. (2007b) since the reported variables values are similar to the optimal solution of $(P1^1)$ but the reported objective function value agrees with $G_0(x)$ of $(P1^2)$.

(iv) The solution reported in Qu et al. (2008) is clearly not optimal for the reported formulation in that paper and for any version considered in our analysis. Note that it is easy to show that x_3 must be equal to 220 in the optimal solution of $(P1^4)$.

Problem 2

This problem is expressed as a signomial geometric program. It has five variables, six constraints, and lower and upper bounds on the variables. Its genealogy can be traced as follows: Qu et al. (2008) cite Rijckaert and Martens (1978) which cite Colville (1970). Moreover, Shen and Jiao (2006) and Shen et al. (2008) cite Shen and Zhang (2004) which cite Rijckaert and Martens (1978). Also, Shen (2005) cites Dembo (1976) as well as Rijckaert and Martens (1978). Finally, Dembo (1976) cites Colville (1970). Note that comparison with Colville (1970) is not possible since neither detailed formulation nor solution is given in this paper.

In Dembo (1976) and Shen (2005), this problem is written as:

$$
(\mathbf{P2})\left\{\begin{array}{ll}
\min & G_0(x) = 5.35785470x_3^2 + 0.83568910x_1x_5 + 37.239239x_1 - 40792.1410 \\
\text{s.t.:} & \\
& G_1(x) = 0.00002584x_3x_5 - 0.00006663x_2x_5 - 0.00000734x_1x_4 \le 1 \\
& G_2(x) = 0.000853007x_2x_5 + 0.00009395x_1x_4 - 0.00033085x_3x_5 \le 1 \\
& G_3(x) = 1330.32937x_2^{-1}x_5^{-1} - 0.42002610x_1x_5^{-1} - 0.30585975x_2^{-1}x_3^2x_5^{-1} \le 1 \\
& G_4(x) = 0.00024186x_2x_5 + 0.00010159x_1x_2 + 0.00007379x_3^2 \le 1 \\
& G_5(x) = 2275.132693x_3^{-1}x_5^{-1} - 0.26680980x_1x_5^{-1} - 0.40583930x_4x_5^{-1} \le 1 \\
& G_6(x) = 0.00029955x_3x_5 + 0.00007992x_1x_3 + 0.00012157x_3x_4 \le 1 \\
& 78.0 \le x_1 \le 102.0 \\
& 33.0 \le x_2 \le 45.0 \\
& 27.0 \le x_i \le 45.0 \quad \text{for } i = 3, 4, 5.
\end{array}\right.
$$

In Rijckaert and Martens (1978), Shen and Zhang (2004), Shen and Jiao (2006), and Shen et al. (2008), the constant in $G_0(x)$ is omitted, the coefficient of the third term in $G_1(x)$ is 0.0000734 instead of 0.00000734, and the coefficients of $G_0(x)$, $G_3(x)$, and $G_5(x)$ are rounded or truncated as follows:

$$
\begin{array}{rcl}
G_0(x) & = & 5.3578x_3^2 + 0.8357x_1x_5 + 37.2392x_1 \\
G_3(x) & = & 1330.3294x_2^{-1}x_5^{-1} - 0.42x_1x_5^{-1} - 0.30586x_2^{-1}x_3^2x_5^{-1} \le 1 \\
G_5(x) & = & 2275.1327x_3^{-1}x_5^{-1} - 0.2668x_1x_5^{-1} - 0.40584x_4x_5^{-1} \le 1.
\end{array}
$$

We refer to $(P2^1)$ for the resulting version of $(P2)$. In Qu et al. (2008), the problem is the same as $(P2^1)$ but there are two typos: the coefficient of the first term in $G_2(x)$ is 0.00085307 instead of 0.000853007 and the coefficient of the last term in $G_6(x)$ is negative instead of positive.

Multiplying $G_3(x)$ by x_2x_5 and $G_5(x)$ by x_3x_5 reformulates $(P2)$ as a quadratic program without the addition of any new variables. Table 2 presents the results reported in the cited papers together with the solutions obtained by QP for the quadratic programming formulations of $(P2)$ and $(P2^1)$. Note that in the computation of $G_0(x)$ value, the constant -40792.1410 is not taken into account, as done for the reported solutions in the cited papers. The computed value of $G_2(x)$ and $G_5(x)$, the only constraints which are tight for at least one optimal solution, is given. One can easily check that the remaining constraints are satisfied for all solutions.

From Table 2, it appears that:

(i) Optimal solutions of $(P2)$ and $(P2^1)$ are very similar.

Table 2: Results for problem 2

Ref.	Reported formulation	Reported optimal solution x_1	x_2	x_3	x_4	x_5	$G_0(x)$	Computed values $G_0(x)$	$G_2(x)$	$G_5(x)$
1	$(P2)$	78	33	29.995510650	45	36.775173970	10126.642520000	10122.430521796	1.000000000	1.000000000
2	$(P2)$	78	32.999999462	29.995510165	44.999998630	36.775175250	10122.430477585	10122.430449341	1.000000002	1.000019949
3	$(P2^1)$	78	33	29.998000000	45	36.767300000	10127.130000000	10122.717434889	0.999826214	1.000042965
4	$(P2^1)$	78	32.999999267	29.995739631	45	36.775328091	10122.493176362	10122.514168062	1.000000000	1.000000000
5	$(P2^1)$	-	-	-	-	-	10122.381121680	-	-	-
6	$(P2^1)$	-	-	-	-	-	10121.794028763	-	-	-
7	$(P2^1)$	78	32.999980000	29.997370000	45	36.775330000	10122.856430000	10123.038349121	0.999979593	0.999867916
QP	$(P2)$	78	33	29.995510652	45	36.775173966		10122.430522242	1.000000000	1.000000000
QP	$(P2^1)$	78	33	29.995740025	45	36.775327094		10122.514229548	1.000000000	0.999980065

1. Dembo (1976)
2. Shen (2005)
3. Rijckaert and Martens (1978)
4. Shen and Zhang (2004)
5. Shen and Jiao (2006)
6. Shen et al. (2008)
7. Qu et al. (2008)

(ii) Reported variables values in Dembo (1976); Shen (2005); Rijckaert and Martens (1978); Shen and Zhang (2004), and Qu et al. (2008) are similar and correspond to feasible solutions of $(P2)$. However, the reported values of $G_0(x)$ in Dembo (1976) and Rijckaert and Martens (1978) do not agree with the reported values of the variables; this not the case (omitting some rounding errors) in Shen (2005); Shen and Zhang (2004), and Qu et al. (2008).

(iii) The reported value of $G_0(x)$ in Shen and Jiao (2006) and Shen et al. (2008) is slightly better than the optimal value obtained with QP. However, it is not possible to check the feasibility of these solutions since values of the variables are not reported.

Problem 3

This problem is expressed as a signomial geometric program. It has three variables, one constraint, and, in some versions, lower and upper bounds on the variables. Its genealogy can be traced as follows: Qu et al. (2008) cite Shen and Zhang (2004) which cite Rijckaert and Martens (1978). Moreover, Jiao et al. (2006); Shen and Jiao (2006), and Shen et al. (2008) cite Shen and Zhang (2004) and Qu et al. (2007b) cite Rijckaert and Martens (1978).

In Rijckaert and Martens (1978), this problem is written as:

$$(\mathbf{P3}) \begin{cases} \min & G_0(x) = 0.5x_1x_2^{-1} - x_1 - 5x_2^{-1} \\ \text{s.t.:} & \\ & G_1(x) = 0.01x_2x_3^{-1} + 0.01x_1 + 0.0005x_1x_3 \leq 1. \end{cases}$$

In Shen and Zhang (2004); Jiao et al. (2006); Shen and Jiao (2006); Qu et al. (2007b); Shen et al. (2008), and Qu et al. (2008), the bounds $70 \leq x_1 \leq 150$, $1 \leq x_2 \leq 30$, and $0.5 \leq x_3 \leq 21$ are added and the second term in $G_1(x)$ is $0.01x_2$ instead of $0.01x_1$, i.e., the variable index is 2 instead of 1. We will refer to $(P3^1)$ as the bounded version of $(P3)$ and $(P3^2)$ as $(P3^1)$ with the wrong $G_1(x)$. Problem $(P3)$ can be reformulated as a nonconvex quadratic program by introducing two variables. Table 3 presents the results reported in the cited papers together with the solutions obtained by QP for the quadratic programming formulations of $(P3^1)$ and $(P3^2)$.

From Table 3, it appears that:

(i) The solution reported in Rijckaert and Martens (1978) is close to the optimal solution of $(P3^1)$.

(ii) The solutions reported in Shen and Zhang (2004); Jiao et al. (2006) and Qu et al. (2007b, 2008) are feasible for $(P3^1)$ and $(P3^2)$, and are close to the optimal solution of $(P3^1)$. It seems that problem $(P3^1)$ is solved in these papers despite the fact that problem $(P3^2)$ is reported.

Table 3: Results for problem 3

Ref.	Reported formulation	Reported optimal solution x_1	x_2	x_3	$G_0(x)$	Computed values $G_0(x)$	$G_1(x)$ of $(P3^1)$	$G_1(x)$ of $(P3^2)$
1	$(P3)$	88.31	7.454	1.311	-83.21	-83.057115640	0.997844566	0.189284566
2	$(P3^2)$	88.724706796	7.672652781	1.317862596	-83.249728406	-83.594492450	1.003930985	0.193410445
3	$(P3^2)$	88.347018980	7.685918099	1.338260065	-83.250249460	-83.250229110	1.000018005	0.193406996
4	$(P3^2)$	-	-	-	-83.249728410	-	-	-
5	$(P3^2)$	-	-	-	-83.249790057	-	-	-
6	$(P3^2)$	88.6274	7.9621	1.3215	-83.6898	-83.689795600	1.005085027	0.198432027
7	$(P3^2)$	88.875643887	7.563758900	1.3124563877	-83.661573642	-83.661593250	1.004709696	0.191590846
QP	$(P3^1)$	88.354285800	7.674941139	1.318103852	-	-83.249732910	1.000000056	0.193206609
QP	$(P3^2)$	150	30	0.5	-	-147.666666700	2.137500000	0.937500000

1. Rijckaert and Martens (1978)
2. Shen and Zhang (2004)
3. Jiao et al. (2006)
4. Shen and Jiao (2006)
5. Shen et al. (2008)
6. Qu et al. (2007b)
7. Qu et al. (2008)

(iii) The values of $G_0(x)$ reported in Shen and Jiao (2006) and Shen et al. (2008) agree with the optimal solution of $(P3^1)$. Again, it seems that problem $(P3^1)$ is solved in these papers despite the fact that problem $(P3^2)$ is reported.

Problem 4

This problem is expressed as a posynomial geometric program. It is based on an example studied by Neghabat and Stark (1972). It has four variables, three constraints, and, in some versions, lower and upper bounds on the variables. Its genealogy can be traced as follows: Qu et al. (2008) cite Rijckaert and Martens (1978). Moreover, Shen and Zhang (2004) and Qu et al. (2007b) cite Rijckaert and Martens (1978) and Shen and Jiao (2006) as well as Shen et al. (2008) cite Shen and Zhang (2004).

In Rijckaert and Martens (1978), this problem is written as:

$$(\mathbf{P4}) \begin{cases} \min & G_0(x) = 168x_1x_2 + 3651.2x_1x_2x_3^{-1} + 3651.2x_1 + 40000x_4^{-1} \\ \text{s.t.:} & \\ & G_1(x) = 1.0425x_1x_2^{-1} \le 1 \\ & G_2(x) = 0.00035x_1x_3 \le 1 \\ & G_3(x) = 1.25x_1^{-1}x_4 + 41.63x_1^{-1} \le 1. \end{cases}$$

In Shen and Zhang (2004); Shen and Jiao (2006); Shen et al. (2008) and Qu et al. (2008), the bounds $40 \le x_1 \le 44$, $40 \le x_2 \le 45$, $60 \le x_3 \le 70$, $0.1 \le x_4 \le 1.4$ are added, the term $3651.2x_1$ in $G_0(x)$ is missing, and $G_2(x) = 0.00035x_1x_2$ instead of $0.00035x_1x_3$. In Qu et al. (2007b), the problem statement is similar except for an additional typo: the coefficient of the second term of $G_0(x)$ is 36512 instead of 3651.2. We don't consider this typo in our analysis. We will refer to $(P4^1)$ as the bounded version of $(P4)$ and $(P4^2)$ as $(P4^1)$ with $G_0(x)$ and $G_2(x)$ of Shen and Zhang (2004); Shen and Jiao (2006); Shen et al. (2008).

Introducing three new variables $y_1 = x_1x_2$, $y_2 = x_3^{-1}$, and $y_3 = x_4^{-1}$, and multiplying $G_0(x)$ by x_3x_4, $G_1(x)$ by x_2, and $G_3(x)$ by x_1 reformulates $(P4)$ as a nonconvex quadratic program. Table 4 presents the results reported in the cited papers together with the solutions obtained by QP for the quadratic programming formulations of $(P4^1)$ and $(P4^2)$. The computed values in the last set of columns are for $(P4)$, the original version of the problem.

From Table 4, it appears that:

(i) Values of variables reported in Dembo (1976) are close to the optimal solution but not precisely the same. The solution is feasible but slightly worse than the optimal solution.

Table 4: Results for problem 4

Ref.	Reported formu-lation	Reported optimal solution					Computed values			
		x_1	x_2	x_3	x_4	$G_0(x)$	$G_0(x)$	$G_1(x)$	$G_2(x)$	$G_3(x)$
1	$(P4)$	43.02	44.85	66.39	1.11	623015	623370.075457636	0.999963211	0.999634230	0.999941887
2	$(P4^2)$	43.013755728	44.814840340	66.423933664	1.107004583	623249.876118100	622990.927217284	1.000602479	1.000000000	1.000000000
3	$(P4^2)$	-	-	-	-	623249.875294750	-	-	-	-
4	$(P4^2)$	-	-	-	-	623249.136172314	-	-	-	-
5	$(P4^2)$	43.0187	44.8491	66.4581	1.1082	142027.91556	623293.496685749	0...999953059	1.000629373	0.999919802
6	$(P4^2)$	43.0899785	44.9997852	66.419945664	1.106998756	468479.996875421	625814.354316774	0.998255934	0.678663922	0.998230910
QP	$(P4^1)$	43.012808311	44.840852664	66.425396760	1.106246649		623249.893341851	1.000000000	1.000000000	1.000000000
QP	$(P4^2)$	43.165467626	45	70	1.228374101		460212.290586926	1.000000000	0.679856115	1.000000000

1. Rijckaert and Martens (1978)
2. Shen and Zhang (2004)
3. Shen and Jiao (2006)
4. Shen et al. (2008)
5. Qu et al. (2007b)
6. Qu et al. (2008)

(ii) The reported variables values in Shen and Zhang (2004) and Qu et al. (2007b) are close to the optimal solution of $(P4^1)$ and are feasible for $(P4)$ and $(P4^1)$. The computed value of $G_0(x)$ is slightly different from the reported value in Shen and Zhang (2004). The reported value of $G_0(x)$ in Qu et al. (2007b) is wrong and is far from the computed value for $(P4)$. It seems that problem $(P4^1)$ is solved in these papers despite the fact that problem $(P4^2)$ is reported.

(iii) The reported value of $G_0(x)$ in Shen and Jiao (2006) and Shen et al. (2008) is close but slightly better than the optimal value for $(P4^1)$ obtained with QP. It seems that problem $(P4^1)$ is solved in these papers despite the fact that problem $(P4^2)$ is reported. However, it is not possible to check the feasibility of these solutions since the values of the variables are not reported.

(iv) The reported variables values in Qu et al. (2008) are close to the optimal solution of $(P4^1)$. The computed value of $G_0(x)$ of $(P4)$ is far from the value reported but slightly worse than the optimal value. Replacing the reported variables values in the wrong $G_0(x)$ presented in the paper gives 468484.224817574 which is close to the reported value and not very far from the optimal value of $(P4^2)$. It is not clear which version of the problem is solved in this paper but it is clear that the reported solution is not optimal for $(P4^2)$ but close (except for the value of $G_0(x)$) to the optimal solution of $(P4^1)$.

Problem 5 This problem is expressed as a signomial geometric program. It has two variables, two constraints, and, in some versions, lower and upper bounds on the variables. Its genealogy can be traced as follows: Qu et al. (2008) cite Peng and Yuan (1997). Moreover, Qu et al. (2006) and Qu et al. (2007a) cite Peng and Yuan (1997).

In Peng and Yuan (1997) this problem is written as:

$$
(\mathbf{P5}) \left\{ \begin{array}{ll} \min & G_0(x) = -4x_2 + (x_1-1)^2 + x_2^2 - 10x_3^2 \\ \text{s.t.:} & \\ & G_1(x) = x_1^2 + x_2^2 + x_3^2 \leq 2 \\ & G_2(x) = (x_1-2)^2 + x_2^2 + x_3^2 \leq 2. \end{array} \right.
$$

In Qu et al. (2006, 2007a), the problem is the same except that the following bounds on variables are added: $2-\sqrt{2} \leq x_1 \leq \sqrt{2}$, $-\sqrt{2} \leq x_2 \leq \sqrt{2}$ and $-\sqrt{2} \leq x_3 \leq \sqrt{2}$. We will refer to $(P5^1)$ as the bounded version of $(P5)$. Note that there is a typo in $G_2(x)$ of Qu et al. (2006): the first term is $(x-2)^2$ instead of $(x_1-2)^2$, i.e., the variable index is missing.

Developing expressions $(x_1-1)^2$ and $(x_1-2)^2$ reformulates $(P5)$ as nonconvex quadratic program. Table 5 presents the results reported in the cited papers together with the solutions obtained by QP for the quadratic programming formulations of $(P5^1)$.

Table 5: Results for problem 5

Ref.	Reported formulation	Reported optimal solution				Computed values		
		x_1	x_2	x_3	$G_0(x)$	$G_0(x)$	$G_1(x)$	$G_2(x)$
1	$(P5)$	-	-	-	-	-	-	-
2	$(P5^1)$	0.99712235	0.18184214	-0.98034321	-	-10.305021809	1.988392354	1.999902954
3	$(P5^1)$	1	0.220971	0.972272	-11.2882	-10.288184237	1.994141025	1.994141025
4	$(P5^1)$	0.99712235	0.18184214	0.98034321	-13.56612456	-10.305021809	1.988392354	1.999902954
QP	$(P5^1)$	1	0.18102669	0.983478767		-10.363640955	2.000001148	2.000001148

1. Peng and Yuan (1997)
2. Qu et al. (2006)
3. Qu et al. (2007a)
4. Qu et al. (2008)

From Table 5, it appears that:

(i) The reported variables values in all papers where the solution is given are not very far but, for some of them, significantly different than the optimal solution of $(P5^1)$ obtained with QP. The computed value of $G_0(x)$ for the solution obtained by QP is better than the computed value for the reported solution in the cited papers but, the solution obtained by QP is slightly infeasible.

(ii) The reported value of $G_0(x)$ in Qu et al. (2006, 2008) is wrong and does not agree with the computed value using the variables values.

3 Conclusion

As, in our opinion, the examples of Section 2 amply illustrate, mutations over time in at least some classes of mathematical programming test problems are not a rare phenomenon. This suggests a few words of caution, which may help in avoiding deterioration of test problems and misinterpretation of results:

(i) When the results of two codes for what is referred to as the same test problem differ significantly, this may due to: (a) errors in one (or both) algorithms used; (b) errors in one (or both) implementations; (c) errors in one (or both) problem formulations; (d) errors in one (or both) data files. It may be tempting to interpret this difference as one's own algorithm being better than previous ones, which usually implies these were incorrect. However, the multiplicity of possible causes suggests it way be rash to jump to such a conclusion.

(ii) Some symptoms of alternate situations are the following: (a) if a better solution than the incumbent is obtained for a problem which has reportedly been solved with the same result by several exact algorithms, a careful check of the identity of formulations and of numerical data appears to be a reasonable first step; (b) if the same optimal value is found for two different versions of the same test problem, it should be checked that those problems are really different (which may happen, e.g., if some added constraints are redundant).

(iii) To detect possible errors, a few easy checks can be made, i.e., substitutions of reported numerical values of variables in the, again reported, objective function and constraints. If objective function values do not agree, clearly there are some errors in formulations or perhaps some typos in numerical values. The same applies if violation of some constraints is larger than some small percentage ϵ (we do not discuss here the fact that many global optimization algorithms provide solutions which do not strictly satisfy all constraints). Note that such checks can be made only if numerical values for all variables are reported (and not as it sometime the case only the optimal objective function value). Moreover, for results to be significant, a sufficient number of decimals should be given.

(iv) To minimize the risk of mutations in test problems, it appears to be worthwhile to use problems from well tested series instead of subsequent versions. Moreover, it is better to use the electronic files (when available) instead of a paper version. Perhaps, as a rule, electronic files of test problems should be *always* made available on the web. Those of the present paper are available in AMPL format at http://neumann.hec.ca/pages/sylvain.perron/. For global optimization, such test problems can be found, e.g., in Floudas et al. (1999). For geometric programming, the best series of problems still appear to be those of Dembo (1976); Rijckaert and Martens (1978).

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