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The Strategic Value of a Seller's Advance Booking Discount Program

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Abstract

In an Advance Booking Discount Program (ABDP), a firm offers a product at a price discount prior to the selling season. In the selling season the product is sold at a regular price. The aim of the paper is to study the strategic value of an ABDP under uncertainty with respect to a specific consumer characteristic. The setup is a duopolistic market, modelled by Hotelling's "linear city" where two firms, A and B, are located at the boundaries of the city. A and B are uncertain about consumers' transportation costs. Firm A only has the option to implement an ABDP.

Three scenarios emerge, each generating a particular outcome. In the first scenario it is not optimal for A to set up an ABDP. The reason typically is that the selling price of B is considerably larger than that of A and there is no incentive to implement the program. In the two other scenarios, A implements an ABDP in which the reduced price enables A to attract customers from B. If consumers' transportation costs are sufficiently low, B actually exits the market. We show that when A does not implement an ABDP, its expected profit increases as uncertainty about consumers' transportation cost increases. We also show that the gain of implementing an ABDP may depend on uncertainty in various ways.

Résumé

Un Programme d'Achat Anticipé (PAA) consiste pour une entreprise à proposer aux consommateurs d'acheter à un prix réduit un produit qui sera livré durant la saison régulière. Le produit est disponible au prix régulier durant la saison. L'objectif de cet article est d'évaluer la valeur stratégique d'un PAA en présence d'incertitude sur une caractéristique des consommateurs.

Le cadre analytique est celui d'un marché duopolistique à la Hotelling où deux firmes, A et B, sont localisées aux deux extrémités de la ligne. Les deux firmes sont incertaines quant au coût de transport des consommateurs. Seule la firme A peut offrir un PAA. Trois scénarios engendrant des gains différents peuvent émerger. Dans le premier, il n'est pas optimal pour A d'offrir un tel programme. La raison est typiquement que le prix régulier au cours de la saison est considérablement plus élevé que le prix réduit. Dans les deux autres scénarios, A implante un PAA pour attirer une partie de la clientèle de B. Si le coût de transport est en fait suffisamment bas, la firme B sort du marché. On montre que quand A n'offre pas un PAA, son profit espéré augmente avec l'incertitude sur le coût de transport des consommateurs. On montre aussi que le gain de proposer un PAA peut dépendre de plusieurs façons de l'incertitude.

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1 Introduction

The paper considers a situation in which a manufacturing firm offers an “Advance Booking Discount Program” (ABDP). The idea of the program is to offer consumers a particular product at a price discount, prior to the “selling season” in which the product is sold at a higher, “regular” price. Customers who accept to participate in the program place an order which is binding; no cancelations or refunds are possible. At the beginning of the selling season the product is delivered to the customers in the program. Consumers who do not participate in the program can purchase the product at the regular price at any time during the selling season.

An incentive for a firm to set up an ABDP is that it may provide advance information about consumer demand which can be useful later on. Furthermore, the firm’s market share could increase if customers of competing firms choose to participate in the program. Another advantage of an ABDP is that the product can be sold over a longer period of time. Moreover, revenues in the program are guaranteed if orders placed under the program are prepaid. The advantage of participating in an ABDP of a consumer is that the purchasing cost is lower and the product is delivered without interruption.

ABDPs have been used in the sales of perishable products that are consumed during a well-defined and concentrated selling season. Tang et al. (2004) and McCardle et al. (2004) provide some examples: Pumpkin pies or fresh turkeys during Thanksgiving, moon cakes during the Chinese mid-Autumn festival, Christmas trees, or new durables with a short selling season and high demand uncertainty (music CDs, computer games). ABDPs are often used in service provision (airlines, hotels, travel, entertainment): See, e.g., Shugan and Xie (2000, 2001, 2004).

The paper is organized as follows. We proceed in this section by a review of the literature on advance booking discount programs, state the aims of the paper, and report our main findings. A formal model of a duopolistic market is set up in Section 2. Section 3 analyzes a benchmark case in which no ABDP is implemented. Section 4 is concerned with the situation where one of the two firms has the option of implementing an ABDP. Our conclusions are found in Section 5.

1.1 Literature review

Weng and Parlar (1999) address the problem where a seller of a product offers a discount to consumers to make them buy earlier. Total sales are random and a certain fraction of sales will be made in the ABDP. The number of customers who will take advantage of the program is a function of the depth of discount. Weng and Parlar assumed that orders within the program are deterministically known. The remaining demand, which occurs during the selling season, is random. There is no explicit time dimension in the model: customers within and outside the program are in the market during a fixed interval of time. Hence, the model essentially is static and there is only one decision maker, the seller.

Tang et al. (2004) extend the setup of Weng and Parlar and assume that the market has two segments: one which buys the product of the seller and one which buys a competing (aggregate) product. Tang et al. suppose that demand within as well as outside the program is random. Moreover, they introduce a fixed cost of implementing the program and analyze also the case where the program is merely an early reservation procedure (where there is no discount). A particular focus is on demand updating, that is, how demand in the program can be used to improve demand forecasts for the selling season. Clearly, this is relevant for the seller's lot sizing decision. The following result is particularly interesting. Denote the random demand in the program by D_1 and let D_2 be demand in the selling season. Then it holds that the variance of the conditional distribution of D_2 given $D_1 = d_1$ is smaller than the variance of the joint distribution of D_1 and D_2 . The intuition here is that the knowledge of sales generated by the program improves the seller's ability to forecast later demands.

Several authors have pointed out that an ABDP can provide demand information by using advance bookings as a leading indicator of total demand and that sellers can use early-booking low prices to generate additional, price-sensitive demand. Chen (2001) suggests an advance-booking discount policy for a monopolist seller when customers select the price to pay by choosing the shipping date of the items bought.

McCardle et al. (2004) abandon the single-firm assumption of the above works and suggest a duopoly game in which each firm can decide to implement an ABDP. The model is a fairly straightforward extension of the one-player setup in Tang et al. (2004) and gives rise to four scenarios. One in which no firm implements a program and one in which both firms implement a program. Not unexpectedly, if implementing a program is costless, there is a unique equilibrium in the duopoly game and in equilibrium both firms offer an ABDP.

There is a sizeable literature in the area of revenue management. These works are concerned with the advance selling of a fixed capacity (seats in airplanes, rooms in hotels). In some situations a seller can increase its capacity, but only by paying a large fixed cost. The fixed capacity feature leads to problems that are essentially different from the one of this paper. Our assumption is that the seller can decide freely on the amount of goods available for sales in the program and during the selling season. Moreover, in our setup the seller does not have to make the capacity decision before launching the program. The reason is that the amount of capacity that is consumed in the program will be endogenously determined as a function of the discount offered in the program.

The advance selling problem has been addressed in marketing literature on price deals and promotions and has relations to contingent claims contracts (insurance), peak load pricing, and state-dependent utility theory (Shugan and Xie (2001)).

1.2 Aim of paper

The paper is positioned in the "intersection" between the Tang et al. (2004) and the McCardle et al. (2004) papers. In the former there is a single firm who offers an ABDP, that is, we have a monopolistic situation. In the latter, there are two firms who compete

in a duopolistic market and each of them can choose to offer an ABDP. In our setup there will be two firms in a duopoly, but one firm only has the option to offer a program. (This corresponds to one of the asymmetric scenarios in McCardle et al. (2004)).

One aim of the paper is to extend the analysis of Tang et al. (2004) (see also McCardle et al. (2004)) by endogenizing the fraction of customers who participate in the ABDP. In these papers it was assumed that the relationship between the fraction of customers who participate in a program and the discount offered by the program is exogenously given and has a specific functional form. In this paper we wish to determine endogenously the number of customers participating in the program. Another difference between the two papers and ours is that we explicitly discount future profits to allow for time-impatience on the part of firms.

Another aim of the paper is to study the impact of uncertainty in another setup than that in Tang et al. (2004) and McCardle et al. (2004). In these papers the assumption is that the joint probability distribution for the random demands in the program and in the selling season is a bivariate normal distribution. In our setup, random demand is modelled by a simple two-point probability distribution. Due to this assumption we are able to derive almost all results analytically. Tang et al. (2004) had, in some instances, to rely on numerical methods to obtain insights.

1.3 Main results

Consider two firms, A and B, who are in the same market. They play a Hotelling “linear city” game (Hotelling (1929)) in which with probability $\frac{1}{2}$ either customers’ transportation cost is low or high. In period one, firm A (only) has the option of implementing an ABDP in which customers are offered a reduced price as an incentive to buy early. To start up such a program, firm A incurs a fixed cost. In period two (the selling season), the firms play a game in which they determine their respective outputs. The normal prices charged in this period are exogenously given. firm A has three decisions to make. First, should an ABDP be implemented? If yes, what is the optimal discount to give the customers in the program? Finally, firm A must determine its optimal output in the duopoly output game played in the selling season. The latter also applies to firm B. It is assumed that firm A has the lowest normal selling price. Consumer demand is determined by the prices and the consumers’ random transportation cost. Demand of firm A decreases when the transportation cost increases, because then demand is less price sensitive.

We show in Section 3 that if firm A does not implement an ABDP, the firm’s optimal expected profit in the output game increases with the uncertainty in the transportation cost. Firm A’s profit increases because the firm gets more customers when uncertainty about the transportation cost increases. The reason is that for firm A demand is decreasing in the transportation cost in a convex way, and increased uncertainty implies that transportation costs can take more extreme values. If transportation costs are high, price differences are less important for consumers in their product choice. An increase in transportation cost then means that the firm having the highest selling price (which by assumption is firm B)

will gain demand at the expense of firm A. Using the same reasoning, firm A will sell more goods when transportation costs are low. Due to the convex dependence of firm A's demand on transportation cost, the latter effect will be dominating.

If firm A chooses to implement an ABDP, we first show that all customers of firm A will join the ABDP. Hence, this means that firm A's sales in period two are zero and information about demand provided by the ABDP is not exploited in fixing the production volume. The conclusion is that the benefit of setting up an ABDP is to increase market share, not to improve demand forecasts for the selling season. In Section 4.1 we determine endogenously how many customers will take part in the program. We show in Section 4.4 that the above result for the optimal expected profit of firm A carries over to the situation with an ABDP (although one subcase requires that the transportation cost is sufficiently low).

We examine the circumstances under which a program should be implemented. If, *ceteris paribus*, the regular price of firm A is low, it does not pay to start the program. Otherwise it may, depending on the fixed cost of setting up the program, be optimal to start the program. Then there are two choices for the optimal discount given to the customers.

In a first case, the competitor's price is "large" and firm A gets a substantial price advantage by implementing the program. Actually, firm A gains all the customers in the market if the transportation cost happens to be low. The optimal price reduction decreases in the competitor's price and increases in firm A's own price. The price reduction increases in the mean transportation cost and decreases with the uncertainty in the transportation cost. The intuition of these results follows from the fact that low transportation costs tend to be low when mean transportation costs are low or when uncertainty is high. Then a small price reduction is enough to attract all customers if the transportation cost is low.

In a second case, the price reduction is larger and by implementing the program, firm A can attract some customers of firm B (but demand of firm B remains positive). The optimal price reduction percentage varies with the competitor's price and own price in the same way as in the first case. However, the price reduction decreases with the mean transportation cost and increases with the uncertainty on the transportation costs. This is the opposite of what happened in the first case. The reason is that price sensitivity is higher when the transportation cost is low such that this is a dominant scenario when it comes to determining the optimal price reduction. When transportation costs are low, a price reduction attracts more customers and a price reduction is more substantial.

In Section 4.6 we study how the decision to start a program is affected by the uncertainty in the transportation cost. For the scenario discussed in the previous paragraph we identify parameter constellations under which the profitability of implementing the program increases, decreases, or first increases and then decreases with the uncertainty. We show that if there is no uncertainty about consumers' transportation cost, the benefit of implementing the program is substantial. The benefit decreases, however, as uncertainty increases and eventually the benefit becomes zero. Then the presence of a fixed start-up cost implies that it is not profitable to start up an ABDP.

2 Model

We consider an ABDP in which firm A can choose to offer a particular product at a price discount, prior to the selling season. If customers accept the offer they make a prepayment at the time of their order. At the beginning of the selling season the product is delivered to those who participate in the program. Consumers who do not want to participate in the program can purchase the product at the regular (and higher price) at any instant of time during the selling season.

Our model is based on Hotelling's "linear city" output (or pricing) game (Hotelling (1929), Tirole (1988)). A linear city of length one is a city which lies on a straight line and there is a population of risk neutral consumers being uniformly distributed over the interval $[0, 1]$. There are two risk neutral firms: firm A sells product A while firm B sells product B. The firms are located at the extremes of the city. (Firms could be located at interior positions, cf. Fudenberg and Tirole (1991). In general, locations at the extremes of the city is advantageous to the firms in that it tends to reduce price competition).

Suppose that firm A is located at $y = 0$, firm B at $y = 1$. Both firms have the same unit cost of production of the good, equal to $c = \text{const.} > 0$. Consumers incur a transportation cost T per unit of distance travelled. Thus, a consumer located at y incurs a cost of Ty when buying from firm A and a cost of $T[1 - y]$ when buying from firm B. A consumer buys either one unit of the good or nothing and derives utility $\bar{s} = \text{const.} > 0$ from the consumption (gross of price and transportation costs) of one product unit. We assume that \bar{s} is a large number such that any consumer prefers to buy A or B to buying nothing. (In this way we do not need to worry about buyers who are not purchasing at all).

Remark 1 *In the above description, consumers are located at different places. An alternative interpretation of the linear city is that customers have heterogenous tastes which lie on a continuum. Firms A and B being at different locations then means that their products have different tastes. The location of a consumer represents her relative preference for the tastes of the two products. Transportation costs reflect the loss of utility suffered by a consumer who does not get her preferred product.*

The linear city model is used in a two-period setup. (For a similar setup, see Shugan and Xie (2001)). In period one, firm A (but not B) has the possibility to implement an ABDP which will be in effect in period one only. The "selling season" is period two in which firms have unit selling prices p_A and p_B , respectively. These prices are the *regular* prices that are assumed exogenously given. If firm A chooses not to implement an ABDP in period one, it charges its regular price in that period two.

Consumers of product A enter the market in period one and can – if they join the program – buy the product of firm A at a reduced price, equal to xp_A , where $x \in (0, 1)$ is the discount offered on the regular price. The reduced price is announced in period one. If a consumer of product A does not buy in period one she wait until period two in which she can buy either product A or product B. Consumers of product B enter the market in period two (or, alternatively, the product of firm B is not available in period one).

To implement the program, firm A must pay a fixed cost $K = \text{const.} > 0$. Firm A is not obliged to produce the goods sold in the program until period two arrives: Customers in the program have prepaid their orders, but delivery is postponed until period two.

A priori, both firms face an uncertainty; they do not know the actual value of the transportation cost T . This cost is a random variable. To make things simple, suppose that T is “high”, equal to $t + w > 0$ with probability $\frac{1}{2}$, or “low” and equal to $t - w > 0$ with probability $\frac{1}{2}$, where $w \in [0, t)$ is called the “uncertainty parameter”. It follows that $E(T) = t$ and $\text{var}(T) = w^2$. The firms must determine their respective quantities q_A and q_B before knowing the actual transportation cost.

A consumer located at $y \in (0, 1)$ derives a net utility of buying product A which equals $\bar{s} - p_A - [t + w]y$ when the transportation cost is high. The net utility is $\bar{s} - p_A - [t - w]y$ if the transportation cost is low. If the transportation cost is high, consumers have a stronger preference for one of the products than when the cost is low. Thus, differences in prices are more important to consumers when preferences are weaker.

In order to avoid less interesting outcomes, we make some assumptions. First, to make production worthwhile, we make

Condition 2 *For both firms, the discounted price exceeds the unit cost.*

By the discounted price we mean the normal price, discounted backwards by one period, *not* the reduced price charged in the ABDP. Denoting the one-period discount factor by $\delta = \text{const.} \in (0, 1)$, the discounted price of firm i equals δp_i , $i \in \{A, B\}$.

If (i) the transportation cost is deterministically known, (ii) prices are endogenously determined, and (iii) an ABDP does not exist, it has been shown (Tirole (1988) or Fudenberg and Tirole (1991)) that the discounted prices simply are $c + T$. We shall study a situation in which prices are not “too different” from $c + T$. More specifically, we make the following assumption.

Condition 3 *Suppose that a firm’s discounted price does not exceed the sum of the unit production cost and two times the maximal transportation cost.*

The two conditions can be formalized as

$$c < \delta p_i < c + 2[t + w], \quad i \in \{A, B\}. \quad (1)$$

Finally we need

Condition 4 *Suppose that for any firm its expected profits will be negative if its output is directed at a market segment where, with probability $\frac{1}{2}$, the customers prefer to buy from either firm.*

The implication is that if firm B serves such a segment, it is suboptimal for firm A to do so, and vice versa. The assumption can also be expressed as

$$\max \{ \delta p_A, \delta p_B \} < 2c. \quad (2)$$

3 No ABDP

This section assumes that either firm A does not have the option of implementing an ABDP, or, if it has, it does not wish to use that option. The latter could be the case if (i) the cost of setting up and running the program is high and the additional revenues generated in the program are insufficient to cover those costs. It could also happen (cf. Shugan and Xie (2001)) in the case where (ii) the firm's production capacity is very small or (iii) when marginal production costs are large. All three cases will be addressed below.

In the absence of an ABDP, firms play in the selling season a one-period game in which they fix their quantities without knowing the transportation cost. First we characterize the demand functions in this game. A demand function can be found by locating the "indifferent consumer" (see, e.g., Rasmusen (1989)). Every individual located to the left (right) of the indifferent consumer buys the product of firm A (B). The location of the indifferent consumer is a value of y which satisfies

$$p_A + Ty = p_B + T[1 - y], \quad (3)$$

that is,

$$y = \frac{T + p_B - p_A}{2T}. \quad (4)$$

Let y_h be the location of the indifferent consumer when the transportation cost is high ($T = t + w$) and let y_l be the location of the indifferent consumer when the transportation cost is low ($T = t - w$). The implication is, using (4), that

$$\begin{aligned} y_h &= \frac{t + w + p_B - p_A}{2[t + w]} \\ y_l &= \frac{t - w + p_B - p_A}{2[t - w]} \end{aligned} \quad (5)$$

and it holds that $y_h (\leq) y_l$ for $p_A (\leq) p_B$.

We confine our interest to the case $p_A < p_B$. The case $p_A = p_B$ is uninteresting because the market is shared equally among the firms, no matter the value of the uncertainty parameter w . We refrain from analyzing the case $p_A > p_B$. Our expectation here is that there is a stronger incentive to start an ABDP. The reason is that firm A could improve its competitive position by lowering the "high" price p_A and offer a substantial price reduction

to customers in the program. To make sure that y_h and y_l are between zero and one it suffices to assume $p_B - p_A \leq t - w$, that is, the price differential does not exceed the low value of the transportation cost. In other words, to have positive demands for both products, the difference in prices must not be too large.

Under the assumption $p_A < p_B$ it holds that consumers with $y \in [0, y_h)$ always prefer product A, consumers with $y \in [y_h, y_l)$ prefer A if the transportation cost is low and B otherwise, whereas consumers with $y \in [y_l, 1)$ always prefer B. Noting that y_l and y_h converge to $\frac{1}{2}$ when the mean transportation cost increases shows that price differences are less important when consumers have high transportation costs (strong preferences).

Denote by $\pi_A(q_A, q_B)$ the profit of firm A and let E_1 represent the expectation operator, given the information available at the start of period one. The expected profit of firm A, which is to be maximized with respect to $q_A \geq 0$, is given by

$$\begin{aligned} E_1 [\pi_A(q_A, q_B)] = & \\ & \min \{y_h, q_A\} \delta p_A + \frac{1}{2} \max \{ \min \{y_l - y_h, q_A - y_h\}, 0 \} \delta p_A + \\ & \frac{1}{2} \max \left\{ \frac{1 - y_h - q_B}{1 - y_h}, 0 \right\} \max \{ \min \{y_l - y_h, q_A - y_h\}, 0 \} \delta p_A + \\ & \max \left\{ \frac{1 - y_l - q_B}{1 - y_l}, 0 \right\} \max \{ \min \{1 - y_l, q_A - y_l\}, 0 \} \delta p_A - cq_A. \end{aligned}$$

This formulation of firm A's objective assumes that the consumers located along the line segment $[0, q_A]$ will be served, given that at least q_A consumers want to buy from firm A. Consumers who would like to buy from firm A, but are located outside the line segment, will not be served by firm A. In a similar fashion, given that at least q_B consumers want to buy from firm B, consumers located in the segment $[1 - q_B, 1]$ will be served by firm B.

The objective consists of four revenue terms and one cost term (cq_A). The first revenue term concerns the consumers in the segment $(0, y_h)$ who will buy from firm A, provided that its output is sufficient. The second revenue term relates to the segment (y_h, y_l) if the transportation cost is low. In this case consumers choose firm A. The third term relates to the segment (y_h, y_l) if the transportation cost is high. Consumers in this segment would like to buy from firm B, but will go to firm A if firm B's output is insufficient. The fourth term relates to the segment $(y_l, 1)$ where consumers prefer to buy from firm B, but choose A if the output of firm B is insufficient.

We turn to the determination of equilibrium values of q_A and q_B . Recall the assumption in (2) and note that since (1) implies $c < \min(\delta p_A, \delta p_B)$, it is optimal for a firm to produce for those consumers who always prefer to buy its product, given it is available. The implications are that

$$q_A \geq y_h, \quad q_B \geq 1 - y_l.$$

Next we need to determine which firm will serve the "intermediate" segment (y_h, y_l) . Consumers prefer A and B with the same probability. For firm A, the expected discounted

revenue per consumer equals $\frac{1}{2}\delta p_A$. Given the assumption in (2), firm A will produce for consumers in the intermediate segment only if firm B does not, and vice versa. The implication is that if firm A chooses $q_A = y_l$, firm B's best reply is $q_B = 1 - y_l$. On the other hand, if firm B selects $q_B = 1 - y_h$, firm A's best reply is $q_A = y_h$. Thus we have two equilibria. In fact, there are many more equilibria since for every $y \in (y_h, y_l)$, it holds that $q_A = y$ and $q_B = 1 - y$ is an equilibrium. We summarize these results in the following proposition.

Proposition 5 *Consider the game where no ABDP is available to firm A. Under the assumption (2) there are infinitely many equilibria such that for every $y \in [y_h, y_l]$, the pair $(q_A = y, q_B = 1 - y)$ is an equilibrium.*

Consider the particular equilibrium in which half of the consumers in the “intermediate” segment (y_h, y_l) are served by firm A, the other half by B. In this equilibrium it holds that

$$\begin{aligned}\hat{q}_A &= y_h + \frac{y_l - y_h}{2} = \frac{y_l + y_h}{2}, \\ \hat{q}_B &= 1 - y_l + \frac{y_l - y_h}{2} = 1 - \frac{y_l + y_h}{2},\end{aligned}\tag{6}$$

yielding the following equilibrium profits of firm A:

$$E_1 [\pi_A(\hat{q}_A, \hat{q}_B)] = [\delta p_A - c] \frac{y_l + y_h}{2} = \frac{\delta p_A - c}{2} \left[1 + \frac{t}{t^2 - w^2} [p_B - p_A] \right],\tag{7}$$

in which we have used (5). It is readily seen that the profit $E_1 [\pi_A(\hat{q}_A, \hat{q}_B)]$ is increasing in w . An intuition follows by noting that

$$\left| \frac{\partial y_l}{\partial w} \right| > \left| \frac{\partial y_h}{\partial w} \right|.$$

The inequality means that if the transportation cost is low, the indifferent consumer moves farther to the right than that the indifferent consumer moves to the left if the transportation cost were high. What drives the result is that the demand of firm A (i.e., the location of the indifferent consumer), is convex decreasing in the transportation cost, cf. (4). In the equilibrium given by (6), firm A expects to get more customers if w increases. Since firm A has the lower output price, it is this firm which benefits from increased uncertainty in the transportation cost.

4 ABDP

This section considers first the situation in which firm A has decided to implement an ABDP. After having studied this case, we proceed by answering the important question: Under which circumstances it is worthwhile for firm A to implement an ABDP?

Consumers who take part in the ABDP pay the price xp_A , where the percentage $x \in (0, 1)$ is a decision variable of firm A. The number of customers who take part in the program is a function of x (the “response” function) and is denoted by $R(x)$. In Tang et al. (2004), function $R(x)$ represents a fraction of demand. These authors assumed that $R(x)$ is an exogenously given function, $R(x) = 1 - ax^f$, where a and f are parameters. We have chosen a different approach and shall determine the function $R(x)$ endogenously.

Recall that the total number of consumers is one. Now, if $R(x)$ consumers choose the program, $1 - R(x)$ consumers will enter period two having bought nothing. In period two they can choose between products A and B. Let $R_h(x)$ denote the demand (number of customers) in the ABDP if the transportation cost is high and let $R_l(x)$ be the demand in the ABDP if the transportation cost is low.

Firm A has the following expected profit, which is to be maximized with respect to $x \in (0, 1), q_A \geq 0$:

$$\begin{aligned} E_1 [\pi_A(q_A, x, q_B)] &= E_1 [\min \{R(x), q_A\} xp_A + \delta R_{IP2} - K - cq_A] = \\ &\quad \frac{1}{2} [\min \{R_h(x), q_A\} xp_A + \delta (R_{IP2} | T = t + w)] + \\ &\quad \frac{1}{2} [\min \{R_l(x), q_A\} xp_A + \delta (R_{IP2} | T = t - w)] - K - cq_A = \\ &\quad \frac{1}{2} [\min \{R_h(x), q_A\} + \min \{R_l(x), q_A\}] xp_A + \\ &\quad \frac{\delta}{2} [R_{IP2} | T = t + w] + \frac{\delta}{2} [R_{IP2} | T = t - w] - K - cq_A, \end{aligned}$$

where R_{IP2} means revenue in period two (which tentatively is left undefined).

The output q_A of firm A is sold partly in the ABDP, partly to customers who wait and buy product A in period two. Note that when firm A must make its decision about x , the sales volume in the program is unknown. On the other hand, customers in the program do not need to have their orders delivered until the end of period one. Hence, assuming instantaneous production, firm A can wait to produce until that instant of time.

The analysis of the ABDP will proceed as follows. In Sections 4.1 through 4.5, the assumption is that firm A already has chosen to start the ABDP. Section 4.1 determines the demand function in the ABDP and Section 4.2 identifies the objective function of firm A as a function of the decision x . Section 4.3 determines optimal values of x in various scenarios and in Section 4.4 we calculate the optimal objective value of firm A. In Section 4.5 we address the duopoly output game. Finally, Section 4.6 identifies the circumstances under which it is optimal for firm A to start an ABDP.

Before doing these analyses, we state a general result.

Proposition 6 *A customer of product A who is located at position y , will enter the ABDP if*

$$xp_A + Ty < \delta p_A + Ty \implies x < \delta.$$

Proof. For a firm A customer, compare her utility $\bar{s} - xp_A - Ty$ of participating in the program, paying xp_A , to her present value utility $\bar{s} - \delta p_A - Ty$ of waiting to buy and paying the regular price p_A in period two. Clearly, if $x < \delta$, the price xp_A paid in the program is lower than the present value, δp_A , of the regular price p_A that is charged in period two. \square

Proposition 2 says that *all* potential customers of product A will enter the ABDP if $x < \delta$. On the other hand, if $x > \delta$, all these individuals choose to wait and buy the product at the regular price in period two. Put in another way: the total demand for product A is either fully satisfied by early sales in the ABDP or by later sales at the normal price. This result is, admittedly, an extreme one as it excludes situations where some consumers of product A choose the program while others wait and buy at the normal price.

The only concern of consumers is utility maximization. Buyers advance purchase only if the present value cost of buying in the program is less than the present value cost of waiting and buying product A later on in period two.

An implication of Proposition 2 is that demand information obtained in the program is not exploited in fixing the production volume in period two: Whenever an ABDP is implemented, sales of product A in period two are zero.

Clearly, consumers may have other objectives than utility maximization. For example, if customers in the program are guaranteed fulfilment of their orders, and it is uncertain whether all demands in the selling season can be satisfied, a consumer may wish to participate in an ABDP, simply to be sure to get the product.

4.1 ABDP Demand Function

For tractability purposes we impose an upper limit on the price differential:

$$p_B - p_A < \frac{[t - w]^2}{t + w}. \tag{8}$$

The upper limit increases in t and decreases in w . Thus, if the mean transportation cost (t) is large, p_B could be considerably larger than p_A . If the uncertainty parameter w in the transportation cost is small, p_B could also be considerably larger than p_A .

The expected demand in the ABDP is $\frac{1}{2} [R_h(x) + R_l(x)]$ and for this quantity we have the following result.

Lemma 7 *Expected demand in the ABDP equals*

$$\frac{1}{2} [R_h(x) + R_l(x)] = \left\{ \begin{array}{c} 0 \\ \frac{1}{2} + \frac{t}{2(t^2 - w^2)} [p_B - p_A] \\ \frac{1}{2} + \frac{t}{2(t^2 - w^2)} [\delta p_B - xp_A] \\ \frac{1}{2} + \frac{\delta p_B - xp_A + t + w}{4[t + w]} \\ 1 \end{array} \right\}$$

$$if \left\{ \begin{array}{l} x > \delta \\ 1 - \frac{(1-\delta)p_B}{p_A} < x \leq \delta \\ \frac{\delta p_B - [t-w]}{p_A} < x \leq 1 - \frac{(1-\delta)p_B}{p_A} \\ \frac{\delta p_B - [t+w]}{p_A} < x \leq \frac{\delta p_B - [t-w]}{p_A} \\ x \leq \frac{\delta p_B - [t+w]}{p_A} \end{array} \right\} .. \quad (9)$$

Proof. See the appendix. \square

In the first line in (9) no consumers choose the ABDP because the price reduction offered in the ABDP is too small (cf. Proposition 2). As of the second line, all consumers of product A choose the program. As of the third line, the program starts to attract customers from firm B. Tang et al. (2004) also noted that lowering the price in the program attracts more customers to firm A and therefore makes a larger portion of this firm's demand certain through the precommitted orders. In our setup Proposition 2 showed that for $x < \delta$ all demands of product A customers will be satisfied in the program and hence is certain.

4.2 Objective function of firm A

We know that if $x > \delta$, an ABDP generates no demand (and should not be implemented). Since we wish to establish firm A's objective function, given it has chosen to implement the program, we only need to consider values of x in the interval $(0, \delta]$. We also know that all demands for product A goes to the program, whenever it exists.

Since demand in the program is known before production takes place, output q_A can be determined such that it equals demand. Then, given the existence of an ABDP, the expected profit function stated in the beginning of Section 4 can be formulated as

$$E_1 [\pi_A(q_A, x, q_B)] = \frac{1}{2} [R_h(x) + R_l(x)] [xp_A - c] - K, \quad (10)$$

and it is to be maximized with respect to $x \in (0, \delta]$, for any feasible pair (q_A, q_B) . Substitution from (9) into (10) yields

$$E_1 [\pi_A(q_A, x, q_B)] = \left\{ \begin{array}{l} \left[\frac{1}{2} + \frac{t}{2(t^2-w^2)} [p_B - p_A] \right] [xp_A - c] - K \\ \left[\frac{1}{2} + \frac{t}{2(t^2-w^2)} [\delta p_B - xp_A] \right] [xp_A - c] - K \\ \left[\frac{1}{2} + \frac{\delta p_B - xp_A + t + w}{4[t+w]} \right] [xp_A - c] - K \\ xp_A - c - K \end{array} \right\}$$

$$if \left\{ \begin{array}{l} 1 - \frac{(1-\delta)p_B}{p_A} < x \leq \delta \\ \frac{\delta p_B - [t-w]}{p_A} < x \leq 1 - \frac{(1-\delta)p_B}{p_A} \\ \frac{\delta p_B - [t+w]}{p_A} < x \leq \frac{\delta p_B - [t-w]}{p_A} \\ 0 < x \leq \frac{\delta p_B - [t+w]}{p_A} \end{array} \right\} .. \quad (11)$$

In the case appearing in the second line of (11), the ABDP attracts some customers from firm B. In the third line, firm A captures the whole market if the transportation cost is low. In the fourth line, firm A captures the whole market whatever the magnitude of the transportation cost. The intuition here is that the value of x decreases as we move from the first to the fourth line in (11), that is, the price reduction is gradually becoming larger and hence the program becomes increasingly attractive for the consumers.

4.3 Optimal price reduction in ABDP

Our task here is to maximize firm A's expected profit with respect to $x \in (0, \delta)$. For this purpose we need to examine the function $E_1 [\pi_A((q_A, x, q_B))]$. For any given pair (q_A, q_B) it is convenient to denote this function by $E_1 [\pi_A(x)]$. The function consists of four segments, defined by the lower bound (zero), the upper bound (δ) and three interior boundaries. Referring to (11) the interior boundaries are

$$x_1 = 1 - \frac{(1 - \delta)p_B}{p_A} \tag{12}$$

$$x_2 = \frac{\delta p_B - [t - w]}{p_A} \tag{13}$$

$$x_3 = \frac{\delta p_B - [t + w]}{p_A}. \tag{14}$$

First we show that the function $E_1 [\pi_A(x)]$ is continuous at the boundary points x_1, x_2, x_3 .

Lemma 8 *The function $E_1 [\pi_A(x)]$ is continuous in x for all $x \in (0, \delta)$.*

Proof. See the appendix. □

Next, we examine the shape of $E_1 [\pi_A(x)]$ on three segments of the x -axis. Our results are stated in Lemmas 5 through 8.

Lemma 9 *In the segment defined by $(x_1, \delta]$ it holds that*

$$E_1 [\pi_A(x)] = \left[\frac{1}{2} + \frac{t}{2(t^2 - w^2)} [p_B - p_A] \right] [xp_A - c] - K$$

and $E_1 [\pi_A(x)]$ is a linearly increasing function.

Proof. Easy and omitted. □

In view of Lemma 5, one might choose $x = \delta$ as a candidate for an optimal x . However, the resulting optimal objective value turns out to be less than the one that could be obtained by charging the normal price. The reason is that the discounted regular price δp_A is equal to the reduced price $x p_A$ in the program. Since the implementation of the program requires the payment of a fixed cost, the choice $x = \delta$ would be suboptimal.

Lemma 10 *In the segment defined by $(x_2, x_1]$ it holds that*

(1) *The expected profit is given by*

$$E_1 [\pi_A(x)] = \left[\frac{1}{2} + \frac{t}{2(t^2 - w^2)} [\delta p_B - x p_A] \right] [x p_A - c] - K \quad (15)$$

(2) *The function $E_1 [\pi_A(x)]$ is*

(3) *If*

$$\delta p_B < 2 [p_B - p_A] + c + t - \frac{w^2}{t},$$

the function $E_1 [\pi_A(x)]$ is increasing

(4) *If*

$$\delta p_B > c + \frac{1}{t} [3t + w] [t - w],$$

the function $E_1 [\pi_A(x)]$ is decreasing and has a local maximum at x_2 , which is given by (13).

(5) *If*

$$2 [p_B - p_A] + c + t - \frac{w^2}{t} < \delta p_B < c + \frac{1}{t} [3t + w] [t - w],$$

the function $E_1 [\pi_A(x)]$ is increasing on (x_2, \hat{x}) and decreasing on $(\hat{x}, x_1]$, where \hat{x} is given by (16) below. The function $E_1 [\pi_A(x)]$ has a local maximum at

$$\hat{x} = \frac{\delta p_B + c + t - w^2/t}{2p_A}. \quad (16)$$

Proof. See the appendix. □

Lemma 11 *In the segment defined by $(x_3, x_2]$ it holds that*

$$E_1 [\pi_A(x)] = \frac{1}{2} \left[1 + \frac{1}{2[t + w]} [\delta p_B - x p_A + t + w] \right] [x p_A - c] - K \quad (17)$$

and the function $E_1 [\pi_A(x)]$ is increasing.

Proof. See the appendix. □

Lemma 12 *In the segment defined by $(0, x_3]$ it holds that*

$$E_1 [\pi_A(x)] = x p_A - c - K,$$

and the function $E_1 [\pi_A(x)]$ is linearly increasing.

Proof. Easy and omitted. □

The above lemmas suggest that we distinguish three scenarios.

Scenario 1. If

$$\delta p_B < 2[p_B - p_A] + c + t - \frac{w^2}{t}, \quad (18)$$

expected profits in the program are increasing in x . The maximal value of x for which a program is profitable is δ . However, we have seen that for $x = \delta$, revenue in the program equals what could be obtained by selling at the normal price. Since a cost of K is incurred by launching the program, it is suboptimal to start an ABDP if the inequality in (18) holds. The scenario occurs if p_A is sufficiently small, $p_A < \frac{1}{2} \left[c + t - \frac{w^2}{t} + p_B(2 - \delta) \right]$. There is no reason for firm A to implement the program since doing so will only make the selling price even smaller.

Scenario 2. If

$$\delta p_B > c + 3t - 2w - \frac{w^2}{t},$$

it may (depending on the magnitude of the fixed cost K) be optimal to start an ABDP with x_2 given by (13). Scenario 2 occurs if p_B is sufficiently large in which case, by implementing the program, firm A will get a substantial price advantage. In fact, the firm will attract all customers in the market if it happens that the transportation cost is low ($T = t - w$).

The optimal price reduction percentage x_2 increases in p_B and decreases in p_A . Both results are intuitive. If the competitor's price p_B is high, the price reduction in the program need not be a dramatic one. On the other hand, if the firm's own (normal) price p_A is high, the price reduction need to be more substantial to attract customers. Using (13) shows that x_2 decreases in the mean transportation cost t . The intuition here is that if this cost is large, a more significant price reduction is needed to attract customers. Finally, x_2 increases in the uncertainty parameter w , which happens if the variance w^2 of T increases. Thus, when uncertainty about the true value of the transportation cost increases, the firm offers a smaller price reduction.

To understand the result for Scenario 2 we note that price reductions have the highest impact (that is, attract the highest number of new customers) if the transportation cost is low ($T = t - w$). When determining the optimal price reduction in the program, the case of a low transportation cost is more important than that of a high transportation cost. In fact, an aim of implementing the program is to attract all customers if the transportation cost happens to be low. Then a lower value of t , and a larger value of w , reduces the transportation cost T . The implication is that a smaller price reduction is needed to attract all customers. Note that x_2 decreases in t and increases in w .

Scenario 3. If

$$2[p_B - p_A] + c + t - \frac{w^2}{t} < \delta p_B < c + 3t - 2w - \frac{w^2}{t}$$

it may (depending on the magnitude of the fixed cost K) be optimal to start an ABDP with \hat{x} given by (16). Implementing the program under such circumstances, firm A can

attract some customers of firm B, but demand of firm B remains positive. The optimal price reduction percentage \hat{x} increases in p_B and decreases in p_A (as was the case for x_2). Moreover, \hat{x} increases in the mean transportation cost t . If this cost is large, the firm will offer a smaller price reduction. (This is the opposite as what was the case for x_2). Finally, \hat{x} decreases in the uncertainty parameter w . (This is also the opposite as what was the case for x_2). When uncertainty about the value of the transportation cost increases, the firm offers a larger price reduction.

To interpret the results in Scenario 3, we focus on the case of a low transportation cost ($T = t - w$). Then the price sensitivity is higher and the low transportation cost becomes dominant in the determination of an optimal price reduction. Under low transportation costs a price reduction will attract more customers and price reductions are more substantial when t is low (and w is high).

It is readily shown that $\hat{x} > x_2$ and hence the price reduction in the ABDP is largest in Scenario 2. In this scenario, firm A can attract all consumers if the transportation cost is low. Scenario 2 is also the one where p_B has the largest value, implying that firm A's price reduction attracts more customers.

We collect the above results in the following proposition.

Proposition 13 *If Scenario 1 applies, that is, if*

$$\delta p_B < 2[p_B - p_A] + c + t - \frac{w^2}{t}$$

it is suboptimal to start an ABDP.

If Scenario 2 applies, that is, if

$$\delta p_B > c + 3t - 2w - \frac{w^2}{t} \tag{19}$$

it may, depending on the fixed cost K , be optimal to start an ABDP with

$$x_2 = \frac{\delta p_B - t + w}{p_A}.$$

If Scenario 3 applies, that is, if

$$2[p_B - p_A] + c + t - \frac{w^2}{t} < \delta p_B < c + 3t - 2w - \frac{w^2}{t} \tag{20}$$

it may, depending on the fixed cost K , be optimal to start an ABDP with

$$\hat{x} = \frac{\delta p_B + c + t - w^2/t}{2p_A}.$$

4.4 Firm A's optimal objective value with an ABDP

Proposition 9 has shown that there are two candidates for an optimal level of the coefficient x . Which one should be chosen depends on the parameter values in the inequalities (19) and (20). The parameters involved are the two regular prices, the production cost, the discount rate and the two parameters of the probability distribution of T . We shall make a sensitivity analysis with respect to the key parameter w , the standard deviation of T . For this purpose we introduce w as an argument of the optimal objective function of firm A.

Lemma 14 *If the inequality*

$$\frac{(t+w)^2}{t^2} \geq 2w$$

is satisfied, the derivative $dE_1[\pi_A(w)]/dw$ is positive in Scenario 2.

Proof. See the appendix. □

Lemma 10 states that when firm A implements the ABDP, its expected profit increases as uncertainty becomes larger (i.e., the parameter w increases). The inequality in the lemma is satisfied for sufficiently small values of t (note that $w < t$).

Lemma 15 *The derivative $dE_1[\pi_A(w)]/dw$ is positive in Scenario 3.*

Proof. See the appendix. □

The lemma shows that the optimal expected profit of firm A in Scenario 3 is increasing in the uncertainty parameter w . This result is true for any feasible value of w .

In Section 3, where there was no ABDP available to firm A, we have seen that the optimal expected profit of firm A is an increasing function of the parameter w for any feasible value of w . Firm A has, by assumption, the lower normal price ($p_A < p_B$) and it stands to benefit from increased uncertainty about the transportation cost, no matter the magnitude of this cost.

When an ABDP is available the results of Lemma 10 and 11 show that in Scenario 2 (for a low expected transportation cost) and in Scenario 3 (always), firm A gains from increased uncertainty.

4.5 Duopoly output game

This section deals with the duopoly output game to be played in period two by firms A and B. Recall the benchmark case of Section 3 where firm A did not implement the program. If firm A chooses to implement the program, Proposition 2 has shown that if firm A selects $x > \delta$, all potential customers of firm A will wait and buy the product in period two. Then there is no demand for product A in the program and A and B play the duopoly game of

Section 3. On the other hand, if firm A selects $x < \delta$, all potential consumers of product A will participate in the program and demand for product A in the duopoly game will be zero.

4.6 Is it optimal to start an ABDP?

To identify the circumstances under which it is optimal for firm A to implement an ABDP we need to compare objective function values. Using (7) shows that if firm A does not start the program, its expected optimal profits equal

$$E_1 [\pi_A(w)] = \frac{1}{2} \left[1 + \frac{t}{t^2 - w^2} [p_B - p_A] \right] [\delta p_A - c]. \quad (21)$$

Now consider the three scenarios listed in Proposition 9. We have shown that it is suboptimal to start an ABDP in Scenario 1 and in that case the value of the objective function is given by (21).

An ABDP started in *Scenario 2* has $x = x_2$ and the objective value is

$$E_1 [\pi_A(w)] = \frac{2t + w}{2[t + w]} [\delta p_B - c - t + w] - K. \quad (22)$$

Using (21) and (22) shows that it is optimal to start the ABDP if

$$\frac{1}{2} \left[1 + \frac{t}{t^2 - w^2} [p_B - p_A] \right] [\delta p_A - c] + K \leq \frac{2t + w}{2[t + w]} [\delta p_B - c - t + w]. \quad (23)$$

On the left-hand side of the inequality in (23), K is the fixed cost K of implementing the program. The first term is the opportunity cost of starting the program.

An ABDP started in *Scenario 3* has $x = \hat{x}$ and objective value is

$$E_1 [\pi_A(w)] = \left[\frac{1}{2} + \frac{t}{2(t^2 - w^2)} (\delta p_B - \hat{x} p_A) \right] (\hat{x} p_A - c) - K. \quad (24)$$

Using (21) and (24) shows that the condition for starting an ABDP is:

$$\frac{1}{2} \left[1 + \frac{t}{t^2 - w^2} [p_B - p_A] \right] [\delta p_A - c] + K \leq \frac{1}{2} \left[1 + \frac{t}{t^2 - w^2} [\delta p_B - \hat{x} p_A] \right] [\hat{x} p_A - c].$$

In what follows we focus on Scenario 3 and wish to assess how the profitability of an ABDP depends on the uncertainty parameter w . For this purpose, let $K(w)$ be the

fixed cost at which firm A is indifferent between starting a program and not starting it. Consequently,

$$K(w) = \frac{1}{2} [\hat{x}p_A - c] - \frac{1}{2} [\delta p_A - c] + \frac{1}{2} \frac{t}{t^2 - w^2} [[\delta p_B - \hat{x}p_A] [\hat{x}p_A - c] - [p_B - p_A] [\delta p_A - c]].$$

For function $K(w)$ we have the following result.

Proposition 16 (1) $K(w)$ is increasing in w if

$$[\delta p_B - c]^2 - t^2 - 4[\delta p_A - c][p_B - p_A] \geq 0$$

(2) $K(w)$ is first decreasing and then increasing in w if

$$[\delta p_B - c]^2 - t^2 - 4[\delta p_A - c][p_B - p_A] < 0 < [\delta p_B - c]^2 - 4[\delta p_A - c][p_B - p_A]$$

(3) $K(w)$ is decreasing in w if $[\delta p_B - c]^2 - 4[\delta p_A - c][p_B - p_A] \leq 0$.

Proof. See the appendix. □

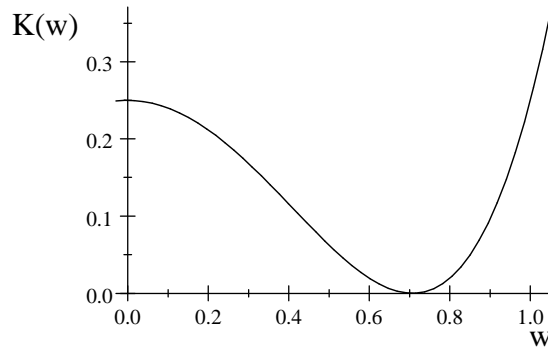
To illustrate Proposition 13, assume that parameters have the values

$$t = \delta = 1, p_B = 4, p_A = 2.75, c = 1.$$

Then

$$\begin{aligned} [\delta p_B - c]^2 - t^2 - 4[\delta p_A - c][p_B - p_A] &= 9 - 1 - 8.75 < 0 \\ [\delta p_B - c]^2 - 4[\delta p_A - c][p_B - p_A] &= 0.25 > 0 \end{aligned}$$

which shows that the example illustrates case (2) of Proposition 12. The figure below depicts the non-monotonic function $K(w)$.



The graph of $K(w)$ shows that if there is no uncertainty about consumers' transportation cost ($w = 0$), the benefit of implementing the program is relatively large. The benefit decreases, however, as w increases and eventually vanishes. Then, for any positive value of K , it is not optimal to implement a program. Afterwards, the benefit increases again. Qualitatively speaking, when it implements the program, firm A would prefer to have a large amount of uncertainty. If this is not attainable, the firm would prefer a little uncertainty only.

5 Conclusions

The paper has studied a two-player, two-period game played by two firms, A and B. The duopolistic market is described by Hotelling's linear city model. Firm A only has the option of implementing in period one an advance booking discount program in which customers are offered a lower price than the regular one as an incentive to buy early. In period two (the selling season), the firms play an output game. The normal prices charged in this period are exogenously fixed.

Firm A has three decisions to make. First, should an ABDP be implemented? Second, what is the optimal discount to offer customers if the program is worthwhile implementing. Third, firm A must determine its optimal output in the duopoly game in period two. Firm B makes one decision only, its output in the game played in period two.

It was assumed that firm A has the lowest normal selling price. Consumer demand was supposed to be determined by a random transportation cost as well as the regular prices. Demand of firm A decreases when the transportation cost increases, because price differences are less important for a customer when her transportation cost is high.

Section 3 demonstrated that if firm A does not implement an ABDP, the firm's optimal expected profit increases with the uncertainty in the transportation cost. The upshot is that the firm with the smaller regular price (i.e., firm A) stands to benefit from increased uncertainty about consumers' transportation cost. This result is a consequence of the fact that firm A's demand is decreasing in the transportation cost in a convex way such that a mean preserving spread raises expected demand.

Given that firm A implements an ABDP, we determined endogenously how many customers will take part in the program. We found that the major purpose of starting a program is to increase the firm's market share by reducing the output price. It was not optimal to implement a program if the selling price of the other firm is relatively high. The reason is that the price differential is already large enough and firm A can capture a considerable share of the market without having to start an ABDP. For an intermediate level of the selling price of firm B, firm A implements an ABDP and captures the whole market if consumers' transportation costs is low. In such a case the difference in regular prices matters to consumers. If the regular price of firm B is low, firm A starts an ABDP in order to gain some of the market share of firm B. However, firm B stays in the market.

We have also shown that under an ABDP the optimal expected profit of firm A increases with the uncertainty on the consumer transportation cost (although in one scenario we need the transportation cost to be sufficiently low).

Finally we showed that the decision to start a program depends significantly on the uncertainty on the transportation cost. Scenarios were identified in which the profitability of starting the program increases, decreases, or first increases and then decreases as uncertainty increases.

6 Appendix

Proof of Lemma 3.

With respect to $R_h(x)$ it holds that consumers located in the interval $[0, y_h)$ will enter the ABDP if

$$xp_A + [t + w]y < \delta p_A + [t + w]y \implies x < \delta.$$

Consumers having a $y \in [y_h, 1]$ will choose the program if

$$xp_A + [t + w]y < \delta p_B + [t + w][1 - y] \implies x < \frac{\delta p_B + [t + w][1 - 2y]}{p_A}. \quad (25)$$

For $R_l(x)$ it holds that consumers located in the interval $[0, y_l)$ will enter the program if

$$x < \delta$$

while consumers having a location at $y \in [y_l, \infty]$ choose the program if

$$x < \frac{\delta p_B + [t - w][1 - 2y]}{p_A}.$$

We define $R_h(x)$ (and, in a similar way, $R_l(x)$) as the sum

$$R_h(x) = R_h^1(x) + R_h^2(x) + R_h^3(x),$$

where $R_h^1(x)$ represents the number of consumers in $[0, y_h)$ who choose the program, $R_h^2(x)$ represents the number of consumers in $[y_h, y_l)$ who choose the program, and $R_h^3(x)$ represents the number of consumers in $[y_l, 1]$ who choose the program. It is readily proved that

$$1/2 [R_h^1(x) + R_l^1(x)] = \left\{ \begin{pmatrix} 0 \\ y_h \end{pmatrix} \right\} \text{ for } x \left\{ \begin{pmatrix} > \\ \leq \end{pmatrix} \right\} \delta \quad (26)$$

and

$$R_l^2(x) = \left\{ \begin{pmatrix} 0 \\ y_l - y_h \end{pmatrix} \right\} \text{ for } x \left\{ \begin{pmatrix} > \\ \leq \end{pmatrix} \right\} \delta. \quad (27)$$

For $R_h^2(x)$ we obtain from (25) that for a consumer to participate in the program it is necessary that

$$\begin{aligned} xp_A + [t+w]y &< \delta p_B + [t+w][1-y] \\ y &< \frac{1}{2[t+w]} [\delta p_B - xp_A + t+w] \end{aligned}$$

and hence

$$\begin{aligned} R_h^2(x) &= \left\{ \begin{array}{c} 0 \\ \frac{1}{2[t+w]} [\delta p_B - xp_A + t+w] - y_h \\ y_l - y_h \end{array} \right\} \\ \text{if } &\left\{ \begin{array}{c} x > \frac{\delta p_B + [t+w][1-2y_h]}{p_A} \\ \frac{\delta p_B + [t+w][1-2y_l]}{p_A} < x \leq \frac{\delta p_B + [t+w][1-2y_h]}{p_A} \\ x \leq \frac{\delta p_B + [t+w][1-2y_l]}{p_A} \end{array} \right\} \dots \end{aligned} \quad (28)$$

In order to calculate $1/2 [R_h^2(x) + R_l^2(x)]$ we need to determine how the x - boundaries

$$\delta, \frac{\delta p_B + [t+w][1-2y_h]}{p_A}, \quad \text{and} \quad \frac{\delta p_B + [t+w][1-2y_l]}{p_A}$$

are related to each other. Here it holds that

$$\begin{aligned} \frac{\delta p_B + [t+w][1-2y_h]}{p_A} &= \frac{\delta p_B + [t+w] \left[1 - \frac{2(t+w+p_B-p_A)}{2[t+w]} \right]}{p_A} = \\ \frac{\delta p_B - p_B + p_A}{p_A} &< \delta, \end{aligned}$$

and hence

$$\frac{\delta p_B + [t+w][1-2y_l]}{p_A} < \frac{\delta p_B + [t+w][1-2y_h]}{p_A} < \delta.$$

Combining (27) and (28) provides

$$\begin{aligned} 1/2 [R_h^2(x) + R_l^2(x)] &= \left\{ \begin{array}{c} 0 \\ 1/2 [y_l - y_h] \\ 1/2 \left[y_l + \frac{1}{2[t+w]} [\delta p_B - xp_A + t+w] \right] - y_h \\ y_l - y_h \end{array} \right\} \\ \text{if } &\left\{ \begin{array}{c} x > \delta \\ \frac{\delta p_B + [t+w][1-2y_h]}{p_A} < x \leq \delta \\ \frac{\delta p_B + [t+w][1-2y_l]}{p_A} < x \leq \frac{\delta p_B + [t+w][1-2y_h]}{p_A} \\ x \leq \frac{\delta p_B + [t+w][1-2y_l]}{p_A} \end{array} \right\} \dots \end{aligned} \quad (29)$$

With respect to $R_l^3(x)$ it holds that the program is chosen if

$$\begin{aligned} x &< \frac{\delta p_B + [t - w][1 - 2y_l]}{p_A} \\ y &< \frac{1}{2[t - w]} [\delta p_B - x p_A + t - w] \end{aligned}$$

and hence one obtains

$$\begin{aligned} R_l^3(x) &= \left\{ \begin{array}{c} 0 \\ \frac{1}{2[t-w]} [\delta p_B - x p_A + t - w] - y_l \\ 1 - y_l \end{array} \right\} \\ \text{if } &\left\{ \begin{array}{c} x > \frac{\delta p_B + [t-w][1-2y_l]}{p_A} \\ \frac{\delta p_B - [t-w]}{p_A} \leq x \leq \frac{\delta p_B + [t-w][1-2y_l]}{p_A} \\ x < \frac{\delta p_B - [t-w]}{p_A} \end{array} \right\} .. \end{aligned}$$

Similar to $R_h^2(x)$ one finds for $R_h^3(x)$ that

$$\begin{aligned} R_h^3(x) &= \left\{ \begin{array}{c} 0 \\ \frac{1}{2[t+w]} [\delta p_B - x p_A + t + w] - y_l \\ 1 - y_l \end{array} \right\} \\ \text{if } &\left\{ \begin{array}{c} x > \frac{\delta p_B + [t+w][1-2y_l]}{p_A} \\ \frac{\delta p_B - [t+w]}{p_A} < x \leq \frac{\delta p_B + [t+w][1-2y_l]}{p_A} \\ x \leq \frac{\delta p_B - [t+w]}{p_A} \end{array} \right\} .. \end{aligned}$$

To calculate $1/2 [R_h^3(x) + R_l^3(x)]$ we need to determine how the x -boundaries

$$\begin{aligned} &\frac{\delta p_B + [t - w][1 - 2y_l]}{p_A}, \frac{\delta p_B - [t - w]}{p_A}, \\ &\frac{\delta p_B + [t + w][1 - 2y_l]}{p_A}, \text{ and } \frac{\delta p_B - [t + w]}{p_A} \end{aligned}$$

are related. Using (8) the following inequalities can be established

$$\begin{aligned} \frac{\delta p_B - [t + w]}{p_A} &< \frac{\delta p_B - [t - w]}{p_A} < \frac{\delta p_B + [t + w][1 - 2y_l]}{p_A} < \\ &\frac{\delta p_B + [t - w][1 - 2y_l]}{p_A} \end{aligned}$$

and we obtain

$$\frac{1}{2} [R_h^3(x) + R_l^3(x)] = \left\{ \begin{array}{c} 0 \\ \frac{1}{2} \left[\frac{1}{2[t-w]} [\delta p_B - x p_A + t - w] - y_l \right] \\ \frac{\delta p_B - x p_A + t - w}{4[t-w]} + \frac{\delta p_B - x p_A + t + w}{4[t+w]} - y_l \\ \frac{1}{2} + \frac{\delta p_B - x p_A + t + w}{4[t+w]} - y_l \\ 1 - y_l \end{array} \right\} \quad (30)$$

$$\text{if } \left\{ \begin{array}{c} x > \frac{\delta p_B + [t-w][1-2y_l]}{p_A} \\ \frac{\delta p_B + [t+w][1-2y_l]}{p_A} < x \leq \frac{\delta p_B + [t-w][1-2y_l]}{p_A} \\ \frac{\delta p_B - [t-w]}{p_A} < x \leq \frac{\delta p_B + [t+w][1-2y_l]}{p_A} \\ \frac{\delta p_B - [t+w]}{p_A} < x \leq \frac{\delta p_B - [t-w]}{p_A} \\ x \leq \frac{\delta p_B - [t+w]}{p_A} \end{array} \right\} \dots$$

Finally, we combine (26), (29), and (30) to derive the desired result. From

$$\frac{\delta p_B - [t+w]}{p_A} < \frac{\delta p_B - [t-w]}{p_A} < \frac{\delta p_B + [t+w][1-2y_l]}{p_A} < \frac{\delta p_B + [t-w][1-2y_l]}{p_A} = \frac{\delta p_B + [t+w][1-2y_h]}{p_A} < \delta$$

we obtain

$$\frac{1}{2} [R_h(x) + R_l(x)] = \left\{ \begin{array}{c} 0 \\ 1/2 [y_l + y_h] \\ \frac{\delta p_B - x p_A + t + w}{4[t+w]} + \frac{\delta p_B - x p_A + t - w}{4[t-w]} \\ \frac{\delta p_B - x p_A + t - w}{4[t+w]} + \frac{\delta p_B - x p_A + t + w}{4[t-w]} \\ \frac{1}{2} + \frac{\delta p_B - x p_A + t + w}{4[t+w]} \\ 1 \end{array} \right\}$$

$$\text{if } \left\{ \begin{array}{c} x > \delta \\ \frac{\delta p_B + [t-w][1-2y_l]}{p_A} < x \leq \delta \\ \frac{\delta p_B + [t+w][1-2y_l]}{p_A} < x \leq \frac{\delta p_B + [t-w][1-2y_l]}{p_A} \\ \frac{\delta p_B - [t-w]}{p_A} < x \leq \frac{\delta p_B + [t+w][1-2y_l]}{p_A} \\ \frac{\delta p_B - [t+w]}{p_A} < x \leq \frac{\delta p_B - [t-w]}{p_A} \\ x \leq \frac{\delta p_B - [t+w]}{p_A} \end{array} \right\} \dots$$

Using this result readily leads to the expression for expected demand that is stated in the proposition. **Q.E.D.**

Proof of Lemma 4.

Continuity of the function $E_1 [\pi_A(x)]$ must be checked at x_1 , x_2 , and x_3 (cf. (12) - (14)). Continuity at x_1 requires

$$\frac{1}{2} [y_l + y_h] [x_1 p_A - c] - K = \left[\frac{\delta p_B - x_1 p_A + t + w}{4[t+w]} + \frac{\delta p_B - x_1 p_A + t - w}{4[t-w]} \right] [x_1 p_A - c] - K,$$

which can be rewritten as

$$[t+w] y_h = [t-w] y_l + w. \quad (31)$$

Employing (5) one can show that (31) holds and hence $E_1 [\pi_A(x)]$ is continuous at x_1 .

Continuity at x_2 requires

$$\left[\frac{\delta p_B - x_2 p_A + t + w}{4[t+w]} + \frac{\delta p_B - x_2 p_A + t - w}{4[t-w]} \right] [x_2 p_A - c] - K = \left[\frac{1}{2} + \frac{\delta p_B - x_2 p_A + t + w}{4[t+w]} \right] [x_2 p_A - c] - K$$

which is equivalent to

$$\frac{1}{2[t-w]} [\delta p_B - x_2 p_A] = \frac{1}{2}.$$

Substitution of (13) into this expression verifies the satisfaction of this equality.

Finally, continuity at x_3 requires

$$\frac{1}{2} \left[1 + \frac{1}{2[t+w]} [\delta p_B - x p_A + t + w] \right] = 1.$$

Combining this expression with (14) shows continuity at x_3 . **Q.E.D.**

Proof of Lemma 6.

Differentiating expected profit with respect to x yields

$$\frac{dE_1 [\pi_A(x)]}{dx} = \frac{1}{2} \left[\frac{1}{2[t+w]} [\delta p_B - x p_A + t + w] + \frac{1}{2[t-w]} [\delta p_B - x p_A + t - w] \right] p_A - \frac{p_A}{2} \left[\frac{t}{t^2 - w^2} \right] [x p_A - c],$$

and by differentiation it follows that

$$\frac{d^2 E_1 [\pi_A(x)]}{dx^2} = -\frac{1}{2} \left[\frac{1}{t+w} + \frac{1}{t-w} \right] p_A^2 < 0.$$

Hence function $E_1 [\pi_A(x)]$ is concave.

Next, we look for an interior maximum in the line segment. From

$$\frac{dE_1 [\pi_A(x)]}{dx} = 0$$

one obtains, denoting the interior value by \hat{x} , that

$$2 - \frac{2t}{t^2 - w^2} [-\delta p_B + 2\hat{x}p_A - c] = 0.$$

Hence

$$\hat{x} = \frac{[\delta p_B + c]t + t^2 - w^2}{2tp_A}.$$

From the above we conclude that in the segment $(x_2, x_1]$:

1. $E_1 [\pi_A(x)]$ is increasing if $x_2 < x_1 < \hat{x}$
2. $E_1 [\pi_A(x)]$ is decreasing if $\hat{x} < x_2 < x_1$
3. $E_1 [\pi_A(x)]$ has a local maximum \hat{x} if $x_2 < \hat{x} < x_1$.

Re 1. This case occurs when $x_1 < \hat{x}$, that is,

$$\frac{\delta p_B + [t-w][1-2y_l]}{p_A} < \frac{[\delta p_B + c]t + t^2 - w^2}{2tp_A}$$

which is equivalent to

$$t\delta p_B < 2t[p_B - p_A] + ct + t^2 - w^2.$$

Re 2. This case occurs when $\hat{x} < x_2$, that is,

$$\frac{[\delta p_B + c]t + t^2 - w^2}{2tp_A} < \frac{\delta p_B - [t-w]}{p_A},$$

which is equivalent to

$$t\delta p_B > ct + [3t+w][t-w].$$

Re 3. This case occurs when the two previous ones do not occur, that is, if

$$2t[p_B - p_A] + ct + t^2 - w^2 < t\delta p_B < ct + [3t+w][t-w].$$

Q.E.D.

Proof of Lemma 7.

Differentiating (17) gives

$$\begin{aligned} \frac{dE_1[\pi_A(x)]}{dx} &= \frac{1}{2} \left[1 + \frac{1}{2[t+w]} [\delta p_B - x p_A + t + w] \right] p_A - \\ &\quad \frac{1}{2} \frac{1}{2[t+w]} p_A [x p_A - c], \end{aligned}$$

from which it can be derived that

$$\frac{d^2 E_1[\pi_A(x)]}{dx^2} = -\frac{p_A^2}{2[t+w]} < 0,$$

which shows that $E_1[\pi_A](x)$ is a strictly concave function.

The extremal point \bar{x} satisfies

$$1 + \frac{1}{2[t+w]} [\delta p_B - 2\bar{x} p_A + t + w + c] = 0,$$

which leads to

$$\bar{x} = \frac{3[t+w] + \delta p_B + c}{2p_A}.$$

Due to the concavity of $E_1[\pi_A(x)]$, this function is increasing on $(x_3, x_2]$, given that $x_2 < \bar{x}$. The implication is that

$$\frac{\delta p_B - [t-w]}{p_A} < \frac{3[t+w] + \delta p_B + c}{2p_A},$$

which can be reduced to

$$\delta p_B < 5t + w + c.$$

Because of (1) and the fact that $w < t$, this inequality is always satisfied. **Q.E.D.**

Firm A's objective value at $x = \hat{x}$.

$$\begin{aligned} E_1[\pi_A](\hat{x}) &= \frac{1}{4[t+w]} [\delta p_B - \hat{x} p_A + t + w] [\hat{x} p_A - c] + \\ &\quad \frac{1}{4[t-w]} [\delta p_B - \hat{x} p_A + t - w] [\hat{x} p_A - c] - K = \\ &\quad \frac{1}{4[t+w]} \left[\delta p_B - \frac{[\delta p_B + c] t + t^2 - w^2}{2t p_A} p_A + t + w \right] [\hat{x} p_A - c] + \\ &\quad \frac{1}{4[t-w]} \left[\delta p_B - \frac{[\delta p_B + c] t + t^2 - w^2}{2t p_A} p_A + t - w \right] [\hat{x} p_A - c] - K = \end{aligned}$$

$$\begin{aligned}
& \frac{1}{8t[t+w]} [t\delta p_B - ct + [t+w]^2] [\hat{x}p_A - c] + \\
& \frac{1}{8t[t-w]} [t\delta p_B - ct + [t-w]^2] [\hat{x}p_A - c] - K = \\
& \frac{t-w}{8t(t^2-w^2)} [t\delta p_B - ct] [\hat{x}p_A - c] + \frac{t-w}{8t(t^2-w^2)} [t+w]^2 [\hat{x}p_A - c] + \\
& \frac{t+w}{8t(t^2-w^2)} [t\delta p_B - ct] [\hat{x}p_A - c] + \frac{t+w}{8t(t^2-w^2)} [t-w]^2 [\hat{x}p_A - c] - K = \\
& \frac{1}{4(t^2-w^2)} [t\delta p_B - ct] [\hat{x}p_A - c] + \frac{t+w}{8t} [\hat{x}p_A - c] + \frac{t-w}{8t} [\hat{x}p_A - c] - K = \\
& \left[\frac{\delta p_B - c}{4(t^2-w^2)} + \frac{1}{4t} \right] [\hat{x}p_A - c] t - K = \\
& \left[\frac{\delta p_B - c}{8(t^2-w^2)} + \frac{1}{8t} \right] [[\delta p_B - c] t + (t^2-w^2)] - K.
\end{aligned}$$

Proof of Lemma 10.

In Scenario 2, the optimal value of the objective function of firm A is

$$E_1 [\pi_A(x_2, w)] = \frac{1}{2} \left[1 + \frac{t}{2(t+w)} [\delta p_B - x_2 p_A + t + w] \right] [x_2 p_A - c] - K.$$

Substituting from (13) yields

$$E_1 [\pi_A(w)] = E_1 [\pi_A(w)] = \frac{t^2 + t + w}{2[t+w]} [\delta p_B - [t-w] - c] - K$$

and differentiation with respect to w provides

$$\frac{dE_1 [\pi_A(w)]}{dw} = -\frac{t^2}{2[t+w]^2} \left[\delta p_B - c - 2t - \frac{(t+w)^2}{t^2} \right].$$

Due to the assumption made in (1), it holds that

$$\frac{(t+w)^2}{t^2} \geq 2w$$

which implies $dE_1 [\pi_A(x_2, w)] / dw > 0$. **Q.E.D.**

Proof of Lemma 11.

In Scenario 3, the optimal value of the objective function of firm A is

$$E_1 [\pi_A(\hat{x}, w)] = \left[\frac{1}{2} + \frac{t}{2(t^2-w^2)} (\delta p_B - \hat{x}p_A) \right] (\hat{x}p_A - c) - K. \quad (32)$$

To assess the sign of the derivative $dE_1 [\pi_A (\hat{x}, w)] / dw$ we use the formula

$$\frac{dE_1 [\pi_A (\hat{x}, w)]}{dw} = \frac{\partial E_1 [\pi_A (\hat{x}, w)]}{\partial w} + \frac{\partial E_1 [\pi_A (\hat{x}, w)]}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial w}.$$

It is readily shown that

$$\frac{\partial E_1 [\pi_A (\hat{x}, w)]}{\partial w} > 0$$

and from Section 4.3 we know that $\partial \hat{x} / \partial w < 0$. Differentiation in (32) provides

$$\begin{aligned} \frac{\partial E_1 [\pi_A (\hat{x}, w)]}{\partial \hat{x}} &= p_A \left[\frac{1}{2} + \frac{t}{2(t^2 - w^2)} [\delta p_B - 2\hat{x}p_A + c] \right] \\ &= \frac{p_A}{2(t^2 - w^2)} [\delta p_B t - 2\hat{x}p_A t + ct + t^2 - w^2] = 0 \end{aligned}$$

and we conclude that

$$\frac{dE_1 [\pi_A (\hat{x}, w)]}{dw} = \frac{\partial E_1 [\pi_A (\hat{x}, w)]}{\partial w} > 0.$$

Q.E.D.

Proof of Proposition 12.

First we calculate

$$K'(w) = \frac{wt}{(t^2 - w^2)^2} [[\delta p_B - \hat{x}p_A] [\hat{x}p_A - c] - [p_B - p_A] [\delta p_A - c]]. \quad (33)$$

Since, according to Proposition 9, it holds that

$$\hat{x}p_A = \frac{1}{2} \left[\delta p_B + c + t - \frac{w^2}{t} \right],$$

we obtain

$$\begin{aligned} \delta p_B - \hat{x}p_A &= \frac{1}{2} \left[\delta p_B - c - t + \frac{w^2}{t} \right], \\ \hat{x}p_A - c &= \frac{1}{2} \left[\delta p_B - c + t - \frac{w^2}{t} \right], \end{aligned}$$

and therefore

$$[\delta p_B - \hat{x}p_A] [\hat{x}p_A - c] = \frac{1}{4} \left[(\delta p_B - c)^2 - \left(t - \frac{w^2}{t} \right)^2 \right].$$

It follows that $[\delta p_B - \hat{x}p_A] [\hat{x}p_A - c]$, and the part in curly brackets in (33), is increasing in w . This establishes the result of the proposition. **Q.E.D.**

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