Les Cahiers du GERAD ISSN: 0711–2440

R&D Equilibrium Strategies with Surfers

F. Ben Abdelaziz, M. Ben Brahim,
G. Zaccour

G–2007–03

January 2007

Les textes publiés dans la série des rapports de recherche HEC n'engagent que la responsabilité de leurs auteurs. La publication de ces rapports de recherche bénéficie d'une subvention du Fonds québécois de la recherche sur la nature et les technologies.

R&D Equilibrium Strategies with Surfers

Fouad Ben Abdelaziz
Mouna Ben Brahim

Larodec Laboratory
ISG, Université de Tunis
Tunisia

Georges Zaccour

GERAD and
Chair in Game Theory and Management
HEC Montréal
Canada
georges.zaccour@gerad.ca

January 2007

Les Cahiers du GERAD

G–2007–03

Abstract

A typical assumption in the game-theoretic literature on research and development (R&D) is that all firms belonging to the industry under investigation pursue R&D activities. In this paper, we assume that the industry is composed of two groups; the first (the investors) is made of firms that have R&D facilities and are involved in this type of activity. The second group corresponds to firms that are inactive in R&D (the surfers). The latter group benefits from its competitors' R&D efforts, thanks to involuntary spillovers. This division of the industry is in line with actual practice, where indeed, not all firms are engaged in costly and risky R&D. We adopt a two-stage game formalism where, in the first stage investors decide on their levels of investment in R&D, and in the second stage, all firms compete à la Cournot in the product market. We characterize and analyze the unique subgame perfect Nash equilibrium.

Key Words: R&D, two-stage games, spillovers, Cournot competition.

Résumé

La littérature ludique sur la recherche et développement (R&D) a typiquement fait l'hypothèse que toutes les firmes de l'industrie investissent dans une telle activité. Dans cet article, on suppose que l'industrie est divisée de deux groupes de firmes. Le premier groupe (les *investisseurs*) possède les facilités requises pour poursuivre des activités de R&D. Le second est formé d'entreprises (les *surfers*) qui ne sont pas actives en R&D. Ce dernier groupe bénéficie néanmoins des investissements de ses concurrents en R&D, à cause de débordement involontaire (spillovers). Cette structure de l'industrie est en fait en lignée avec ce qu'on observe dans la réalité. On adopte le formalisme des jeux à deux étages où, au premier étage les investisseurs décident des montants investis en R&D, et au deuxième, toutes les firmes se font concurrence à la Cournot dans le marché du produit. On caractérise et analyse l'unique équilibre parfait de Nash.

Mots clés : R&D, jeux à deux étages, effets de débordement, concurrence à la Cournot.

Acknowledgments: Research supported by NSERC, Canada.

1 Introduction

Following the seminal paper by d'Aspremont and Jacquemin (Ref. 1), a significant body of literature has developed with the aim of characterizing cooperative and noncooperative research and development (R&D) strategies in oligopolistic industries (see, e.g., Refs 2, 3, 4, 5, and 6)[1]. These (and other papers in this area) share the following features: (i) The model adopted is a two-stage game where the firms decide on their R&D expenditures in one stage and on their output levels in a second stage. (ii) R&D efforts are process-oriented, that is, they are aimed at reducing the unit production cost of the homogenous product. (iii) Each firm leaks part of its knowledge to competitors and, similarly, benefits from its competitors' R&D efforts. In reality, this spillover can be voluntary or involuntary. Voluntary spillover corresponds to the case where the firms decide to enter into a research joint venture to maximize the impact of their R&D dollars, without however, cooperating in the product market in the second stage. Involuntary spillover may be due to, e.g., reverse engineering or industrial espionage. (iv) All players are assumed to be active in R&D.

The main contribution of this paper is in analyzing R&D equilibrium strategies in a setting where not all firms belonging to the industry are active in R&D. We retain all other above-mentioned features, namely a two-stage model, process R&D and presence of spillovers. The latter are well documented in practice, and the industrial-organization literature has shown that they indeed affect the firms' R&D and output decisions (see, e.g., Refs. 8, 9, 10, 11). Although the two-stage game paradigm is memoryless and hence does not accommodate for experience effects in R&D investment and spillover, we shall nevertheless follow the literature and stick to it, mainly for its tractability and to allow us to isolate the impact of having noninvestors in R&D. (For an analysis of dynamic R&D games with experience effects, see, e.g., Refs. 12 and 13).

Research and development has been acknowledged as an important source of wealth for an economy, as well as a source of profit for firms engaged in such an activity. Still, it has been empirically observed that not all firms conduct R&D, even in high-tech sectors. One can put forward some common-sense (probably highly correlated) explanations for this, among them the lack of resources, especially in small- and medium-size firms, the long investment-recovery times-back, the inherently risky nature of R&D, the availability of alternatives (e.g., outsourcing, licensing), etc. In any event, these firms must develop means to avoid being too outdistanced by innovating firms, in terms of unit-production costs. Our assumption here is that an inactive firm can still benefit, albeit less so than an active one, from the knowledge produced by firms engaged in R&D. Our objectives are to:

[1]The game theoretical literature on R&D is huge and several surveys are available. A recent one, which covers almost all of the facets of this literature is Silipo (Ref. 7).

1. Characterize subgame-perfect Nash equilibria when some firms are not engaged in R&D;
2. Compare the output strategies and payoffs of investors and noninvestors.

The rest of this paper is organized as follows: Section 2 introduces a parsimonious model to investigate the above-mentioned setting. Section 3 characterizes the subgame-perfect Nash equilibrium and conducts some sensitivity analyses. Section 4 compares players' strategies and payoffs for the two types of players. Section 5 briefly concludes.

2 Model

Let $\mathcal{N} = \{1, \ldots, N\}$ be the set of firms making up the industry and producing a homogenous product. Following an established tradition in R&D literature, we consider a two-stage model where, in the first stage, the firms decide on their levels of investment in R&D, and they compete à la Cournot in the product market in the second stage. Denote by q_j the output level of firm $j \in \mathcal{N}$ and by $Q = \sum_{j=1}^{N} q_j$ the total production. For simplicity's sake, the inverse demand is assumed affine and given by $p = a - Q$, where $a > 0$.

Contrary to previous studies, we consider that the industry is made up of two heterogenous subsets of firms, $\mathcal{I} = \{1, \ldots, I\}$ and $\mathcal{S} = \{1, \ldots, S\}$, with $\mathcal{I} \cap \mathcal{S} = \emptyset$. Subset $\mathcal{I}$ consists of firms that have R&D facilities (laboratory, scientific and technical personnel, etc.) and are active in R&D. Subset $\mathcal{S}$ is made up of firms that do not pursue these types of activity for any of the reasons mentioned above. We shall refer to subset $\mathcal{I}$ as the $\mathcal{I}$nvestors (or $\mathcal{I}$nnovators) and to subset $\mathcal{S}$ as the $\mathcal{S}$urfers. The R&D efforts are aimed at reducing the production cost and are not perfectly appropriable, i.e., a firm that is active in R&D helps (involuntarily) its competitors to also reduce their cost. An investor, as well as a surfer, benefits from all investors' R&D efforts at zero cost. The assumption of zero cost finds support in Ref. 14 where it is argued that inventing something new is costly, but copying it is costless. However, the two types of firms differ in their capacity to capture others' R&D results.[2] Indeed, we assume, not unrealistically, that by having a research facility, an investor is better placed than a surfer to absorb the knowledge produced by the innovators.

Let x_i be the investment in R&D by firm $i, i \in \mathcal{I}$, and denote by X_k the total level of knowledge available to firm $k \in \mathcal{N}$, that is, the level taking into account the spillovers, i.e.,

[2]Normally, one would introduce a function transforming R&D dollars into knowledge. To keep the model parsimonious, we are assuming that investment is equal to knowledge.

$$X_k = \begin{cases} x_k + \beta \sum_{j \in I, j \neq k} x_j, & k \in \mathcal{I} \\ \gamma \sum_{j \in I} x_j, & k \in \mathcal{S} \end{cases}, \tag{1}$$

where $0 \leq \gamma < \beta \leq 1$. The parameters β and γ measure the capacity of the two types of firms to benefit from the others' R&D investments. The inequality $\gamma < \beta$ reflects the assumption made above regarding the two types of firms. The actual values of these parameters are ultimately an empirical matter.[3]

The unit-production cost of firm k depends on the total available knowledge and is denoted by $c_k(X_k)$, with $c_k(0) > 0$ and $c_k'(X_k) \leq 0$. To keep the model simple, we assume that $c_k(X_k)$ can be approximated by an affine function (at least in the range of R&D expenditures that might be part of a Nash equilibrium),[4] i.e.,

$$c_k(X_k) = c_0 - c_1 X_k,$$

where X_k is defined in (1). Equivalently, the cost functions can be written as

$$c_k(x) = \begin{cases} c_0 - c_1 \left(x_k + \beta \sum_{j \in I, j \neq k} x_j \right), & k \in \mathcal{I} \\ c_0 - c_1 \left(\gamma \sum_{j \in I} x_j \right), & k \in \mathcal{S} \end{cases}, \tag{2}$$

where $x = (x_1, \ldots, x_I)$. The parameter c_0 is interpreted as the initial cost, that is, the unit cost in the absence of any improvement in the production process due to R&D ($c_0 = c_k(0)$). The parameter c_1 measures the speed at which the initial unit cost decreases thanks to the acquired knowledge. We assume $c_0 < a$, i.e., that the initial cost is lower than the consumer's willingness-to-pay, and that the unit cost is positive for $x > 0$. Note that the parameters of the cost functions are not firm-specific. This reflects the idea that the industry is using the same production technology, which is consistent with the homogenous good assumption. Therefore, the difference in the firms' unit-production costs is solely due to their investment in R&D and their capacity to capture others' R&D efforts.

Assuming a profit-maximization behavior, the optimization problem of firm k reads as follows:

$$\pi_k = \begin{cases} \max_{q_k, x_k} (a - \sum_{j=1}^{N} q_j - c_k(x)) q_k - x_k, & k \in \mathcal{I} \\ \max_{q_k} (a - \sum_{j=1}^{N} q_j - c_k(x)) q_k, & k \in \mathcal{S} \end{cases}. \tag{3}$$

[3] In a setting with differentiated products, the level of spillover would depend on the degree of substitutability between them (see, e.g., Ref. 15). Here, the assumption is that the industry produces a homogenous good.

[4] A full characterization of Nash equilibrium for the non-linear case where $C_k(X_k) = c_0 - \sqrt{X_k}$ is available from the authors upon request. Although this specification is more realistic, the drawback is that it does not allow much insight in terms of sensitivity analysis of equilibrium strategies.

3 Symmetric Nash Equilibrium

We suppose that the two-stage game is played à la Nash, i.e., in each stage the mode of play is simultaneous and noncooperative. To obtain a subgame-perfect equilibrium, we first solve the second stage and obtain quantities as functions of investment levels in R&D. Next, we solve the first stage. We follow the literature and focus on symmetric equilibrium. The second-stage equilibrium output levels, as functions of R&D investments, as well as profits are characterized in the following proposition.

Proposition 1 *Assuming an interior solution, firm k's output level and profit are given by*

$$q_k(x) = \frac{\left[a - Nc_k(x) + \sum_{j \in \mathcal{N}, j \neq k} c_j(x)\right]}{N+1}, k \in \mathcal{N}, \tag{4}$$

$$\pi_k(x) = \begin{cases} q_k^2(x) - x_k, & k \in \mathcal{I} \\ q_k^2(x), & k \in \mathcal{S} \end{cases}. \tag{5}$$

Proof. Assuming an interior solution, first-order equilibrium conditions are given by

$$\frac{\partial \pi_k}{\partial q_k} = a - \sum_{j=1}^{N} q_j - c_k(x) - q_k = 0, \quad k \in \mathcal{N}. \tag{6}$$

In matrix form, the above system reads

$$\begin{pmatrix} 2 & 1 & . & . & . & 1 \\ 1 & 2 & 1 & . & . & 1 \\ .. & . & . & . & . & . \\ .. & . & . & . & . & . \\ .. & . & . & . & . & . \\ 1 & 1 & . & . & . & 2 \end{pmatrix} \begin{pmatrix} q_1 \\ .. \\ .. \\ .. \\ .. \\ q_n \end{pmatrix} = \begin{pmatrix} a - c_1(x) \\ .. \\ .. \\ .. \\ .. \\ a - c_n(x) \end{pmatrix}.$$

Straightforward computations lead to the result. Substituting for $q_k(x)$ in (3) yields the profits as functions of x. ■

Remark 1 *It is easy to check that an investor and a surfer outputs are related as follows:*

$$q_i(x) - q_s(x) = c_s(x) - c_i(x), i \in \mathcal{I}, s \in \mathcal{S}. \tag{7}$$

Indeed, using (6) for $i \in \mathcal{I}, s \in \mathcal{S}$ leads to

$$
\begin{aligned}
q_i(x) - q_s(x) &= \frac{\left[a - Nc_i(x) + \sum_{j\in\mathcal{N}, j\neq i} c_j(x)\right]}{N+1} \\
&\qquad\qquad - \frac{\left[a - Nc_s(x) + \sum_{j\in\mathcal{N}, j\neq s} c_j(x)\right]}{N+1} \\
&= \frac{-Nc_i(x) + Nc_s(x) - c_i(x) + c_s(x)}{N+1} = c_s(x) - c_i(x).
\end{aligned}
$$

We now turn to solving the first-stage equilibrium. In this stage, the surfers are not active players and competition takes place only among investors. Note, however, that the latter are affected, profit-wise, by spillovers from their R&D efforts to the surfers.

Proposition 2 *Assuming an interior solution, the unique symmetric subgame-perfect equilibrium is given by*

$$
\begin{aligned}
x &= \frac{(N+1)^2 - 2c_1 Z(a - c_0)}{2c_1^2 YZ}, \\
q_i &= \frac{N+1}{2c_1 Z}, \quad i \in \mathcal{I}, \\
q_s &= \frac{1}{2c_1 YZ}\left[Y(N+1) - W\left((N+1)^2 - 2c_1 Z(a - c_0)\right)\right], \quad s \in \mathcal{S},
\end{aligned}
$$

where

$$
\begin{aligned}
W &= 1 + \beta(I-1) - \gamma I, \\
Y &= (S+1)(1 + \beta(I-1)) - \gamma IS, \\
Z &= N - \beta(I-1) - \gamma S.
\end{aligned}
$$

Proof. The optimization problem of investor $i \in \mathcal{I}$ reads as follows:

$$
\max_{x_i} \pi_i(x) = q_i^2(x) - x_i.
$$

Assuming an interior solution, first-order equilibrium conditions are given by

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial x_i} &= \frac{\partial\left(q_i^2(x) - x_i\right)}{\partial x_i} = 0, \quad i \in \mathcal{I}, \\
&= \frac{2q_i(x)}{N+1}\left[-N\frac{\partial c_i(x)}{\partial x_i} + \sum_{j\in\mathcal{I}, j\neq i}\frac{\partial c_j(x)}{\partial x_i} + \sum_{s\in\mathcal{S}}\frac{\partial c_s(x)}{\partial x_i}\right] - 1 = 0, \ i \in \mathcal{I}.
\end{aligned}
$$

Considering a symmetric solution, i.e., imposing $x_i = x$ for all $i \in \mathcal{I}$, we obtain

$$\left[-N\frac{\partial c_i(x)}{\partial x_i} + \sum_{j\in\mathcal{I},j\neq i}\frac{\partial c_j(x)}{\partial x_i} + \sum_{s\in\mathcal{S}}\frac{\partial c_s(x)}{\partial x_i}\right] = c_1 Z,$$

where

$$Z = N - \beta(I-1) - \gamma S.$$

For a symmetric solution, we have

$$q_i(x) = \frac{1}{N+1}(a - c_0 + c_1 x Y), \tag{8}$$

where

$$Y = (S+1)(1+\beta(I-1)) - \gamma I S.$$

The equilibrium condition for an investor becomes

$$\frac{2c_1 Z}{(N+1)^2}(a - c_0 + c_1 x Y) - 1 = 0,$$

and thus

$$x = \frac{(N+1)^2 - 2c_1 Z(a - c_0)}{2c_1^2 Y Z}.$$

Substituting for x in $q_i(x)$, we obtain the equilibrium quantity in terms of the parameters, i.e.,

$$q_i = \frac{N+1}{2c_1 Z}, i \in \mathcal{I}.$$

For surfers, we use (7)

$$q_s(x) = q_i(x) + c_i(x) - c_s(x), \quad i \in \mathcal{I}, s \in \mathcal{S},$$

which is equivalent to

$$q_s(x) = q_i(x) - c_1 x W, \quad i \in \mathcal{I}, s \in \mathcal{S}, \tag{9}$$

where

$$W = 1 + \beta(I-1) - \gamma I.$$

Substituting for x gives

$$q_s = \frac{1}{2c_1 Y Z}\left[Y(N+1) - W\left((N+1)^2 - 2c_1 Z(a - c_0)\right)\right], \quad s \in \mathcal{S}.$$

■

Substituting for the equilibrium output and R&D levels, we obtain the following equilibrium payoffs:

$$\pi_i = \frac{(Y-2Z)(N+1)^2 + 4c_1Z^2(a-c_0)}{4c_1^2YZ^2}, \quad i \in \mathcal{I}, \tag{10}$$

$$\pi_s = \frac{1}{4c_1^2Y^2Z^2}\left(Y(N+1) - W\left((N+1)^2 - 2c_1Z(a-c_0)\right)\right)^2, \quad s \in \mathcal{S}. \tag{11}$$

The two above propositions assume an interior solution, which translates into the following conditions on the model's parameters (note that $q_i > 0$ for all parameters' values and that W, Y and Z are positive[5]):

$$\begin{aligned} x &> 0 \Leftrightarrow (N+1)^2 - 2c_1Z(a-c_0) > 0, \\ q_s &> 0 \Leftrightarrow Y(N+1) - W\left((N+1)^2 - 2c_1Z(a-c_0)\right) > 0. \end{aligned}$$

Further, to have an economically meaningful solution, we require that the price, the production costs and the profits to be nonnegative. Since the surfer's production cost is necessarily higher than that of the investor, it suffices to check the positivity of the investors' costs. Noting that the surfer's payoff is nonnegative for all parameters' values (see (11)), we are left with the following conditions:

$$\begin{aligned} c_i &\geq 0 \Leftrightarrow 2c_0c_1YZ - (1+\beta(I-1))\left((N+1)^2 - 2c_1Z(a-c_0)\right) \geq 0, \\ &\Leftrightarrow (N+1)^2 - 2c_1Z(a-c_0) \leq \frac{2c_0c_1YZ}{(1+\beta(I-1))}, \\ P &\geq 0 \Leftrightarrow 2c_1Z(aY - aW + c_0W) - (N+1)(N(Y-W) - W) \geq 0, \\ &\Leftrightarrow (N+1)^2 - 2c_1Z(a-c_0) \geq \frac{Y(N(N+1) - 2ac_1Z)}{W}, \\ \pi_i &\geq 0 \Leftrightarrow (Y-2Z)(N+1)^2 + 4c_1Z^2(a-c_0) \geq 0, \\ &\Leftrightarrow (N+1)^2 - 2c_1Z(a-c_0) \leq \frac{Y(N+1)^2}{2Z}. \end{aligned}$$

The interior solution conditions and the above three conditions can be compacted as follows:

---

[5] To see it, rewrite W, Y and Z as follows:

$$\begin{aligned} W &= 1 - \beta + I(\beta - \gamma), \\ Y &= (S+1)(1-\beta) + I(\beta(S+1) - \gamma S), \\ Z &= N - \beta I - \gamma S + 1. \end{aligned}$$

Positivity follows from the assumption that $0 \leq \gamma \leq \beta \leq 1$.

$$\max\left(0,\frac{Y\left(N\left(N+1\right)-2ac_1Z\right)}{W}\right)<\left(N+1\right)^2-2c_1Z\left(a-c_0\right)$$
$$<\min\left(\frac{Y\left(N+1\right)}{W},\frac{2c_0c_1YZ}{1+\beta\left(I-1\right)},\frac{Y\left(N+1\right)^2}{2Z}\right).$$

The following proposition provides a sensitivity analysis of R&D strategy with respect to demand, cost and spillover parameters.

Proposition 3 *The symmetric investment in R&D varies with the model's parameters as follows:*

$$\text{(i)}\quad \frac{\partial x}{\partial a} = -\frac{1}{c_1Y}<0,$$
$$\text{(ii)}\quad \frac{\partial x}{\partial c_0} = \frac{1}{c_1Y}>0,$$
$$\text{(iii)}\quad \frac{\partial x}{\partial c_1} = \frac{1}{ZY}\left(\frac{Z\left(a-c_0\right)c_1-\left(N+1\right)^2}{c_1^3}\right)<0,$$
$$\text{(iv)}\quad \frac{\partial x}{\partial \beta} = -\frac{\left(S+1\right)\left(I-1\right)}{Y}\left(x+\frac{\left(N+1\right)^2}{2c_1^2Z^2}\right)<0,$$
$$\text{(v)}\quad \frac{\partial x}{\partial \gamma} = \frac{IS}{Y}\left(x+\frac{\left(N+1\right)^2}{2c_1^2Z^2}\right)>0.$$

Proof. Straightforward derivations lead to the above expressions. The signs of these derivatives are obvious (the sign of $\frac{\partial x}{\partial c_1}$ follows from the assumption of an interior solution, more specifically that x is positive). ■

These results are rather intuitive, Indeed, the higher the willingness-to-pay (a) the less is the pressure to reduce cost, and hence, to invest in R&D. On the other hand, the higher the initial cost (c_0), the better the prospect (or reward) offered by conducting R&D. As mentioned previously, the parameter c_1 measures the speed at which effective knowledge translates into a reduction of the unit-production cost. Consequently, the higher is this speed, the lower is the level of R&D effort needed to optimize profit. The most interesting result is the opposite signs of the derivatives of x with respect to the spillover parameters. The negative sign of the derivative of x with respect to β has a free-riding flavor. Indeed, the higher the technological spillover inflow, the less a firm needs to invest in R&D to achieve the same cost reduction. On the other hand, a high γ reduces the competitive advantage of an investor. Therefore, the latter is bound to invest more in R&D to counter this effect and keep its competitive lead.

4 Comparison

If all firms invest in R&D and decide to enter a research joint venture, we recover the cooperative R&D case analyzed by Kamien et al. (Ref. 2). Indeed, it suffices to set $\mathcal{S} = \emptyset$ (and $\mathcal{I} = \mathcal{N}$) and $\beta = 1$ in the proposition to obtain their result, i.e.,

$$\begin{aligned} x^* &= \frac{(N+1)^2 - 2c_1(a-c_0)}{2c_1^2 N}, \\ q_i^* &= \frac{N+1}{2c_1}, \quad i \in \mathcal{N}, \\ \pi_i^* &= \frac{(N-2)(N+1)^2 + 4c_1(a-c_0)}{4c_1^2 N}, \quad i \in \mathcal{N}. \end{aligned}$$

If all firms invest in R&D but play both stages noncooperatively, then it suffices to set $\mathcal{S} = \emptyset$ (and $\mathcal{I} = \mathcal{N}$) in the proposition to get the resulting equilibrium, i.e.,

$$\begin{aligned} x^{nc} &= \frac{(N+1)^2 - 2c_1(a-c_0)\tilde{Z}}{2c_1^2 \tilde{Y}\tilde{Z}}, \\ q_i^{nc} &= \frac{N+1}{2c_1\tilde{Z}}, \quad i \in \mathcal{N}, \\ \pi_i^{nc} &= \frac{\left(\tilde{Y} - 2\tilde{Z}\right)(N+1)^2 + 4c_1^2\tilde{Z}(a-c_0)}{4c_1^2\tilde{Y}\tilde{Z}^2}, i \in \mathcal{I}, \end{aligned}$$

where

$$\begin{aligned} \tilde{Y} &= (1 + \beta(N-1)), \\ \tilde{Z} &= N - \beta(N-1). \end{aligned}$$

The above equilibrium can be seen as an interesting benchmark to ours, since the only difference between the two lies in the presence of surfers in our setting. To compare the R&D investments in the two equilibria, we compute the difference

$$x - x^{nc} = \frac{1}{2c_1^2 YZ\tilde{Y}\tilde{Z}}\left[\left(\tilde{Y}\tilde{Z} - YZ\right)(N+1)^2 - 2c_1 Z\tilde{Z}(a-c_0)\left(\tilde{Y} - Y\right)\right].$$

We observe that

$$\begin{aligned} \tilde{Y} - Y &= -S(1 + \beta(I-1) - \gamma I) < 0, \\ \tilde{Z} - Z &= -S(\beta - \gamma) < 0. \end{aligned}$$

Therefore, substituting Z for $\tilde{Z}$ in the first bracketed term leads to

$$\begin{aligned} x - x^{nc} &< \frac{1}{2c_1^2 YZ\tilde{Y}\tilde{Z}} \left[\left(\tilde{Y}Z - YZ \right) (N+1)^2 - 2c_1 Z\tilde{Z} (a - c_0) \left(\tilde{Y} - Y \right) \right] \\ &= \frac{\left(\tilde{Y} - Y \right)}{2c_1^2 Y\tilde{Y}\tilde{Z}} \left[(N+1)^2 - 2c_1 \tilde{Z} (a - c_0) \right] < 0. \end{aligned}$$

The negativity follows from the (implicit) assumption that $x^{nc} > 0$. The conclusion here is that innovators invest less in R&D when there are surfers in the industry. Consequently, the total knowledge produced by the whole industry is also lower. Note that Kamien et al. (Ref. 2) established that noncooperative R&D levels are lower than their cooperative counterparts. Therefore, we clearly have $x < x^{nc} < x^*$.

In terms of outputs, it can be readily seen from (9) that $q_s(x) < q_i(x), \forall i \in \mathcal{I}, \forall s \in \mathcal{S}$, for all admissible values of x. This can be explained by the cost advantage that the investor has over the surfer, which is the result of a higher level of spillover inflow and of the investor's own R&D investment. Note that the difference in outputs can be expressed, after a straightforward substitution in (9) of the investors and surfers unit costs given in (2), as follows:

$$q_i(x) - q_s(x) = c_1 \left((\beta - \gamma) X + (1 - \beta) x \right), \quad \forall i \in \mathcal{I}, \forall s \in \mathcal{S},$$

where $X = Ix$, that is, the total knowledge produced by the industry. It can be readily seen that this difference is increasing in R&D investment, in β and c_1, and decreasing in γ. Further, an investor's output is increasing in x. Indeed, differentiating (8) yields

$$\frac{dq_i(x)}{dx} = \frac{c_1 Y}{N+1} > 0, \quad i \in \mathcal{I}.$$

Thus, the higher the level of R&D, the lower is the unit-production cost and the higher is the output. For a surfer, we use (9) to obtain

$$\begin{aligned} \frac{dq_s(x)}{dx} &= \frac{c_1 I}{N+1} \left(\gamma (I+1) - (1 + \beta (I-1)) \right), \ s \in \mathcal{S}, \\ \frac{dq_s(x)}{dx} &> 0 \Leftrightarrow \gamma > \frac{\beta (I-1) + 1}{I+1}, \ s \in \mathcal{S}. \end{aligned}$$

If there is only one investor in the industry (β is naturally equal to one in this case), then γ must be at least greater than 1/2 to have $\frac{dq_s(x)}{dx} > 0$. Further, for $\beta > 1/2$, the right-hand side of the above inequality is increasing in I. The conclusion therefore is that it takes a "low" β and a small number of investors for us to observe a positive relationship between a surfer's output and x. Although the result for a surfer is not clear-cut, we can still show that the total quantity put on the market increases with the level of x, independently of the parameters' values. Indeed, differentiating $Q = Iq_i + Sq_s$ with respect to x, we get

$$\frac{dQ(x)}{dx} = \frac{c_1 I}{N+1}\left[1 + \beta(I-1) - \gamma S\right] > 0.$$

The above results show that the presence of surfers in the industry therefore has a dual impact. First, in terms of R&D, the total available knowledge is lower, not only because surfers are not contributing to create knowledge, but also because they are pushing the innovators to invest less in R&D. Second, this leads to a lower total available knowledge than the one we would witness if the surfers were also investors. The consequence for consumers is a higher product price and a loss in welfare.

We now turn to a profit comparison. From (5) and (7), we have

$$\pi_i(x) - \pi_s(x) = (c_s(x) - c_i(x))\,(q_i(x) + q_s(x)) - x.$$

Therefore, the above difference is positive if

$$(c_s(x) - c_i(x))\,q_i(x) > x - (c_s(x) - c_i(x))\,q_s(x).$$

The term $(c_s(x) - c_i(x))$ corresponds to the incremental cost advantage for an investor with respect to the "original" situation in which x is equal to zero, and the unit cost is c_0 for all players. The above condition can therefore be interpreted as follows: for a given investment in R&D, an investor achieves a higher profit than does a surfer, if the incremental revenue obtained from this investment is higher than its cost minus the incremental revenue spilled over to a surfer.

To obtain the difference in players' payoffs in terms of the parameters, we use (10)-(11) to get

$$\begin{aligned}\pi_i - \pi_s &= -\frac{(N+1)^2 - 2c_1 Z(a-c_0)}{(2c_1 YZ)^2}\left[2Y\left(Z - (N+1)W\right)\right.\\ &\quad \left. + W^2\left((N+1)^2 - 2c_1 Z(a-c_0)\right)\right].\end{aligned}$$

Under the assumption of an interior solution (namely $x > 0$), the term $(N+1)^2 - 2c_1 Z(a-c_0)$ is positive. Therefore, the condition for having $\pi_i > \pi_s$, i.e., an investor having a higher profit than a surfer, reads as follows

$$\pi_i > \pi_s \Leftrightarrow (N+1)^2 - 2c_1 Z(a-c_0) < \frac{2Y((N+1)W - Z)}{W^2}.$$

To recapitulate, the presence of surfers in an industry leads to lower individual investments in R&D, and consequently, to a lower collective level of knowledge and a higher product price. The comparison of surfer and investor profits is not conclusive, the result being dependent on the model's parameters.

5 Conclusion

This paper characterized a subgame Nash equilibrium of a two-stage game in which the players determine their R&D efforts in the first stage, and compete à la Cournot in the second. The originality of this work lies in the assumption that not all players conduct R&D, which seems to be valid in practice. Our study is an exploratory analysis of the impact of having two types of players in an industry, and relies on a number of simplifying assumptions, which would be worth relaxing in future investigations. First, we assumed given the type of each player. One could add an initial stage in which each player chooses whether or not to build a research facility (i.e., laboratory, hiring scientific personnel, etc.) at a certain fixed cost. This would endogenize the number of players conducting R&D instead of having it be exogenous. Second, although we distinguished between the two types of players in terms of their spillover parameters, it may be the case, as argued by Cohen and Levinthal (Ref. 16), that a firm still needs to make a minimal investment in R&D, to be able to absorb its competitors' R&D. Third, we adopted a deterministic model for simplicity. The extension to a setting where R&D produces uncertain results is clearly of interest. Finally, we followed the literature and focused on symmetric R&D equilibrium among investors. An investigation of asymmetric equilibria is worth conducting.

References

1. D'ASPREMONT, C., and JACQUEMIN, A., *Cooperative and noncooperative R&D in duopoly with spillovers*, American Economic Review, Vol. 78, pp. 1133–1137, 1988.
2. KAMIEN, M.I., MULLER, E., and ZANG, I., *Research joint ventures and R&D cartels*, American Economic Review, Vol. 82, pp. 1293–1306, 1992.
3. SUZUMURA, S., *Cooperative, and noncooperative R&D in an oligopoly with spillovers*, American Economic Review, Vol. 82, pp. 1307–1320, 1992.
4. AMIR, R., *Modeling imperfectly appropriable R&D via spillovers*, International Journal of Industrial Organization, Vol. 18, pp. 1013–1032, 2000.
5. HINLOOPEN, J., *Strategic R&D cooperatives*, Research in Economics, Vol. 54, pp. 153–185, 2000.
6. KAMIEN, M.I., and ZANG, I., *Meet me halfway: Research joint venture and absorptive capacity*, International Journal of Industrial Organization, Vol. 18, pp. 1895–1912, 2000.
7. SILIPO, D.G., *The economics of cooperation and competition in research and development: A survey*, Dipartimento di Economia e Statistica -Università della Calabria, Discussion Paper no. 37, 2003.
8. STEWART, M.B., *Noncooperative oligopoly and preemptive innovation without winner-take-all*, Quarterly Journal of Economics, Vol. 98, pp. 681–694, 1983.

9. JAFFE, A.B., *Technological opportunity and spillovers of R&D: Evidence from firms patents, profits and market value.* American Economic Review, Vol. 76, pp. 984–1001, 1986.
10. HENRIQUES, I., *Cooperative and noncooperative R&D in duopoly with spillovers: Comment*, American Economic Review, Vol. 80, pp. 638–640, 1990.
11. AMIR, R., and WOODERS, J., *One way spillovers, endogenous innovator-imitator roles and research joint ventures*, Games and Economic Behavior, Vol. 31, pp. 1–25, 2000.
12. PETIT, M.L., and TOLWINSKI, B., *R&D cooperation or competition*, European Economic Review, Vol. 43, pp. 185–208, 1999.
13. BRETON, M., VENCATACHELLUM, D., and ZACCOUR, G., *Dynamic R&D with strategic behavior*, Computers & Operations Research, Vol. 33, pp. 426–437, 2006.
14. ARROW, K.J., *Economic welfare and the allocation of resources for inventions*, in Nelson, R.R. (ed.), The rate and direction of incentive activity, Princeton, N.J., Princeton University Press, 1962.
15. BRETON, M., TURKI, A., and ZACCOUR, G., *Dynamic model of R&D, spillovers and efficiency of Bertrand and Cournot equilibria*, Journal of Optimization Theory and Applications, Vol. 123, pp. 1–25, 2004.
16. COHEN, W., and LEVINTHAL, D., *Innovation and learning: the two faces of R&D*, Economic Journal, Vol. 99, pp. 569–596, 1989.
17. SALANT, S.W., and SHAFFER, G., *Optimal asymmetric strategies in research joint ventures*, International Journal of Industrial Organization, Vol. 16, pp. 195–208, 1998.