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Abstract

This paper deals with the problem of determining optimal reservoir daily operating policy over a one-year period. This problem is stochastic since the daily reservoir inflows are random and cannot be predicted far in advance. The aim of the paper is to show that optimal reservoir operating policy changes with the way the problem is solved and the information that is taken into account. The paper first shows that the operating policy determined using Stochastic Dynamic Programming greatly improves when the multi-lag autocorrelation of the inflows is included. Next, it shows that the operating policy improves with the number of days that the inflows are assumed to be known in advance. Finally, the paper shows that a better operating policy may be obtained by solving the optimization problem with Sampling Dynamic Programming. Numerical results are presented, compared and analyzed.

Résumé

Cet article porte sur la détermination de la règle optimale de gestion journalière d'un réservoir sur un horizon d'une année. Le problème est stochastique puisque les apports au réservoir sont aléatoires et ne sont généralement connus qu'une ou quelques journées à l'avance. Le but de l'article est de montrer que la règle optimale de gestion du réservoir change avec la façon de modéliser le problème et de prendre en compte l'information. Le problème est tout d'abord solutionné sans tenir compte de la corrélation temporelle des apports. Puis il est solutionné en prenant en compte la corrélation entre les apports d'un jour donné et ceux des 1, 2, . . . , n jours précédents. L'article montre que la règle de gestion s'améliore avec la valeur de n. Le problème d'optimisation stochastique journalier est solutionné par la suite en supposant que les apports sont connus 1, 2, . . . , m jours d'avance. Les résultats montrent que la règle optimale de gestion s'améliore aussi avec la valeur de m. Finalement, l'article montre qu'une meilleure solution peut, dans certains cas, être obtenue en utilisant le *Sampling Dynamic Programming* pour résoudre le problème. Des résultats numériques sont présentés, comparés et analysés.

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Introduction

The first application of Dynamic Programming (DP) to the reservoir management problem dates back more than sixty years, when the first paper on the subject was published by Massé in 1946. The paper was in fact published six years before the first paper by Bellman (1952) on DP, so that the first DP application probably involved a reservoir management problem. Since Massé (1946), several papers have been published on the subject. In the survey by Yakowitz (1982) on DP application to water resources, 45 papers dealing with reservoir management are reviewed. About the same number of papers are cited by Yeh (1985) in this state-of-the-art review. Labadie (2004) lists 27 papers published since 1980.

The paper's contribution is not to show how to determine the optimal reservoir operating policy using DP since this was done a long time ago. Rather, it consists in showing that the optimal operating policy changes with the way the problem is solved and the information that is taken into consideration. For instance, the daily inflow can be assumed to be an independent random variable when solving the DP problem. However, if the inflow is not independent but correlated to the previous day's inflow, a better operating policy will be obtained if this correlation is taken into account. The inflow on day t may be correlated not only to the inflow of the preceding day but to those of several previous days. The paper shows that the solution improves when the multi-lag autocorrelation is taken into consideration.

When the reservoir's operating policy is determined on a daily time basis, it is usually assumed that the inflow of day t is known at the beginning of that day and, hence, when the reservoir's operating policy is set for that day. The possibility that the inflow of day $t+1$ may also be known at the beginning of day t is never considered. This inflow is always supposed to be known in probability only at the beginning of day t. In practice, however, the inflows are often known many days in advance. In fact, there are continuous improvements in weather forecasting, with forecasts now made for periods of four to ten days. If the inflow is known for more than one day in advance, a better operating policy will be obtained if this is taken into account. This paper shows that the results improve significantly with the number of days that the inflows are known in advance. Finally, the paper shows that a better operating policy may be obtained by solving the problem with Sampling Dynamic Programming (Kelman 1990) instead of Stochastic Dynamic Programming.

Problem formulation

The problem consists in determining the optimal operating policy of a reservoir supplying a hydroelectric powerplant. The problem is solved on a daily time basis for a period of one year. The problem is stochastic since the reservoir inflows are random and cannot be predicted long in advance.

The objective of the problem is to maximize the expected revenues from the sale of the hydroelectric energy produced by the powerplant, and more specifically to:

$$\text{maximize } E\left\{\sum_{t=1}^{T} \upsilon_t \cdot \eta\left(S_t, S_{t+1}\right) \cdot g_t\left(R_t\right)\right\} + \Phi\left(S_{T+1}\right) \tag{1}$$

where $E\{\}$ represents the mathematical expectation, S_t the reservoir content in hm^3 at the beginning of day t, R_t the reservoir release in m^3/s in day t, υ_t the value of the MWh produced by the powerplant in day t, and where $T = 365$. The function $g_t\left(R_t\right)$ gives the energy generated by the powerplant in day t as a function of R_t and with the water head equal to H^{ref} m. The function $\eta\left(S_t, S_{t+1}\right)$ corrects the generation when the water head is different from H^{ref}.

$$\eta\left(S_t, S_{t+1}\right) = \frac{h\left([S_t + S_{t+1}]/2\right)}{H^{ref}}, \tag{2}$$

where $h()$ denotes the water head in m as a function of the reservoir content. Equation (2) supposes that the generation increases linearly with the water head, which is almost true. The function $\Phi\left(S_{T+1}\right)$ in (1) represents the expected value of the water stored in the reservoir at the end of the year.

The function $g_t\left(R_t\right)$ used in this paper is piecewise linear, concave and non-decreasing, like the one shown in Figure 1 for a powerplant consisting of four generating units. Point 1 in this figure corresponds to the optimal generation of the most efficient generating unit, point 2 to the optimal generation of the two most efficient units, point 3 to the optimal generation of the three most efficient units, and point 4 to the optimal generation of the four units. The generation is said to be optimal when the generating units operate at maximum efficiency. The generation is smaller, however, than the capacity of the four units, which explains why there are three intersection points beyond point 4 in Figure 1. These three points give the maximum generation values obtained with outflows equal to u_5, u_6 and u_7 m^3/s. The powerplant generation is not really a piecewise linear function. Between points 0 and 1, for instance, the generation increases nonlinearly, first in a convex way and afterward in a concave way. The real curve is in fact S-shaped. Straight lines are still used between intersection points in Figure 1 because they simplify the problem and give good results. The discharges determined with the optimization model generally correspond to the intersection points of the piecewise linear function, which is very similar to the solutions found by Linear Programming for piecewise linear problems. The reason may be that problem (1)–(5) is almost a piecewise linear problem and would in fact be one without function $\eta\left(S_t, S_{t+1}\right)$ in (1). The solution for problem (1)–(5) with the piecewise linear function $g_t\left(R_t\right)$ is excellent since it generally adjusts the reservoir discharge to u_1, u_2, u_3, ..., u_7 m^3/s, and hence to the most productive discharges.

The curve in Figure 1 may change throughout the year since the number of available generating units may not always be the same because of maintenance. The powerplant's maximal outflow, denoted by R_t^{turb}, may therefore change with time. When the reservoir release R_t is greater than R_t^{turb}, spillage occurs and less energy is generated because S_{t+1} and $\eta\left(S_t, S_{t+1}\right)$ diminish.

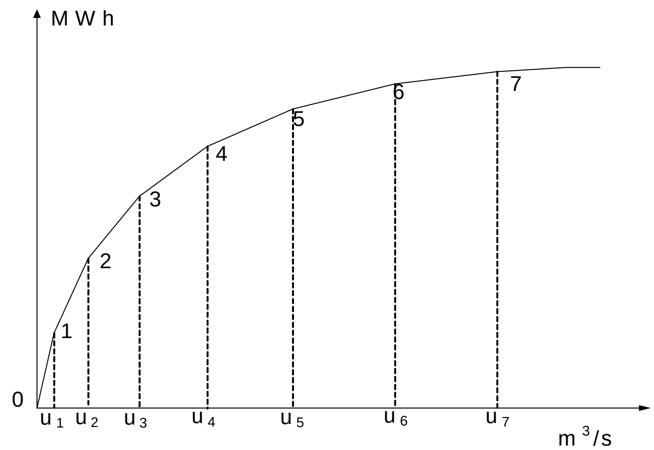

Figure 1: Generation vs outflow

The optimal reservoir operating policy is the policy that maximizes the expected revenues while respecting the following constraints:

$$S_{t+1} = S_t + (Q_t - R_t) \cdot c\,; \qquad t = 1, 2, \ldots, T \qquad (3)$$

$$S_{t+1}^{\min} \le S_{t+1} \le S_{t+1}^{\max}\,; \qquad t = 1, 2, \ldots, T \qquad (4)$$

$$R_t^{\min} \le R_t \le R_t^{\max}\,; \qquad t = 1, 2, \ldots, T \qquad (5)$$

where $c = 0.0864$. This factor converts the m^3/s for one day into hm^3. The variable Q_t represents the reservoir inflow on day t. There are lower and upper bounds in (4) and (5) on variables S_{t+1} and R_t because there are usually environmental, recreational or economic constraints to satisfy. For instance, the reservoir may have to be maintained above a certain level during the summer to allow for boating and recreational activities, but not too high because flooding might occur. The reservoir discharge may also have to be maintained above a certain level during certain periods of the year for environmental or economic reasons.

Stochastic Dynamic Programming

The stochastic optimization problem defined in the preceding section may not be solvable because there is a probability that constraints (4) and (5) could be violated. The problem can easily be made solvable, however, by changing the deterministic constraints for probabilistic constraints or by replacing constraints (4) and (5) with a penalty function in the objective function. This second approach is used in this paper and problem (1)–(5) is rewritten as:

$$\text{maximize } E\left\{\sum_{t=1}^{T} \upsilon_t \cdot \eta\left(S_t, S_{t+1}\right) \cdot g_t\left(R_t\right) - L_t\left(S_{t+1}, R_t\right)\right\} + \Phi\left(S_{T+1}\right) \tag{6}$$

subject to constraint (3) and the two following inequalities:

$$0 \leq S_{t+1} \leq S^{cap} \tag{7}$$

$$R_t \geq 0 \tag{8}$$

where $L_t\left(S_{t+1}, R_t\right)$ is the penalty function,

$$L_t\left(S_{t+1}, R_t\right) = e_{1,t} \cdot \max\left(0, S_{t+1}^{\min} - S_{t+1}\right) + e_{2,t} \cdot \max\left(0, S_{t+1} - S_{t+1}^{\max}\right) + e_{3,t} \cdot \max\left(0, R_t^{\min} - R_t\right) + e_{4,t} \cdot \max\left(0, R_t - R_t^{\max}\right). \tag{9}$$

Constraints (7) and (8) imply that all the surplus water is spilled when the reservoir becomes full, and more specifically when $S_{t+1} = S^{cap}$. The value of $\max(0, \omega)$ in (9) is equal to ω when $\omega > 0$ and to 0 otherwise. The value of $L_t\left(S_{t+1}, R_t\right)$ is therefore equal to zero when constraints (4) and (5) are satisfied and increases linearly when the constraints are violated. The only difficulty with penalty function (9) is to correctly adjust the parameters $e_{i,t}$, $i = 1, \ldots, 4$, $t = 1, \ldots, T$.

To simplify the presentation, let:

$$B_t\left(S_t, S_{t+1}, R_t\right) = \upsilon_t \cdot \eta\left(S_t, S_{t+1}\right) \cdot g_t\left(R_t\right) - L_t\left(S_{t+1}, R_t\right) \tag{10}$$

The optimization problem can then be rewritten as:

$$\text{maximize } E\left\{\sum_{t=1}^{T} B_t\left(S_t, S_{t+1}, R_t\right)\right\} + \Phi\left(S_{T+1}\right) \tag{11}$$

subject to constraints (3), (7) and (8). This optimization problem can be solved with DP, and more specifically with the following recursive equation:

$$F_t\left(S_t\right) = \underset{Q_t}{E}\left\{\max_{R_t}\left[B_t\left(S_t, S_{t+1}, R_t\right) + F_{t+1}\left(S_{t+1}\right)\right]\right\}. \tag{12}$$

Equation (12) must be solved backwards in time, beginning on day T with $F_{T+1}\left(S_{T+1}\right) = \Phi\left(S_{T+1}\right)$. The solution must, of course, respect constraints (3), (7) and (8). When the function $\Phi\left(S_{T+1}\right)$ is not known, one may proceed as follows: set $\Phi\left(S_{T+1}\right) = 0 \; \forall S_{T+1}$ and solve equation (12) backwards in time for a period of a year. Next, fix $\Phi\left(S_{T+1}\right) = F_1\left(S_{T+1}\right) \; \forall S_{T+1}$, and solve the recursive equation again for a year. Repeat the procedure until the marginal value of $\Phi\left(S_{T+1}\right)$ is the same in two consecutive iterations. The optimal operating policy determined with (12) is a function of S_t and Q_t since Q_t is assumed to be known when R_t is fixed.

The inflow Q_t is usually correlated with the inflow of the preceding day. This correlation varies throughout the year and can be very high at certain time, such as during Canadian winters. When this correlation is included in the optimization model, the variance of the random variables becomes smaller and better results are obtained. The recursive equation then becomes:

$$F_t\left(S_t, Q_{t-1}\right) = \underset{Q_t|Q_{t-1}}{E} \left\{ \max_{R_t} \left[B_t\left(S_t, S_{t+1}, R_t\right) + F_{t+1}\left(S_{t+1}, Q_t\right) \right] \right\} \tag{13}$$

The variable Q_{t-1} is considered to be a state variable in equation (13) because Q_t is a function of this variable. This function is usually assumed to be as follows:

$$Q_t = b_{0,t} + b_{1,t} \cdot Q_{t-1} + b_{2,t} \cdot \zeta_t \tag{14}$$

where $b_{0,t}$, $b_{1,t}$ and $b_{2,t}$ are parameters and ζ_t is a random variable with the mean equal to zero. When the variables Q_t and Q_{t-1} are normally distributed, the variable $b_{2,t} \cdot \zeta_t$ is also normally distributed with a mean equal to zero and a standard deviation equal to:

$$b_{2,t} = \sigma_t \sqrt{\left(1 - \rho_{t,t-1}^2\right)} \tag{15}$$

The symbol σ_t in (15) denotes the standard deviation of Q_t, and $\rho_{t,t-1}$ the correlation coefficient between Q_t and Q_{t-1} . The variance of the random variable is therefore equal to σ_t^2 in equation (12) and to $\sigma_t^2 \cdot \left(1 - \rho_{t,t-1}^2\right)$ in equation (13). If $\rho_{t,t-1} = 0.90$, as is often the case in winter, the variance of the random variable in problem (13) is only equal to 19% of the variance in problem (12), which may have an important effect on the operating policy.

The inflow Q_t may be correlated not only to Q_{t-1}, but to Q_{t-2}, Q_{t-3}, …,Q_{t-n}, where n may be as large as 15. The relation between Q_t and the inflows of the n preceding days is usually assumed to be linear and equal to:

$$Q_t = b_{0,t} + b_{1,t} \cdot Q_{t-1} + b_{2,t} \cdot Q_{t-2} + \ldots + b_{n,t} \cdot Q_{t-n} + b_{n+1,t} \cdot \zeta_t \tag{16}$$

When the variables Q_{t-1}, Q_{t-2}, Q_{t-3}, …,Q_{t-n} are all normally distributed, the variable $b_{n+1,t} \cdot \zeta_t$ is also normally distributed with a mean of zero and a standard deviation equal to:

$$b_{n+1,t} = \sigma_t \sqrt{\left(1 - \Upsilon_t^2\right)} \tag{17}$$

where Υ_t^2 is the coefficient of determination (Fiering and Jackson 1971). Υ_t^2 is equal to $\rho_{t,t-1}^2$ when $n = 1$ and increases with the value of n, at least for a while. The idea is to increase n as long as Υ_t^2 increases and this improves the reservoir's operating policy determined with the following equation:

$$F_t\left(S_t, Q_{t-1}, \ldots, Q_{t-n}\right) = \underset{Q_t|Q_{t-1},\ldots,Q_{t-n}}{E} \left\{ \max_{R_t} \Big[B_t(S_t, S_{t+1}, R_t) + F_{t+1}(S_{t+1}, Q_t, \ldots, Q_{t-n+1}) \Big] \right\} \quad (18)$$

It is well known that a recursive equation like (18) cannot be solved in a reasonable time when the number of state variables exceeds four. Turgeon (2005) has recently shown that the problem can be solved approximately for any value of n with two state variables only, S_t and M_t, where:

$$M_t = b_{1,t} \cdot Q_{t-1} + b_{2,t} \cdot Q_{t-2} + \ldots + b_{n,t} \cdot Q_{t-n} \quad (19)$$

Substituting (19) into (16) gives:

$$Q_t = b_{0,t} + M_t + b_{n+1,t} \cdot \zeta_t \quad (20)$$

Recursive equation (18) can then be approximated by the following equation:

$$F_t\left(S_t, M_t\right) = \underset{Q_t|M_t}{E} \left[\underset{M_{t+1}[M_t,Q_t}{E} \left\{ \max_{R_t} \left[B_t\left(S_t, S_{t+1}, R_t\right) + F_{t+1}\left(S_{t+1}, M_{t+1}\right) \right] \right\} \right] \quad (21)$$

Turgeon's paper shows how to determine the conditional distribution of Q_t in terms of M_t and that of M_{t+1} in terms of M_t and Q_t.

Sampling Dynamic Programming

The reservoir's optimal operating policy was determined in the preceding section based on the probability distributions or the conditional daily inflow probability distributions. In this section, it is determined from the historical inflow scenarios or the scenarios generated by autoregressive equation (16). Let N be the number of available scenarios and Q_t^n the inflow on day t corresponding to scenario n. If one disregards the serial correlation of the inflows, the reservoir's optimal operating policy can be determined by solving one of the following two recursive equations:

$$F_t\left(S_t, n\right) = \max_{R_t} \left[B_t\left(S_t, S_{t+1}, R_t\right) + \sum_{m=1}^{N} F_{t+1}\left(S_{t+1}, m\right) \cdot p_t(m) \right] \quad (22)$$

$$F_t\left(S_t, n\right) = \max_{R_t} \left[B_t\left(S_t, S_{t+1}, R_t\right) + \frac{1}{N} \sum_{m=1}^{N} F_{t+1}\left(S_{t+1}, m\right) \right] \quad (23)$$

The function $F_t\left(S_t, n\right)$ represents the expected profit between the beginning of day t and the end of the horizon when reservoir storage at the beginning of day t is equal to S_t, the forecasted inflow for day t corresponds to that of scenario n, and the optimal operating policy is applied every day. The function $p_t(m)$ in (22) represents the probability that the inflow on day t corresponds to that of scenario m. This probability can be determined with a histogram when there are sufficient data. Otherwise, it must be determined from the probability distribution that best fits the inflow data of day t. In (23), the problem is simplified since the same probability $1/N$ is assigned to each of the N scenarios. However, this approach should only be used when N is large and historical scenarios are used. There is no advantage to using (23) for synthetic inflows with known probability distributions.

There is very little difference between recursive equations (22) and (12) given that (12) can also be written as follows:

$$F_t\left(S_t, Q_t\right) = \max_{R_t} \left[B_t\left(S_t, S_{t+1}, R_t\right) + \underset{Q_{t+1}}{E} \left\{ F_{t+1}\left(S_{t+1}, Q_{t+1}\right) \right\} \right] \tag{24}$$

To solve (24) in a reasonable period of time, the variables S_t, S_{t+1}, Q_t and Q_{t+1} must be discretized. Let $q_{t+1,1}$, $q_{t+1,2}$, …, $q_{t+1,J}$ be the J values of Q_{t+1} used to solve (24). The only difference between (22) and (24) is therefore that (22) is solved for Q_{t+1} equal to Q_{t+1}^1, Q_{t+1}^2, …, Q_{t+1}^N and (24) for Q_{t+1} equal to $q_{t+1,1}$, $q_{t+1,2}$, …, $q_{t+1,J}$. Although these values are different, the result will be approximately the same since the probability distribution of Q_{t+1} is the same for the two problems. Consequently, there is no advantage to using (22) instead of (12) to determine the reservoir's operating policy.

The correlation between the inflows of days t and $t-1$ can be taken into account with the scenarios by solving the following recursive equation instead of (22):

$$F_t\left(S_t, n\right) = \sum_{m=1}^{N} \left\{ \max_{R_t} \left[B_t\left(S_t, S_{t+1}, R_t\right) + F_{t+1}\left(S_{t+1}, m\right) \right] \right\} \cdot p_t\left(m|n\right) \tag{25}$$

where $p_t\left(m|n\right)$ is the probability that the inflow on day t corresponds to that of scenario m given that the inflow on day $t-1$ was equal to that of scenario n.

Mixed Dynamic Programming

The word "mixed" here means a mixture of deterministic and stochastic dynamic programming to solve the reservoir's management problem. This method is used when the inflows are known for more than a day in advance and, more specifically, for j days in advance where j may vary throughout the year. In this case, the optimal reservoir's operating policy, without autocorrelation, can be determined by solving the following recursive equation for $t = 1$, $j+1$, $2j+1$, $3j+1$, …, etc.:

$$F_t\left(S_t, n\right) = \max \left[\sum_{i=0}^{j-1} B_{t+i}\left(S_{t+i}, S_{t+i+1}, R_{t+i}\right) + \sum_{m=1}^{N} F_{t+j}\left(S_{t+j}, m\right) \cdot p_t(m) \right] \tag{26}$$

Solving equation (26) can be shown to be equivalent to solving one of the two following equations, according to the value of t:

$$F_t(S_t, n) = \max_{R_t} \left[B_t(S_t, S_{t+1}, R_t) + \sum_{m=1}^{N} F_{t+1}(S_{t+1}, m) \cdot p_t(m) \right]; \quad \text{if } t = k \cdot j \qquad (27a)$$

$$F_t(S_t, n) = \max_{R_t} \left[B_t(S_t, S_{t+1}, R_t) + F_{t+1}(S_{t+1}, n) \right]; \quad \text{otherwise} \qquad (27b)$$

where k is a positive integer. A mixture of stochastic and deterministic DP is used here since (27a) is stochastic whereas (27b) is deterministic and solved with the inflows of scenario n. The lag-one inflow autocorrelation can be taken into account by replacing (27a) by (25). The problem can also be solved by taking into account the correlation between the average inflow of the next j days with that of the past j days. For $j = 7$, for instance, this would be equivalent to taking into account the correlation between the average inflows of weeks k and $k-1$. Let $\tilde{p}_k(m|n)$ denote the probability that the average inflow in period k is equal to that of scenario m given that it was equal to that of scenario n in period $k-1$. This autocorrelation can be taken into account by simply replacing equation (27a) with the following equation:

$$F_t(S_t, n) = \sum_{m=1}^{N} \left\{ \max_{R_t} \left[B_t(S_t, S_{t+1}, R_t) + F_{t+1}(S_{t+1}, m) \right] \right\} \cdot \tilde{p}_k(m|n) \qquad (28)$$

Numerical results

It was shown in the preceding sections that the optimal reservoir operating policy can be determined with nine different recursive equations, and more precisely with equations (12), (13), (18), (21), (22), (23), (25), (27) and (28). These recursive equations are all easy to program and, except for (18), can be solved in a few minutes by a computer. If the objective is to find the best reservoir operating policy, these recursive equations should all be programmed and their results compared. This is what this section does: compare the operating policies determined by the recursive equations for a 107-MW hydroelectric powerplant fed by a 537.5-hm^3 reservoir.

A set of one hundred inflow scenarios was used to solve the optimization problems. Figure 2 gives the minimum, average and maximum inflows in this set for each day of the year. The average annual inflow and standard deviation are 75 m^3/s and 44 m^3/s respectively. The Figure 3 correlograms show that the inflows are autocorrelated, which is usually the case when the problem is solved on a daily basis. According to these correlograms, the daily inflow may not only be correlated to the inflow of the preceding day but to the inflows of many previous days. The inflow data probability distributions are skew in each day of the year, meaning that the inflows are not normally distributed. The skewness coefficients vary throughout the year from 0.20 to 6.97, the average being 1.73. Since it is recommended that the variables in autoregressive equations (14) and (16) be normally distributed, the inflow data were normalized, i.e. transformed into data that are normally

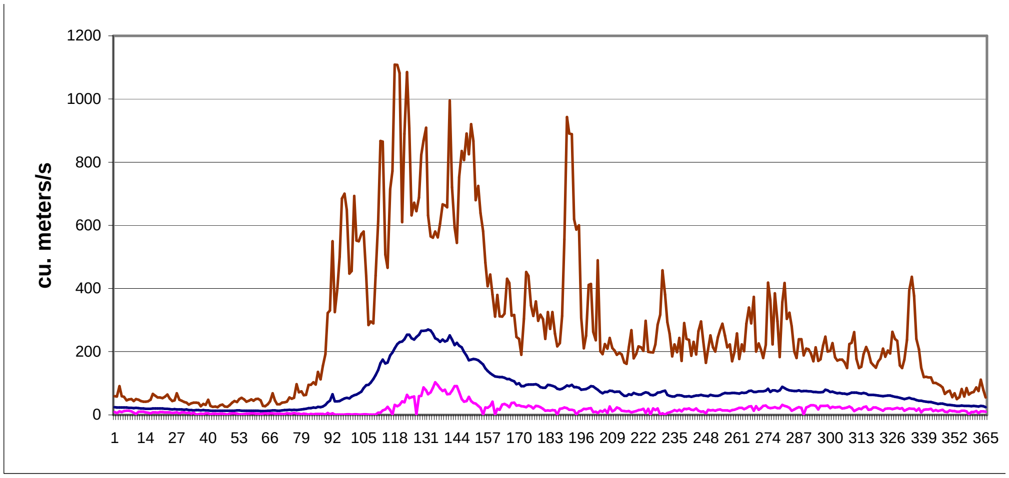

Figure 2: Minimum, mean and maximum daily inflows

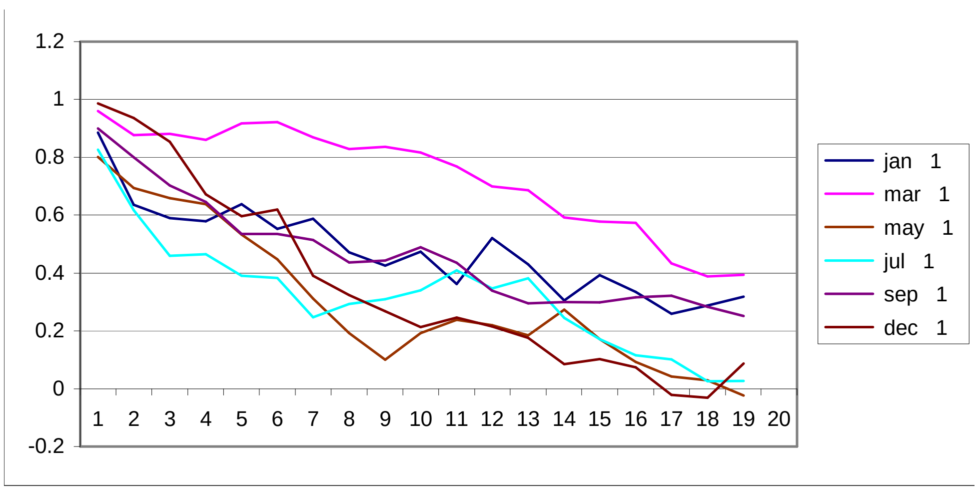

Figure 3: Correlograms of six different days

distributed. The transformations were done using the logarithmic and gamma functions. The normalized data are represented by the variable Y_t below, where $Y_t = \mathrm{N}_t\left(Q_t\right)$.

The powerplant's daily generation is supposed to be a piecewise linear function of the outflow, like the one shown in Figure 1. The values of the outflow and the generation at the six intersection points of this function are given in Table 1. The generation corresponds to a water head of 118.6 m, which ranges from 111 to 122 m depending on the reservoir content.

Table 1: Generation vs outflow

Intersection points	Outflow m^3/s	Generation MWh
1	33.5	35.06
2	67.0	67.96
3	100.2	98.55
4	105.3	102.22
5	110.4	105.08
6	115.5	106.97

The objective of the optimization problem is the same as that of problem (6)–(9). The value of the MWh generated on day t, represented by υ_t in (6), was set to 1.30 for the days in January and February, 1.20 for the days in March and December, 1.10 for the days in April and November, and to 1.0 for the other months of the year. There is no constraint regarding the maximum reservoir level other than $S_t \leq S^{cap}\,\forall t$, so that $e_{2,t}$ was set to zero in equation (9). However, there are constraints on the minimum and maximum reservoir discharges and minimum reservoir level in the summer. These constraints require that $50 \leq R_t \leq 450\ \forall t$ and $S_t \geq 433$ for $166 \leq t \leq 258$. They are taken into account by penalty function $L_t\left(S_{t+1}, R_t\right)$. The coefficient $e_{1,t}$ in $L_t\left(S_{t+1}, R_t\right)$ was set equal to 12 for $166 \leq t \leq 258$ and to zero for the rest of the year. The coefficients e_{3t} and $e_{4,t}$ were set to 10 and 20 for all t values.

The variables S_t and Q_t in recursive equation (12) were discretized so as to solve the problem in a reasonable amount of time. The equation was in fact solved for 15 different values of Q_t and 35 equidistant values of S_t in the interval $[0, S^{cap}]$. The values of Q_t were set equal to $\mathrm{N}_t^{-1}\left(Y_t\right)$ for Y_t equal to:

$$(\text{the mean of } Y_t) + \lambda_i \cdot (\text{the standard deviation of } Y_t),$$

with $\lambda_i =$ -2.5, -2.0, -1.5, -1.25, -1.0, -0.75, -0.35, 0, 0.35, 0.75, 1.0, 1.25, 1.5, 2.0 and 2.5. Tests made with a greater number of values of λ_t, and hence of Q_t, have not produced much better results, as Table 2 shows. The results presented in this table were obtained by simulating the reservoir's operating policy found by (12) over a period of one hundred years. The simulations were done with the inflows of the 100 scenarios. Columns 2, 3, and 4 in Table 2 give the mean annual values of the energy produced, the penalty cost and the profit. Columns 5, 6 and 7 give the number of years and days for which the constraints on the maximum and minimum discharge and on the minimum reservoir level were not respected. A flood is supposed to occur when the reservoir discharge exceeds $R_t^{\max}$.

The results presented in Table 2 do not take into account the serial inflow correlation. When this correlation is taken into account, the results improve considerably, as Table 3 shows. The first column in this table gives the number η of preceding days to which the inflow of day t is correlated. When η is increased from 0 to 1, the profit increases by

Table 2: Results for different numbers of λ_t

Number of λ_t	Revenue generated	Penalty cost	Profit	Number of years (days)		
				$R_t^{\max}$	$R_t^{\min}$	$S_t^{\min}$
15	680164	72007	608157	5 (26)	36 (412)	27 (325)
23	680283	71883	608400	5 (26)	35 (406)	27 (325)
35	680284	71871	608413	5 (26)	35 (405)	27 (325)
51	680225	71711	608514	5 (26)	35 (403)	27 (325)

Table 3: Results for different numbers of lags

Number of lags η	Revenue generated	Penalty cost	Profit	Number of years (days)		
				$R_t^{\max}$	$R_t^{\min}$	$S_t^{\min}$
0	680164	72007	608157	5 (26)	36 (412)	27 (325)
1	658421	33558	624863	5 (25)	8 (68)	6 (27)
2	665082	34620	630462	5 (25)	8 (89)	7 (42)
4	665941	34529	631412	5 (25)	8 (98)	7 (43)
6	666373	33815	632558	5 (24)	9 (97)	7 (42)
7	666297	33060	633237	5 (24)	8 (96)	8 (40)
10	666380	33113	633267	5 (24)	8 (93)	8 (40)
13	666178	32388	633790	5 (24)	8 (88)	8 (40)

2.7%, which is not negligible. The most interesting result, however, is that the numbers of days where violations of the constraints occur diminish drastically in columns (6) and (7). The profit increases very little afterwards with the value of η. However, considering that computing time increases very little also, the value of η should be increased as long as the profit increases. The results in Table 3 were also obtained by simulating the reservoir's operating policy over a period of 100 years with the inflow scenarios. The operating policy was determined with recursive equation (13) for $\eta = 1$ and with equation (21) for $\eta > 1$. The fact that the number of days in columns 6 and 7 are smaller for $\eta = 1$ than for $\eta > 1$ may look strange, but it is not so. The objective of the problem is not to minimize the number of violations but to maximize the profits, which increase with the value of η in Table 3. Furthermore, the penalty cost in column 3 is not only a function of the number of violations but also of the severity, so that a severe violation in one day may be more costly than small violations over several days. This explains why the penalty cost is smaller for $\eta = 13$ than for $\eta = 1$, even though the number of days of violation is greater.

Table 4 presents the results obtained with the Sampling Dynamic Programming method. The results in the first line were obtained by simulating the reservoir operating policy

Table 4: Sampling dynamic programming results

Eqn no.	No. of lags	Revenue generated	Penalty cost	Profit	Number of years (days)		
					$R_t^{\max}$	$R_t^{\min}$	$S_t^{\min}$
(22)	0	679498	75377	604121	4 (22)	40 (482)	31 (375)
(23)	0	679183	76579	602604	5 (24)	40 (490)	30 (368)
(25)	1	662238	31668	630570	3 (20)	8 (86)	7 (31)

determined by recursive equation (22) over a period of 100 years. This equation was solved for 35 equidistant values of S_t in the interval $[0, S^{cap}]$ and for Q_t equal to the values of the 100 scenario inflows in day t. The results in the second line were obtained with the operating policy determined by recursive equation (23), and those in the third line with the operating policy determined by equation (25), as the first column of the table indicates. The results of the two first lines show that it is preferable to determine and use the probability distribution of the 100 inflows on day t than to assign the same probability 1/100 to each inflow. Strangely enough, the results of the two first lines are not as good as those in Table 3, which were obtained with the operating policy determined by recursive equation (12). In other words, in this case, Sampling Dynamic Programming does not perform as well as Stochastic Dynamic Programming. This is probably due to the fact that the serial correlation of the inflows has not been taken into account. To disregard the autocorrelation when it is high – and Figure 3 shows that it is high here – may certainly yield a poor operating policy. When the lag-one autocorrelation is taken into account, Sampling Dynamic Programming gives better results than Stochastic Dynamic Programming. Comparing the results in the last line of Table 4 to those of the second line in Table 3 shows that the profit is large, the penalty cost smaller and floods occur in three years only when Sampling Dynamic Programming is used.

Table 5 presents the results obtained with Mixed Dynamic Programming and more specifically with recursive equations (27a) and (27b). The probability $p_t\,(m)$ in (27a) has, however, been replaced with $p_t\,(m|n)$ to take account of the correlation between the inflows of days t and $t-1$. The first column in Table 5 gives the number of days, j, for which the inflows are supposed to be known in advance. This number is the same for the entire year. The results show very clearly that the profits increase with the value of j. The last line in Table 5 presents the results obtained with recursive equation (28), and more specifically with conditional probabilities that take into account the correlation between the average inflows of two consecutive five-day periods. These results are simply bad when compared to those of line 5, which were obtained with the operating policy determined with (27a). These results show that it is preferable to take into account the correlation between the inflows of two consecutive days than the correlation between the average inflows of two consecutive five-day periods.

Table 5: Mixed dynamic programming results

Number of days j	Revenue generated	Penalty cost	Profit	Number of years (days)		
				$R_t^{\max}$	$R_t^{\min}$	$S_t^{\min}$
1	662238	31668	630570	3 (20)	8 (86)	7 (31)
2	659248	24946	634302	4 (19)	8 (74)	1 (2)
3	666612	24856	641756	3 (18)	8 (65)	2 (3)
4	662406	18806	643600	2 (13)	6 (67)	1 (2)
5	670050	18359	651691	2 (12)	9 (101)	2 (5)
5 ave	679017	51718	627299	3 (18)	33 (417)	12 (49)

Conclusion

The problem of determining the optimal daily operating policy of a reservoir with DP may seem trivial, but it is not as this paper has shown. The optimization problem can of course be easily solved with DP, but the results will be good only if the problem's mathematical model is good. The paper shows, for instance, that the profits are larger and the constraints on the reservoir content and discharge better respected when the inflow autocorrelation is taken into account. In Table 3, the number of years, in a simulation of 100 years where the constraint on the minimum discharge is not respected, decreases from 36 to 8 when the lag-one autocorrelation is taken into account, which is enormous. The reservoir operating policy also improves significantly when the model takes into account the fact that the inflows are often known more than one day in advance.

The paper has shown that nine different recursive equations can be used to determine the optimal reservoir operating policy. These equations can very easily be programmed, and except for equation (18), solved in a few minutes of computing time. For this reason, we consider it important to compare the results obtained with these different equations before choosing the one to be implemented in practice.

Appendix I. References

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Appendix II. Notation

The following symbols are used in the paper:

$E\{\}$	=	mathematical expectation
S_t	=	reservoir content at the beginning of day t
$S_t^{\min}$	=	lower bound on the reservoir content
$S_t^{\max}$	=	upper bound on the reservoir content
S^{cap}	=	reservoir storage capacity
R_t	=	reservoir discharge in day t
$R_t^{\min}$	=	lower bound on the reservoir discharge
$R_t^{\max}$	=	upper bound on the reservoir discharge
R_t^{turb}	=	powerplant maximal outflow
υ_t	=	the value of the MWh produced in day t
$g_t(R_t)$	=	energy generated by the powerplant in day t
$n(S_t, S_{t+1})$	=	function of the water head
$h(S_t)$	=	water head as a function of reservoir content
H^{ref}	=	reference head
c	=	conversion factor
$\Phi(S_{T+1})$	=	value of the water in storage at the end of the horizon
Q_t	=	reservoir inflow on day t
Y_t	=	normalized inflow
M_t	=	conditional mean
$L_t(S_{t+1}, R_t)$	=	penalty function
σ_t	=	standard deviation of Q_t
Υ_t	=	coefficient of determination
$\rho_{t,t-1}$	=	coefficient of correlation between Q_t and Q_{t-1}
ζ_t	=	standard normal random variable
$p_t(m)$	=	probability that the inflow on day t will be equal to the inflow of scenario m
$p_t(m\|n)$	=	conditional probability of the inflow on day t
$F_t(S_t, Q_{t-1})$	=	expected profit between the beginning of day t and the end of the horizon when reservoir storage is equal to S_t at the beginning of day t and the inflow in day $t-1$ is equal to Q_{t-1}