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Abstract

We propose a game-theoretic model in which one national-brand manufacturer, acting as a leader, maximizes her own profit and one retailer, selling the national brand and her private label and acting as a follower, maximizes her category profit. We characterize the resulting Stackelberg equilibrium in terms of the amount of shelf space allocated to these brands as well as their prices. The results suggest that the allocation of the shelf space depends on the quality of the private label. In our framework, quality is measured by the baseline sales (or brand equity), the degree of brand substitution and the price positioning.

Key Words: Shelf Space Allocation, Marketing Channel, Private Labels, Game theory, Stackelberg Equilibrium.

Résumé

Dans cet article, nous considérons un canal de distribution constitué par un fabricant d'une marque nationale et un détaillant qui vend la marque nationale en plus de sa marque privée. Le modèle que nous proposons tient compte des effets prix des deux marques ainsi que de l'effet direct de l'exposition de chacune d'elles. Le jeu est à la Stackelberg, avec le manufacturier comme leader qui cherche à maximiser son propre profit et le détaillant comme suiveur qui maximise le profit de toute la catégorie. Le détaillant choisit l'espace à allouer à chacune des marques ainsi que leur prix, et le manufacturier le prix de transfert. Les résultats suggèrent que l'allocation de l'espace linéaire entre les deux marques dépend de la qualité de la marque privée. Dans le cadre de notre étude, la qualité est mesurée en terme de préférence intrinsèque (ou image de marque), le degré de substitution entre les deux marques et le positionnement prix.

Mots clés : Allocation d'espace, circuit de distribution, marque privée, théorie des jeux, équilibre de Stackelberg.

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1 Introduction

Shelf space is one of the retailer's most important assets. It is a limited resource that must be optimally divided among the different categories and their various brands. The issue of shelf-space allocation and its impact on retailers' performance has attracted the attention of both marketing and operations research scholars. Schematically speaking, the literature can be divided into three streams.

A first, which is empirical in nature, verified that shelf space has indeed a positive impact, albeit with a decreasing marginal effect, on the retailer's sales and profitability (see, e.g., Curhan (1973), Drèze et al. (1994), Desmet and Renaudin (1998)) and tackled the joint problem of item selection and pricing using either experimentation (McIntyre and Miller (1999)) or conjoint analysis (Green and Savitz (1994)). The second stream developed mathematical programming models to provide optimal shelf-space allocation policies for retailers (e.g., Anderson and Amato (1974), Hansen and Heinsbroek (1979), Corstjens and Doyle (1981, 1983), Bultez and Naert (1988), Zufryden (1986), Urban (1998), Yang (2001)). However, all of these studies had a vested interest in the retailer's perspective and disregarded the interdependence with manufacturers. Closer to this paper, a third stream of studies of recent vintage considered the allocation problem within the framework of the strategic interaction between the partners in a marketing channel, typically formed of two competing manufacturers and a retailer (see, Martín-Herrán and Taboubi (2005a,b), Martín-Herrán et al. (2005a,b)). An assumption in this last group of studies is that the manufacturer can influence, by her wholesale price or advertising policy, the retailer's shelf-space allocation. The result is no longer an optimal solution to a mathematical programming problem but rather, equilibrium in a noncooperative game.

In this game-theoretic literature the roles of the two levels (manufacturing and retailing) of the marketing channel are clear cut. Indeed, the retailer is offering an outlet to competing brands produced and promoted by their owners. An interesting question is how the problem of shelf-space allocation is affected when the retailer is also a competitor of the national-brand manufacturers? This is the main topic of this paper.

Nowadays, private labels (PLs) account for 14% of total retail sales in US supermarkets. Their share ranges from 20% to 45% of total retail sales in the UK, Belgium, Germany, Spain and France.¹ Store brands are the only brands for which the retailer is responsible not only for promotion, shelf placement and pricing but also for deciding on its exact positioning in the product space which includes packaging, precise quality, etc. (Morton and Zettelmeyer (2004)).

Why are retailers introducing private labels? The primary-and rather straightforward-reason is that offering an additional brand to national ones allows a retailer to increase her sales by reaching a larger array of consumers' preferences. Morton and Zettelmeyer

¹ Figures given by the Private Label Manufacturers Association or PLMA. (See Website at www.plma.com).

(2004) state that a retailer's control over brand positioning in the product space enables her to carry substitutes to key national brands (NBs) or to mimic leading ones. This will decrease the added value of these NBs, which in turn, allow the retailer to have better supply terms with NB manufacturers. Therefore, a second strategic role of private labels is to increase the bargaining power of the retailer with respect to manufacturers of national brands (see also Narasimhan and Wilcox (1998)).

The impact of private labels on vertical strategic interactions in marketing channels has been the topic of many studies. For instance, Cotterill and Putsis (2001) and Putsis and Dhar (1998) focused on the vertical strategic interaction between manufacturers and retailers across multiple product categories. They found that the nature of competitive interaction is idiosyncratic to the category and that there is no consistent pattern of competition even across marketing instruments. Raju *et al.* (1995) and Narasimhan and Wilcox (1998) analyzed the impact of private-label introduction on equilibrium pricing strategies and corresponding profits. Raju *et al.* (1995) found that category profits are higher when the price competition between NBs is low, when the price competition between the PL and NBs is high, and when the number of NBs is high in a channel composed of competing manufacturers and a common retailer. More specifically, Narasimhan and Wilcox (1998) showed that the introduction of the private label not only shifts some surplus from the manufacturer to the retailer but also to the consumer, in the context of one manufacturer and one retailer. Karray and Zaccour (2004) characterized the circumstances under which a manufacturer can mitigate her losses, at least in part, by implementing a cooperative advertising program.

In this paper, we propose a game-theoretic model involving a national-brand manufacturer and a retailer selling her PL along with the NB. The strategic variables are the following: the wholesale price for the manufacturer, the retail price for both brands, and the shelf space allocated to each brand by the retailer. We assume that the manufacturer is the leader in a Stackelberg game and characterize the resulting equilibrium in terms of prices, shelf space and profits for both players.

The rest of the article is organized as follows. In Section 2, we develop a model for the channel under study. In Section 3, we derive the Stackelberg equilibrium. In Section 4, we look at the players' strategies and profits for different scenarios characterizing the quality of the private label. In Section 5, we conclude.

2 The Model

We consider a retailer R offering two brands within a given product category. One brand is produced and supplied by a national manufacturer M at a wholesale price w . The second brand is the retailer's private label, produced by another manufacturer who does not play any strategic role in our framework. We therefore assume, without any loss of generality, that the retailer's purchasing cost of the private brand is zero. This assumption has been

common in the literature (see, e.g., Cotterill and Putsis (2001) and Raju et al. (1995)) and does not affect the qualitative results of the paper.

The retailer controls the shelf space allocated to each brand. We normalize the total shelf space available for this product's category to one. Denote by $S \geq 0$ the share of this space dedicated to the national brand. Assuming that the total shelf space is allocated, the share of the private label is thus $1 - S$.

We suppose that the demand for each brand depends on the price of each brand and on the exposure each receives, as measured by shelf space. The following functional forms are assumed:

$$\begin{aligned} D_n &= (\alpha_n - p_n + \psi_n p_s) S, \\ D_s &= (\alpha_s - p_s + \psi_s p_n) (1 - S), \end{aligned}$$

where α_n and α_s are positive parameters and $\psi_n \in [0, 1)$ and $\psi_s \in [0, 1)$. The parameters α_n and α_s represent the baseline sales (or brand equity) of the national and private brand, respectively. The range of values for ψ_n and ψ_s are chosen so that these cross-price parameters are at most equal to the direct price effects, here taken as equal to one. This is rather a standard assumption in economics (see, e.g., Cotterill et al. (2000)). It can be readily seen that the demand for each brand is increasing with its shelf space. The rationale is that if a product is given large shelf space, it increases the probability of its being noticed by the consumer and of being selected (see, e.g., Martín-Herrán and Taboubi (2005a), Yang and Chen (1999), Bultez and Naert (1988) and Corstjens and Doyle (1981)). Further, each brand's demand is increasing in competing brand's price and decreasing in its own price. These assumptions on the effect of pricing are standard. The demand specification indicates that the function is multiplicatively separable into prices and shelf space. This implies that the marginal price effect on demand depends on the shelf space allocated to the brand. Such specification has been used extensively in the literature (see, e.g., Corstjens and Doyle (1981, 1983) and Zufryden (1986)).

Empirical studies such as those of Cotterill et al. (2000), Cotterill and Putsis (2000) and Sethuraman (1995) as well as theoretical ones, e.g., Blattberg and Wisniewski (1989) and Bronnenberg and Wathieu (1996), have shown that the national brand cross-price effect ψ_s is higher than the private label cross-price effect ψ_n when the PL is of lower quality. Theoretically, the inequality could be the other way around if the quality gap between brands were not sufficiently large in comparison with the price gap (Bronnenberg and Wathieu (1996)). We shall nevertheless suppose here that the PL is, at best, of the same quality as the NB and hence adopt the assumption that $\psi_s \geq \psi_n$.

We shall assume that the following rule links the retail price of both brands:

$$p_s = \gamma p_n, \quad 0 \leq \gamma < 1.$$

The above rule implies that the retailer reduces her pricing-decision problem to the determination of the retail price of the national brand, with the implicit assumption that γ is

based on the results of marketing research or simply on an established tradition. The rule also says that the retailer sells her private label at a lower price than that of the national brand. This last assumption is often assumed in the literature and is largely supported empirically.² Narasimhan and Wilcox (1998) provide the rationale for this assumption by arguing that consumers usually perceive the national brand as a higher-quality, less-risky product than the private label. Further, Raju et al. (1995) have found that the average price of the private labels was lower than the price of national brands for over 95% of the 426 product categories they examined. What remains to be settled is the actual value of γ . Parker and Kim (1997) and Nogales and Suarez (2005) have found it to be generally in the range of $[0.60; 0.90]$. Hence, we will make the distinction between low-quality private labels (low γ) and higher-quality private labels (high γ),³ and we shall discuss the impact of γ on the results.

Making the change of variable $p_s = \gamma p_n$, the demand functions can be then written as

$$D_n = (\alpha_n - \beta_n p_n) S \quad (1)$$

$$D_s = (\alpha_s + \beta_s p_n) (1 - S) \quad (2)$$

where $\beta_n = 1 - \psi_n \gamma > 0$ and $\beta_s = \psi_s - \gamma$. Given the empirically established ranges for the cross-price effect ψ_s and the private-label pricing parameter γ , it is justified to assume that $\beta_s < 0$.

Following Raju et al. (1995) and Lal (1990), we assume that the NB baseline sales are higher than the PL ones, i.e., $\alpha_n > \alpha_s$. We normalize α_n to one. At equal prices, this assumption means that consumers prefer the national brand to the private label (Narasimhan and Wilcox (1998)). We further suppose that the retailer cannot afford but to offer the national brand, i.e., $S > 0$. The reason could be related for instance to the desire to have a good relationship with the manufacturer that also supplies the retailer with other products, etc. We therefore impose a lower bound on the shelf space allocated to this brand, which we denote S^{\min} . This bound could be defined in terms of the size of one facing unit of the brand or a minimal number of facings which is necessary to the brand to be visually noticed. Another issue involved in retailing is the number of brands offered within a product category. Given that consumers have different preferences and usually seek variety, retailers may reject from the outset a solution that excludes some brands from

² However, Sethuraman (1992) leaves open the issue of premium PLs that are of higher quality and are more expensive than NBs. In fact, while this practice is fairly common in Europe, it has gained importance in the US market only in the last few years.

³ Meza and Shudir (2003) explained that retailers may use either a differentiation strategy or an imitation strategy when they introduce store brands. Differentiation can be achieved by offering low quality PLs that includes white-label generics, for which retailers cannot establish loyalties (Parker and Kim (1997)), or distinct second-tier PLs (low γ in our study) for which they can improve image but reach consumers more sensitive to prices (Nogales and Suarez (2005)). Differentiation can also be achieved through high-quality PL's that target segments different from those targeted by the NBs. (These are not the concern of this paper). Finally, imitation strategy is the more used strategy nowadays and consists of introducing me-too brands that compare to popular NBs (high γ in our study).

the shelf. Therefore, we should normally also assume that a lower bound is also in force for the private label. Equivalently, the shelf space for the national brand is upper bounded by S^{\max} . The shelf-space constraint is thus given by

$$S^{\min} \leq S \leq S^{\max}.$$

Assuming that the manufacturer and the retailer are profit maximizers, their objectives read as follows:

$$\begin{aligned} \max_w \pi_M &= w(1 - \beta_n p_n) S, \\ \max_{S, p_n} \pi_R &= (p_n - w)(1 - \beta_n p_n) S + \gamma p_n (\alpha_s + \beta_s p_n)(1 - S). \end{aligned}$$

Note that the retailer does not incur a shelf-space cost, which is a simplifying assumption. However, since there is no reason to believe that such cost varies across brands, we can assume it to be equal to zero without any loss of generality.

The game is played à la Stackelberg with the manufacturer as leader and the retailer as follower. This amounts to saying, not unrealistically, that the manufacturer can influence the retailer's shelf-space decisions. This can be done through a series of instruments, e.g., shelf-space allowance. Here, we assume that the wholesale price could be seen as the tool for the manufacturer to capture the right shelf-space share. As usual in Stackelberg information structure games, the sequence of events is as follows: The manufacturer (leader) first announces her wholesale price strategy. The retailer reacts to this information by choosing the shelf-space allocation as well as the retail price of the national brand. Next the manufacturer chooses the optimal wholesale price. The Stackelberg equilibrium solution has often been adopted in the literature dealing with marketing channels. (See, e.g., Choi (1991, 1996) for static games and Jørgensen et al. (2000, 2001, 2003) for differential games).

3 Stackelberg Equilibrium

To determine the reaction function of the retailer to the manufacturer's transfer price w , we need to solve the following optimization problem:

$$\begin{aligned} \max_{S, p_n \geq 0} \pi_R &= [(p_n - w)(1 - \beta_n p_n) - \gamma p_n (\alpha_s + \beta_s p_n)] S + \gamma p_n (\alpha_s + \beta_s p_n) \\ \text{subject to} &: S^{\min} \leq S \leq S^{\max}. \end{aligned}$$

First-order optimality conditions are

$$\frac{\partial \pi_R}{\partial p_n} = 0 \Leftrightarrow p_n = \frac{1}{2} \left(\frac{S(1 + w\beta_n) + \gamma\alpha_s(1 - S)}{S\beta_n - \gamma\beta_s(1 - S)} \right), \quad (3)$$

$$S = \begin{cases} S^{\max}, & \text{if } Z > 0 \\ S^{\min}, & \text{if } Z < 0 \end{cases}, \quad (4)$$

where

$$Z = [(p_n - w)(1 - \beta_n p_n) - \gamma p_n (\alpha_s + \beta_s p_n)]. \quad (5)$$

Define by

$$p_n^{\max}(w) = \frac{1}{2} \left(\frac{S^{\max}(1 + w\beta_n) + \gamma\alpha_s(1 - S^{\max})}{S^{\max}(\beta_n + \gamma\beta_s) - \gamma\beta_s} \right), \quad (6)$$

$$p_n^{\min}(w) = \frac{1}{2} \left(\frac{S^{\min}(1 + w\beta_n) + \gamma\alpha_s(1 - S^{\min})}{[S^{\min}(\beta_n + \gamma\beta_s) - \gamma\beta_s]} \right), \quad (7)$$

$$Z^{\max}(w) = [(p_n^{\max}(w) - w)(1 - \beta_n p_n^{\max}(w)) - \gamma p_n^{\max}(w)(\alpha_s + \beta_s p_n^{\max}(w))], \quad (8)$$

$$Z^{\min}(w) = [(p_n^{\min}(w) - w)(1 - \beta_n p_n^{\min}(w)) - \gamma p_n^{\min}(w)(\alpha_s + \beta_s p_n^{\min}(w))]. \quad (9)$$

The reaction function to the leader's announcement of the wholesale price reads as follows:

$$(p_n(w), S(w)) = \begin{cases} (p_n^{\max}(w), S^{\max}), & \text{for } Z^{\max}(w) > 0 \\ (p_n^{\min}(w), S^{\min}), & \text{for } Z^{\min}(w) < 0 \end{cases}.$$

Therefore, we have to consider two cases, depending on whether the shelf space allocated to the national brand is at its maximal or minimal value.

Substituting in (8) for p_n^{\max} by its value from (6), leads to

$$Z^{\max}(w) = \frac{L_1^{\max} w^2 + L_2^{\max} w + L_3^{\max}}{4[S^{\max}(\beta_n + \gamma\beta_s) - \gamma\beta_s]^2} \quad (10)$$

where

$$\begin{aligned} L_1^{\max} &= \beta_n^2 S^{\max} [S^{\max}(\beta_n + \gamma\beta_s) - 2\gamma\beta_s], \\ L_2^{\max} &= -2[S^{\max}(S^{\max}(\beta_n + \gamma\beta_s) - 2\gamma\beta_s)(\beta_n + \gamma(2\beta_s + \beta_n\alpha_s)) \\ &\quad + \beta_s\gamma^2(2\beta_s + \beta_n\alpha_s)], \\ L_3^{\max} &= [S^{\max} + \gamma\alpha_s(1 - S^{\max})][S^{\max}(\gamma\beta_s + \beta_n)(1 - \gamma\alpha_s) \\ &\quad - \gamma(2\beta_s + \beta_n\alpha_s - \gamma\alpha_s\beta_s)]. \end{aligned}$$

Similarly, substituting in (9) for p_n^{\min} by its value from (7) gives

$$Z^{\min}(w) = \frac{L_1^{\min} w^2 + L_2^{\min} w + L_3^{\min}}{4[S^{\min}(\beta_n + \gamma\beta_s) - \gamma\beta_s]^2} \quad (11)$$

where

$$L_1^{\min} = \beta_n^2 S^{\min} [S^{\min}(\beta_n + \gamma\beta_s) - 2\gamma\beta_s],$$

$$\begin{aligned}
L_2^{\min} &= -2 \left[S^{\min} (S^{\min} (\beta_n + \gamma\beta_s) - 2\gamma\beta_s) (\beta_n + \gamma (2\beta_s + \beta_n\alpha_s)) \right. \\
&\quad \left. + \beta_s\gamma^2 (2\beta_s + \beta_n\alpha_s) \right], \\
L_3^{\min} &= \left[S^{\min} + \gamma\alpha_s (1 - S^{\min}) \right] \left[S^{\min} (\gamma\beta_s + \beta_n) (1 - \gamma\alpha_s) \right. \\
&\quad \left. - \gamma (2\beta_s + \beta_n\alpha_s - \gamma\alpha_s\beta_s) \right].
\end{aligned}$$

Note that, since the denominators of Z^{\max} and Z^{\min} are positive, their signs are those of their numerators.

To wrap up, the result shows that the allocation of shelf space depends on a simple, comparative marginal profitability rule. Indeed, the optimality condition, with respect to shelf space can be stated as follows:

$$Z^{\max} > 0 \Leftrightarrow [(p_n - w) (1 - \beta_n p_n)] > [\gamma p_n (\alpha_s + \beta_s p_n)] \Leftrightarrow S = S^{\max}, \quad (12)$$

$$Z^{\min} < 0 \Leftrightarrow [(p_n - w) (1 - \beta_n p_n)] < [\gamma p_n (\alpha_s + \beta_s p_n)] \Leftrightarrow S = S^{\min}, \quad (13)$$

where $[(p_n - w) (1 - \beta_n p_n)]$ represents the marginal contribution to profit of the shelf space allocated to the national brand and $[\gamma p_n (\alpha_s + \beta_s p_n)]$ represents its private label counterpart. This result is intuitive and has been actually prescribed in a game setting in Martín-Herrán et al. (2005b) where, however, the shelf-space variable is continuous. This comparative-profitability rule is also implemented in the decision-making support systems that are available to retailers to assist them in optimizing their shelf space.⁴ Further, the conditions in (12) and (13) show that the allocation of shelf space depends on all of the model's parameters and on the manufacturer's strategy. Therefore, the wholesale price can be seen as a device used by the manufacturer to influence the retailer's shelf-space decision. In the literature, one usually optimizes the retailer's profit, and take the transfer price for granted. Here, the interaction between the shelf-space decision and the transfer price is explicit and strategic.

Before stating the Stackelberg equilibrium results, it is insightful to analyze the retailer's reaction function. Differentiating the retail price of the national brand with respect to the manufacturer's transfer price w gives:

$$\frac{dp_n^{\max}(w)}{dw} = \frac{1}{2} \left(\frac{\beta_n S^{\max}}{S^{\max} \beta_n - \gamma \beta_s (1 - S^{\max})} \right) > 0, \quad (14)$$

$$\frac{dp_n^{\min}(w)}{dw} = \frac{1}{2} \left(\frac{\beta_n S^{\min}}{S^{\min} \beta_n - \gamma \beta_s (1 - S^{\min})} \right) > 0. \quad (15)$$

Recalling that the price of the store brand is given by $p_s = \gamma p_n$, we have

$$\frac{dp_s^{\max}(w)}{dw} = \gamma \frac{dp_n^{\max}(w)}{dw} > 0, \quad (16)$$

⁴ Examples of such support systems are APOLLO, COSMOS, OBM, PROGALI, SLIM, SPACEMAN, etc.

$$\frac{dp_s^{\min}(w)}{dw} = \gamma \frac{dp_n^{\min}(w)}{dw} > 0. \quad (17)$$

The above results show (i) that there is vertical strategic complementarity, i.e., increasing (decreasing) the wholesale price leads to an increase (decrease) in the retail price; and, (ii) that there is horizontal strategic complementarity, which is a direct consequence of the retailer's pricing rule $p_s = \gamma p_n$.

We now turn to the manufacturer's optimization problem, which is given by

$$\max_{w \geq 0} \pi_M = w(1 - \beta_n p_n(w)) S(w),$$

where

$$(p_n(w), S(w)) = \begin{cases} (p_n^{\max}(w), S^{\max}), & \text{for } Z^{\max}(w) > 0 \\ (p_n^{\min}(w), S^{\min}), & \text{for } Z^{\min}(w) < 0 \end{cases},$$

and $p_n^{\max}(w), p_n^{\min}(w), Z^{\max}(w)$ and $Z^{\min}(w)$ are given by (6), (7), (10) and (11). The following proposition characterizes the unique Stackelberg equilibrium.

Proposition 1 *Stackelberg equilibrium is given by*

$$(p_n, S, w) = \begin{cases} (p_n^{\max}, S^{\max}, w^{\max}), & \text{for } Z^{\max}(w^{\max}) > 0 \\ (p_n^{\min}, S^{\min}, w^{\min}), & \text{for } Z^{\min}(w^{\min}) < 0 \end{cases},$$

where

$$w^{\max} = \frac{S^{\max} \beta_n + \gamma(2\beta_s + \beta_n \alpha_s)(S^{\max} - 1)}{2\beta_n^2 S^{\max}}, \quad (18)$$

$$p_n^{\max} = \frac{1}{4\beta_n} \left(\frac{3S^{\max} \beta_n + \gamma(\beta_n \alpha_s - 2\beta_s)(1 - S^{\max})}{\gamma\beta_s(S^{\max} - 1) + S^{\max} \beta_n} \right), \quad (19)$$

$$w^{\min} = \frac{S^{\min} \beta_n + \gamma(2\beta_s + \beta_n \alpha_s)(S^{\min} - 1)}{2\beta_n^2 S^{\min}}, \quad (20)$$

$$p_n^{\min} = \frac{1}{4\beta_n} \left(\frac{3S^{\min} \beta_n + \gamma(\beta_n \alpha_s - 2\beta_s)(1 - S^{\min})}{\gamma\beta_s(S^{\min} - 1) + S^{\min} \beta_n} \right). \quad (21)$$

The values of $Z^{\max}(w^{\max})$ and $Z^{\min}(w^{\min})$ are given by inserting in (10) and (11) w^{\max} and w^{\min} , respectively.

Proof. After inserting the retailer's reaction function in the manufacturer's optimization problem, the latter becomes

$$\max \pi_M = wS \left(1 - \frac{1}{2} \beta_n \left(\frac{S[1 + w\beta_n - \gamma\alpha_s] + \gamma\alpha_s}{S(\beta_n + \gamma\beta_s) - \gamma\beta_s} \right) \right)$$

$$\text{where } S = \begin{cases} S^{\max}, & \text{for } Z^{\max} > 0 \\ S^{\min}, & \text{for } Z^{\min} < 0 \end{cases}.$$

The first-order optimality condition is given by

$$\frac{d\pi_M}{dw} = (S\beta_n - \gamma\beta_n\alpha_s - 2\gamma\beta_s + 2\gamma S\beta_s + \gamma S\beta_n\alpha_s) - (2S\beta_n^2)w = 0,$$

$$\text{where } S = \begin{cases} S^{\max}, & \text{for } Z^{\max} > 0 \\ S^{\min}, & \text{for } Z^{\min} < 0 \end{cases}.$$

A full characterization of the conditions under which the S^{\max} or S^{\min} is chosen is provided in the Appendix. \square

The retail price (in both scenarios, i.e., p_n^{\max} and p_n^{\min}) is strictly positive. The condition for having a positive wholesale price is given by

$$S^k\beta_n + \beta_n\gamma\alpha_s S^k - 2\gamma\beta_s + 2\gamma\beta_s S^k - \beta_n\gamma\alpha_s > 0, \quad k = \max, \min. \quad (22)$$

Inserting the equilibrium strategies p_n^k and w^k , $k = \max, \min$ in the objective functions of the retailer and the manufacturer, and assuming that all conditions are satisfied in each scenario, then the equilibrium profits are, for $k = \max, \min$, as follows:

$$\pi_M^k = \frac{[\beta_n S^k + \gamma(S^k - 1)(2\beta_s + \alpha_s\beta_n)]^2}{8\beta_n^2 [S^k(\beta_n + \gamma\beta_s) - \gamma\beta_s]},$$

$$\pi_R^k = \frac{(S^k)^2 X_1 - 2S^k X_2 - X_3}{16\beta_n^2 [S^k(\beta_n + \gamma\beta_s) - \gamma\beta_s]}$$

where

$$X_1 = \beta_n^2(1 - \gamma\alpha_s)^2 - 12\gamma[(\beta_n + \gamma\beta_s)(\alpha_s\beta_n + \beta_s)],$$

$$X_2 = \gamma[\gamma\beta_n^2\alpha_s^2 - 12\gamma\beta_n\alpha_s\beta_s - 12\gamma\beta_s^2 - 7\beta_n^2\alpha_s - 6\beta_n\beta_s],$$

$$X_3 = \gamma^2[12\beta_n\alpha_s\beta_s + 12\beta_s^2 - \beta_n^2\alpha_s^2].$$

4 Simulation Results

The objective of this section is to shed some light on the relationship between profitability, shelf-space allocation, and the quality of the private label. The expressions of the strategies and the profits do not allow for much analytical analysis. We shall therefore conduct some numerical simulations to achieve our goal.

The model has six parameters, i.e., $\alpha_s, \gamma, \psi_s, \psi_n, S^{\min}$ and S^{\max} . To be able to derive some qualitative results from the experiments, we need to organize them in an insightful

manner. We shall have two series of scenarios. In the first one, the private label is assumed to be of “low” quality and in the second series to be of “high” quality. In our framework, the quality of the private label can be characterized in terms of (i) its price positioning (i.e., in terms of the γ parameter); (ii) its brand equity or baseline sales (i.e., in terms of the α_s parameter); and finally (iii) the cross price substitution.

We suppose that a low-quality PL would be priced at a “significantly” lower price than the NB. One way of operationalizing this is to choose a value for γ that is in the lower end of the interval $[0.60, 0.90]$ found in the literature. For a high-quality PL, we will pick up a value for γ in the upper end of this interval.

The brand equity α_n of the national brand has been normalized to one. For the private label, we divide the range of values for α_s into two subintervals. For $\alpha_s \in [0, 0.50)$, we shall say that the brand equity of the PL is low, whereas for $\alpha_s \in [0.50, 1)$ we refer to the store brand as enjoying high brand equity. Actually, in each scenario, a particular value of α_s is chosen in the appropriate subinterval.

Given our pricing-rule assumption for the private label, cross-price parameters are less important in our framework. We shall therefore fix them in each scenario as follows: When the PL is of low quality, we select values for the cross-price parameters ψ_s and ψ_n such that $\psi_s \gg \psi_n$. As an illustration, we take $\psi_s = 1.5\psi_n$. In the case of high quality, we assume that the two parameters’ values are close to, and higher than, the case where the PL is of low quality (Sayman et al. (2002)). As an illustration we take $\psi_s = 0.35$ and $\psi_n = 0.27$.

Finally, we have to provide values for the minimum and maximum shares for the national brand. As reference values, we take $S^{\min} = 0.25$ and $S^{\max} = 0.70$. Note that all the results provided are in general robust to important changes in these values, i.e., to more or less or minus 20%.

Remark 1 *When conducting numerical simulations, one has to check for the nonnegativity of demands, prices and profits as well as other conditions involved in the derivation of equilibrium (see the Appendix). This renders the calibration problem far from being a trivial exercise and means that for some combinations of parameters’ values, there may not be an economically meaningful solution.*

4.1 Low-Quality Private Label

In this series of experiments, the private label is assumed to be of low quality. The reference values for the parameters are as follows

$$\alpha_s = 0.35; \gamma = 0.60; \psi_n = 0.1; \psi_s = 0.25; S^{\max} = 0.70; S^{\min} = 0.25.$$

4.1.1 Scenario 1: Varying Brand Equity of PL The results of varying α_s are provided in Table 1. For low values of α_s , the equilibrium solution is $(p_n^{\max}, S^{\max}, w^{\max})$

Table 1: Results for different values of α_s (PL of low-quality)

α_s	0.30	0.39	α_s	0.45	0.49
p_s^{\max}	0.4760	0.4794	p_s^{\min}	0.4921	0.4990
p_n^{\max}	0.7934	0.7990	p_n^{\min}	0.8201	0.8316
w^{\max}	0.5927	0.5804	w^{\min}	0.8141	0.7758
Retailer's profit	0.0389	0.0540	Retailer's profit	0.0605	0.0775
Manufacturer's profit	0.1055	0.1012	Manufacturer's profit	0.0466	0.0423

Table 2: Results for different values of γ (PL of low-quality)

γ	0.60	0.64	0.68
p_s^{\max}	0.4779	0.5100	0.5420
p_n^{\max}	0.7965	0.7969	0.7970
w^{\max}	0.5859	0.6050	0.6260
Retailer's profit	0.0473	0.0401	0.0320
Manufacturer's profit	0.1031	0.1076	0.1127

and for high values of α_s it becomes $(p_n^{\min}, S^{\min}, w^{\min})$. For $\alpha_s \in [0.30; 0.49]$ the retailer's profits are monotonically increasing in her private's brand equity. It is the other way around for the manufacturer. One interpretation of this is that, when the store brand does not have enough loyal of a customer base, the retailer will find it optimal to allocate the maximum level of the shelf space to the NB. However, when the retailer benefits from a more loyal customer base (the threshold is at $\alpha_s = 0.4$), the equilibrium solution is changed. Hence, the retailer becomes powerful, which may lead the NB manufacturer to fear its opportunism by limiting the proportion allocated to her brand (Anderson et al. (2001)). In fact, by allocating S^{\min} to the NB in such a context, the manufacturer offers her brand at a higher level of transfer price than when she benefits from S^{\max} , but she tends always to decrease it as the loyalty to the PL increases. Indeed, this situation seems to threaten her as her profits are on the decline. As a consequence, the NB's margin decreases sharply (from 0.2186 for $\alpha_s = 0.39$ to 0.0060 for $\alpha_s = 0.45$) but increases again as loyalty to the PL improves (0.0558 for $\alpha_s = 0.49$).

4.1.2 Scenario 2: Varying Price Positioning of PL The results in Table 2 show that the equilibrium solution for the different values of γ is $(p_n^{\max}, S^{\max}, w^{\max})$. They also show that it is not in the best interest of the retailer to position the PL closer to the NB when her label is of low quality (as characterized by the reference values of the parameters in this series of scenarios). Indeed, the retailer's profit decreases sharply in γ . Note also that, whereas the retail price of the NB varies little with γ , the transfer price is increasing and hence the retailer's margin on this brand is squeezed.

Table 3: Results for different values of α_s (PL of high-quality)

α_s	0.75	0.85	0.95
p_s^{\min}	0.7671	0.7919	0.8168
p_n^{\min}	1.0228	1.0559	1.0890
w^{\min}	0.9840	0.8430	0.7019
Retailer's profit	0.1979	0.2624	0.3278
Manufacturer's profit	0.0454	0.0333	0.0231

4.2 High-Quality Private Label

For this series of scenarios, in which the private label is of high quality, the reference values for the parameters are

$$\alpha_s = 0.85; \gamma = 0.75; \psi_n = 0.27; \psi_s = 0.35; S^{\max} = 0.70; S^{\min} = 0.25.$$

4.2.1 Scenario 3: Varying Brand Equity of PL The results in Table 3 show that, for all values considered for α_s , the equilibrium is $(p_n^{\min}, S^{\min}, w^{\min})$. Thus, contrary to the corresponding scenario where the PL is of low quality, the equilibrium here is always to allocate the minimum shelf space to the NB. Note that increasing the brand equity of the PL leads to a sharp decrease in the transfer price. In parallel, the retail prices are increasing. Hence, by selling a high-quality store-brand, the retailer is in a much better position to negotiate good deals on the national brand. These findings suggest that the retailer should invest in the brand equity of her private label. To enrich the analysis, we also investigated the case where the PL's cross-price effect could exceed the NB's cross-price effect and the results could be summarized as follows: (i) the impact of varying the PL's brand equity on the resulting equilibrium seems to be the same as stated in this scenario except that the optimal prices are higher; and, (ii) the result found in Bronnenberg and Wathieu (1996) is verified because the scenario does work for high values of α_s ($\alpha_s \in [0.86; 1]$) and a low value of γ ($\gamma = 0.75$).

4.2.2 Scenario 4: Varying Price Positioning of PL The equilibrium solution in this scenario is again given by $(p_n^{\min}, S^{\min}, w^{\min})$. (See Table 4). The results show that the closer the positioning of the private label to the national brand, the lower is the retailer's profit. The transfer price is increasing rapidly in γ , as does the manufacturer's profit. Hence, the manufacturer seems to retaliate as she is disfavored by the retailer. These results show that it is a bad strategy for the retailer to mimic the national brand too much. We also verified in this scenario the case where the PL's cross-price effect could exceed the NB's cross-price effect and the results could be summarized as follows: (i) the impact of varying the PL's price positioning on the resulting equilibrium seems to be the same as that stated in this scenario except that the optimal prices are higher; and, (ii)

Table 4: Results for different values of γ (PL of high-quality)

γ	0.75	0.77	0.79
p_s^{\min}	0.7920	0.8098	0.8275
p_n^{\min}	1.0559	1.0516	1.0475
w^{\min}	0.8430	0.9381	1.0402
Retailer's profit	0.2624	0.2527	0.2418
Manufacturer's profit	0.0333	0.0392	0.0457

the result found in Bronnenberg and Wathieu (1996) is again verified because the scenario does work for low values of γ ($\gamma \in [0.75; 0.8]$) and a high value of α_s ($\alpha_s = 0.95$).⁵

4.3 Testable Conjectures

The numerical simulations are meant, as stated previously, to shed some light on the relationships between shelf-space decisions and the quality of the private label. These experiments are thus seen as an exploratory study that needs to be complemented by a descriptive one, in order to validate or refute these results. To simplify what should be a next step in the analysis of these relationships, we rephrase our results (or expectations) in the forms of testable conjectures.

Conjecture 1 *A retailer devotes a small shelf space to the store brand when the latter is of low quality, unless it has a relatively high brand equity, and devotes a large shelf space when the PL is of high quality.*

Since our model assumes a category with two brands, a large shelf-space for the PL was synonymous with the largest part of it. The above conjecture attempts to generalize for a category with more than two brands. This conjecture can be tested by conducting a survey on a number of product categories in different stores (supermarkets, pharmacies, hardware, etc.) as well as in different countries. Indeed, a recent case study in Spanish retail industry was conducted recently (Nogales and Suarez (2005)) and investigated whether the space occupied by store brands was out of proportion with their market shares. Through direct shelf observation, they found that store brands occupied more than 75% of the space in some outlets and categories in 2003, but less space when there were strong leaders or a greater differentiation in the NBs. Note that if we interpret the PL's brand equity as its reputation, our results (and conjecture) would also be also in line with Sethuraman (1992) and Hoch and Banerji (1993) who refuted the common perception that a PL's primary attraction was the substantial price discount relative to the NB, and supported the notion that perceived quality is a much more important as determinant of PL success.

⁵ To investigate the qualitative difference between taking into account the standard asymmetry and the case where the reverse could occur, many of the parameters in this scenario would have to be changed in order to obtain consistent results.

Conjecture 2 *The closer the retail price of the PL is to the retail price of the NB,*

- (i) *the higher is the transfer price asked for by the NB manufacturer.*
- (ii) *the lower is the retailer's margin on the NB.*

According to Meza and Sudhir (2003), the retailer should behave strategically whenever she faces an attractive segment of the market,⁶ and on the contrary, should not pay much attention to less attractive ones. More specifically, imitated brands in the attractive segment will be disfavored through increased margins (the wholesale prices are lower for the leading NBs and retail prices are higher) and less frequent promotions in order to support the store brand. By contrast, the non-imitated brands in the attractive segment (the brands with second-highest share) are treated more favorably after the introduction of store brands in order to reduce the threat of retaliation from manufacturers (both prices are higher but frequent promotions should accompany the retail prices). As the authors just investigated one category (the cereal category), more studies are needed before being able to generalize the results.

Conjecture 3 *Both brands' prices are higher when the PL is of high quality.*

The above conjectures can be tested by choosing, for instance, product categories that include some recognized low-quality store brands and premium PLs. Clearly, the prices must be normalized for the experiment to be valid.

Conjecture 4 *A price positioning as close as possible to the NB's price is not always beneficial for the retailer if the PL is of high quality*

One explanation could be that the NB still offers a high margin and hence, competing against it is detrimental (Corstjens and Lal (2000)) even if it could minimize the double marginalization problem in some instances (Sayman et al. (2002)). In the latter study, it is also claimed that competing against the leading NB when there is a price-sensitive segment may not be optimal and, in some categories, consumers may prefer to buy the "real thing" rather than the "copycat." Moreover, in the empirical part of this study, the authors do not find that price differential manipulations (15% to 30%) have a significant impact, and advised caution about using superficial appearance cues. They specify that explicit targeting (as opposed to ambiguous targeting) influences only physical similarity and not perceptions about the product's quality or its comparative quality. Hence, the question about the right appearance cues as well as the right positioning, and whether cues should be a combination of signals or a single one, is still open.

⁶ Attractive segments are NBs with large market shares, a portion of which can be stolen, and those that have not created enough branded variants.

5 Concluding Remarks

To the best of our knowledge, this paper is the first attempt to simultaneously tackle pricing strategies and shelf-space allocation in the context of private labels. We distinguished between different parameters characterizing the quality of the private label, i.e., brand equity, price-positioning and price substitution. Our results seem to show that these parameters do not similarly affect the performance of each channel's member. This implies that the quality of a (store) brand cannot be fully captured by only one parameter.

This study contains a number of shortcomings that deserve future investigation. First, the model is linear in the shelf space allocated to each brand. It is of interest to study the case where the shelf space has a non-linear impact on demand. Second, private labels are becoming more and more popular and their quality is improving. It is thus also relevant to examine the case of premium PL brands, as highlighted by Sethuraman (1992). Third, in the same spirit, it would be interesting to consider goodwill (or brand equity) as a stock in which the retailer can invest. This would require the elaboration of a dynamic model with the advantage of allowing for a full characterization of the relationship between shelf-space allocation and the (controlled) quality of the PL. Fourth, our model does not account for competition between retailers or manufacturers. According to Sethuraman (1992) and Raju et al. (1995), the competition at each level of the distribution channel leads to different impacts. Although it is realistic to introduce both levels of competition (manufacturing and retailing), the resulting model would surely be analytically intractable and one should resort to numerical simulations to analyze the impact of competition. Fifth, we made the choice of a game played à la Stackelberg with the manufacturer as leader. Cotterill and Putsis (2001) advised that multiple forms of vertical strategic interactions be considered in order to produce more general results by comparing the different scenarios. The reason behind this is that the vertical interaction is idiosyncratic to the category. Hence, future research should compare our findings to those in the case of a Nash game or retailer Stackelberg game. Finally, we did not make the assumption of fixed total demand, which is suitable in a context where the focus is on category expansion (see, e.g., Putsis and Dhar (2001) and Cotterill and Putsis (1999)). However, the shelf-space battle is more akin to a zero-sum game situation and hence it is necessary to link the change in the amount of shelf space allocated to each brand and the exact proportion of consumers that will switch from one brand to another.

6 Appendix

6.1 Retailer's Problem

We have to solve the following retailer's problem:

$$\text{Max}\pi_R = \text{Max} \left(\left[(p_n - w)(1 - \beta_n p_n) - \gamma p_n (\alpha_s + \beta_s p_n) \right] S + \gamma p_n (\alpha_s + \beta_s p_n) \right)$$

subject to

$$S^{\min} \leq S \leq S^{\max}$$

Maximizing the retailer's profit with respect to p_n gives

$$\frac{\partial \pi_R}{\partial p_n} = [(1 - 2\beta_n p_n + w\beta_n) - \gamma(\alpha_s + 2\beta_s p_n)] S + \gamma(\alpha_s + 2\beta_s p_n) = 0. \quad (23)$$

The profit function being linear in S , the optimality condition is thus

$$S = \begin{cases} S^{\max}, & \text{if } Z > 0 \\ S^{\min}, & \text{if } Z < 0 \end{cases},$$

where

$$Z = (p_n - w)(1 - \beta_n p_n) - \gamma p_n (\alpha_s + \beta_s p_n)$$

Assuming that $S(\beta_n + \gamma\beta_s) - \gamma\beta_s \neq 0$, then the retailer's reaction function to the manufacturer's transfer price w (either in the case of S^{\max} or S^{\min}) is obtained from the first order conditions (23):

$$p_n(w) = \frac{1}{2} \left(\frac{S[1 + w\beta_n - \gamma\alpha_s] + \gamma\alpha_s}{S(\beta_n + \gamma\beta_s) - \gamma\beta_s} \right)$$

Inserting the above in the expression of Z leads to

$$Z = \frac{(L_1 w^2 + L_2 w + L_3)}{4[S_n(\beta_n + \gamma\beta_s) - \gamma\beta_s]^2}$$

where

$$\begin{aligned} L_1 &= \beta_n^2 S [S(\beta_n + \gamma\beta_s) - 2\gamma\beta_s], \\ L_2 &= -2[S(S(\beta_n + \gamma\beta_s) - 2\gamma\beta_s)(2\gamma\beta_s + \beta_n + \gamma\beta_n\alpha_s) \\ &\quad + \beta_s\gamma^2(2\beta_s + \beta_n\alpha_s)], \\ L_3 &= [S + \gamma\alpha_s(1 - S)][S(\gamma\beta_s + \beta_n)(1 - \gamma\alpha_s) \\ &\quad - \gamma(2\beta_s + \beta_n\alpha_s - \gamma\alpha_s\beta_s)]. \end{aligned}$$

The sign of expression Z is the sign of its numerator which is a polynomial equation. If $L_1 \neq 0$, i.e., $S(\beta_n + \gamma\beta_s) \neq 2\gamma\beta_s$, then the two roots are as follows:

$$\begin{aligned} w_1 &= A(B + \sqrt{C}) \\ w_2 &= A(B - \sqrt{C}) \end{aligned}$$

where

$$\begin{aligned} A &= \frac{1}{\beta_n^2 S [S(\beta_n + \gamma\beta_s) - 2\gamma\beta_s]} \\ B &= S^2(\beta_n + \gamma\beta_s)(\beta_n + 2\gamma\beta_s + \gamma\beta_n\alpha_s) \\ &\quad - 2\beta_s S \gamma(\beta_n + 2\gamma\beta_s + \gamma\beta_n\alpha_s) + \beta_s \gamma^2(2\beta_s + \beta_n\alpha_s) \\ C &= \gamma(S\beta_n - \gamma\beta_s + \gamma S\beta_s)^2 \left[\gamma(2\beta_s + \beta_n\alpha_s)^2 - 8\beta_s S \gamma(\beta_s + \beta_n\alpha_s) \right. \\ &\quad \left. + 4S^2(\beta_n + \gamma\beta_s)(\beta_s + \beta_n\alpha_s) \right] \end{aligned}$$

We have different possibilities for the choice of S^{\max} (for which $Z = Z^{\max} > 0$) versus S^{\min} (for which $Z = Z^{\min} < 0$). The superscript max indicates that expressions (A, B, C, w_1, w_2 and Z) are computed with S^{\max} and the superscript min indicates that expressions (A, B, C, w_1, w_2 and Z) are computed with S^{\min} . Note also that A^{\max} and A^{\min} are always positive if $\beta_s < 0$ and we require that $S(\beta_n + \gamma\beta_s) \neq \gamma\beta_s$ and $S(\beta_n + \gamma\beta_s) \neq 2\gamma\beta_s$ for either $S = S^{\max}$ or $S = S^{\min}$. The following tables summarize the conditions under which the choice for S is S^{\max} or S^{\min} .

Choice of S^{\max}	
Cases	Transfer price's condition for $Z^{\max} > 0$
$C^{\max} < 0$ and $A^{\max} > 0$	$w \in [0, +\infty)$
$C^{\max} = 0$ and $A^{\max} > 0$	$w \in [0, +\infty) \setminus \left\{ \frac{B^{\max}}{A^{\max}} \right\}$ when $B^{\max} \geq 0$
$C^{\max} > 0$ and $A^{\max} > 0$ and $B^{\max} > 0$ and $B^{\max} - \sqrt{C^{\max}} > 0$	$w \in [0, w_2^{\max} [\cup] w_1^{\max}, +\infty)$
$C^{\max} > 0$ and $A^{\max} > 0$ and $B^{\max} \geq 0$ and $B^{\max} - \sqrt{C^{\max}} < 0$	$w \in] w_1^{\max}, +\infty)$
$C^{\max} > 0$ and $A^{\max} > 0$ and $B^{\max} \leq 0$ and $B^{\max} + \sqrt{C^{\max}} > 0$	$w \in] w_1^{\max}, +\infty)$
$C^{\max} > 0$ and $A^{\max} > 0$ and $B^{\max} < 0$ and $B^{\max} + \sqrt{C^{\max}} < 0$	$w \in [0, +\infty)$
$C^{\max} > 0$ and $A^{\max} < 0$ and $B^{\max} \geq 0$ and $B^{\max} - \sqrt{C^{\max}} < 0$	$w \in [0, w_2^{\max} [$
$C^{\max} > 0$ and $A^{\max} < 0$ and $B^{\max} \leq 0$ and $B^{\max} + \sqrt{C^{\max}} > 0$	$w \in [0, w_2^{\max} [$
$C^{\max} > 0$ and $A^{\max} < 0$ and $B^{\max} < 0$ and $B^{\max} + \sqrt{C^{\max}} < 0$	$w \in] w_1^{\max}, w_2^{\max} [$

In each case, the retailer's reaction function is:

$$p_n^{\max}(w) = \frac{1}{2} \left(\frac{S^{\max} [1 + w\beta_n - \gamma\alpha_s] + \gamma\alpha_s}{S^{\max} (\beta_n + \gamma\beta_s) - \gamma\beta_s} \right).$$

Choice of S^{\min}	
Cases	Transfer price's condition for $Z^{\min} < 0$
$C^{\min} < 0$ and $A^{\min} < 0$	$w \in [0, +\infty)$
$C^{\min} = 0$ and $A^{\min} < 0$	$w \in [0, +\infty) \setminus \left\{ \frac{B^{\min}}{A^{\min}} \right\}$ when $B^{\min} \leq 0$
$C^{\min} > 0$ and $A^{\min} > 0$ and $B^{\min} > 0$ and $B^{\min} - \sqrt{C^{\min}} > 0$	$w \in]w_2^{\min}, w_1^{\min}[$
$C^{\min} > 0$ and $A^{\min} > 0$ and $B^{\min} \geq 0$ and $B^{\min} - \sqrt{C^{\min}} < 0$	$w \in [0, w_1^{\min}[$
$C^{\min} > 0$ and $A^{\min} > 0$ and $B^{\min} \leq 0$ and $B^{\min} + \sqrt{C^{\min}} > 0$	$w \in [0, w_1^{\min}[$
$C^{\min} > 0$ and $A^{\min} < 0$ and $B^{\min} > 0$ and $B^{\min} - \sqrt{C^{\min}} > 0$	$w \in [0, +\infty)$
$C^{\min} > 0$ and $A^{\min} < 0$ and $B^{\min} \geq 0$ and $B^{\min} - \sqrt{C^{\min}} < 0$	$w \in]w_2^{\min}, +\infty)$
$C^{\min} > 0$ and $A^{\min} < 0$ and $B^{\min} \leq 0$ and $B^{\min} + \sqrt{C^{\min}} > 0$	$w \in]w_2^{\min}, +\infty)$
$C^{\min} > 0$ and $A^{\min} < 0$ and $B^{\min} < 0$ and $B^{\min} + \sqrt{C^{\min}} < 0$	$w \in [0, w_1^{\min}[\cup]w_2^{\min}, +\infty)$

In each case, the retailer's reaction function is

$$p_n^{\min}(w) = \frac{1}{2} \left(\frac{S^{\min} [1 + w\beta_n - \gamma\alpha_s] + \gamma\alpha_s}{S^{\min} (\beta_n + \gamma\beta_s) - \gamma\beta_s} \right)$$

6.2 The Manufacturer's Problem

We have to solve the following manufacturer's problem:

$$\text{Max} \pi_M = \text{Max} [w(1 - \beta_n p_n(w)) S]$$

subject to

$$S = \begin{cases} S^{\max}, & \text{if } Z^{\max} > 0 \\ S^{\min}, & \text{if } Z^{\min} < 0 \end{cases}.$$

Substituting for $p_n(w)$ in the manufacturer's optimization problem, the first-order optimality condition

$$\frac{d}{dw} \left[wS \left(1 - \frac{1}{2} \beta_n \left(\frac{S[1 + w\beta_n - \gamma\alpha_s] + \gamma\alpha_s}{S(\beta_n + \gamma\beta_s) - \gamma\beta_s} \right) \right) \right] = 0,$$

leads to

$$(S\beta_n - \gamma\beta_n\alpha_s - 2\gamma\beta_s + 2\gamma S\beta_s + \gamma S\beta_n\alpha_s) - (2S\beta_n^2)w = 0.$$

If $S \neq 0$, the optimal w is given by

$$w^* = \frac{S\beta_n + \gamma(2\beta_s + \beta_n\alpha_s)(S - 1)}{2\beta_n^2 S}.$$

Substituting for w^* in the retailer's reaction function yields the equilibrium price strategy, which takes this form:

$$p_n^* = \frac{1}{4\beta_n} \left(\frac{3S\beta_n + \gamma(\beta_n\alpha_s - 2\beta_s)(1 - S)}{S(\beta_n + \gamma\beta_s) - \gamma\beta_s} \right)$$

To recapitulate:

$S = S^{\max}$
$w^{\max} = \frac{S^{\max}\beta_n + \gamma(2\beta_s + \beta_n\alpha_s)(S^{\max} - 1)}{2\beta_n^2 S^{\max}}$
$p_n^{\max} = \frac{1}{4\beta_n} \left(\frac{3S^{\max}\beta_n + \gamma(\beta_n\alpha_s - 2\beta_s)(1 - S^{\max})}{S^{\max}(\beta_n + \gamma\beta_s) - \gamma\beta_s} \right)$
$S = S^{\min}$
$w^{\min} = \frac{S^{\min}\beta_n + \gamma(2\beta_s + \beta_n\alpha_s)(S^{\min} - 1)}{2\beta_n^2 S^{\min}}$
$p_n^{\min} = \frac{1}{4\beta_n} \left(\frac{3S^{\min}\beta_n + \gamma(\beta_n\alpha_s - 2\beta_s)(1 - S^{\min})}{S^{\min}(\beta_n + \gamma\beta_s) - \gamma\beta_s} \right)$

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