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# Comparison of ILP Formulations for the RWA Problem

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## Abstract

We present a review of the various integer linear programming (ILP) formulations that have been proposed for the routing and wavelength assignment problem in WDM optical networks with a unified and simplified notation under asymmetrical assumptions on the traffic. We show that all formulations proposed under asymmetrical traffic assumptions, both link and path formulations, are equivalent in terms of the upper bound value provided by the optimal solution of their linear programming relaxation, although their number of variables and constraints differ. We also propose some improvements for some of the formulations that result in the elimination of potential looping lightpaths and lead to further reductions in the number of variables and constraints. We next discuss the easiness of adding a constraint on the number of hops (i.e., how to take into account the node/link attenuating effect) depending on the formulations.

Under the objective of minimizing the blocking rate, we propose an experimental comparison of the best lower and upper bounds that are available. We then discuss the easiness of exact ILP solution depending on the formulations. We solve exactly for the first time some RWA (Routing and Wavelength Assignment) instances, including those proposed by Krishnaswamy and Sivarajan (2001), with a proof of the optimality. The conclusion is that LP relaxations bounds often provide solutions with a value very close to the optimal ILP one.

**Key Words:** WDM network, network dimensioning, RWA problem, integer programming, bounds, hop constraints, optimal solution.

## Résumé

Nous présentons une synthèse des différentes formulations de programmation en nombres entiers (PNE) qui ont été proposées pour le problème de routage de d'affectation de longueurs d'onde dans les réseaux optiques WDM, appelé problème RWA, avec une notation unifiée et simplifiée sous des hypothèses de trafic asymétrique. Nous montrons que toutes les formulations proposées avec des hypothèses de trafic asymétrique, à la fois les formulations en termes de liens et de chemins, sont équivalentes en termes des bornes supérieures fournies par la valeur optimale de leur relaxation linéaire, bien que leurs nombres de variables et de contraintes varient. Nous proposons également diverses améliorations pour certaines des formulations, et celles-ci conduisent à l'élimination de boucles dans les chemins optiques virtuels et à des réductions supplémentaires dans les nombres de variables et de contraintes. Nous discutons ensuite de la facilité à ajouter une contrainte sur le nombre de sauts (pour tenir compte de l'effet d'atténuation des noeuds et des liens) pour les différentes formulations.

Avec l'objectif de minimiser le taux de blocage, nous proposons une comparaison expérimentale des meilleures bornes inférieures et supérieures disponibles. Nous discutons ensuite de la facilité de la résolution en nombres entiers dépendant des formulations. Nous résolvons exactement pour la première fois certains exemples de problèmes RWA, incluant les problèmes proposés par Krishnaswamy et Sivarajan (2001), avec une preuve d'optimalité. La conclusion de notre étude est que les bornes de relaxation continue (programmation linéaire) fournissent souvent des solutions avec une valeur très proche de celle de la valeur optimale en nombres entiers.

**Mots Clefs:** Réseau WDM, dimensionnement de réseaux, problème RWA, programmation en nombres entiers, bornes, contraintes de sauts, solution optimale.

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## 1 Introduction

The WDM (Wavelength Division Multiplexing) optical networks offer the promise of providing the high bandwidth required by the increasing multimedia communication applications, see, e.g., Ramaswami and Sivarajan [1] for a general reference on optical networks. This has led to a wide interest in the RWA (Routing and Wavelength Assignment) problem defined as follows: given the physical structure of a network and a set of requested connections, select a suitable routing path and wavelength for each connection so that no two paths sharing a link on the same fiber, are assigned the same wavelength.

Many papers have already appeared on the RWA problem, proposing various heuristic scheme solutions under different assumptions on the traffic patterns, availability of the converters, and objectives, cf. the surveys of Dutta and Rouskas [2] and Zang, Jue and Mukherjee [3] for a summary of the works until 2000, and Jaumard, Meyer and Thiongane [4] for a recent survey on symmetrical systems under various objectives. The most often studied objectives are the minimization of the number of wavelengths (called min-RWA problem), the maximization of the number of accepted connections (called max-RWA problem) (Krishnaswamy and Sivarajan [5]), the minimization of the congestion (Krishnaswamy and Sivarajan [6]) and the minimization of the multiplexing costs.

Several types of heuristics and metaheuristics have been proposed. For the most efficient or recent ones, see, e.g., various greedy heuristics (Banerjee and Mukherjee [7], Banerjee, Yoo and Chen [8], Chlamtac, Ganz and Karmi [9], Zhang and Acampora [10]) and different metaheuristics: Tabu Search (Jaumard and Hemazro [11] for non uniform traffic, Noronha and Ribeiro [12] for uniform traffic), Simulated Annealing (Katangur, Pan and Fraser [13]) or genetic algorithms (Ali, Ramamurthy and Deogun [14]; Qin, Siew and Li [15]; Banerjee, Mehta and Pandey [16]). The reader can refer to Hyttiä and Virtamo [17] for a comparison of some of them.

With respect to exact solutions, the RWA problem has been formulated as an integer programming problem but most of the times those formulations have not been used for developing solution schemes except for some rounding off procedures. We review those formulations for static traffic models, focusing on the max-RWA problem with asymmetrical traffic matrices. There are two classes of formulations, those with link variables (see [5], [18]), and those with path variables (see [19], [20], [21], [22]). We compare the optimal values of their linear programming relaxations (or LP relaxations for short) and show that they are all leading to the same upper bound. We also propose some further improvements for some of the formulations that eliminate potential looping lightpaths and lead to some reductions in the number of variables and constraints. However, the number of variables and constraints of these formulations differ and the ease of solving the corresponding integer linear programming formulations (or ILP for short) varies: while some of them can be solved using ILP software (e.g. with the CPLEX libraries of ILOG inc. [23]), others are just intractable as soon as the size of the instances is increasing. For the formulations with path variables, some attempts have been made to solve them using decomposition techniques

such as column generation (Ramaswami and Sivarajan [22], Lee, Lee and Park [24]) or Lagrangean relaxation (Saad and Luo [21]). Moreover, if additional constraints need to be considered such as hop or signal regeneration constraints (Ye *et al.* [25], Ali, Ramamurthy and Deogun [26]), some formulations are easier to adapt than others.

We next compare these upper bounds provided by the LP relaxations with the optimal ILP solutions obtained using the CPLEX-MIP software of ILOG [23]. Indeed, using the ILP formulation with the smallest number of variables and constraints, we solve several RWA instances exactly for the first time, with a proof of optimality, including the instances of Krishnaswamy [27], studied, e.g., in Krishnaswamy and Sivarajan [5].

The paper is organized as follows. In the next section, we present a more formal statement of the RWA problem with the objective of minimizing the blocking rate and define notation that will be used throughout the paper. Section 3 is devoted to the models using link variables. We present three different formulations: the first two have been initially proposed by Krishnaswamy and Sivarajan [5], and the third one, a source formulation, has been studied by several authors (Coudert and Rivano [28]; Tornatore, Maier and Pattavina [20], Krishnaswamy and Sivarajan [6]). We show that their LP relaxations provide the same upper bounds and discuss how to improve them in terms of reducing their number of variables and constraints. In Section 4, we study the path ILP formulations, show that their LP relaxations provide the same upper bound than the link formulations of the previous sections. Further comparisons are conducted in Section 5 on the overall set of formulations, where we discuss the easiness of taking into account additional constraints. Computational experience are reported in Section 6. Conclusions are drawn in the last section.

## 2 Statement of the RWA Problem - Notation

We assume that the optical network is represented by a multigraph  $G = (V, E)$  with a node set  $V = \{v_1, v_2, \dots, v_n\}$  where each node is associated with a node of the physical network, and an arc set  $E = \{e_1, e_2, \dots, e_m\}$  where each arc is associated with a link and a fiber of the physical network: the number of arcs from  $v_i$  to  $v_j$  is equal to the number of fibers supporting traffic from  $v_i$  to  $v_j$ , see Figure 1 for an illustration: the two arcs from  $v_2$  to  $v_1$  indicate that there are two fibers. Note however that most of the networks today are with only one fiber per link. The traffic is defined by a set  $K$  of connections, where each connection has a capacity corresponding to the capacity of a wavelength. All wavelengths are assumed to have the same capacity. We denote by  $\Lambda$  the set of wavelengths that we assume of cardinality  $W = |\Lambda|$ . Moreover, we consider only single-hop connections, as we do not assume that node converters or signal regeneration schemes are available.

The RWA problem can then be formally stated as follows: given a multigraph  $G$  corresponding to a WDM optical network, and a set  $K$  of requested connections, find a suitable lightpath  $(p, \lambda)$  for each (accepted) connection where  $p$  is a routing path and  $\lambda$  a wavelength, so that no two paths sharing an arc of  $G$  are assigned the same wavelength.

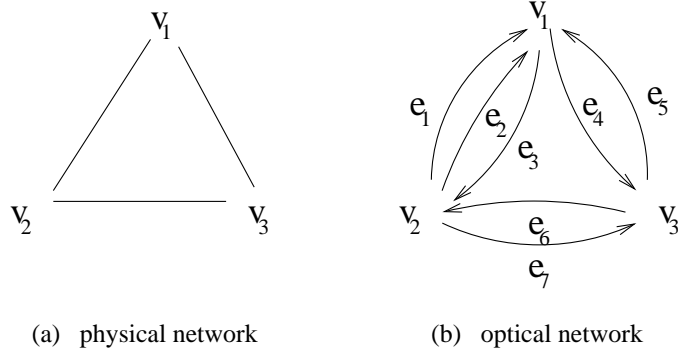


Figure 1: Multigraphs and Multifiber Optical Networks.

As we consider asymmetrical traffic matrices, connections and therefore links are directional. Denote by  $T$  a traffic matrix. Each element  $T_{ij}$  defines the number of requested connections from  $v_i$  to  $v_j$ . For each connection  $k$ , let  $s_k$  and  $d_k$  denote the source and destination nodes respectively.

We define below the main variables that will be used in the different formulations presented in the forthcoming sections. The first set of variables will be used in the so-called link formulations, while the second set of variables will be used in the so-called path formulations.

First set of variables:

$$\begin{aligned}
 x_k &= \begin{cases} 1 & \text{if connection } k \text{ is accepted} \\ 0 & \text{otherwise,} \end{cases} \\
 x_k^\lambda &= \begin{cases} 1 & \text{if wavelength } \lambda \text{ supports connection } k \\ 0 & \text{otherwise,} \end{cases} \\
 x_{ke}^\lambda &= \begin{cases} 1 & \text{if wavelength } \lambda \text{ supports connection } k \\ & \text{on arc } e \\ 0 & \text{otherwise.} \end{cases}
 \end{aligned}$$

Second set of variables:

$$x_p^\lambda = \begin{cases} 1 & \text{if there is a lightpath defined with path } p \\ & \text{and wavelength } \lambda \\ 0 & \text{otherwise.} \end{cases}$$

We will denote by  $\omega^+(v_i)$  (resp.  $\omega^-(v_i)$ ) the set of outgoing (resp. incoming) links at node  $v_i$ .

### 3 Link ILP Formulations

As already mentioned in the Introduction, there are two classes of formulations. We present below the formulations of the first class, i.e., those with variables associated with links. There are mainly three formulations, depending on whether the connections are taken into account individually (paragraph 3.1), or grouped with respect to their source and destination nodes (paragraph 3.2), or grouped only with respect to their source (paragraph 3.4).

#### 3.1 First Formulation of Krishnaswamy and Sivarajan

In [5], Krishnaswamy and Sivarajan present two formulations of the RWA problem with the objective of maximizing the number of connections. We present below the first formulation, denoted by (KS1), in which each connection is considered individually.

$$\max \quad z_{\text{KS1}}(x) = \sum_{k \in K} x_k$$

subject to:

$$\sum_{e \in \omega^+(v_i)} x_{ke}^\lambda = \sum_{e \in \omega^-(v_i)} x_{ke}^\lambda \quad k \in K, \lambda \in \Lambda, v_i \in V \setminus \{s_k, d_k\} \quad (1)$$

$$\sum_{e \in \omega^+(s_k)} x_{ke}^\lambda - \sum_{e \in \omega^-(s_k)} x_{ke}^\lambda = x_k^\lambda \quad k \in K, \lambda \in \Lambda \quad (2)$$

$$\sum_{e \in \omega^-(d_k)} x_{ke}^\lambda - \sum_{e \in \omega^+(d_k)} x_{ke}^\lambda = x_k^\lambda \quad k \in K, \lambda \in \Lambda \quad (3)$$

$$\sum_{k \in K} x_{ke}^\lambda \leq 1 \quad e \in E, \lambda \in \Lambda \quad (4)$$

$$\sum_{\lambda \in \Lambda} x_k^\lambda = x_k \quad k \in K \quad (5)$$

$$x_{ke}^\lambda \leq x_k^\lambda \quad k \in K, e \in E, \lambda \in \Lambda \quad (6)$$

$$x_k, x_k^\lambda, x_{ke}^\lambda \in \{0, 1\} \quad k \in K, e \in E, \lambda \in \Lambda. \quad (7)$$

The first three sets of constraints correspond to the wavelength continuity constraints, i.e., a unique wavelength is assigned to connection  $k$  throughout its lightpath and are analogous to the flow conservation constraints in a multicommodity flow problem. Constraints (4) are the clash constraints: they ensure that no two lightpaths sharing the same arc  $e$  (i.e., the same link and fiber) can be assigned the same wavelength. Constraints (5) guarantee that if a lightpath is defined for connection  $k$ , exactly one wavelength is assigned to it. Finally, constraints (6) ensure consistency between variables  $x_{ke}^\lambda$  and  $x_k^\lambda$ .

Even if it does not affect its value, the optimal solution of (KS1) may contain looping lightpaths involving either the origin or the destination nodes of the connections. In order to eliminate them, we introduce a slight variation in formulation (KS1), leading to a new formulation denoted by (RWA). It consists in replacing constraints (2) and (3) by constraints (2a) and (3a) and adding constraints (8).

$$\max \quad z_{\text{RWA}}(x) = \sum_{k \in K} x_k$$

subject to:

$$\sum_{e \in \omega^+(v_i)} x_{ke}^\lambda = \sum_{e \in \omega^-(v_i)} x_{ke}^\lambda \quad k \in K, \lambda \in \Lambda, v_i \in V \setminus \{s_k, d_k\} \quad (1)$$

$$\sum_{e \in \omega^+(s_k)} x_{ke}^\lambda = x_k^\lambda \quad k \in K, \lambda \in \Lambda \quad (2a)$$

$$\sum_{e \in \omega^-(d_k)} x_{ke}^\lambda = x_k^\lambda \quad k \in K, \lambda \in \Lambda \quad (3a)$$

$$\sum_{e \in \omega^-(s_k)} x_{ke}^\lambda = \sum_{e \in \omega^+(d_k)} x_{ke}^\lambda = 0 \quad k \in K, \lambda \in \Lambda \quad (8)$$

$$\sum_{k \in K} x_{ke}^\lambda \leq 1 \quad e \in E, \lambda \in \Lambda \quad (4)$$

$$\sum_{\lambda \in \Lambda} x_k^\lambda = x_k \quad k \in K \quad (5)$$

$$x_{ke}^\lambda \leq x_k^\lambda \quad k \in K, e \in E, \lambda \in \Lambda \quad (6)$$

$$x_k, x_k^\lambda, x_{ke}^\lambda \in \{0, 1\} \quad k \in K, e \in E, \lambda \in \Lambda. \quad (7)$$

Note that when solving the above mathematical program, constraints (8) should be eliminated after having set to 0 and eliminated all variables  $x_{ke}^\lambda$  appearing in them. Moreover, observe that the variables  $x_k$  are not necessary: for more efficient solution they can be omitted when solving the (RWA) ILP with the objective being rewritten:

$$z'_{\text{RWA}}(x) = \sum_{k \in K} \sum_{\lambda \in \Lambda} x_k^\lambda, \quad (9)$$

and constraints (5) replaced by

$$\sum_{\lambda \in \Lambda} x_k^\lambda \leq 1 \quad k \in K. \quad (5a)$$

Let  $\Omega_{\text{LP}}(\text{KS1})$  and  $\Omega_{\text{LP}}(\text{RWA})$  be the feasible domains of the LP relaxations of (KS1) and (RWA), i.e., of (KS1) and (RWA) where the constraints (7) have been replaced by:

$$0 \leq x_k, x_k^\lambda, x_{ke}^\lambda \leq 1 \quad k \in K, e \in E, \lambda \in \Lambda.$$



Denote by  $\bar{z}_{\text{RWA}}$  and  $\bar{z}_{\text{KS1}}$  the optimal values of the LP relaxations of formulations (KS1) and (RWA).

We first show that:

**Theorem 1.** *The optimal values of the LP relaxations of formulations (KS1) and (RWA) are equal:*

$$\bar{z}_{\text{KS1}} = \bar{z}_{\text{RWA}}.$$

*Proof.* It is easy to see that  $\Omega_{\text{LP}}(\text{RWA}) \subseteq \Omega_{\text{LP}}(\text{KS1})$ , hence that  $\bar{z}_{\text{RWA}} \leq \bar{z}_{\text{KS1}}$ .

Let us now consider an optimal solution of (KS1), say  $x = (x_k, x_k^\lambda, x_{ke}^\lambda)$ , such that there exists at least one connection  $k \in K$  and a wavelength  $\lambda \in \Lambda$  satisfying  $\sum_{e \in \omega^-(s_k)} x_{ke}^\lambda \neq 0$  or

$\sum_{e \in \omega^+(d_k)} x_{ke}^\lambda \neq 0$ . Let us assume that  $\sum_{e \in \omega^-(s_k)} x_{ke}^\lambda \neq 0$ . This implies that connection  $k$  has a looping lightpath on its source node  $s_k$ . Removing the circuit leads to no change on the objective function value as  $x_k = \sum_{\lambda \in \Lambda} x_k^\lambda$  remains unchanged. Thus, by removing all circuits in all established connections of the optimal solution  $x$ , we obtain an optimal solution of (KS1) which is feasible for (RWA), therefore  $\bar{z}_{\text{RWA}} \geq \bar{z}_{\text{KS1}}$ .

We can then conclude that both values  $\bar{z}_{\text{RWA}}$  and  $\bar{z}_{\text{KS1}}$  are equal.  $\square$

### 3.2 Second Formulation of Krishnaswamy and Sivarajan

Let us now examine the second formulation of Krishnaswamy and Sivarajan [5] denoted by (KS2), in which connections are grouped with respect to their origin and destination pair  $(v_s, v_d)$  of nodes. Let  $K_{sd}$  denote the set of connections between a pair of origin and destination nodes  $v_s$  and  $v_d$ . New variables  $y_{sd} = \sum_{k \in K_{sd}} x_k$  and  $y_{sde}^\lambda = \sum_{k \in K_{sd}} x_{ke}^\lambda, y_{sde}^\lambda \in \{0, 1\}$  are introduced, where  $y_{sd}$  is the number of accepted connections from  $v_s$  to  $v_d$  and  $y_{sde}^\lambda = 1$  if a connection from  $v_s$  to  $v_d$  uses wavelength  $\lambda$  on link  $e$  and 0 otherwise. Note that, for a given  $e, \lambda$ , at most one connection can be supported.

$$\max \quad z_{\text{KS2}}(y) = \sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} y_{sd}$$

subject to:

$$\sum_{e \in \omega^+(v_i)} y_{sde}^\lambda = \sum_{e \in \omega^-(v_i)} y_{sde}^\lambda \quad \lambda \in \Lambda, v_i \in V \setminus \{v_s, v_d\},$$

$$(v_s, v_d) \in V \times V : T_{sd} > 0 \quad (10)$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_s)} y_{sde}^\lambda = \sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda = y_{sd} \quad (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (11)$$

$$\sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} y_{sde}^\lambda \leq 1 \quad e \in E, \lambda \in \Lambda \quad (12)$$

$$y_{sd} \in \{0, 1, \dots, T_{sd}\} \quad (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (13)$$

$$y_{sde}^\lambda \in \{0, 1\} \quad e \in E, \lambda \in \Lambda, \quad (v_s, v_d) \in V \times V : T_{sd} > 0. \quad (14)$$

Constraints (10) and (11) are the wavelength continuity constraints while (12) correspond to the clash constraints. However this formulation is incomplete. Indeed it allows the existence of looping lightpaths passing through the origin and/or the destination of a connection, and contrarily to what happened for formulation (KS1), the objective function value is modified. There are two ways to correct the formulation. Either replace (11) by

$$\begin{aligned} \sum_{\lambda \in \Lambda} \left( \sum_{e \in \omega^+(v_s)} y_{sde}^\lambda - \sum_{e \in \omega^-(v_s)} y_{sde}^\lambda \right) = \\ \sum_{\lambda \in \Lambda} \left( \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda - \sum_{e \in \omega^+(v_d)} y_{sde}^\lambda \right) = y_{sd} \\ (v_s, v_d) \in V \times V : T_{sd} > 0 \end{aligned}$$

or add to (11) the constraints

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_s)} y_{sde}^\lambda = \sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_d)} y_{sde}^\lambda = 0 \quad (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (15)$$

as for the addition of constraints (8) with respect to constraints (2) and (3) in formulation (KS1). Note that the first proposal may allow the existence of paths from  $v_d$  to  $v_s$  since  $\sum_{e \in \omega^+(v_s)} y_{sde}^\lambda - \sum_{e \in \omega^-(v_s)} y_{sde}^\lambda$  may be negative for some  $\lambda$ . These paths can however be canceled with paths from  $v_s$  to  $v_d$ , leading to an optimal solution for the RWA problem. However we prefer and retain the second solution, as it allows the elimination of some variables  $y_{sde}^\lambda$ . This leads to the following (RWA\_sd) formulation:

$$\max \quad z_{\text{RWA\_sd}}(y) = \sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} y_{sd}$$

subject to:

$$\begin{aligned} \sum_{e \in \omega^+(v_i)} y_{sde}^\lambda = \sum_{e \in \omega^-(v_i)} y_{sde}^\lambda \quad \lambda \in \Lambda, v_i \in V \setminus \{v_s, v_d\}, \\ (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (10) \end{aligned}$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_s)} y_{sde}^\lambda = \sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda = y_{sd} \quad (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (11)$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_s)} y_{sde}^\lambda = \sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_d)} y_{sde}^\lambda = 0 \quad (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (15)$$

$$\sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} y_{sde}^\lambda \leq 1 \quad e \in E, \lambda \in \Lambda \quad (12)$$

$$y_{sd} \in \{0, 1, \dots, T_{sd}\} \quad (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (13)$$

$$y_{sde}^\lambda \in \{0, 1\} \quad e \in E, \lambda \in \Lambda, \quad (v_s, v_d) \in V \times V : T_{sd} > 0. \quad (14)$$

Again constraints (15) should be eliminated when solving the above formulation after having set to 0 and eliminated the variables  $y_{sde}^\lambda$  appearing in them. Observe that the variables  $y_{sd}$  are actually not necessary; for more efficiency they can be omitted when solving the (RWA\_sd) ILP with the objective function being rewritten:

$$z'_{\text{RWA\_sd}}(y) = \sum_{\lambda \in \Lambda} \sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} \sum_{e \in \omega^+(v_s)} y_{sde}^\lambda \quad (16)$$

and the constraints (11) replaced by

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_s)} y_{sde}^\lambda = \sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda \leq T_{sd} \quad (v_s, v_d) \in V \times V : T_{sd} > 0. \quad (11a)$$

Although this last formulation does not explicitly provide the set of lightpaths between a pair  $(v_s, v_d)$  of origin and destination nodes, it ensures that exactly  $y_{sd} \leq T_{sd}$  lightpaths will exist between  $v_s$  and  $v_d$ . Even if the lightpaths are not explicitly provided in the solution of (RWA\_sd), they can easily be deduced from an iterative process: see, e.g., the proof of Theorem 2.

### 3.3 Comparison of the LP Relaxations of the Formulations (RWA) and (RWA\_sd)

Let us compare the LP relaxations of formulations (RWA) and (RWA\_sd). We denote by  $\bar{z}_{\text{RWA}}$  and  $\bar{z}_{\text{RWA\_sd}}$  their optimal values.

**Theorem 2.** *The optimal values of the LP relaxations of the ILP formulations (RWA) and (RWA\_sd) are equal:*

$$\bar{z}_{\text{RWA}} = \bar{z}_{\text{RWA\_sd}}.$$

*Proof.* Let us first show that starting from a solution of  $\Omega_{\text{LP}}(\text{RWA})$  we can deduce a solution for  $\Omega_{\text{LP}}(\text{RWA\_sd})$  with the same value and therefore that  $\bar{z}_{\text{RWA}} \leq \bar{z}_{\text{RWA\_sd}}$ . Given

$(x_k, x_k^\lambda, x_{ke}^\lambda) \in \Omega_{\text{LP}}(\text{RWA})$ , we define

$$\begin{aligned} y_{sd} &= \sum_{k \in K_{sd}} x_k, & (v_s, v_d) \in V \times V : T_{sd} > 0 \\ y_{sde}^\lambda &= \sum_{k \in K_{sd}} x_{ke}^\lambda, & (v_s, v_d) \in V \times V : T_{sd} > 0, \quad e \in E. \end{aligned}$$

Observe first that (1) in (RWA)  $\implies$  (10) in (RWA<sub>sd</sub>). Consider  $\lambda \in \Lambda$ ,  $(v_s, v_d) \in V \times V : T_{sd} > 0$  and  $v_i \in V \setminus \{v_s, v_d\}$ . We have:

$$\begin{aligned} (1) &\implies \sum_{e \in \omega^+(v_i)} x_{ke}^\lambda = \sum_{e \in \omega^-(v_i)} x_{ke}^\lambda \quad k \in K_{sd} \\ &\implies \sum_{k \in K_{sd}} \left( \sum_{e \in \omega^+(v_i)} x_{ke}^\lambda \right) = \sum_{k \in K_{sd}} \left( \sum_{e \in \omega^-(v_i)} x_{ke}^\lambda \right) \\ &\implies \sum_{e \in \omega^+(v_i)} \left( \sum_{k \in K_{sd}} x_{ke}^\lambda \right) = \sum_{e \in \omega^-(v_i)} \left( \sum_{k \in K_{sd}} x_{ke}^\lambda \right). \end{aligned}$$

Using the relation  $y_{sde}^\lambda = \sum_{k \in K_{sd}} x_{ke}^\lambda$ , we derive (10).

Let us next show that (2a), (3a), (5) in (RWA)  $\implies$  (11) in (RWA<sub>sd</sub>). Let  $(v_s, v_d) \in V \times V : T_{sd} > 0$ ,  $\lambda \in \Lambda$ . Considering constraints (2a) for  $k \in K_{sd}$  and summing them, we obtain:

$$\sum_{k \in K_{sd}} \left( \sum_{e \in \omega^+(s_k)} x_{ke}^\lambda \right) = \sum_{e \in \omega^+(s_k)} \left( \sum_{k \in K_{sd}} x_{ke}^\lambda \right) = \sum_{k \in K_{sd}} x_k^\lambda,$$

or equivalently,

$$\sum_{e \in \omega^+(s_k)} y_{sde}^\lambda = \sum_{k \in K_{sd}} x_k^\lambda. \quad (17)$$

Moreover, summing (5) over  $k \in K_{sd}$ , we get

$$\sum_{k \in K_{sd}} \sum_{\lambda \in \Lambda} x_k^\lambda = \sum_{k \in K_{sd}} x_k.$$

Summing constraints (17) over  $\lambda$  leads then to

$$\sum_{e \in \omega^+(s_k)} \sum_{\lambda \in \Lambda} y_{sde}^\lambda = \sum_{k \in K_{sd}} x_k = y_{sd}.$$

Similarly, using constraints (3a) and (5) in (RWA), we obtain

$$\sum_{e \in \omega^-(d_k)} \sum_{\lambda \in \Lambda} y_{sde}^\lambda = \sum_{k \in K_{sd}} x_k = y_{sd}.$$

Hence, constraints (11) in (RWA\_sd) are satisfied. Using the decomposition  $K = \bigcup_{(v_s, v_d): T_{sd} > 0} K_{sd}$ , it is easy to see that (4) in (RWA)  $\implies$  (12) in (RWA\_sd). Finally, summing the constraints (8) over  $k \in K_{sd}$ ,  $\lambda \in \Lambda$  leads to constraints (15). This completes the first part of the proof.

We next establish the reverse inequality.

Let  $(y_{sd}, y_{sde}^\lambda)$  be a solution from  $\Omega_{LP}(\text{RWA\_sd})$ . Let  $(v_s, v_d)$  be fixed. We first show how to represent the vector  $y_{sde}^\lambda$  ( $\lambda \in \Lambda, e \in E$ ) by a set of paths from  $v_s$  to  $v_d$ . Define  $\beta = y_{sd}$  and  $\beta_e^\lambda = y_{sde}^\lambda$  for  $\lambda \in \Lambda$  and  $e \in E$ . Using (10), (11) and (15), we have

$$\sum_{e \in \omega^+(v_i)} \beta_e^\lambda = \sum_{e \in \omega^-(v_i)} \beta_e^\lambda, \quad \lambda \in \Lambda, v_i \in V \setminus \{v_s, v_d\} \quad (18)$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_s)} \beta_e^\lambda = \beta \quad (19)$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_s)} \beta_e^\lambda = 0. \quad (20)$$

As long as  $\beta > 0$ , there exist  $\tilde{e}_1 \in \omega^+(v_s)$  and  $\tilde{\lambda}_1 \in \Lambda$  such that  $\beta_{\tilde{e}_1}^{\tilde{\lambda}_1} > 0$ . Assume that  $\tilde{e}_1 = (v_s, v_{i_1})$ . By (18) for  $v_i = v_{i_1}$  and  $\lambda = \tilde{\lambda}_1$ , there exist  $\tilde{e}_2 \in \omega^+(v_{i_1})$  such that  $\beta_{\tilde{e}_2}^{\tilde{\lambda}_1} > 0$ . Denote by  $v_{i_2}$  the vertex such that  $\tilde{e}_2 = (v_{i_1}, v_{i_2})$ . We repeat this process until one of the following 2 cases occur: a) we revisit an already explored vertex or b) we reach  $v_d$ . Note that one of these 2 cases necessarily occur after at most  $n$  steps. In the case a), we have generated a circuit. Denote by  $c$  this circuit and by  $E(c)$  the set of corresponding arcs. Let  $\delta = \min_{e \in E(c)} \{\beta_e^{\tilde{\lambda}_1}\}$ . Observe that  $\delta > 0$  by construction. We redefine  $\beta_e^{\tilde{\lambda}_1} \leftarrow \beta_e^{\tilde{\lambda}_1} - \delta$  for  $e \in E(c)$ . Note that the resulting vector  $(\beta, \beta_e^\lambda)$  still satisfies (18)-(20). Since at least one  $\beta_e^\lambda$  has been set to 0, the case a) can appear at most  $mW$  times.

Assume now that the case b) occurred. The process has defined a path  $p$  from  $v_s$  to  $v_d$ , associated with wavelength  $\tilde{\lambda}_1$ . Let  $E(p)$  be the set of arcs of path  $p$ . We define  $\delta = \min \left\{ \beta, \min_{e \in E(p)} \{\beta_e^{\tilde{\lambda}_1}\} \right\}$ . Again  $\delta > 0$  by construction. We redefine  $\beta \leftarrow \beta - \delta$  and  $\beta_e^{\tilde{\lambda}_1} \leftarrow \beta_e^{\tilde{\lambda}_1} - \delta$  for  $e \in E(p)$ . Note that the resulting vector  $(\beta, \beta_e^\lambda)$  still satisfies (18)-(20), and that either  $\beta = 0$  or at least one  $\beta_e^\lambda$  has been set to 0. Hence step b) cannot occur more than  $mW$  times.

At the end of this process, we have a collection  $\{p : p \in \mathcal{L}\}$  of paths  $p$  from  $v_s$  to  $v_d$  with wavelength  $\lambda(p)$  and value  $\delta(p)$  such that

$$y_{sd} = \sum_{p \in \mathcal{L}} \delta(p), \quad (21)$$

$$y_{sde}^\lambda \geq \sum_{p \in \mathcal{L}: e \in p, \lambda(p) = \lambda} \delta(p), \quad \lambda \in \Lambda, \quad e \in E \quad (22)$$

(the inequality in (22) comes from the deletion of circuits with nonnegative values; note that by (15), no circuit going through  $v_s$  is possible, hence the equality in (21)).

We will now redistribute the paths of  $\mathcal{L}$  among the connections of  $K_{sd}$ . This redistribution will involve the splitting of some paths. Consider a numbering of the paths of  $\mathcal{L}$ , i.e.,  $\mathcal{L} = \{p_1, p_2, \dots, p_r\}$  with  $r = |\mathcal{L}|$ . Assume that  $K_{sd} = \{k_1, \dots, k_q\}$  where  $q = |K_{sd}|$ . We define the values of the variables  $x_k$ ,  $k \in K_{sd}$  as follows:

$$x_{k_t} = \begin{cases} 1 & t = 1, 2, \dots, \lfloor y_{sd} \rfloor \\ y_{sd} - \lfloor y_{sd} \rfloor & t = \lfloor y_{sd} \rfloor + 1 \\ 0 & t = \lfloor y_{sd} \rfloor + 2, \dots, q. \end{cases}$$

Note that

$$\sum_{k \in K_{sd}} x_k = y_{sd}. \quad (23)$$

Given  $x_{k_1}$ , we determine  $\ell(k_1)$  such that:

$$\sum_{\ell=1}^{\ell(k_1)-1} \delta(p_\ell) < x_{k_1} \leq \sum_{\ell=1}^{\ell(k_1)} \delta(p_\ell).$$

If  $x_{k_1} < \sum_{\ell=1}^{\ell(k_1)} \delta(p_\ell)$ , we create a new lightpath  $(p, \lambda(p))$  that is identical to  $p_{\ell(k_1)}$  in terms

of the arcs and such that  $\lambda(p) = \lambda(p_{\ell(k_1)})$  and  $\delta(p) = \sum_{\ell=1}^{\ell(k_1)} \delta(p_\ell) - x_{k_1}$ . We next modify the

value of  $p_{\ell(k_1)}$  as follows:  $\delta(p_{\ell(k_1)}) \leftarrow x_{k_1} - \sum_{\ell=1}^{\ell(k_1)-1} \delta(p_\ell)$ . This new path  $p$  is added at the end of the list  $\mathcal{L}$  and  $r$  is increased by 1. Note that we still have

$$y_{sd} = \sum_{p \in \mathcal{L}} \delta(p).$$

By definition,

$$x_{k_1} = \sum_{\ell=1}^{\ell(k_1)} \delta(p_\ell).$$

The same procedure is applied iteratively for the next values of  $k_t \in \{k_1, \dots, k_q\}$  such that  $x_{k_t} > 0$ ; we obtain  $\ell(k_t)$  such that (with the convention  $\ell(k_0) = 0$ ):

$$\sum_{\ell=\ell(k_{t-1})+1}^{\ell(k_t)-1} \delta(p_\ell) < x_{k_t} \leq \sum_{\ell=\ell(k_{t-1})+1}^{\ell(k_t)} \delta(p_\ell).$$

We have

$$x_{k_t} = \sum_{\ell=\ell(k_{t-1})+1}^{\ell(k_t)} \delta(p_\ell), \quad t = 1, \dots, q. \quad (24)$$

Now let us define  $x_{ke}^\lambda$ :

$$x_{k_t e}^\lambda = \sum_{\substack{\ell=\ell(k_{t-1})+1, \\ \ell: e \in p_\ell, \lambda(\ell)=\lambda}}^{\ell(k_t)} \delta(p_\ell) \quad \lambda \in \Lambda, \quad t = 1, \dots, q, \quad e \in E. \quad (25)$$

It follows that for any  $k_t \in K_{sd}$ :

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_s)} x_{k_t e}^\lambda = x_{k_t}.$$

We define

$$x_{k_t}^\lambda = \sum_{e \in \omega^+(v_s)} x_{k_t e}^\lambda.$$

Clearly (5) is satisfied. For any  $\lambda \in \Lambda$ , since each path of  $\{p \in \mathcal{L} : \lambda(p) = \lambda\}$  satisfies constraints (1), (8) and

$$\sum_{e \in \omega^+(s_k)} x_{ke}^\lambda = \sum_{e \in \omega^-(s_k)} x_{ke}^\lambda,$$

it follows that (1), (2a), (3a) and (8) are satisfied. Considering that

$$x_{k_t}^\lambda = \sum_{\substack{\ell=\ell(k_{t-1})+1, \\ \ell: \lambda(\ell)=\lambda}}^{\ell(k_t)} \delta(p_\ell), \quad \lambda \in \Lambda, \quad t = 1, \dots, q.$$

Comparing this last equality with (24), we conclude that constraints (6) are satisfied.

To show (4), observe that

$$\sum_{k \in K} x_{ke}^\lambda = \sum_{(v_s, v_d) \in V \times V} \sum_{k \in K_{sd}} x_{ke}^\lambda$$

and that

$$\sum_{k \in K_{sd}} x_{ke}^\lambda = \sum_{t=1}^q x_{k_t e}^\lambda = \sum_{\substack{\ell=1, \\ \ell: e \in p_\ell, \lambda(\ell)=\lambda}}^{r'} \delta(p_\ell) \leq y_{sde}^\lambda \quad (26)$$

where  $r'$  is the cardinality of  $\mathcal{L}$  after possibly splitting some paths. The inequality in (26) follows from (22). Inequality (4) then follows from (12).

Finally equality (23) ensures that the value of the objective function is preserved. We are now able to conclude that the reverse inequality  $\bar{z}_{\text{RWA}} \geq \bar{z}_{\text{RWA\_sd}}$  holds.  $\square$

The last part of the proof of Theorem 2 uses a technique that is very similar to the relationship between link and path formulations for flow problems: see, e.g., Ahuja, Magnanti and Orlin [29, p. 80–81]. As noted by several authors, see, e.g., [28], the RWA problem can also be formulated as a multicommodity flow problem in a layered graph constructed as follows for the (RWA\_sd) formulation: we consider  $W$  copies of the graph  $G$ , each copy corresponding to one wavelength. For each pair  $(v_i, v_j)$  of origin/destination nodes, we add an extra source node  $v_{ij}^s$  that is connected to each copy of node  $v_i$  and similarly we add an extra destination node  $v_{ij}^d$  that is connected to each copy of node  $v_j$ , see Figure 2 for an illustration. All arc capacities in the copies of  $G$  are equal to 1. A similar layered graph can be associated with the (RWA) formulation with extra source nodes  $v_{k_1}^s, v_{k_2}^s, \dots, v_{k_{|K|}}^s$  and extra destination nodes  $v_{k_1}^d, v_{k_2}^d, \dots, v_{k_{|K|}}^d$ .

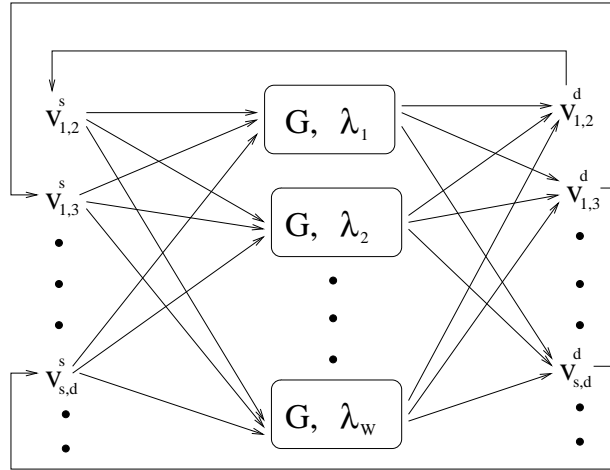


Figure 2: General shape of the flow network of the (RWA\_sd) formulation.

Note also that the second part of the proof of Theorem 2 provides a method to determine the associated lightpaths from an optimal solution  $(y_{sd}, y_{sde}^\lambda)$  of (RWA\_sd).

### 3.4 Source Formulations

Source formulations, in which connections are grouped with respect to their source nodes, have been investigated in three papers: Krishnaswamy and Sivaraman [6], Tornatore, Maier and Pattavina [20], Coudert and Rivano [30]. Although no study exists for destination



formulation, they can be defined in a similar fashion than the source formulations. Depending on the traffic matrices, they even may be more economical formulations than the source formulations, in terms of the number of variables.

Let us consider the source formulation proposed by Coudert and Rivano [30], denoted by (CR), and rearrange it so as to minimize the number of constraints and variables, as well as change the notation in order to unify it with the formulations of the previous sections. Let  $K_s$  denote the set of connections with origin  $v_s$  and  $D_s$  the set of destination nodes for connections originating from  $v_s$ . New variables  $y_{se}^\lambda = \sum_{k \in K_s} x_{ke}^\lambda \in \{0, 1\}$  are introduced, where  $y_{se}^\lambda = 1$  if a connection originating from  $v_s$  uses wavelength  $\lambda$  on link  $e$  and  $y_{se}^\lambda = 0$  otherwise. Note that, for a given  $e, \lambda$ , at most one connection can be supported.

We have slightly modified the constraints of the original (CR) model as we do not consider wavelength conversions. Note also that the graph model considered by Coudert and Rivano is an adaptation of the layered graph described at the end of the previous section, see their paper [30] for more details. A difference exists in the modeling of a multifiber network, instead of using a multigraph as we did, they remain with a simple graph and modify the clash constraints: the limit on the number of times a wavelength is used on a physical link is set to the number of fibers on that link.

The (CR) model needs two additional sets of variables:  $\alpha_s^\lambda$  and  $\alpha_{sd}^\lambda$  that define the number of connections from  $v_s$ , respectively from  $v_s$  to  $v_d$ , that use the wavelength  $\lambda$ . Let  $T_s = \sum_{d: v_d \in D_s} T_{sd}$ .

$$\max \quad z_{\text{CR}}(y) = \sum_{v_s, v_d \in V: T_{sd} > 0} y_{sd}$$

subject to:

$$\sum_{e \in \omega^+(v_i)} y_{se}^\lambda - \sum_{e \in \omega^-(v_i)} y_{se}^\lambda = 0 \quad \lambda \in \Lambda, v_s \in V : T_s > 0, v_i \in V \setminus (D_s \cup \{v_s\}) \quad (27)$$

$$\sum_{e \in \omega^+(v_s)} y_{se}^\lambda - \sum_{e \in \omega^-(v_s)} y_{se}^\lambda = \alpha_s^\lambda \quad \lambda \in \Lambda, v_s \in V : T_s > 0 \quad (28)$$

$$\sum_{e \in \omega^+(v_d)} y_{se}^\lambda - \sum_{e \in \omega^-(v_d)} y_{se}^\lambda = -\alpha_{sd}^\lambda \quad v_s \in V : T_s > 0, v_d \in D_s, \lambda \in \Lambda \quad (29)$$

$$\sum_{\lambda \in \Lambda} \alpha_{sd}^\lambda = y_{sd}, \quad v_s \in V : T_s > 0, v_d \in D_s \quad (30)$$

$$\sum_{\lambda \in \Lambda} \alpha_s^\lambda = \sum_{v_d \in D_s} y_{sd} \quad v_s \in V : T_s > 0 \quad (31)$$

$$\sum_{v_s \in V: T_s > 0} y_{se}^\lambda \leq 1 \quad e \in E, \lambda \in \Lambda \quad (32)$$

$$y_{sd} \in \{0, 1, \dots, T_{sd}\} \quad v_s, v_d \in V : T_{sd} > 0 \quad (33)$$

$$y_{se}^\lambda \in \{0, 1\} \quad e \in E, v_s \in V : T_s > 0, \lambda \in \Lambda \quad (34)$$

$$\alpha_s^\lambda \in \mathbb{N} \quad v_s \in V : T_s > 0, \lambda \in \Lambda \quad (35)$$

$$\alpha_{sd}^\lambda \in \mathbb{N} \quad v_s, v_d \in V : T_{sd} > 0, \lambda \in \Lambda. \quad (36)$$

Constraint (27) expresses the flow conservation for connections originating from a node  $v_s$  at any node  $v_i$  corresponding to a node of the physical network, with  $v_i \neq v_s$  and  $v_i \notin D_s$ . Constraints (28) and (29) express the flow conservation at the origin  $v_s$  and at the destination nodes of  $v_s$  respectively. Constraints (30) take care of the connections from  $v_s$  to  $v_d$  that are assigned a wavelength  $\lambda$  on one of the fibers, while (31) take care of the outflows routed on lightpaths with  $\lambda$  and that are originating from  $v_s$ . Finally, constraint (32) expresses that there is at most one connection per arc and wavelength.

The number of variables of the formulation of Coudert and Rivano can be reduced with the elimination of the  $\alpha_s^\lambda$  and  $\alpha_{sd}^\lambda$  variables. First note that by summing (29) over  $v_d \in D_s$  and the resulting equality with (27) and (28), we get the equality

$$\alpha_s^\lambda = \sum_{v_d \in D_s} \alpha_{sd}^\lambda, \quad \lambda \in \Lambda, v_s \in V : T_s > 0.$$

Hence (28) can be replaced by (3.4). Constraints (31) can be removed as they are redundant with (30) and (3.4). Summing constraints (29) over  $\lambda$  and using (30) leads to the replacement of (30) by

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_d)} y_{se}^\lambda - \sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{se}^\lambda = -y_{sd} \quad v_s \in V : T_s > 0, v_d \in D_s. \quad (37)$$

At this point, variables  $\alpha_{sd}^\lambda$  and  $\alpha_s^\lambda$  appear only in (29) and (3.4). Since  $y_{se}^\lambda \in \{0, 1\}$  by (34), it follows that  $\alpha_{sd}^\lambda$  and  $\alpha_s^\lambda$  are integral. Adding inequalities that guarantee the nonnegativity of  $\alpha_{sd}^\lambda$  allows the elimination of the variables  $\alpha_{sd}^\lambda$  and  $\alpha_s^\lambda$ , leading to a more economical formulation with flows aggregated according to their origin:

$$\max \quad z_{\text{RWA-}s}(y) = \sum_{v_s, v_d \in V : T_{sd} > 0} y_{sd}$$

subject to:

$$\sum_{e \in \omega^+(v_i)} y_{se}^\lambda - \sum_{e \in \omega^-(v_i)} y_{se}^\lambda = 0 \quad \lambda \in \Lambda, v_s \in V : T_s > 0, v_i \in V \setminus (D_s \cup \{v_s\}) \quad (27)$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_d)} y_{se}^\lambda - \sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{se}^\lambda = -y_{sd} \quad v_s \in V : T_s > 0, v_d \in D_s \quad (37)$$

$$\sum_{e \in \omega^+(v_d)} y_{se}^\lambda - \sum_{e \in \omega^-(v_d)} y_{se}^\lambda \leq 0 \quad \lambda \in \Lambda \quad v_s \in V : T_s > 0, v_d \in D_s \quad (38)$$

$$\sum_{v_s \in V : T_s > 0} y_{se}^\lambda \leq 1 \quad e \in E, \lambda \in \Lambda \quad (32)$$

$$y_{sd} \in \{0, 1, \dots, T_{sd}\} \quad v_s, v_d \in V : T_{sd} > 0 \quad (39)$$

$$y_{se}^\lambda \in \{0, 1\} \quad e \in E, v_s \in V : T_s > 0, \lambda \in \Lambda. \quad (34)$$

**Theorem 3.** *The optimal values of the LP relaxations of (RWA<sub>sd</sub>) and (RWA<sub>s</sub>) are equal:*

$$\bar{z}_{\text{RWA}_{sd}} = \bar{z}_{\text{RWA}_s}.$$

*Proof.* Let us first show that  $\bar{z}^{\text{RWA}_s} \geq \bar{z}^{\text{RWA}_{sd}}$  starting from a solution of  $\Omega_{\text{LP}}(\text{RWA}_{sd})$  and deducing from it a solution for  $\Omega_{\text{LP}}(\text{RWA}_s)$  with same value. Let  $(y_{sd}, y_{sd}^\lambda) \in \Omega_{\text{LP}}(\text{RWA}_{sd})$ . Define

$$y_{se}^\lambda = \sum_{v_d \in D_s} y_{sde}^\lambda, \quad \lambda \in \Lambda, v_s \in V : T_s > 0, e \in E.$$

Let us show that  $(y_{sd}, y_{se}^\lambda) \in \Omega_{\text{LP}}(\text{RWA}_s)$ .

$$\begin{aligned} & \sum_{e \in \omega^+(v_d)} y_{se}^\lambda - \sum_{e \in \omega^-(v_d)} y_{se}^\lambda \\ &= \sum_{e \in \omega^+(v_d)} \sum_{v_{d'} \in D_s} y_{sd'e}^\lambda - \sum_{e \in \omega^-(v_d)} \sum_{v_{d'} \in D_s} y_{sd'e}^\lambda \\ &= \sum_{v_{d'} \in D_s} \left( \sum_{e \in \omega^+(v_d)} y_{sd'e}^\lambda - \sum_{e \in \omega^-(v_d)} y_{sd'e}^\lambda \right) \\ &= - \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda \leq 0 \end{aligned} \quad (40)$$

where we used (10) for  $v_{d'} \neq v_d$  and (15) for  $v_{d'} = v_d$ . Hence constraints (38) are satisfied. Summing (40) over  $\lambda$  and using (11) shows that (37) are satisfied. (27) follows easily from (10). Finally, using (12), we get

$$\sum_{v_s \in V : T_s > 0} y_{se}^\lambda = \sum_{v_s \in V : T_s > 0} \sum_{v_d \in D_s} y_{sde}^\lambda = \sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} y_{sde}^\lambda \leq 1,$$

which shows that constraints (32) are also satisfied. Hence  $(y_{sd}, y_{se}^\lambda) \in \Omega_{\text{LP}}(\text{RWA}_s)$ . Noting that the objective value remains unchanged, we get the inequality  $\bar{z}^{\text{RWA}_s} \geq \bar{z}^{\text{RWA}_{sd}}$ .

Let us now establish the reverse inequality. We use a technique similar to the one used for proving Theorem 2. Let  $v_s \in V$  be fixed. We first show how to represent the vector

$y_{se}^\lambda$  ( $\lambda \in \Lambda, e \in E$ ) by a set of paths from  $v_s$  to the destination nodes of  $v_s$ . Define  $\beta_d = y_{sd}$  for  $v_d \in D_s$  and  $\beta_e^\lambda = y_{se}^\lambda$  for  $\lambda \in \Lambda$  and  $e \in E$ . By (27), (37) and (38), we have

$$\sum_{e \in \omega^+(v_i)} \beta_e^\lambda = \sum_{e \in \omega^-(v_i)} \beta_e^\lambda \quad \lambda \in \Lambda, \quad v_i \in V \setminus (D_s \cup \{v_s\}) \quad (41)$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} \beta_e^\lambda - \sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_d)} \beta_e^\lambda = \beta_d \quad v_d \in D_s \quad (42)$$

$$\sum_{e \in \omega^+(v_d)} \beta_e^\lambda - \sum_{e \in \omega^-(v_d)} \beta_e^\lambda \leq 0 \quad \lambda \in \Lambda, v_d \in D_s. \quad (43)$$

Let  $v_{\tilde{d}} \in D_s$  such that  $\beta_{\tilde{d}} > 0$ . Then there exist  $\tilde{e}_1 \in \omega^-(v_{\tilde{d}})$  and  $\tilde{\lambda}_1 \in \Lambda$  such that  $\beta_{\tilde{e}_1}^{\tilde{\lambda}_1} > 0$ . Assume that  $\tilde{e}_1 = (v_{i_1}, v_{\tilde{d}})$ . By (41) or (43) depending whether  $v_{i_1}$  is one of the destination nodes of  $v_s$  or not, there exist  $\tilde{e}_2 \in \omega^-(v_{i_1})$  and  $\tilde{\lambda}_1 \in \Lambda$  such that  $\beta_{\tilde{e}_2}^{\tilde{\lambda}_1} > 0$ . Denote by  $v_{i_2}$  the vertex such that  $\tilde{e}_2 = (v_{i_2}, v_{i_1})$ . We repeat this process until one of the following 2 cases occur: a) we revisit an already explored vertex or b) we reach  $v_s$ . Note that one of these 2 cases necessarily occur after at most  $n$  steps. In the case a), we have generated a circuit. Denote by  $c$  this circuit and by  $E(c)$  the set of corresponding arcs. Let  $\delta = \min_{e \in E(c)} \{\beta_e^{\tilde{\lambda}_1}\}$ .

Observe that  $\delta > 0$  by construction. We redefine  $\beta_e^{\tilde{\lambda}_1} \leftarrow \beta_e^{\tilde{\lambda}_1} - \delta$  for  $e \in E(c)$ . Note that the resulting vector  $(\beta_d, \beta_e^\lambda)$  still satisfies (41)-(43). Since at least one  $\beta_e^\lambda$  has been set to 0, the case a) can appear at most  $mW$  times.

Assume now that the case b) occurred. The process has defined a path  $p$  from  $v_s$  to  $v_{\tilde{d}}$ . Let  $E(p)$  be the set of corresponding arcs. We define  $\delta = \min\{\beta_{\tilde{d}}, \min_{e \in E(p)} \{\beta_e^{\tilde{\lambda}_1}\}\}$ . Again  $\delta > 0$  by construction. We redefine  $\beta_{\tilde{d}} \leftarrow \beta_{\tilde{d}} - \delta$  and  $\beta_e^{\tilde{\lambda}_1} \leftarrow \beta_e^{\tilde{\lambda}_1} - \delta$  for  $e \in E(p)$ . Note that the resulting vector  $(\beta_d, \beta_e^\lambda)$  still satisfies (41)-(43), and that either  $\beta_{\tilde{d}} = 0$  or at least one  $\beta_e^\lambda$  has been set to 0. Hence step b) cannot occur more than  $mW$  times. We repeat this for each  $d$  such that  $\beta_d > 0$ .

At the end of all this process, we have a collection  $\left\{p : p \in \bigcup_{v_d \in D_s} \mathcal{L}_d\right\}$  of paths  $p$  originating at  $v_s$  with wavelength  $\lambda(p)$  and value  $\delta(p)$  such that

$$y_{sd} = \sum_{p \in \mathcal{L}_d} \delta(p), \quad (44)$$

$$y_{se}^\lambda \geq \sum_{p \in \mathcal{L}: e \in p, \lambda(p) = \lambda} \delta(p), \quad \lambda \in \Lambda, \quad e \in E \quad (45)$$

where  $\mathcal{L}_d$  is the set of paths from  $v_s$  to  $v_d$  for  $v_d \in D_s$  (the inequality in (45) comes from the deletion of circuits with nonnegative values). Define

$$y_{sde}^\lambda = \sum_{p \in \mathcal{L}_d: \lambda(p) = \lambda, e \in p} \delta(p), \quad \lambda \in \Lambda, e \in E. \quad (46)$$

Note that

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} \sum_{p \in \mathcal{L}_d: \lambda(p)=\lambda, e \in p} \delta(p) = \sum_{p \in \mathcal{L}_d} \delta(p),$$

hence from (44) we deduce

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda = y_{sd}.$$

Since each path  $p \in \mathcal{L}_d$  such that  $\lambda(p) = \lambda$  satisfies the constraints (10), (15) and

$$\sum_{e \in \omega^+(v_s)} y_{sde}^\lambda = \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda,$$

it follows that (10), (11) and (15) are satisfied. Since

$$\begin{aligned} \sum_{(v_s, v_d) \in V \times V: T_{sd} > 0} y_{sde}^\lambda &= \sum_{v_s \in V: T_s > 0} \sum_{p \in \mathcal{L}: e \in p, \lambda(p) = \lambda} \delta(p) \\ &\leq \sum_{v_s \in V: T_s > 0} y_{se}^\lambda \end{aligned}$$

by (45), it follows from (32) that (12) are satisfied. Hence  $(y_{sd}, y_{sde}^\lambda)$  is a feasible solution to  $\Omega_{LP}(\text{RWA}_{sd})$ . Since the objective value depends only on the variables  $y_{sd}$ , the value of the objective function remains unchanged. Hence the result.  $\square$

### 3.5 Comparison of the Three Link Formulations

Although the three formulations (RWA), (RWA<sub>sd</sub>) and (RWA<sub>s</sub>) differ with respect to the analytical expression of their constraints, they are equivalent in terms of the upper bounds provided by their LP relaxations. The comparison of their number of variables and constraints is summarized in Table 1. For the comparison, we consider the most economical formulations in terms of variables, i.e.,

- formulation (RWA) in which variables  $x_k$  have been eliminated and constraints (5) replaced by (5a), leading to the objective function  $z'_{\text{RWA}}$  as defined in (9);
- formulation (RWA<sub>sd</sub>) in which variables  $y_{sd}$  have been eliminated and constraints (11) replaced by (11a), leading to the objective function expression  $z'_{\text{RWA}_{sd}}$  as defined in (16);
- formulation (RWA<sub>s</sub>) in which variables  $y_{sd}$  have been eliminated and constraints (37) replaced by

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{se}^\lambda - \sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_d)} y_{se}^\lambda \leq T_{sd} \quad v_s \in V : T_s > 0, v_d \in D_s, \quad (37a)$$

leading to the objective function

$$z'_{\text{RWA}_s}(y) = \sum_{v_s, v_d \in V: T_{sd} > 0} \left( \sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{se}^\lambda - \sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_s)} y_{se}^\lambda \right). \quad (47)$$

In order to evaluate the number of variables and constraints of these formulations, we assume that there is at least one connection request for each pair of origin-destination. This implies that  $|K| \geq n(n-1)$ . An upper bound on  $|K|$  is obtained by noting that  $T_{sd}$  can be bounded from above by  $mW$ . Indeed  $mW$  is the maximum number of lightpaths that can be established on the network. If  $T_{sd} > mW$ , we can reject without loss of generalization  $T_{sd} - mW$  connections. We deduce  $|K| \leq n(n-1)mW$ . The number of variables and constraints for each link formulation is reported in Table 1. We observe that both in terms of variables and constraints the source formulation (RWA\_s) is the best one, followed by (RWA\_sd). Formulation (RWA) is the worst one, except if we need to consider hop constraints, see Section 5 for more details.

# (binary) variables	
(RWA)	$ K W(1+m)$ $= O(mW K )$
(RWA_sd)	$mn(n-1)W$ $= O(n^2mW)$
(RWA_s)	$nmW$ $= O(nmW)$
# constraints	
(RWA)	$(n+m+2)W K  + mW +  K $ $= O((n+m)W K )$
(RWA_sd)	$n(n-1)(n-2)W + 2n(n-1) + mW$ $= O((n^3+m)W)$
(RWA_s)	$n(n-1)(W+2) + mW$ $= O((n^2+m)W)$

Table 1: Number of Variables and Constraints in the Different Formulations for the Asymmetrical RWA Problem.

## 4 Path ILP Formulations

In the previous section, we discussed upper bounds derived from LP (i.e. linear programs) relaxations of formulations of the RWA problem using link variables. In this section, we discuss other upper bounds derived from formulations using path variables.

#### 4.1 PATH Formulation

Let  $\mathcal{P}$  be the overall set of elementary (e.g., with no loop) paths from  $v_s$  to  $v_d$  for all pairs  $(v_s, v_d)$  of source and destination nodes such that  $T_{sd} > 0$ . It can be decomposed as follows:  $\mathcal{P} = \bigcup_{sd} \mathcal{P}_{sd}$  where  $\mathcal{P}_{sd}$  corresponds to the set of potential paths for node pair  $v_s, v_d$ . We also define the parameters

$$a_{ep} = \begin{cases} 1 & \text{if arc } e \text{ belongs to path } p \\ 0 & \text{otherwise.} \end{cases}$$

for all  $p \in \mathcal{P}$  and  $e \in E$ . We then have this following path formulation, denoted PATH, that has been already proposed by several authors, see, e.g., Lee *et al.* [19] or Saad and Luo [21]:

$$\max \quad z_{\text{PATH}}(x) = \sum_{\lambda \in \Lambda} \sum_{p \in \mathcal{P}} x_p^\lambda$$

subject to:

$$\sum_{p \in \mathcal{P}} a_{ep} x_p^\lambda \leq 1 \quad e \in E, \lambda \in \Lambda \quad (48)$$

$$\sum_{\lambda \in \Lambda} \sum_{p \in \mathcal{P}_{sd}} x_p^\lambda \leq T_{sd} \quad (v_s, v_d) \in V \times V : T_{sd} > 0 \quad (49)$$

$$x_p^\lambda \in \{0, 1\} \quad p \in \mathcal{P}, \lambda \in \Lambda. \quad (50)$$

Constraints (48) correspond to the clash constraints, i.e., they express that there is at most one lightpath going through each pair  $(e, \lambda)$ , and (49) are the demand constraints.

While the (PATH) formulation may seem to suffer from a large number of variables, it can be solved by column generation techniques in which only a small subset of columns (i.e. variables) need to be on hand at each iteration, see [31] for more details. Moreover additional constraints like the hop constraints (see Section 5) can reduce the number of paths that have to be considered.

#### 4.2 A Relaxation of the PATH Formulation

Let us consider a relaxation of the (PATH) formulation, denoted by (R $\lambda$ -PATH), in which we care only for identifying a path for each connection request that satisfies the link capacities, but do not assign any wavelength to it. We introduce the set of variables  $x_p$  for  $p \in \mathcal{P}$  where  $x_p$  defines the number of connections carried out by path  $p$ . An upper bound can be obtained by solving the following integer linear program, denoted by (R $\lambda$ -PATH)

$$\max \quad z_{\text{R}\lambda\text{-PATH}}(x) = \sum_{p \in \mathcal{P}} x_p$$

subject to:

$$\sum_{p \in \mathcal{P}} a_{ep} x_p \leq W \quad e \in E \quad (51)$$

$$\sum_{p \in \mathcal{P}_{sd}} x_p \leq T_{sd} \quad (v_s, v_d) \in V \times V \quad (52)$$

$$x_p \in \{0, 1, \dots, W\} \quad p \in \mathcal{P}, \quad (53)$$

where the first constraint reinforces the capacity constraint for each link: no more than  $W$  lightpaths per link, the second constraint corresponds to satisfying at most the requested connections, and then the integrality condition for variables  $x_p$ . This problem is sometimes referred to as the circuit-switched routing problem, see, e.g., Ramaswami and Sivaraman [22].

### 4.3 Comparison of the Upper Bounds Provided by the Path Formulations

**Theorem 4.** *Assume that  $\mathcal{P}_{sd}$  contains all possible paths from  $v_s$  to  $v_d$  for  $(v_s, v_d) \in V \times V$  such that  $T_{sd} > 0$ . Then the optimal values of the LP relaxations of (RWA<sub>sd</sub>) and (PATH) are equal:*

$$\bar{z}_{\text{RWA}_{sd}} = \bar{z}_{\text{PATH}}.$$

*Proof.* Let us first show that  $\bar{z}_{\text{RWA}_{sd}} \geq \bar{z}_{\text{PATH}}$ . We denote by  $\Omega_{\text{LP}}(\text{RWA}_{sd})$  and  $\Omega_{\text{LP}}(\text{PATH})$  the feasible domain of the LP relaxations of (RWA<sub>sd</sub>) and (PATH) respectively. Consider a feasible solution  $x_p^\lambda, p \in \mathcal{P}, \lambda \in \Lambda$  of  $\Omega_{\text{LP}}(\text{PATH})$  and let us derive from it a feasible solution from  $\Omega_{\text{LP}}(\text{RWA}_{sd})$  with an identical value. We define the variables  $y_{sd}$  and  $y_{sde}^\lambda$  as follows:

$$y_{sd} = \sum_{\lambda \in \Lambda} \sum_{p \in \mathcal{P}_{sd}} x_p^\lambda, \quad (v_s, v_d) \in V \times V : T_{sd} > 0$$

$$y_{sde}^\lambda = \sum_{p \in \mathcal{P}_{sd}} a_{ep} x_p^\lambda \quad (v_s, v_d) \in V \times V : T_{sd} > 0,$$

$$\lambda \in \Lambda, e \in E.$$

Let us consider constraint (48) of the (PATH) formulation. We have:

$$\begin{aligned} \sum_{p \in \mathcal{P}} a_{ep} x_p^\lambda &= \sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} \sum_{p \in \mathcal{P}_{sd}} a_{ep} x_p^\lambda \\ &= \sum_{(v_s, v_d) \in V \times V : T_{sd} > 0} y_{sde}^\lambda \leq 1, \quad e \in E, \lambda \in \Lambda. \end{aligned}$$



Hence constraints (12) are satisfied. Consider next constraint (49) of the (PATH) formulation. As

$$\sum_{e \in \omega^+(v_s)} a_{ep} = 1 \quad p \in \mathcal{P}_{sd} \quad (54)$$

$$\sum_{e \in \omega^-(v_s)} a_{ep} = 1 \quad p \in \mathcal{P}_{sd}, \quad (55)$$

we deduce:

$$\sum_{\lambda \in \Lambda} \sum_{e \in \omega^+(v_s)} y_{sde}^\lambda = \sum_{\lambda \in \Lambda} \sum_{e \in \omega^-(v_d)} y_{sde}^\lambda = \sum_{\lambda \in \Lambda} \sum_{p \in \mathcal{P}_{sd}} x_p^\lambda = y_{sd},$$

hence constraints (11) are satisfied. By definition of an elementary path, constraint (15) is satisfied. At last, due to path flow conservation, we easily deduce that constraints (10) are also satisfied, hence the variables  $(y_{sd}, y_{sde}^\lambda)$  define a feasible solution to  $\Omega_{LP}(RWA_{sd})$ , with same objective value.

The reverse inequality has essentially been shown in the second part of the proof of Theorem 2.  $\square$

The following result was proved by Ramaswami and Sivaraman [22].

**Theorem 5.** *The optimal values of the LP relaxations of (R $\lambda$ \_PATH) and (PATH) are equal:*

$$\bar{z}_{R\lambda\_PATH} = \bar{z}_{PATH}.$$

*Proof.* Clearly (R $\lambda$ \_PATH) is a relaxation of (PATH), hence we deduce  $\bar{z}_{R\lambda\_PATH} \geq \bar{z}_{PATH}$ . Consider now a feasible solution  $(x_p)$  of the continuous relaxation of (R $\lambda$ \_PATH). We define

$$x_p^\lambda = \frac{x_p}{W}, \quad \lambda \in \Lambda, \quad p \in \mathcal{P}.$$

Clearly  $(x_p^\lambda)$  is a feasible solution to  $\Omega_{LP}(PATH)$  with same objective value. Hence  $\bar{z}_{PATH} \geq \bar{z}_{R\lambda\_PATH}$ .  $\square$

In the sequel, we will denote by  $\bar{z}$  the common value of the continuous relaxation of all formulations considered in this paper, and by  $z_{R\lambda\_PATH_\ell}^*$  the upper bound obtained by solving optimally the formulation (R $\lambda$ \_PATH) with the set of paths restricted to those of length  $\leq \ell$ .

## 5 Further Comparisons and Hop Constraints

For, e.g., delay consideration, it may be interesting to bound the number of hops of the lightpaths. Assuming that the number of hops of a lightpath is defined as the number of links in the path, observe that not all formulations considered in this paper allow the inclusion of this constraint.

Let  $H$  be the upper bound on the number of hops. The formulation that is the most easy to modify is the (PATH) formulation: instead of considering all possible paths, consider only the paths with number of hops less than or equal to  $H$ .

The hop constraints can be included in formulation (RWA) through the following constraints

$$\sum_{\lambda \in \Lambda} \sum_{e \in E} x_{ke}^{\lambda} \leq H, \quad k \in K. \quad (56)$$

In contrast, it is not possible to include the hop constraints in formulations (RWA\_sd) and (RWA\_s) as the variables in these formulations do not refer to an unique path.

## 6 Computational Experience

We compare the values of the upper bounds discussed in the previous sections together with the optimal ILP value, denoted by  $z^*$  on eight RWA problem instances, six with non-uniform traffic matrices and two with uniform traffic.

We consider three optical networks, two of them widely used in the literature, the NSF and the EON networks in addition to the Brazil network available, e.g., in [12]. The NSF network is a network with 14 nodes and 21 optical links, with a maximum of 4 links per node, and is, e.g. described in [6]. Miyao and Saito [32] consider a slight variation of the NSF network where they have added one link in order to make each local degree more than two. The EON network is described in, e.g., Mahony *et al.*; it is a network with 20 nodes and 39 optical links, with a maximum of 7 links per node. Fumagalli *et al.* [33] consider a slight variation of the EON network where they have removed one node of degree 2 but have added two links. The Brazil network contains 27 nodes and 70 optical links with a maximum of 10 links per node.

We used six non-uniform traffic matrices. Two of them, matrices  $T^1$  and  $T^4$  come from Krishnaswamy [27]. They correspond to 268 connections for the NSF instance, and 374 for the EON one. We generated a second traffic matrix,  $T^2$ , for the NSF network with around twice the number of connections than in the Krishnaswamy one, with a random number of connections between 0 and 6 for each pair of origin and destination nodes. Similarly, we randomly generated a traffic matrix  $T^6$  for the Brazil network with a random number of connections between 0 and 3 for each pair of origin and destination nodes. All traffic matrices are available in [34]. The last two non-uniform traffic matrices  $T^3$  and  $T^5$  come from [32] and [33] respectively.

We observe that on all 8 instances, the three values,  $\bar{z}$ , the LP relaxation upper bound,  $z_{R\lambda\_PATH\_l}^*$ , the optimal ILP value of the  $\lambda$ -relaxation of the PATH formulation introduced in Section 4.2 and  $z^*$ , the ILP optimal value for the original RWA problem (computed using CPLEX-MIP [23] and the formulation (RWA\_s)) are very close and equal most of the time. Indeed the only instance for which we observe a difference between  $\bar{z}$  and  $z_{R\lambda\_PATH\_l}^*$  in our experiments is for the Brazil network with non-uniform traffic and 10 wavelengths. For a

difference between  $z_{R\lambda\_PATH}^*$  and  $z^*$ , let us illustrate it on the following small example. Let us consider the physical network represented on Figure 3 together with the traffic matrix of Figure 4, the routing provided by  $z_{R\lambda\_PATH}^*$  do not lead necessarily to feasible RWA solution. We assume that  $W = 2$ . Plain arrows corresponds to directional fibers. Dashed and dotted arrows to lightpaths, the dashed ones to those with wavelength  $\lambda_1$ , the dotted ones to those with wavelength  $\lambda_2$ . Observe that connection  $(v_5, v_2)$  cannot be assigned a wavelength, whereas it is possible to find 5 routing paths satisfying the capacity constraints for each arc, leading to  $\bar{z} = 5$ ,  $z_{R\lambda\_PATH}^* = 5$  and  $z^* = 4$ .

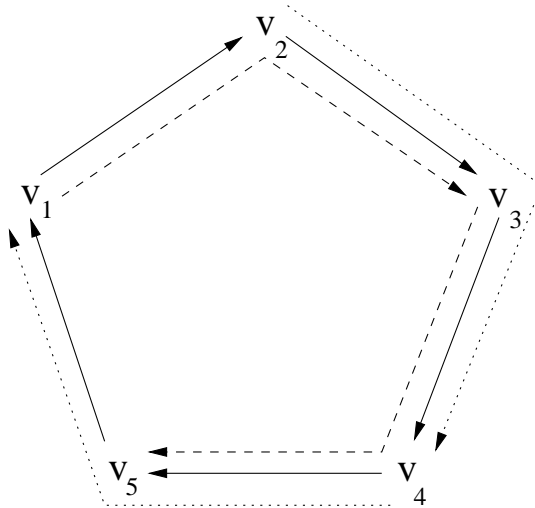


Figure 3: A small network.

	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$v_1$	-	0	1	0	0
$v_2$	0	-	0	1	0
$v_3$	0	0	-	0	1
$v_4$	1	0	0	-	0
$v_5$	0	1	0	0	-

Figure 4: Traffic matrix of the example

Even if the optimal values of the LP relaxation and the ILP formulations are almost always equal, it does not entail that rounding off procedures on the optimal LP relaxation solutions lead to the optimal or near optimal ILP solutions. For instance, looking at the results obtained by Krishnaswamy and Sivarajan [5] on formulations (KS1) and (KS2), we observe that most integer solutions derived from rounding off procedures do not provide the optimal ILP solutions except for some rare cases, and that gaps between the rounding off and the optimal ILP solutions vary from 1 to 6%.

In terms of computational effort,  $\bar{z}$  is the easiest value to compute as it corresponds to the solution of a LP program. The most economical formulation to compute it, as well as  $z^*$ , corresponds to the (RWA\_s) formulation if no hop constraints are considered. Moreover, the solution of  $z_{R\lambda\_PATH\_l}^*$  is also easy to compute in practice even if it corresponds to the optimal solution of an ILP program: a very limited number of nodes need to be developed in the branch-and-bound search tree when using the CPLEX-MIP package. The computation of  $z^*$  requires a couple on minutes for all instances except for those involving the Brazil network that required between 1 and 2 hours for uniform traffic, and from 1 hour (10 wavelengths) to 7 days (14 wavelengths) for non uniform traffic. All experiments have been done on a Linux Dell machine with a Pentium 4 and the CPLEX-MIP version 8.1 of ILOG [23].

<b>nsf Network - Krishnaswamy non-uniform traffic data [27]</b>						
# wavelengths	Non-Uniform Traffic - Matrix $T^1$			Uniform Traffic		
	$\bar{z}$	$z_{R\lambda\_PATH\_5}^*$	$z^*$	$\bar{z}$	$z_{R\lambda\_PATH\_5}^*$	$z^*$
10	198.0	198.0	198.0	164.0	164.0	164.0
12	218.0	218.0	218.0	180.0	180.0	180.0
14	238.0	238.0	238.0	182.0	182.0	182.0
16	258.0	258.0	258.0	182.0	182.0	182.0
18	267.0	267.0	267.0	182.0	182.0	182.0
$\geq 20$	268.0	268.0	268.0	182.0	182.0	182.0

<b>nsf network - Traffic data <math>T^2</math></b>			
# wavelengths	Non-Uniform Traffic		
	$\bar{z}$	$z_{R\lambda\_PATH\_5}^*$	$z^*$
26	439.0	439.0	439.0
28	453.0	453.0	453.0
30	467.0	467.0	467.0
32	481.0	481.0	481.0

<b>nsf network and traffic data <math>T^3</math> adapted from [32]</b>			
# wavelengths	Non-Uniform Traffic		
	$\bar{z}$	$z_{R\lambda\_PATH\_5}^*$	$z^*$
10	172.0	172.0	172.0
12	190.0	190.0	190.0
14	208.0	208.0	208.0

<b>eon network - Krishnaswamy traffic data <math>T^4</math> [27]</b>						
Non-Uniform Traffic						
# wavelengths	$\bar{z}$	$z_{R\lambda\_PATH\_7}^*$	$z^*$			
10	285.0	285.0	285.0			
12	317.0	317.0	317.0			
14	337.0	337.0	337.0			
16	350.0	350.0	350.0			
18	362.0	362.0	362.0			
20	370.0	370.0	370.0			
22	374.0	374.0	374.0			
24	374.0	374.0	374.0			
<b>eon network and traffic data <math>T^5</math> adapted from [33]</b>						
Non-Uniform Traffic						
# wavelengths	$\bar{z}$	$z_{R\lambda\_PATH\_7}^*$	$z^*$			
10	393.0	393.0	393.0			
12	447.0	447.0	447.0			
14	498.0	498.0	498.0			
<b>brazil network</b>						
Non-Uniform Traffic - <b>Matrix <math>T^6</math></b>						
# wavelengths	$\bar{z}$	$z_{R\lambda\_PATH\_7}^*$	$z^*$	Uniform Traffic		
				$\bar{z}$	$z_{R\lambda\_PATH\_7}^*$	$z^*$
10	721.5	721.0	721.0	560.0	560.0	560.0
12	800.0	800.0	800.0	614.0	614.0	614.0
14	872.0	872.0	872.0	630.0	630.0	630.0

Table 2: Comparison of the Upper Bounds with the Optimal ILP Values

## 7 Conclusion

Although several ILP formulations have been proposed for the RWA problem in the literature, we show in this paper that their LP relaxations all provide the same upper bounds. Moreover, the optimal values of the LP relaxations are most of the time very close or equal to the optimal values of the ILP formulations leading to some hopes that it might be no

so difficult to solve exactly some instances, with a proof of the optimality of the solutions. Comparing the values of some of the upper bounds with heuristic solutions available in the literature for some of the instances (e.g., [11] recently improved by [35]), we observe that some heuristics do provide some optimal solutions for some instances, but without any optimality proof.

An important side contribution of the paper is a review of the various integer linear programming (ILP) formulations of the RWA problem considering the objective of minimizing the blocking rate with asymmetrical traffic models, with a unified and simplified notation. We show that depending on whether some constraints are considered or not (e.g., the hop constraints), the best formulation is not the same. Moreover, we show that further improvements are possible for some formulations in order to reduce their number of variables and constraints. A complementary paper on the particular case of symmetrical traffic models is also available from the same authors, see [4], where we consider several objectives, not only the blocking rate. For asymmetrical traffic matrices, further work is needed to study the ILP formulations with other objectives such as the cost of the multiplexing equipments or the minimum number of wavelengths per link (i.e., transport capacities).

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