Generating Redundantly Rigid Frameworks in 2-dimension

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Combinatorial rigidity plays an important role in designing rigid frameworks and analyzing rigidity of frameworks. In designing such frameworks, it is crucial to take into account redundancy of frameworks when we consider failure of components. However, the theoretical work on how we design frameworks with certain redundauncy has not been studied well. This is our motivation.

A bar-joint framework is modeled as a graph $G = (V, E)$ by neglecting coordinates of joints such that a vertex and an edge represent a joint and a bar connecting two joints. The graphs is called a bar-joint graph. We study redundantly rigid bar-joint graphs in 2-dimension. In 2-dimension, Laman theorem tells us whether a given framework is rigid or not can be combinatorially characterized. A framework with n points is called minimally rigid if it is rigid and consists of $2n - 3$ bars. A graph $G = (V, E)$ is called k-edge-rigid if deletion of any $k - 1$ edges from G still preserves the rigidity of G. For $k = 2$, Garcia et al. studied how to augment Laman graph (minimally rigid) graph to a 2-edge-rigid graph with minimum number of added edges.

We will propose an algorithm for enumerating 4-regular 3-edge-rigid simple graphs $G = (V, E)$ for all $|V| \geq 5$. Since vertex degree is at least 4 for any 3-edge-rigid graph, graphs we construct have minimum number of edges among all 3-edge-rigid graphs for a fixed number of vertices. Starting with K_5, the algorithm perfoms an X-replacement operation to a 4-regular 3-edge-rigid simple graph with n vetices that (i) chooses 2 vertex disjoint edges, (ii) adds a new vertex v, and (iii) connects v to endvertices of chosen 2 edges. We prove that this operation produces another 4-regular 3-edge-rigid simple graph with $n + 1$ vertices. Conversely, we show that any 4-regular 3-edge-rigid simple graph with at least 5 vertices can be constructed by a series of X-replacements.

We also mention an algorithm for generating 6-regular 5-edge-rigid graphs which is based on what we call $*$-operation. However, it is not yet proved whether the algorithm generates all 6-regular 5-edge-rigid graphs.

We will finally address many important open problems such as (1) Minimum-cost redundancy augmentation from Laman graph to k-edge-rigid graph. (2) Combinatorial characterization of k-vertex rigid graphs. (3) Minimum-cost redundancy augmentation from minimally rigid body-hinge (or panel-hinge) graphs in 3-dimension to k-hinge-rigid or k-body-rigid (or k-panel-rigid) graphs.

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