19th International Symposium on Dynamic Games and Applications

July 25-28, 2022

INTERNATIONAL PROGRAM COMMITTEE

Steve Alpern, UK
Eitan Altman, France
Rabah Amir, USA
Joe Apaloo, Canada
Martino Bardi, Italy
Michèle Breton, Canada
Mark Broom, City, UK (co-chair)
Joel Brown, USA
João Correia da Silva, Portugal
Ross Cressman, University, Canada
Jozsef Garay, Hungary
Elena Gubar, Russia
Astrid Hilbert, Sweden
Anna Jaskiewicz, Poland
Vlastimil Krivan, Czech Republic
Peter Markowich, Saudi Arabia
Guiomar Martín-Herrán, Spain
Vladimir Mazalov, Russia
Elena Parilina, Russia
Valerii Patsko, Russia
Leon Petrosyan, Russia
Alberto A. Pinto, Portugal(co-chair)
David Ramsey, Poland
Jérôme Renault, France
Joana Resende, Portugal
Francisco Silva, France
Katerina Stankova, The Netherlands
Tristan Tomala, France
Corinne Touati, France
Georges Zaccour, Canada
Jorge Zubelli, Brazil

ORGANIZING COMMITTEE

Alberto Pinto (chair)
Elvio Accinelli Gamba
Diogo Aguiar Gomes
João Paulo Almeida
Alberto Alvarez López
António Miguel Arantes da Silva
Gian Italo Bischi
Fabio Chalub
João Costa Freitas
Rui Manuel da Costa Miranda
Max Oliveira De Souza
Dalila B.M.M. Fontes
Sílvio M.A. Gama
Filomena Garcia
Ignacio García Jurado
Carlos Hervés-Beloso
Penelope Hernandez
José María Labeaga Azcona
Fabio Lamantia
Fernando Lobo Pereira
Filipe Martins
José Martins
Elisabetta Michetti
Bruno Oliveira
António M. Osório Costa
Jorge Oviedo
Luis Quintas
Renato Soeiro
Carlos Ramos
Delfim F. M. Torres
Marta Cristina Vieira Faias Mateus

PLENARY SPEAKERS

Vivek Borkar, India
Arne Traulsen, Germany

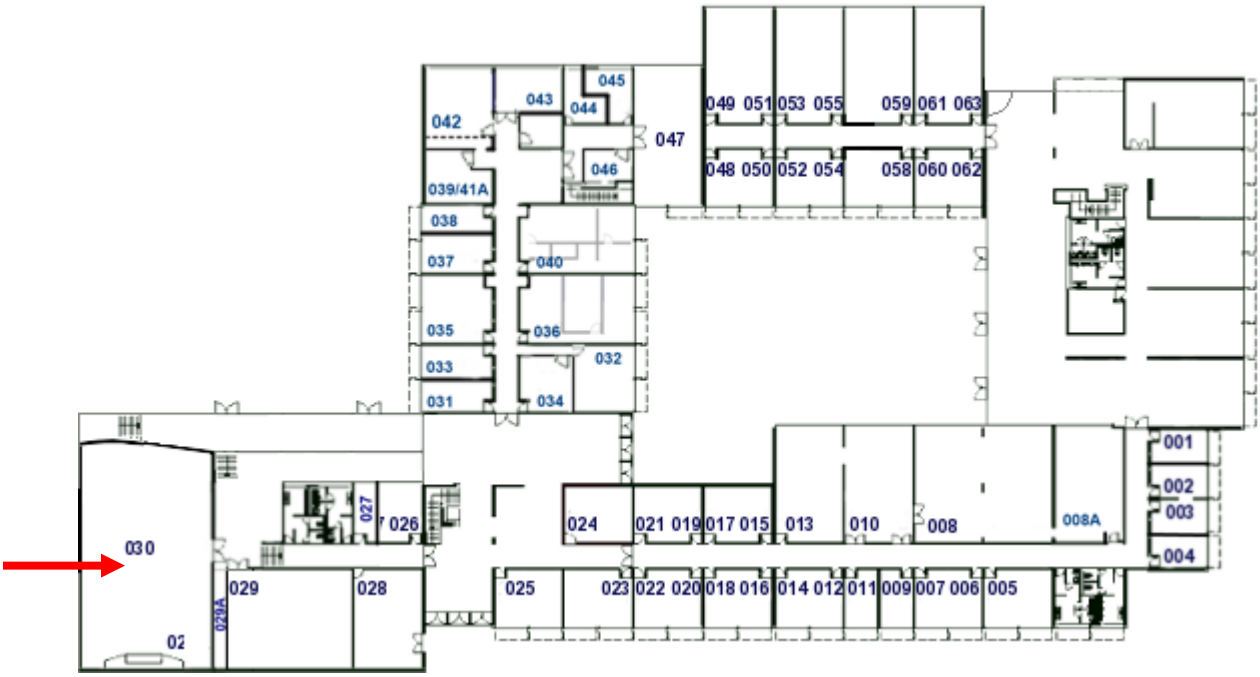
TUTORIALS

Margarida Carvalho, Canada
Thomas Lidbetter, USA

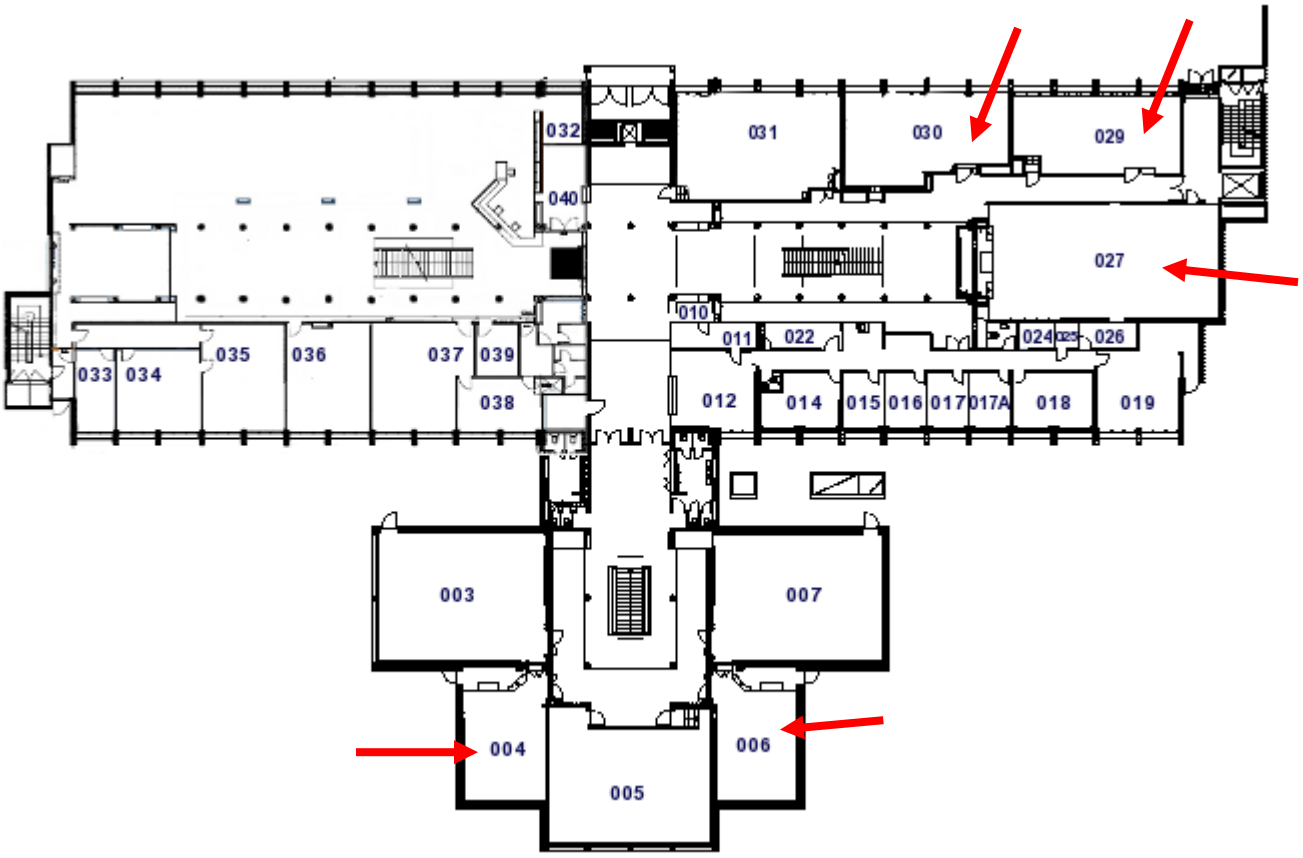
PRACTICAL INFORMATION

Conference Venue

Porto University
Edifício de Ciência de Computadores (FC6)



Edifício de Matemática (FC1)

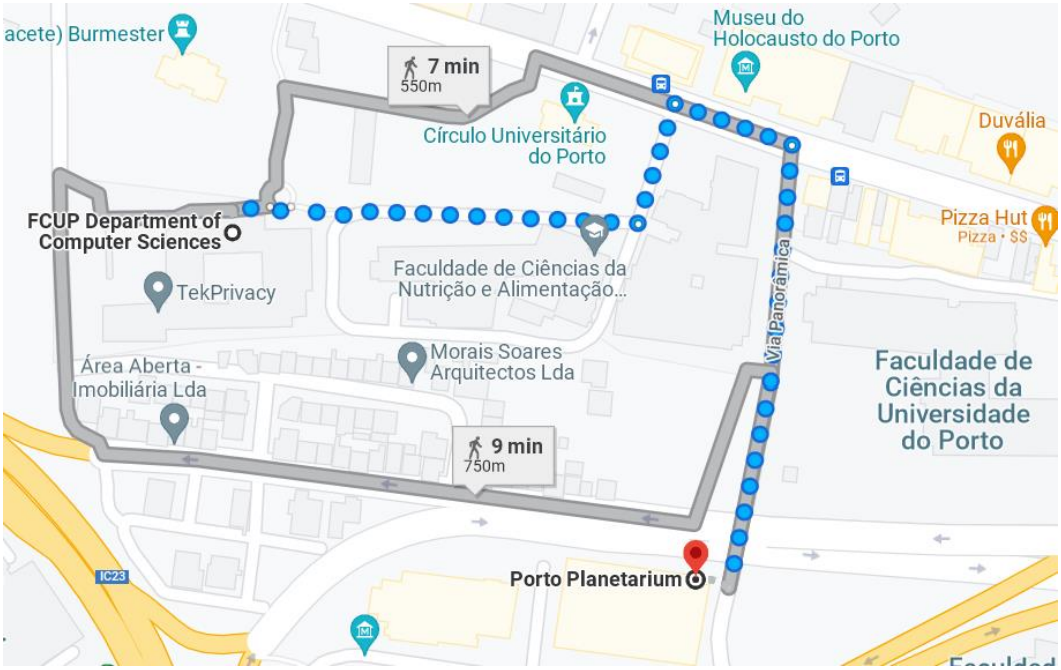


Coffee Breaks are at the bar of DM and FCUP.

The lunches, cocktail and farewell will be in the bar of the planetarium.

Planetário do Porto - Centro Ciência Viva
Rua das Estrelas, 4150-762 Porto
22 608 9800

Walking path between the venue of the conference, DM, FCUP, and the bar of the planetarium.



PROGRAM OVERVIEW

Mon, July 25	FC6 Edifício de Ciência de Computadores FC6 030
8:00 – 9:00	Registration
9:00 – 9:15	Welcome address
09:15 – 10:15	Plenary I Chair: Mark Broom **Arne Traulsen** *Stochastic evolutionary game dynamics in finite populations*
10:15 – 10:45	Coffee Break
10:45 – 12:15	Tutorial I Chair: Alberto Pinto **Margarida Carvalho** *Integer programming tools for games*
12:15 – 14:00	Lunch
14:00 – 15:30	Tutorial II Chair: Vladimir Mazalov **Thomas Lidbetter** *An introduction to search games*
15:30 – 16:00	Coffee Break
16:00 – 17:00	Plenary II Chair: Georges Zaccour **Vivek Borkar** *Some interesting dynamics motivated by reinforcement learning*
17:05 – 18:00	General Assembly of ISDG All participants in the Symposium are welcome to attend
18:30	Welcome Cocktail

MFG: Mean Field Games
EG: Evolutionary Games
DGEM: Dynamic Games in Economics and Management
DGM: Dynamic Games Methodology
ERE: Environmental and Resource Economics
SPG: Search and Patrolling Game

Tue, July 26	FC1 Edifício de Matemática			
	FC1 029	**FC1 030**	**FC1 004**	**FC1 006**
08:30 – 10:10	**MFG 1** Chair: Silva Alvarez, F.J. Bonesini, O Mészáros, A.R. Lavigne, P Silva Alvarez, F.J.	**EG 1** Chair: Stankova, K. Pires, D. Glynatsi, N. Chalub, F. Cressman, R.	**DGEM 1** Chair: Navas, J. Cabo, F. Tidball, M. Mohamadichamgavi, J. Navas, J.	**DGM 1** Chair: de Zeeuw, A. Petrosjan, L. Zaccour, G. Sedakov, A. de Zeeuw, A.
10:10 – 10:40	Break			
10:40 – 12:20	**MFG 2** Chair: Firoozi, D. Mendico, C. Graber, J. Gutierrez, J. Firoozi, D.	**EG 2** Chair: Apaloo, J. Ramirez, M.A. Murali, D. Schreiber, A. Couto, M.	**DGEM 2** Chair: Petkov, V. Malladi, Suraj. Dragicevic, A. Sun, P. Petkov, V.	**Young Scholar Presentations** Chair: Broom, M. Bukkuri, A. Kouhkouh, H. Sadana, U.
12:20 – 14:00	Lunch			
14:00 – 15:40	**MFG 3** Chair: Tonon, D. Ricciardi, M. Mazanti, G. Duisembay, S Tonon, D.	**EG 3** Chair: Krivan, V. Aizouk, R. Gubar, E. Zubelli, J. Martins, J.	**DGEM 3** Chair: Breton, M. Li, Y. Stępniak, P. Bajoori, E. Breton, M.	**DGM 2** Chair: Parilina, E. Ramos, C. de la Cruz, O. Ballester Granell, M.A. Parilina, E.
15:40 – 16:10	Break			
16:10 – 17:50	**MFG 4** Chair: Gomes, D. Marzufero, L. Calzola, E. Gomes, D.	**EG 4** Chair: Cressman, R. Stankova, K, Apaloo, J. Salvioli, M. Brown, J.	**DGEM 4** Chair: Pinto, A. Baha, A. Soeiro, R. Almeida, J.P. Pinto, A.	**DGM 3** Chair: Wagener, F. Matychyn, I. Ramsey, D. Wagener, F.

Wed, July 27	FC1 Edifício de Matemática			
	FC1 029	FC1 030	FC1 004	FC1 006
09:00 – 10:40	**MFG 5** Chair: Basar, T. Daudin, S. Vasileiadis, A. Basar, T.	**EG 5** Chair: Brown, J. Vincent, T. Scaramangas, A. McNickle, G. Gokhale, C.	**DGEM 5** Chair: Buratto, A. De Giovanni, P. Lu, L. Fakhrabadi, M. Buratto, A.	**DGM 4** Chair: Turova, V. Von Moll, A. Turetsky, V. Mazalov, V. Turova, V.
10:40 – 11:10	Break			
11:10 – 12:50	**MFG 6** Chair: Cirant, M. Ferreira, R. Mimikos-Stamatopoulos, M. Firoozi, D. Cirant, M.	**EG 6** Chair: Broom, M. Bayer, P. Wu, Z. Gubar, E. Accinelli, E.	**ERE 1** Chair: Sbragia, L. Pająk, M De Giovanni, D. Alberdon, J. Sbragia, L.	
13:00 – 14:00	Meeting of the Executive Board of ISDG			
Afternoon	Visit to a Porto Cellar: Details to come later			
19:00	Symposium Dinner			

Thu, July 28	FC1 Edifício de Matemática			
	FC1 029	FC1 030	FC1 004	FC1 006
09:00 – 10:40	**MFG 7** Chair: Malhamé, R. Ghilli, D. Séguret, A. Festa, A. Malhamé, R.	**EG 7** Chair: Gubar, E. Sacco, A. Argasinski, K. Krivan, V. Broom, M.	**ERE 2** Chair: Martín-Herrán, G. de Frutos Cachorro, J. Morales, J.R. Mañó-Cabello, C. Wiszniewska-Matyszkiel, A.	**SPG 1** Chair: Angelopoulos, S. Zeng, L. Brethouwer, J.T. Lidbetter, T. Angelopoulos, S.
10:40 – 11:10	Break			
11:10 – 12:25	**Stochastic Games** Chair: Kleshnina, M. Szajowski, K. Palmowski, Z. Kleshnina, M.	**Microeconomic Agents** Chair: Álvarez-López, A.A. Sérgio, N. Martín García, R. Álvarez-López, A.A.	**ERE 3** Chair: Chenavaz, R. Zaccour, G. Chenavaz, R.	**SPG 2** Chair: Lidbetter, T. Clarkson, J. Fokkink, R. Papadaki, K.
12:25 – 14:00	Lunch			
14:00 – 15:30	Isaacs Awards Presentations Room: FC1 027			
15:30 – 16 :30	Farewell drinks			

DETAILED PROGRAM

July 25, 2022

09:00 AM Welcome address

Plenary I

Location: FC6 Edifício de Ciência de Computadores
Chair: Broom, Mark

09:15 AM Stochastic evolutionary game dynamics in finite populations

Traulsen, Arne

Evolutionary game theory studies the dynamics of a population in which reproductive success is tied to the success in a game. While the standard model to capture the dynamics is based on ordinary differential equations and thus deterministic, stochastic evolutionary game dynamics presents a powerful alternative. While in the limit of large population size, the usual replicator dynamics can be recovered, in other cases such as the weak selection regime or the weak mutation regime, new results can be achieved that allow a more complete characterization of the evolutionary dynamics in a game. However, changing the dynamics implies that conventional wisdom has to be re-considered. For example, the iterated removal of dominated strategies is no longer meaningful when weak selection is considered. This can be relevant in models for the evolution of cooperation, where the inclusion of seemingly paradoxical strategies can have an impact on the long term outcome.

10:15 AM Coffee Break

Tutorial I

Location: FC6 030 - FC6 Edifício de Ciência de Computadores
Chair: Pinto, Alberto

10:45 AM Integer programming tools for games

Carvalho, Margarida

This tutorial aims to highlight recent advances in the use of integer programming (combinatorial optimization) for the computation of equilibria. To this end, we will start by reviewing the fundamental background related to solving integer programs, namely, polyhedral theory and optimality conditions. Then, we will cover the use of integer programming in the context of bilevel programs (Stackelberg games). Finally, two algorithms to compute Nash equilibria heavily based on integer programming will be presented. Our running example throughout the tutorial will be the class of games where Stackelberg leaders compete.

12:15 PM Lunch

Tutorial II

Location: FC6 030 - FC6 Edifício de Ciência de Computadores
Chair: Mazalov, Vladimir

02:00 PM **An introduction to search games**

Lidbetter, Thomas

Search games are typically played between a Searcher and a Hider. The Hider wishes to remain hidden and the Searcher wishes to find him. We discuss the classic search game on a network introduced by S. Gal in the 1970's, and present the solution to the game on so-called weakly Eulerian networks (networks consisting of a number of Eulerian cycles that, when reduced to a single point, leave a tree). We then discuss more recent variations to the classic model, including variable speed networks, where the time to traverse an arc depends on the direction of travel; and expanding search, where the Searcher can instantaneously return to any point of the network already visited.

03:30 PM **Coffee Break**

Plenary II

Location: FC6 030 - FC6 Edifício de Ciência de Computadores
Chair: Zaccour, Georges

04:00 PM **Some interesting dynamics motivated by reinforcement learning**

Borkar, Vivek

I shall discuss some related (and not so related) dynamics, old and new, that have found uses in reinforcement learning, with a view to alert the dynamic games community about their potential usefulness in dynamic games.

05:05 PM **General Assembly of ISDG**

06:30 PM **Welcome Cocktail**

July 26, 202

MFG 1 Mean Field Games 1

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Silva Alvarez, Francisco José

08:30 AM On correlated equilibria and mean field games in progressive strategies

Bonesini, Ofelia; Campi, Luciano; Fischer, Markus

In Game Theory, Correlated Equilibria are a generalization of Nash Equilibria introduced to consider the possibility of a correlation between the strategies of the players. We study these equilibria in the context of N-player and mean-field games. This work aims at extending the results in [Correlated Equilibria and Mean-Field Games: a simple model (2020); L. Campi, M. Fischer], relaxing the hypothesis that the strategies used by the players are restricted, i.e. they only depend on the state of the player himself. Instead, we consider more general deviations that also depend on the states of the other players through the empirical measure. This generalization is highly non-trivial and introduces several technical difficulties in the problem. Our first concern is to provide a good definition for the concept of Correlated solution in the mean-field context. The consistency of this definition is then checked proving that a correlated solution for the MFG can be used to build εN-Correlated Equilibria for the N-player game, with an infinitesimal sequence εN. Finally, we display an example of a two-state MFG possessing correlated solutions with non-deterministic flow of measures that satisfies all the assumptions.

08:55 AM A variational approach to first order kinetic mean field games with local couplings

Mészáros, Alpár R.

In this talk we describe how to construct global in time weak solutions to a first order mean field games system involving kinetic transport operators, where the costs are local (hence non-regularizing) functions of the density variable with polynomial growth. We show the uniqueness of these solutions on the support of the agent density. Such systems arise naturally in models involving control on the acceleration. This is achieved by characterizing solutions through two convex optimization problems in duality. As part of our approach, we develop tools for the analysis of mean field games on a non-compact domain by variational methods. We introduce a notion of `reachable set', built from the initial measure, that allows us to work with initial measures with or without compact support. In this way we are able to obtain crucial estimates on minimizing sequences for merely bounded and continuous initial measures with finite first velocity moment. These are then carefully combined with L^1-type averaging lemmas from kinetic theory to obtain pre-compactness for the minimizing sequence. Finally, under stronger convexity and monotonicity assumptions on the data, we prove higher order Sobolev estimates of the solutions. The talk is based on a joint work with M. Griffin-Pickering (Durham University). Keywords: kinetic mean field games; convex duality; weak solutions; averaging lemmas.

09:20 AM Generalized conditional gradient and learning in potential mean field games

Lavigne, Pierre

We apply the generalized conditional gradient algorithm to potential mean field games and we show its well-posedeness. It turns out that this method can be interpreted as a learning method called fictitious play. More precisely, each step of the generalized conditional gradient method amounts to compute the best-response of the representative agent, for a predicted value of the coupling terms of the game. We show the convergence of the method, that is to say we show the convergence of the potential cost, the exploitability and the variables of the problem (distribution, congestion, price, value function and control terms). We also provide convergence rates. We study two learning sequences: 1/(k+1) used in the fictitious play literature and 2/(k+2) well known in the Frank-Wolfe literature. In both cases we are able to recover expected sublinear convergence rates for those algorithms. In addition we show that Quasi-Armijo-Goldstein

backtracking line search yields linear convergence rates. We provide numerical experiments and illustrate the aforementioned convergence rates for two problems: a congestion mean field game (interactions through the law of states) and a Cournot mean field game (interactions through the law of controls).

09:45 AM **Approximation of deterministic mean field games with control-affine dynamics**

Silva Alvarez, Francisco José; Gianatti, Justina

In this talk we study the numerical approximation of deterministic Mean Field Games where the dynamics of a typical agent is non-linear with respect to the state variable and affine with respect to the control variable. Particular instances of the problem considered here are MFGs with control on the acceleration. Our main result is the convergence of solutions of this approximation towards MFG equilibria.

EG 1 Evolutionary Games 1

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Stankova, Katerina

08:30 AM **More can be Better: An analysis of single mutant fixation probability functions under 2x2 games**

Pires, Diogo; Broom, Mark; Erovenko, Igor

The self-organization of collective behaviour is a topic of interest in numerous research fields, and in its context, evolutionary game theory has proved to be a powerful probing tool. Even though initial evolutionary models with frequency-dependent fitness assumed infinite populations, it has been shown that the stability of a strategy may depend not only on the game's payoff matrix but on the size of a finite population. To perform a systematic analysis of 2x2 games in well-mixed finite populations, we start by proving that 9 of the 24 possible payoff orderings always lead to single mutant fixation probability functions decreasing monotonically with population size as they trivially do under fixed fitness scenarios. However, we observe a diversity of fixation functions with increasing regions under 12 other orderings, which included anti-coordination games (e.g. Hawk-Dove/Snowdrift game), the fixation of unbeatable strategies (e.g. Defectors in the Prisoner's Dilemma), and the fixation of Stag Hunters under that game (the only exception in coordination games). Fixation functions that increase from a global minimum to a finite asymptotic value are pervasive. These may have been easily concealed by the weak selection limit. We prove under which payoff matrices it is possible to have fixation increasing for the smallest populations and find three different ways this can happen. Finally, we describe two distinct fixation functions having two local extremes and associate them with transitions from ones with one global minimum.

08:55 AM **Reactive-n strategies of direct reciprocity**

Glynatsi, Nikoleta

In both social and biological interactions, how does cooperation emerge? Why do cells sacrifice themselves, and why do humans behave in an altruistic manner? Direct reciprocity is one of the fundamental mechanisms that explain cooperation. Direct reciprocity is based on the idea that individuals are more likely to cooperate if they can expect their beneficiaries to remember and to return their cooperative acts in future. Most of the existing models, however, consider subjects who can only choose from a restricted set of representative strategies or who react to the outcome of the very last round only. There are exceptions with a number of models exploring the emergence of reciprocity when subjects' memory is longer or not restricted at all. To contribute to the ongoing discussion in this work, we consider subjects with long memory but that do not consider their own actions, thus, they choose from the set of reactive strategies. We explore reactive-n strategies for the prisoner's dilemma, and we characterise cooperative Nash Equilibria (NE). Moreover, with numerical analysis we explore which of the cooperative NE can evolve.

09:20 AM **From fixation probabilities to d-player games: an inverse problem in evolutionary dynamics**

Fabio, Chalub

The probability that the frequency of a particular trait will eventually become equal to 1 is a central issue in the study of population evolution. Its computation, once we are given a stochastic finite population model without mutations and a (possibly frequency dependent) fitness function, is straightforward. Here we address the inverse problem: from a given fixation pattern and population size, we want to infer what is the game being played by the population. We show that any fixation function can be approximated, with arbitrary precision, using d-player game theory, provided d is large enough. The pay-off matrix that emerges naturally from the approximating game will provide useful information about the individual interaction structure that is not itself apparent in the fixation pattern. Joint work with Max Souza. Published in Bulletin of Mathematical Biology (2019) 81:4625–4642.

09:45 AM **Cooperation in Social Dilemmas with Opting Out**

Cressman, Ross; Krivan, Vlastimil

Abstract: Evolutionary game theory was developed under a number of simplifying assumptions. One that is not often explicitly stated is that each interaction among individuals takes the same amount of time no matter what strategies these individuals use. When interaction time is strategy-dependent, it is more natural to take individual fitness as the payoff received per unit time. For instance, two Hawks interacting in the standard two-player Hawk-Dove game are assumed to engage in a fight over a resource with fixed payoff, implying that they may be involved in fewer interactions than Doves who avoid such contests. The same issues arise in repeated games when the number of rounds that the two players interact is under the players' control (e.g. when players can opt out against defection in the repeated Prisoner's Dilemma (PD) game). The talk will first analyze how the resulting individual fitnesses affect the evolutionary outcome (e.g. the evolutionarily stable strategy (ESS) and Nash equilibrium (NE)) in the PD game with opting out. The analysis will then be extended to the multi-player Public Goods Game (PGG). It is shown that cooperation can coexist with defection in the PD and PGG games, a result consistent with empirical evidence from game experiments based on the corresponding opting out game.

DGEM 1 Dynamic Games in Economics and Management 1

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Navas, Jorge

08:30 AM **Positional effects in public good provision. Strategic interaction and inertia**

Cabo, Francisco; Tidball, Mabel; Jean-Marie, Alain

Consumption satisfaction depends on other factors apart from the inherent characteristic of commodities. Among them, positional concerns are central in behavioral economics. Individuals enjoy returns from the ranking occupied by the consumed item. In public good, agents obtain satisfaction from their relative contribution. We analyze how positional preferences for voluntary contribution to a public good favor players' contributions and could lead to social welfare improvements. A two-player public good game is analyzed, first a one-shot game and later a simple dynamic game with inertia. Homogeneous and non-homogeneous individuals are considered and particular attention is given to the transition path.

08:55 AM **Positional and conformist effects in public good provision. Strategic interaction and inertia**

Tidball, Mabel; Cabo, Francisco; Jean-Marie, Alain

Consumption satisfaction depends on other factors apart from the inherent charac- teristic of commodities. Leibenstein (1950) studies how the consumer's demand reacts to other factors

different from the inherent characteristic of a specific commodity. In particular, he highlights the desire of some consumers to be "in style", and the attempt by other for exclusiveness. The desire of some people to conform with the others, to be fashionable or stylish provokes a "bandwagon effect": the demand for a commodity increases due to the fact that others are consuming this same commodity. Conversely, the desire of some other people to be exclusive, or different from the "common hence" provokes the "snob" effect: the demand for a commodity decreases due to the fact that others are consuming it. This paper analyzes positional concerns and conformism in a model of private contributions to public good. A positional consumer gets joy when his/her relative contribution to the public good is higher than the average contribution by others. A conformist consumers feels better if his behavior fits the average behavior in society, i.e., near the average contribution by others. The well-being of a conformist consumer decreases both when his contribution is above or below the average contribution by others. We analyze how positional preferences and conformism for voluntary contributions to a public good favor players' contributions and under which conditions can lead to social welfare improvements. A two-player public good game between a positional and a conformist is analyzed, first in a one-shot game and later in a simple dynamic game with inertia, placing particu- lar attention to the transition path. Homogeneous and non-homogeneous individuals are considered regarding the players' valuation of the public good.

09:20 AM **Asymmetric investment in Public Goods Game according to the importance of agents in the population**

Mohamadichamgavi, Javad

Understanding the condition of the emergence of cooperative behavior in social dilemmas remains a challenging issue. Two simple games, the prisoners' dilemma(PDG) and the snowdrift game are widely used to study the emergence of cooperative behaviors. These games are generally considered to describe pair interactions between individuals. However, group interaction is sometimes interesting when individual effort in teamwork for a common benefit. The public goods game(PGG), the so-called multi-person version of PDG, considers group interaction between agents In general, this game represents the N-players interaction game, and all N players independently and simultaneously have an opportunity to decide whether to cooperate or defect. In the simplified version of the public goods game (PGG), each agent participates in a single-group game constituted by the focal agent and its nearest neighbors at each time step. All contributions multiply by r, which is an enhancement factor that determines the synergistic effect of the public good game and divide between all agent equally. Finally, each agent can update its strategy according to different update rules and its payoff. Generally, in PGG, each player is unaware of the contribution of other players. In this project, We study asymmetric investment in public goods games in structured populations where agents invest in the pool depending on their importance in the population obtained by different centrality measures. Centrality measures determine the importance of nodes in networks, and each centrality leads to a node ranking based on its definition. Four cases are selected out of various existing centrality measures: degree, betweenness, eigenvector, and closeness. The simulation results show that asymmetric investment can affect the amount of cooperators in different ways where important or unimportant agents determined by different centralities contribute more to the pool. In Barabasi-Albert networks, if unimportant agents based on degree, betweenness, and eigenvector centrality invest more than others the cooperation promotes, even for a small value of enhancement factor while closeness centrality can not help to persist cooperative behaviors. In the case of Small-World networks, just contribution according to importance obtained by betweenness centrality leads to the emergence of cooperation.

09:45 AM **Regime switching in dynamic games with hyperbolic discounting**

Navas, Jorge; Marín-Solano, Jesús

Dynamic optimization problems with endogenous regime shifts have been widely studied in the literature. In the case of dynamic games, the characterization of optimal switching times as Markovian strategies presents some difficulties, derived from the fact that the strategies of one player over a regime also affect the timing of transition of the other player to a new regime. Some of these situations have been addressed for different classes of strategies, such as mixed open-

loop / feedback or piecewise closed loop strategies. In the case of time inconsistent preferences, problems with one decision maker can be seen as (dynamic) sequential games with many agents. This introduces additional problematic issues in the search of "optimal" switching times. In this work, our aim is to extend previous results on regime shifts previously derived, independently, in the literature of dynamic games and non-constant discounting. Both discrete and continuous time settings are explored. The main results are illustrated with a resource extraction model with technology adoption. In this model, we analyze first the problem with one decision maker with time-inconsistent preferences, and extend later on the analysis to the case of two players. Keywords: Technology adoption; Regime shift; Switching time; Non-constant discounting; Dynamic games

DGM 1 Dynamic Games Methodology 1

Location: FC1 006 - FC1 Edifício de Matemática
Chair: de Zeeuw, Aart

08:30 AM Characteristic Functions in Cooperative Differential Games on Networks

Petrosyan, Leon; Pankratova, Yaroslavna

The differential cooperative games on networks with nonnegative payoffs are considered. In these games feed-back strategies of players include additional possibility of cutting connections with neighbors at each time instant. The payoff of each player depends upon his behavior and the behavior of his neighbors. The cooperative version of the game with transferable payoffs is investigated and two types of characteristic function are introduced. In the first case, the characteristic function for each coalition is computed in a classical way as lower value of the game played by coalition S and the coalition N/S of left-out players. Since left out players can cut connections with players from S, the lower value of this game is equal to the maximal joint payoff of players in S. In second scenario, the values of characteristic function for each coalition is supposed to be equal to the payoff of players in this coalition under cooperation without payments induced by actions of players outside the coalition. This essentially simplifies the computation of the characteristic function and the Shapley Value and guarantees the convexity of the game under consideration. The results are demonstrated by examples. References 1. Petrosyan, L. A., Yeung, D. & Pankratova, Y. B., Cooperative Differential Games with Partner Sets on Networks 2021, Trudy Instituta Matematiki i Mekhaniki UrO RAN. 27, 3, p. 286-295 2. Tur, A. V. & Petrosyan, L. A., Cooperative Optimality Principles in Differential Games on Networks 2021, Automation and Remote Control. 82, 6, p. 1095-1106 Keywords: differential network game, cooperation, characteristic function, the Shapley Value

08:55 AM Payment Schemes for Sustaining Cooperation in Dynamic Games

Zaccour, Georges; Parilina, Elena

It is a challenge to sustain cooperation in a finite-horizon dynamic game. Since players generally have an incentive to deviate to their noncooperative strategies in the last stage, a backward induction argument leads them to defect from cooperation in all stages. In this paper, we propose two payment schemes having some desirable properties, namely, individual rationality and stability, which ensure that the players cooperate throughout the entire planning horizon. The setup and the results are general, that is, they do not rest on particular specifications of the payoff functionals or the state dynamics. We illustrate our results with a linear-quadratic dynamic game of pollution control.

09:20 AM Influence in social networks with stubborn agents: from competition to bargaining

Kareeva, Yulia; **Sedakov, Artem**; Zhen, Mengke

The literature on game theoretic models of opinion dynamics in social networks mainly focuses on the Nash equilibrium, which reflects a competitive situation between influencing agents called players. In some real-world situations, however, players engage in negotiations over a game and

thus a different type of solution should be considered to account for possible outcomes. In this paper, we examine an opinion dynamics game based on the Friedkin-Johnsen model for which we characterize the Pareto frontier, including the Nash bargaining solution. Next, we will analyze the behavior of this solution when there are changes in the susceptibility of noninfluencing agents with respect to their initial opinions. We will also demonstrate how the Nash equilibrium outcome differs from the outcome prescribed by the Nash bargaining solution. Keywords: Social networks; opinion dynamics; Friedkin-Johnsen model; discrete-time games; equilibrium; bargaining.

09:45 AM Differential Games

de Zeeuw, Aart

The methodology of differential games is a combination of optimal control theory and game theory. It is the natural framework for economic analysis with strategic interaction and dynamical optimization. The theory started its development in the early seventies, and it gradually found its way into economics. The purpose of this paper is to make the theory and applications of differential games easily accessible by explaining the basics and by developing some characteristic applications. The core of the theory focuses on the open-loop and the multiple Markov-perfect Nash equilibria that use the maximum principle and dynamic programming as the techniques for solving the optimal control problems. The applications are the game of international pollution control and the game of managing a lake, which is an example of an ecological system with tipping points. Finally, the discovery of time-inconsistency in the open-loop Stackelberg equilibrium had a huge impact on macroeconomics, since policy making under rational expectations is a Stackelberg differential game.

10:10 AM Break

MFG 2 Mean Field Games 2

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Firoozi, Dena

10:40 AM Asymptotic behavior of solutions to first-order mean field games.

Mendico, Cristian

The analysis of the ergodic behavior of solutions to Hamilton-Jacobi-Bellmann equations has a long history going back to the seminal paper by [Lions, P.-L., Papanicolaou, G. and Varadhan, S.R.S]. Since this work, the subject has grown very fast and when the Hamiltonian is of Tonelli type a large number of results have been proved. However, few results are available if the Hamiltonian fails to be Tonelli, i.e., the Hamiltonian is neither strictly convex nor coercive with respect to the momentum variable. More recently, such problems have been investigated in case of first-order mean field games, i.e., when the Hamilton-Jacobi equation is coupled with a Fokker-Plank equation that describes the evolution of players. In this talk, I will present some recent results obtained in collaboration with Piermarco Cannarsa and Pierre Cardaliaguet concerning the long time-average behavior of solutions to the classical mean field game system and also of solutions to the mean field game system with control of acceleration. Notice that, the first belongs to the Tonelli framework and the second to the non-Tonelli framework, previously described. I will present the results obtained but I will particularly emphasize the relevant differences between the two settings which lead the way to new interesting problems in mean field games but also in pure optimal control.

11:05 AM A link between some mean field games and conservation laws

Graber, Jameson; Meszaros, Alpar

We consider a class of first order mean field games for which the master equation can be reduced to a scalar transport equation on the space of probability measures. We prove the existence of classical solutions under a monotonicity assumption on the initial datum, which is different from Lasry-Lions or displacement monotonicity. Under certain assumptions on the cost functional, the transport equation

reduces further to a conservation law. We show that the relationship between entropy solutions and Nash equilibria is not straightforward: assuming uniform convexity of a one-dimensional flux function, the entropy solution gives an equilibrium, but in general it might fail to do so.

11:30 AM **A Mean Field Game price formation model with common noise**

Gutierrez, Julian; Gomes, Diogo; Ribeiro, Ricardo

We consider a market where a finite number of players trade an asset whose supply is a stochastic process. The price formation problem consists of finding a price process that ensures that when agents act optimally to minimize their trading costs, the market clears, and supply meets demand. This problem arises in market economies, in- cluding electricity generation from renewable sources in smart grids. Our model includes noise in the supply side, which is counterbalanced in the consumption side by storing energy or reducing the demand according to a dynamic price process. By solving a con- strained minimization problem, we prove that the Lagrange multiplier corresponding to the market-clearing condition defines the solution of the price formation problem. For the linear-quadratic structure, we characterize the price process using optimal control techniques, and we include two numerical approaches for the price computation.

11:55 AM **Optimal Design of Renewable Energy Certificate markets: A Principal-Agent Mean Field Game Approach**

Firoozi, Dena; Shrivats, Arvind; Jaimungal, Sebastian

Principal agent games are a growing area of research which focuses on the optimal behaviour of a principal and an agent, with the former contracting work from the latter, in return for providing a monetary award. While this field canonically considers a single agent, the situation where multiple agents, or even an infinite amount of agents are contracted by a principal are growing in prominence and pose interesting and realistic problems. Here, agents form a Nash equilibrium among themselves, and a Stackelberg equilibrium between themselves as a collective and the principal. We apply this framework to the problem of implementing Renewable Energy Certificate (REC) markets, where the principal requires regulated firms (power generators) to pay a non-compliance penalty which is inversely proportional to the amount of RECs they have. RECs can be obtained by generating electricity from clean sources or purchasing on the market. The agents react to this penalty and optimize their behaviours to navigate the system at minimum cost. In the agents' model we incorporate market clearing as well as agent heterogeneity. For a given market design, we find the Nash equilibrium among agents using techniques from mean field games. We then use techniques from extended McKean-Vlasov control problems to solve the principal (regulators) problem, who aim to choose the penalty function in such a way that balances environmental and revenue impacts optimally. We find through these techniques that the optimal penalty function is linear in the agents' terminal RECs.

EG 2 Evolutionary Games 2

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Apaloo, Joseph

10:40 AM **Chaos and noise: disorder in population dynamics**

Ramirez, Maria Alejandra; Traulsen, Arne

Evolutionary game dynamics is a framework used to model the evolution of strategies in a population. For finite populations, dynamics that appear to lack a pattern or principle of organisation can arise from sources like demographic noise and chaos. The former refers to the stochasticity caused by the probabilistic nature of birth and death events, while the latter is related to deterministic dynamical complexity. Currently, the effect of noise on dynamics displaying complex behaviour is not yet understood. Therefore, we analyse the interplay between complexity, coming from chaos, and stochasticity, coming from demographic noise. For this, we compare the dynamics that arises from a chaotic deterministic system with the dynamics of the system when

demographic noise is added. In particular, our analysis focuses on quantifying the dynamics' relevant characteristics via numerical measures. For example, we use tools from evolutionary game theory and chaos theory to quantify the fixation time and the fractal dimension of the system. Our results confirm the intuitive idea that, for small population sizes, the system's stochasticity dominates the dynamics. On the other hand, for large enough population sizes, the population dynamics is strongly influenced by the so-called underlying deterministic skeleton, which can exhibit chaotic behaviour. Overall, our results can help to understand dynamically complex systems affected by demographic noise. Precisely, we found that focusing on the deterministic skeleton can be beneficial to describe and predict complex dynamics of a large but finite population. Key words: Chaos - Demographic Noise - Population Dynamics - Evolutionary Game Theory

11:05 AM **Stable stochastic games and population dynamics**

Murali, Divya; Rao, K. S. Mallikarjuna; Shaiju, A. J.

In Hofbauer and Sandholm (2007,2009), a class of population games called stable games was introduced as an extension of negative semi-definite games. Using this definition of stable games and the framework of evolutionary stochastic games, we define stable stochastic games and study their relations with associated state games. Evolutionary stochastic games were first introduced in Flesch et al. (2013), as a multi state generalisation of the classical evolutionary games. In this model, the evolutionary game theoretic approach was introduced into stochastic games, through which evolutionarily stable strategy (ESS), and replicator dynamics for stochastic games were defined. Here, we define Perturbed Best Response (PBR) dynamics and Brown-von Neumann-Nash (BNN) dynamics for a stochastic game. We show that for stable stochastic games, the set of all population distributions inducing Nash equilibrium (NE) strategies is asymptotically stable with respect to PBR and BNN dynamics. Further, this dynamic stability is illustrated through examples.

11:30 AM **Sampling Dynamics and Stable Mixing in Hawk–Dove Games**

Arigapudi, Srinivas; **Schreiber, Amnon**; Heller, Yuval

The hawk–dove game admits two types of equilibria: an asymmetric pure equilibrium in which players in one population play "hawk" and players in the other population play "dove," and an inefficient symmetric mixed equilibrium, in which hawks are frequently matched against each other. The existing literature shows that populations will converge to playing one of the pure equilibria from almost any initial state. By contrast, we show that plausible sampling dynamics, in which agents occasionally revise their actions by observing either opponents' behavior or payoffs in a few past interactions, can induce the opposite result: convergence to one of the inefficient mixed stationary states.

11:55 AM **Introspection dynamics: A simple model of counterfactual learning in asymmetric games**

Couto, Marta; Giaimo, Stefano; Hilbe, Christian

Typically, evolutionary game theory captures strategic interactions with symmetric games. In these games, individuals are assumed to have access to the same set of strategies, and they experience the same payoff consequences. As a result, they can learn more profitable strategies by imitation. However, interactions are often asymmetric. In that case, imitation may be infeasible (because individuals differ in the strategies they can use), or it may be undesirable (because individuals differ in their incentives to use a strategy). Here, we consider an alternative learning process which applies to arbitrary asymmetric games – introspection dynamics. According to this dynamics, individuals regularly compare their present strategy to a randomly chosen alternative from their set of possible strategies. If the alternative strategy yields a payoff advantage, it is more likely adopted. In this work, we formalise introspection dynamics for 2-player games. Conveniently, the proposed learning process can be translated into a Markov chain. Thus, we derive simple and explicit formulas for the long-run strategy distribution. We then apply these results to several well-known social dilemmas. In particular, we analyse the asymmetric

volunteer's timing dilemma. In this game, individuals can choose when (earlier or later) to volunteer for a cooperative action. We show that the player with the lowest contribution cost tends to cooperate without delay. Keywords: evolutionary game theory, counterfactual learning, myopic updating, asymmetric games, social dilemmas

DGEM 2 Dynamic Games in Economics and Management 2

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Petkov, Vladimir

10:40 AM An Economic Model of Prior-Free Spatial Search

Malladi, Suraj

We propose a model of sequential spatial search with learning. There is a mapping from a space of technologies (or products) to qualities that is unknown to the searcher. The searcher can learn various points on this mapping through costly experimentation. She cares both about the technology that she adopts as well as the best one available, as would a firm in an innovation race or an online shopper concerned with missing a good deal. She does not have a prior over mappings but knows only that neighboring technologies in attribute space are similar in quality. We characterize optimal search strategies when the searcher worries about worst-case mappings at every step of the way. These are mappings that trigger wild-goose chases: excessive search with relatively poor discoveries to show for it. We derive comparative statics that match patterns observed in empirical studies on spatial search. Finally, we apply the results to the problem of optimal search space design faced by online platforms.

11:05 AM Stochastic Exchange Networks with Risk Sharing

Dragicevic, Arnaud

Following the models of exchange networks, subject to stochastic matching between the pairs of agents, we analyze the dynamics of bargaining under risk sharing in an atomistic-type market structure. Convergent expectations, divergent expectations and social preferences of model-players are three scenarios addressed in the study. The extension of earlier works lies in the consideration of stochastic shocks, introduced through a Poisson process, which can put the coordination within the decentralized trading mechanism at risk. By means of Pareto weights, agents can nevertheless apply a risk-sharing protocol so as to mitigate against the shocks. In all three scenarios, the model outcomes show that the allocations are not the Nash bargaining solutions. That being stated, the results also show that, on condition that the time length is sufficiently long, the dynamics systematically converges to a fixed point situated a little away from the balanced outcome, that is, from the Nash equilibrium. The small gap reveals the risk premium to hedge against the mutual risk.

11:30 AM Dynamic network formation with ordered partitioning and incomplete information

Sun, Ping; Parilina, Elena

We introduce a dynamic network formation model with incomplete information and asymmetric players who are partitioned into ordered groups. At each stage, a pair of players is randomly selected to update the link connecting them. The feature of the model is that the cost of a link depends on the distance between the labels of the groups players belong to. As a result, information about the other player’s label is significant in the decision to form a link. Assuming incomplete information, players initially do not know each other’s labels, but learn them upon being connected. Comparing the topologies of stable networks for cases of complete and incomplete information indicates that the presence of the latter can have deep implications. Furthermore, our simulation experiments are focused on three issues: the time it takes to reach a stable network, the heterogeneity index, and the impact of the group size gap.

11:55 AM **Recursive representation of dynamic intra-personal games of generalized hyperbolic discounters**

Petkov, Vladimir

Our paper examines the Markov-perfect equilibrium (MPE) in intra-personal dynamic games of sophisticated decision-makers with generalized hyperbolic time preferences. When such decision-makers have homothetic instantaneous utilities, their intra-personal games usually feature equilibria involving linear strategies. In previous research, we obtained conditions for the MPE strategies that were derived from the one-shot deviation principle. However, our approach was only applicable to simple games with a single state variable. In the current project, we characterize these linear MPE by formulating the decision-maker's problem recursively. This method allows us to study intra-personal games with more complicated state dynamics. The recursive formulation of the dynamic problem uses Laplace transforms to represent the generalized hyperbolic discount function as an integral of weighted exponential functions. This enables us to express the current agent's continuation payoff by integrating a continuum of continuation value functions (one for each exponential discount factor). In some cases, we find a closed-form solution for this integral, and thus we provide an analytic expression for the corresponding Euler equation. In other cases a closed-form solution is not available, but we can compute the MPE strategies numerically. Our analysis is applied to an extension of Laibson's consumption-savings model. In this application, a generalized hyperbolic consumer with constant absolute risk aversion instantaneous utility chooses his consumption over time. In contrast to the standard setting, we assume that the interest rate follows a three-state Markovian process. This modification invalidates Laibson's equivalence result: the equilibrium strategies of a generalized hyperbolic consumer can no longer be replicated with quasi-hyperbolic or exponential discounting. We use our approach to compute the MPE strategies in this model. This allows us to evaluate when quasi-hyperbolic discounting can be used as a good approximation to generalized hyperbolic discounting.

Young Scholar Best Paper Award

Location: FC1 006 - FC1 Edifício de Matemática
Chair: Broom, Mark

10:40 AM **Integrating Eco-Evolutionary Dynamics into Matrix Population Models for Structured Populations**

Bukkuri, Anuraag; Brown, Joel

Recent evidence suggests that a polyaneuploid cancer cell (PACC) state may play a key role in the adaptation of cancer cells to stressful environments and in promoting therapeutic resistance. The PACC state allows cancer cells to pause cell division and to avoid DNA damage and programmed cell death. Transition to the PACC state may also lead to an increase in the cancer cell's ability to generate heritable variation (evolvability). One way this can occur is through evolutionary triage. Under this framework, cells gradually gain resistance by scaling hills on a fitness landscape through a process of mutation and selection. Another way this can happen is through self-genetic modification whereby cells in the PACC state find a viable solution to the stressor and then undergo depolyploidization, passing it on to their heritably resistant progeny. Here, we develop a stochastic model to simulate both of these evolutionary frameworks. We examine the impact of treatment dosage and extent of self-genetic modification on eco-evolutionary dynamics of cancer cells with aneuploid and PACC states. We find that under low doses of therapy, evolutionary triage performs better whereas under high doses of therapy, self-genetic modification is favored. This study has implications for teasing apart these biological hypotheses in the context of cancer and is the first modeling comparison of Mendelian and non-traditional forms of inheritance.

11:05 AM Viscous ergodic Mean-Field Games in non-compact setting

Kouhkouh, Hicham

We address the problem of existence of solutions to ergodic mean-field games in the whole space with unbounded and merely measurable data, taking advantage from non-degeneracy and a recurrence condition. The method we use is different from classical approaches. It relies on duality theory and optimization in abstract Banach spaces together with maximal dissipativity of diffusion operators. Keywords: Duality, ergodic Mean-Field Games, invariant measures, optimization, weak solutions.

11:30 AM Feedback Nash equilibria in differential games with impulse control

Sadana, Utsav; Reddy, Puduru Viswanadha; Zaccour, Georges

We study a class of deterministic finite-horizon two-player nonzero-sum differential games where players are endowed with different kinds of controls. We assume that Player 1 uses piecewise-continuous controls, while Player 2 uses impulse controls. For this class of games, we seek to derive conditions for the existence of feedback Nash equilibrium strategies for the players. More specifically, we provide a verification theorem for identifying such equilibrium strategies, using the Hamilton-Jacobi-Bellman (HJB) equations for Player 1 and the quasi-variational inequalities (QVIs) for Player 2. Further, we show that the equilibrium number of interventions by Player 2 is upper bounded. Furthermore, we specialize the obtained results to a scalar two-player linear-quadratic differential game. In this game, Player 1's objective is to drive the state variable towards a specific target value, and Player 2 has a similar objective with a different target value. We provide, for the first time, an analytical characterization of the feedback Nash equilibrium in a linear-quadratic differential game with impulse control. We illustrate our results using numerical experiments.

12:20 PM Lunch

MFG 3 Mean Field Games 3

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Tonon, Daniela

02:00 PM The Master Equation in a Bounded Domain with Neumann Conditions

Ricciardi, Michele

In this talk we investigate the well-posedness of the Master Equation of Mean Field Games in a framework of Neumann boundary condition. The definition of solution is closely related to the classical one of the Mean Field Games system, but the boundary condition here leads to two Neumann conditions in the Master Equation formulation, for both space and measure. The global regularity of the linearized system, which is crucial in order to prove the existence of solutions, is obtained with a deep study of the boundary conditions and the global regularity at the boundary of a suitable class of parabolic equations. Eventually, we use these results in order to prove the convergence problem of the Nash equilibria in the N-players system towards the strategies of the Mean Field Games system.

02:25 PM Minimal-time mean field games with state constraints

Mazanti, Guilherme; Sadeghi Arjmand, Saeed

In this talk, we are interested in a mean field game (MFG) model inspired by the analysis of crowd motion in which agents evolve in a bounded domain Ω and wish to reach a given target set in minimal time. Interaction between agents is modelled through their dynamics and is nonlocal in nature, through the assumption that an agent's maximal speed is determined by an averaged density of agents around their position. During their movement, agents are subject to the state

constraint of always remaining inside the domain Ω, as well as a control constraint, and the final time in their optimization criterion is free. Existence of equilibria of such games in a Lagrangian framework can be established through a classical fixed-point argument based on Kakutani's fixed point theorem. However, proving that such equilibria can be described through an MFG system of PDEs is a much harder task: the velocity field appearing in the continuity equation involves the normalized gradient of the value function of the optimal control problem solved by each agent. Due to the state constraint, this value function is not necessarily semiconcave, and hence existence of such a normalized gradient is hard to ensure. In this talk, after recalling results on the existence of Lagrangian equilibria, we will provide an alternative characterization of the velocity field in the continuity equation, based on a weak notion of "direction of steepest descent" of the value function that does not required differentiability. Thanks to this characterization, one can provide a proper meaning to the continuity equation in the MFG system, which is the main result presented in the talk. The results presented here come from a joint work with Saeed Sadeghi Arjmand.

02:50 PM **Particle approximation of one-dimensional Mean-Field-Games with local interactions**

Duisembay, Serikbolsyn

We study a particle approximation for one-dimensional first-order Mean-Field-Games (MFGs) with local interactions with planning conditions. Our problem comprises a system of a Hamilton-Jacobi equation coupled with a transport equation. As we deal with the planning problem, we prescribe initial and terminal distributions for the transport equation. The particle approximation builds on a semi-discrete variational problem. First, we address the existence and uniqueness of a solution to the semi-discrete variational problem. Next, we show that our discretization preserves some previously identified conserved quantities. Finally, we prove that the approximation by particle systems preserves displacement convexity. We use this last property to establish uniform estimates for the discrete problem. We illustrate our results for the discrete problem with numerical examples.

03:15 PM **A comparison principle for Hamilton-Jacobi equations on infinite dimensional spaces**

Tonon, Daniela

In this talk, we present a comparison principle for the Hamilton Jacobi (HJ) equation corresponding to linearly controlled gradient flows of an energy functional defined on a metric space. The main difficulties are given by the fact that the geometrical assumptions we require on the energy functional do not give any control on the growth of its gradient flow nor on its regularity. Therefore this framework is not covered by previous results on HJ equations on infinite dimensional spaces (whose study has been initiated in a series of papers by Crandall and Lions). Our proof of the comparison principle combines some rather classical ingredients, such as Ekeland’s perturbed optimization principle, with the use of the Tataru distance and of the regularizing properties of gradient flows in evolutional variational inequality formulation, that we exploit for constructing rigorous upper and lower bounds for the formal Hamiltonian. Our abstract results apply to a large class of examples, including gradient flows on Hilbert spaces and Wasserstein spaces equipped with a displacement convex energy functional satisfying McCann’s condition. However, with respect to the existing literature about the Master equation in Mean Field Games our assumptions have a different nature. Nevertheless, some ideas could be of use for further studies.

EG 3 Evolutionary Games 3

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Krivan, Vlastimil

02:00 PM The game-theoretical modelling of a dynamically evolving network: revisiting the target sequence 111

Aizouk, Raneem; Broom, Mark

In this talk we considered a model of a dynamically evolving network of interactions between a group of individuals, where each individual has an optimum level of social engagement with other group members. A randomly selected individual will form or break a link to obtain the required number of contacts. These interactions were formulated as a graph realisation problem. A game theoretical version was known from previous work to be very complex for all but almost trivial cases, with the exception of an example with three players considered by Broom and Cannings. we will revisit this game-theoretical version of the model, where individuals strategically choose the specific link to form or break and we found that even this is more complex than previously thought. We find a general expression for the payoff functions for all possible strategy combinations. In addition to the three Nash equilibria previously found, we find a set of six more. The considerations of all possibilities prove to be infeasible, leaving the possibility of more solutions open.

02:25 PM Analysis of epidemic waves in a two-virus SIRS model in a scale-free network

Gubar, Elena; Tainitskiy, Vladislav

This study presents a modification of the classical susceptible-infected-recovered model (SIRS) of two viruses that spread in a scale-free network. Previous studies and real cases show that many viruses cannot be completely eradicated, and they constantly provoke new waves of epidemics. The annual influenza outbreaks or several waves of novel coronavirus confirm this fact. In current models, we describe the case of multiple epidemic waves, caused by the same viruses during the several outbreaks. We present the basic SIRS epidemic model on the scale free network for two-virus case, formulate the optimal control problem with the assumption that the structure of optimal control strategies depends on the basic reproduction number. We find optimal control strategies under the special conditions, which protect the population and analyze the properties of the system stationary stets and its key coefficients. The series of numerical experiments presents different scenarios.

02:50 PM Estimating and forecasting epidemiological dynamics and public-health response with a spatiotemporal approach.

Albani, Vinicius; **Zubelli, Jorge**

We use an SEIR-type meta-population model to simulate and monitor epidemiological evolution from spatial as well as temporal data. Although our implementation is performed using data for the COVID-19 pandemics, it can be readily adapted to other situations. The basic model consists of seven categories, namely, susceptible (S), exposed (E), three infective classes, recovered (R), and deceased (D). We define these categories for different age and sex groups in various spatial locations. Therefore, the resulting model contains all epidemiological classes for each age group, sex, and location. The mixing between them is accomplished by means of time-dependent infection rate matrices. The model is calibrated with the curve of daily new infections in New York City and its boroughs, including census data, and the proportions of infections, hospitalizations, and deaths for each age range for NYC and its boroughs. We finally obtain a model that matches the reported curves and predicts accurate infection information for different locations and age classes. It also allows scenario generation, simulations, and optimization of different mitigation strategies.

03:15 PM **Evolutionary vaccination strategies for the reinfection SIRI model**

Martins, José; Pinto, Alberto

In this work, we study the evolution of vaccination decisions in a homogeneous population depending on the morbidity risks of the vaccine, the morbidity risks of the disease, and also depending on the decisions of all other individuals. In 2017 \cite{Martins2017}, Martins and Pinto introduced the evolutionary vaccination dynamics of the population vaccination strategy for the basic reinfection SIRI model. We analyze the changes provoked in the vaccination dynamics when the morbidity risks also evolve with the course of the disease.

DGEM 3 Dynamic Games in Economics and Management 3

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Breton, Michèle

02:00 PM **Investigating the Asymmetric Behavior of Oil Price Volatility Using Support Vector Regression**

Hyunjoo Kim, Karlsson**; Yushu, Li**

This paper investigates the asymmetric behavior of oil price volatility using different types of Asymmetric Power ARCH (APARCH) model. We compare the estimation and forecasting performance of the models estimated from the maximum likelihood estimation (MLE) method and support vector machine (SVM) based regressions. Combining nonparametric SVM method with parametric APARCH model not only enables to keep interpretations of the parametric models but also leads to more precise estimation and forecasting results. Daily or weekly oil price volatility is investigated from March 8, 1991 to September 13, 2019. This whole sample period is split into four sub-periods based on the occurrence of certain economic events, and we examine whether the asymmetric behavior of the volatility exists in each sub-period. Our results indicate that SVM regression generally outperforms the other method with lower estimation and forecasting errors, and it is more robust to the choice of different APARCH models than the MLE counterparts are. Besides, the estimation results of the SVM based regressions in each sub-period show that the ARCH models with asymmetric power generally perform better than the models with symmetric power when the data sub-period includes large swings in oil price. The asymmetric behavior of oil price volatility, however, is not detected when the analysis is done using the whole sample period. This result underscores the importance of identifying the dynamics of the dataset in different periods to improve estimation and forecasting performance in modelling oil price volatility. This paper, therefore, examines volatility behavior of oil price with both methodological and economic underpinnings.

02:25 PM **Perpetual American cancellable options and Levy-type last passage times**

Stępniak, Paweł; Palmowski, Zbigniew

In this talk we give the explicit price of the perpetual American cancellable options where the cancellation is realized at the last hitting time. We consider the price of the underlying being the spectrally negative Levy process. The proof is based on guess and verify arguments. Keywords: optimal stopping American options verification method Levy process

02:50 PM **Optimal delegated search with learning and no monetary transfers**

Bajoori, Elnaz; Wirtz, Julia

A principal delegates the search for the cheapest price to an agent with private information about the price distribution. We do not allow for any monetary transfers to incentivize the agent. The optimal pooling search rule features strictly increasing thresholds, which reflect the principal's updated belief about the price distribution. We show that the pooling rule can be improved upon by a separating menu of search rules with fixed thresholds and a minimum number of offers. Then, we find conditions under which either rule is preferred. Finally, we characterize the optimal separating search rule with a minimum number of offers.

03:15 PM The impact of safety covenants in syndicated loan agreements

Breton, Michèle; Nabassaga, Tiguene

We propose a stochastic dynamic game model of syndicated loan contract adjustments in the presence of a safety covenant. The model accounts for the lender's right to punish (increase the interest payments or the collateral) or tolerate any breach of the covenant, and for the borrower's flexibility in adjusting its investment and risk-taking strategy. We consider a Stackelberg setting under two possible information structures; in the first case, the lender uses a feedback strategy (variable spread), while in the second case, the lender's strategy is open-loop (performance pricing). Our numerical experiments show that, while a safety covenant improves the loan value in most states, it can have an adverse effect when bankruptcy risk becomes important. Additional investigation shows that the lender can optimally tolerate some technical default to prevent this adverse effect.

DGM 2 Dynamic Games Methodology 2

Location: FC1 006 - FC1 Edifício de Matemática
Chair: Parilina, Elena

02:00 PM Nonlinearity, control and decision

Ramos, Carlos

Nonlinearity is related with the notions of control and decision making, in a certain perspective. Several ilustrating examples are discussed arising from well known simple models from biomathematics and computation. The relation between the three concepts is developed in the context of symbolic dynamics and iterated maps of the interval. Certain idealized games are described.

02:25 PM Hamilton and D´Hondt differences in the Spanish parliament context

de la Cruz, Omar; Tomé Bermejo, Fernando; Ramiro Moreno, Rafael

This paper analyses the voting behavior of the parliamentary groups in the Regional Assembly of Madrid to appoint the Bureau, its representative body, in all the legislatures since its inception (1983–2021). To this end, the actual result of the voting is compared with the mock result attained by following a Nash equilibrium (NE) and a d'Hondt (d'H) allocation in each vote. But the result of a d'Hondt allocation varies based on the number of stages in which the voting is performed (President, Vice-President, and Secretaries), so a bias towards disproportionality could exist as measured by the absolute index of disproportionality which calculates the number of seats non-proportionally allocated. The results show that, in view of the hypothesis on the importance of the number of seats, the NE was only followed in four of the 12 Legislatures for Vice-Presidents (it was always followed for Secretaries). Thus, parliamentary groups could gain more seats by modifying their strategies. Additionally, the absolute rate of disproportionality and the number of seats non-proportionally allocated indicate that, in general, parliamentary groups obtain voting results that are less disproportionate than they could be (due to the number of voting stages)

02:50 PM Convergence conditions of market algorithms

Ballester Granell, Miguel Ángel; Hernandez Rojas, Penélope

The rise of complex algorithms during the last decades has influenced many sectors. In the field of dynamic pricing, different iterative algorithms, such as Q-Learning, have been used in order to find optimal pricing policies. Therefore, optimization problems can be solved by means of iterative algorithms. Such algorithms consider also Markov Decision Processes to capture the uncertainty in the dynamics of the environment, as the markets or the strategic-nature of the economic agents. Therefore, the achievement of the optimal solution or policy entails a learning framework with a convergence property. To be precise, departing from an analytical mathematical perspective, we focus on contraction mappings in a complete metric space. Adding a learning procedure, we place

emphasis on stochastic iterative algorithms. Consequently, it is worth wondering whether these algorithms and the environments they interact in meet the required conditions to guarantee their convergence to a steady policy, and whether that policy is actually the optimal policy. In this paper, we characterize a set of conditions that can allow to categorize algorithms based on their convergent behavior. Specially, we focus on the convergence conditions of reinforcement learning models when they use neural networks as function approximations, such as the case of Deep Q-Learning. Finally, we apply the above characterization to answer the question regarding algorithms that intentionally or unintentionally collude in a competitive market. Keywords: algorithms, markov decision processes, q-learning, neural networks, collusion

03:15 PM **Sustainable Cooperation in Dynamic Games on Event Trees with Players' Asymmetric Beliefs**

Parilina, Elena; Zaccour, Georges

We built a time-consistent cooperative solution for the class of dynamic games played over event trees in the context where the tree structure is given, but the players have different beliefs about the transition probabilities between nodes. Our three-step approach is as follows. First, we consider three alternative methods for aggregating the players' beliefs and assume that the players agree to adopt one of them if they decide to cooperate. Second, we determine the Nash bargaining outcomes for the whole duration of the game. Finally, to insure sustainability of cooperation throughout the whole duration of the game, we propose two time-consistent decompositions over nodes of each player's cooperative share, namely a proportion-consistent and a node-consistent allocation schemes. We illustrate our results with a simple Cournot oligopoly with capacity constraints.

03:40 PM Break

MFG 4 Mean Field Games 4

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Gomes, Diogo

04:10 PM **On a pursuit-evasion game between a single player and a population of agents**

Marzufero, Luciano; Bagagiolo, Fabio; Capuani, Rossana

We study a finite-horizon differential game of pursuit-evasion type, with a single player and a mass of agents. The mass of agents directly controls its evolution, given by a transport equation, in an almost centralized manner. Using also the concept of non-anticipating strategies, we derive an infinite dimensional Isaacs equation and we study the existence and uniqueness of the solution. The future goal of the work is to address a mean-field type pursuit-evasion game between two populations.

04:35 PM **A second order accurate Lagrange-Galerkin scheme for a class of Fokker-Planck equations and applications to Mean Field Games**

Calzola, Elisa; Carlini, Elisabetta

We propose a second order Lagrange-Galerkin method to approximate the solution of the linear Fokker-Planck (FP) equation with constant positive diffusion. The scheme is constructed starting from the representation formula for the FP equation. Firstly, we discretize the solution of the stochastic differential equation for the characteristic curves using the Crank-Nicolson method, then we consider symmetric Lagrange interpolation basis functions of odd degree and a suitable quadrature method for the approximation of the integrals in the representation formula. We have proved mass conservation, consistency, L2-stability, and convergence of the exactly-integrated scheme. Finally, we couple this scheme with a second order semi-Lagrangian discretization of the Hamilton-Jacobi-Bellman equation, in order to obtain an approximation method for the solution of Mean Field Games.

05:00 PM **On weak strong uniqueness for mean-field games**

Gomes, Diogo; Rita, Ferreira; Vardan, Voskanyan

Monotonicity conditions for mean-field games are an essential tool to establish the uniqueness of solutions. Monotonicity conditions combined with Minty's method can also be used to establish the existence of solutions. Unfortunately, these solutions are not regular enough for the standard uniqueness proof to work. In this talk, we present some recent results on the strong-weak uniqueness problem; that is, whether the existence of a strong solution implies uniqueness among all weak solutions.

EG 4 Evolutionary Games 4

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Cressman, Ross

04:10 PM **Validating game-theoretic models of cancer and its treatment**

Stankova, Katerina

We will explore how to validate game-theoretic models of cancer and its treatment through in-vitro and in-vivo data. Further, we will demonstrate how cancer treatment can be optimized with respect to various objectives of the physician.

04:35 PM **Multi-trait Model of Cancer Evolution showing Coadaptation between Aggressiveness and Co-option of Fibroblasts**

Apaloo, Joseph; Brown, Joel

Cancer cells inhabit complex tumor ecosystems that include a diversity of normal cell types, blood vasculature and extra-cellular matrix. Some of the normal cells represent threats to cancer cells in the form of immune cells. Other cells such as fibroblasts can either be a threat by inhibiting cancer cells growth or they can facilitate the survival and proliferation of cancer cells by providing protection and key resources. In normal tissues, fibroblasts play key roles in creating the extracellular matrix. They play critical roles in wound healing. Understanding the tumor microenvironments of cancer cells is seen as a new research frontier and essential to understanding and successfully treating cancer. Towards this goal, we start by presenting a simple model of wound healing and tissue homeostasis where epithelial cells signal and control the proliferation of fibroblasts. We then apply the model to oncogenesis consisting of an epithelial cell clade becoming cancerous. The model follows the early evolution of cancer cells that includes adaptations by the cancer cells to better exploit their microenvironment (aggressiveness trait) as well as a signaling strategy that allows them to manipulate the dynamics of cancer associated fibroblasts (co-option trait). Once cancerous, the model is an evolutionary game. The fitness of a cancer cell is influenced by: 1) its aggressiveness and co-option traits, 2) the traits of neighboring cancer cells, and 3) the number of fibroblasts in its neighborhood as influenced by other cancer cells' co-option trait. The model of wound healing shows stable dynamics back towards the whole tissue homeostasis following a perturbation event. Notably the fibroblasts initially overshoot their equilibrium level thus speeding wound healing. With cancer initiation, the model shows an increase in the density of cancer cells above that of the normal epithelial cells, an increased level of aggressiveness, and a higher level of signaling that then supports a higher density of fibroblasts. At the ESS, the level of aggressiveness by the cancer increases their level of competition with each other, while the evolution of increased co-option of fibroblasts by the cancer cells provides a public good indirectly enhancing the fitness of neighboring cancer cells. The competition between cancer cells can lead to speciation resulting in two coexisting cancer "species" at the ESS: one that has a high level of aggressiveness and low level of co-option, and vice-versa for the other. The model suggests entry points for therapies that can target the cancer cells' aggressiveness or co-option traits, or that directly target the fibroblasts.

05:00 PM **Is the Success of Adaptive Therapy in Metastatic Castrate-Resistant Prostate Cancer Influenced by Cell-Type-Dependent Production of Prostate-Specific Antigen?**

Salvioli, Monica; Vandelaer, Len; Schneider, Katharina; Cavill, Rachel; Stankova, Katerina

Prostate-specific antigen (PSA) is the most common serum marker for prostate cancer. It is used to detect prostate cancer, to assess responses to treatment and recently even to determine when to switch treatment on and off in adaptive therapy protocols. However, the correlation between PSA and tumor volume is poorly understood. There is empirical evidence that some cancer cell types produce more PSA than others. Still, recent mathematical cancer models assume either that all cell types contribute equally to PSA levels, or that only specific subpopulations produce PSA at a fixed rate. Here, we compare time to competitive release of the PSA-based adaptive therapy protocol by Zhang et al. with that of the standard of care based on continuous maximum tolerable dose under different assumptions on PSA production. In particular, we assume that androgen dependent, androgen producing, and androgen independent cells may contribute to the PSA production to different extents. Our results show that, regardless the assumption on how much each type contributes to PSA production, the time to competitive release is always longer under adaptive therapy than under the standard of care. However, in some cases, e.g., if the androgen-independent cells are the only PSA producers, adaptive therapy protocol by Zhang et al. cannot be applied, because the PSA value never reaches half of its initial size and therefore therapy is never discontinued. Furthermore, we observe that in the adaptive therapy protocol, the number of treatment cycles and their length strongly depend on the assumptions about the PSA contribution of the three types. Our results support the belief that a better understanding of patient-specific PSA dynamics will lead to more successful adaptive therapies.

05:25 PM **Game Theory models significantly prolong response to Abiraterone in metastatic castrate resistant prostate cancer**

Brown, Joel; Zhang, Jingsong; Cunningham, Jessica J.; Gatenby, Robert

Abiraterone acetate is an effective treatment for metastatic castrate resistant metastatic prostate cancer but evolution of resistance inevitably leads to progression. Six years ago, we initiated a pilot clinical trial. A game theory model with three cancer cell types was used to create a rule of thumb for an adaptive therapy regimen. In the adaptive therapy cohort (17 patients), abiraterone was stopped when PSA (a biomarker of overall tumor burden) was <50% of pretreatment value and resumed when PSA returned to baseline. We could compare trial results to a contemporaneous cohort (16 patients) who received continuous abiraterone therapy as standard of care. The adaptive therapy patients had significantly improved median Time To Progression (TTP) (33.5 months) and median overall survival (58.5 months) compared to 14.3 and 31.3 months, respectively, in the SOC cohort. We can use each patient's data to parameterize and create a game personalized to that patient. The analyses of the patient-specific games demonstrate key outcomes: 1) all adaptive therapy patients would have done worse on standard of care, 2) all standard of care patients would have done better with adaptive therapy, 3) adaptive therapy would have been even more effective had we been able to switch therapies more precisely, and 4) in many cases, with more precise drug switching, a patient's disease burden might be controlled indefinitely. The application of game theory models to patient care may permit the physician to anticipate and steer the ecological and evolutionary dynamics of the cancer for substantially better patient outcomes.

DGEM 4 Dynamic Games in Economics and Management 4

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Pinto, Alberto

04:10 PM Dynamic monitoring of adaptive crime

Baha, Alae

I study the problem of monitoring in a dynamic setting, in which the monitor's ability to detect misbehavior is endogenous: In addition to choosing the amount of fraud, a fraudster can privately develop a hiding technology that makes misbehavior undetectable, and the inspector can invest in R\&D to recover her detection ability. In equilibrium, the inspector invests whenever she is sufficiently confident to be lagging technologically. However, too much deterrence of detectable fraud (e.g., high fines or more monitoring) induces the fraudster to invest in hiding technologies, which triggers an arms race and can increase the average quantity of misbehavior. The optimal policy trades off less misbehavior, when detectable, with shorter technological cycles (and higher spending in R&D). The model has applications to digital security, drug smuggling, money laundering, and tax evasion.

04:35 PM Shared subgame-perfect equilibria in price competition duopolies with network effects

Soeiro, Renato; Pinto, Alberto

The study of network effects in models of price competition is usually a perturbation of an (equilibrium) demand function, which is either a primitive or a specification arising from a standard model. When network effects are positive and depend on the aggregate of choices, such a construction is necessary to control the bandwagon effect and avoid ending up in a corner solution. We show that in duopolies where network effects depend on a partition into groups, some influence structures produce a local (pure price) duopoly equilibrium where both firms profit. The local solution can be part of a subgame-perfect equilibrium if firms' are afraid that a price outside the interval that supports the local optimum will tip the market to their rival. It may be particularly suitable for some fickle popularity phenomena. It also shows it is possible to take network effects as the microeconomic foundation for demand and then perturb that solution with other effects if deemed necessary.

05:00 PM A Hotelling game in networks with uncertainty on costs

Almeida, João P.

We develop a theoretical framework to study the location-price competition in a Hotelling- type network game, extending the Hotelling model with linear transportation costs from a line (city) to a network (town). We show the existence of a pure Nash equilibrium price if, and only if, some explicit conditions on the production costs and on the network structure hold. Furthermore, we prove that the localizations of the local firms are optimal at the cross-roads of the town. In addition, we extend the results to the case were Firms know their production costs but are uncertain about the production costs of the competitor firms.

05:25 PM Cicles of corruption and democracy

Pinto, Alberto; Oliveira, Bruno

In this paper we propose a game theoretic model with three populations, namely a government, officials who serve the state, and citizens, to analyse the evolution of corruption in a society. The influence of democracy in corruption is modelled through the action of the citizens who exercise influence in the government because of their elective power since corruption causes a great displeasure in the citizens which can result in a vote against a ruler elite that promotes or is an accomplice to corruption. When immersed in a society in which corruption is a common

occurrence, citizens may behave in a complacent manner with corruption because of a lack of valid alternatives to this behaviour even if they oppose corruption. Indeed, this complacent behaviour may also be observed in democratic societies and can lead to periods of growing and diminishing corruption. We are thus able to get a better understanding of some causes for the evolution of corruption and how the evolution may be halted and the effects of democracy and influence in this.

DGM 3 Dynamic Games Methodology 3

Location: FC1 006 - FC1 Edifício de Matemática
Chair: Wagener, Florian

04:10 PM Game Problems for Fractional-Order Nonstationary Systems

Matychyn, Ivan; Onyshchenko, Viktoriia

Fractional differential equations (FDEs) provide a powerful tool to describe memory effect and hereditary properties of various materials and processes. While linear systems of FDEs represent a fairly well investigated field of research, relatively few papers deal with nonstationary fractional-order systems described by linear FDEs with variable coefficients. Meanwhile, a number of real-life systems and processes can be described by linear FDEs with variable coefficients, e.g. linearized aircraft models, linearized models of population restricted growth, models related to the distribution of parameters in the charge transfer and the diffusion of the batteries etc. Linear differential equations with variable coefficients arise in a natural way when modeling RLC-circuits with variable capacitance or inductance. With the advent of electronic components like super-capacitors (also called ultracapacitors) and fractances, one should employ fractional differential equations for circuit models. This provides motivation for research on FDEs with variable coefficients as well as related control and game problems. Explicit solutions to linear systems of differential equations provide basis to perform stability analysis and to solve control and game-theoretical problems. Explicit solutions to linear systems of differential equations are usually expressed in terms of state transition matrix. In the case of FDEs with constant coefficients the state transition matrix can be represented using the matrix Mittag-Leffler function. Recently explicit solutions for the linear systems of FDEs with variable coefficients were obtained by the authors in terms of generalized Peano--Baker series. This paper deals with differential games described by the systems of linear FDEs with variable coefficients involving Riemann--Liouville and Caputo derivatives. The game problem is treated from convex-analytical viewpoint using the method of resolving functions. On the basis of the resolving functions method sufficient conditions for the finite-time game termination from given initial states are derived. Theoretical results are supported by an illustrative example.

04:35 PM Game Theoretical Models of Choosing a Valuable Good in the Age of Big Data

Ramsey, David

When looking for a unique value good from a large set of offers (e.g. a flat in a city or a second-hand car), consumers can obtain information about a large number of offers with very little effort via the Internet. This information might be rich enough to indicate that an offer is potentially attractive, but is not sufficient to accurately assess the attractiveness of an offer. Since exhaustive search of the available offers is excessively costly, decision makers (DMs) use heuristic procedures, such as shortlist formation. Using the information from the Internet, DMs select a relatively small set of potentially attractive offers to investigate more closely (in practice, by physically viewing an offer). Such procedures reduce the search costs, while ensuring that the offer selected is attractive. Since such consumer choices are made at the level of households, models of such search should take into account game theoretic aspects and/or group decision procedures. This talk presents two game theoretic models of such a search procedure. According to the first of these models, the players are assigned different roles in the search procedure, i.e. one player chooses the shortlist and then the second player selects an offer from this shortlist. According to the second model, each player is equally involved in both rounds of the selection procedure. Conclusions regarding the appropriateness of these two procedures are made. Key Words: consumer decisions, heuristic procedures, imperfect information, search problem

05:00 PM **Differential games of public investment: Markovian best responses in the general case**

Jaakkola, Niko; **Wagener, Florian**

We define a differential game of public investment with a discontinuous Markovian strategy space. The best response correspondence for the game is well-behaved: a best response exists and maps a profile of opponents' strategies back to the strategy space. Our chosen strategy space thus makes the differential game well-formed as a normal form game, resolving a long-standing open problem in the literature. We provide a user-friendly necessary and sufficient condition for constructing the best response. Our methods do not require recourse to specific functional forms. Our theory has general applications, including to problems of noncooperative control of stock pollutants, harvesting of natural resources, and joint investment problems.

July 27, 2022

MFG 5 Mean Field Games 5

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Basar, Tamer

09:00 AM Optimal Control of the Fokker-Planck equation with state constraint in the Wasserstein Space

Daudin, Samuel

We consider an optimal control problem for the Fokker-Planck equation subject to a state constraint in the space of probability measures equipped with the L^2-Wasserstein distance. The first order necessary optimality conditions associated to this optimization problem give rise to a second order mean field game (MFG) system which has a potential structure. Since the problem is subject to the state constraint, additional unknowns (beside the usual density and value function variables) appear in the MFG system, which can be seen as the corresponding Lagrange multipliers. We will explain how to derive the optimality conditions and what we can deduce for the corresponding solutions. In particular we will show that -despite the presence of the state constraint- optimal controls are Lipschitz continuous in time.

09:25 AM Exploration Noise for learning Linear Quadratic Mean Field Games.

Delarue, François; **Vasileiadis, Athanasios**

Keywords: Mean Field Games, Reinforcement Learning, Common noise, Stochastic Control, Fictitious Play From their conception, Mean Field Games (MFGs) have known a rapid development in an increasing number of directions. Recently, they have attracted a lot of attention from the Reinforcement Learning (RL) community for their capacity to solve problems of increasing complexity. In a general RL paradigm an agent interacts with her environment through her actions in order to fulfil her objective. In response to her action she receives information about her state in the environment and a reward or cost specific for her task. A key concept is the balance between exploration of the environment and exploitation of it. In this talk we will try to bridge MFGs and RL. In a Linear Quadratic MFG example we will investigate this dilemma with common noise serving as a form of exploration. We will show convergence without any monotonicity condition or potential structure of our fictitious play scheme, and illustrate by numerical examples.

09:50 AM Incentive Designs for Stochastic Stackelberg Games in the High Population Regime

Sanjari, Sina; Bose, Subhonmesh; **Başar, Tamer**

Incentive Designs for Stochastic Stackelberg Games in the High Population Regime Sina Sanjari, Subhonmesh Bose, and Tamer Başar* (sanjari, boses, basar1)@illinois.edu University of Illinois Urbana-Champaign Abstract Incentive design problems are hierarchical decision-making problems between a leader and a collection of followers with possibly different goals, where the leader has dynamic information on the actions of the followers, or a function thereof, as well as some private observations. In this paper we study such dynamic games in the stochastic setting and with a high (finite as well as infinite) population of followers. Our goal is to study the extent to which the leader can craft a soft policy that induces a desired (targeted) behavior among the followers who operate under Nash equilibrium. By “soft”, we mean a policy that is smooth in the followers' actions, as opposed to a “threat policy” which is not smooth. Such targeted outcome is called “team optimal” if inducement leads to overall optimization of the leader’s expected cost with full cooperation of all the followers under the given information structure. We first show that in the finite but high population regime, and when the leader’s cost function is strictly convex and a few other sufficient conditions hold, there exists an incentive policy for the leader that induces an approximate (to any degree of approximation) team-optimal behavior, with a symmetric

approximate Nash best response among the followers. Next, leveraging functional analytic tools, we show that there exists an incentive policy, affine in the dynamic part of the leader's information, comprising partial information on the actions taken by the followers. We then turn to asymptotic analysis of incentive design problems, i.e., when the follower population is driven to infinity. In the infinite follower population game, we establish that there does not exist a smooth finite-energy incentive strategy, meaning that a mean-field limit for such games is not well defined. To arrive at a meaningful mean-field variant, we introduce an additional intermediate player, called the major follower, and analyze the setting where the leader only provides an incentive for the major follower alone, who in turn influences the remaining (minor) followers through her policy in a Nash game. For this class of games, we establish the existence of incentive policies for the leader and the major follower with finitely many minor followers that lead to inducement of targeted behavior. We also prove that if the leader's incentive policy with finitely many minor followers converges as their population size grows, then the limit defines a target-achieving incentive policy for the corresponding mean-field Stackelberg game. Finally, we show the existence of a randomized incentive policy for this mean-field Stackelberg game, which in turn provides an approximate incentive policy for the corresponding finite-population counterpart. Keywords: Incentive designs, stochastic Stackelberg games, mean-field games, dynamic information structure

* Author for correspondence: Tamer Başar, Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, 1308 West Main St., Urbana, IL, 61801, USA basar1@illinois.edu

EG 5 Evolutionary Games 5

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Brown, Joel

09:00 AM Darwin's finches and Darwinian dynamics: using evolutionary games to understand competitive pressures and ecological risks among co-occurring species.

Vincent, Tania L.S.; Apaloo, Joseph

The combination of an evolutionary game model and pertinent field data from Darwin’s finches (Geospiza spp.) from 15 islands on the Galapagos archipelago allowed us to take a snapshot in time of the evolutionary and ecological forces predicted to be in play on the extant species. Key inputs to the model include beak “strategies” and the empirical relationships between those strategies and the food resources available on each island, and a key assumption is that modeled strategies only interact via competition for food. Our model produced evolutionarily stable adaptive landscapes that could be compared to the “extant strategy" landscapes for each island at the time the field data were collected. This allowed us to predict which islands were “stable,” which islands could be invaded and by whom, which species were under evolutionary pressure, and which species were likely to go extinct

09:25 AM Ecological and evolutionary stability in Batesian mimicry systems

Scaramangas, Alan; Broom, Mark

The term aposematism (or "warning colouration" as first characterized by Alfred Russel Wallace in 1877) describes the process by which defended organisms (animals or plants) advertise their unprofitability to potential predators. Previous work has explored the relationship between evolutionarily stable levels of signalling and defence within the context of a game-theoretical, prey-predator setup in which the prey population consisted of a single species. The focus of this talk would be on more recent efforts to extend the mentioned predator-prey description to systems in which the prey population consists of two species: a model and a mimic. This modification leads us naturally into the celebrated adaptive mechanism named after Henry Walter Bates, Batesian mimicry. In Batesian mimicry complexes individuals from a palatable (mimic) species resemble individuals from an unpalatable (model) species to gain protection against predators. While there is ample empirical evidence to suggest that individuals from one species may gain selective advantage by resembling individuals from another, the mathematical modelling of Batesian

mimicry is rather limited. We predict that models and mimics can co-exist along a continuum of solutions (representing the conspicuousness, noxiousness, and average mimic-to-model proportion) that are both ecologically and locally evolutionarily stable. We establish several results that confirm both common sense intuition and a considerable body of related works. For instance, that the success of the mimic depends critically on the response of the predator and that this is likelier when the model is sufficiently aversive or that the relationship between the "mimetic load" and the equilibrium conspicuousness is an increasing one.

09:50 AM A model of plant community dynamics rooted in evolutionary game theory

McNickle, Gordon; Halloway, Abdel

Modern ecological theory shows that species coexistence can be decomposed into two main components: low fitness differences and high niche differences generated by equalizing and stabilizing mechanisms respectively. And yet, modern coexistence theory remains implicit in form; it does not say what the equalizing and stabilizing mechanisms are, only that they exist. We propose that evolutionary game theory can explain equalizing and stabilizing mechanisms by rooting them in functional traits. We demonstrate this by analyzing a game-theoretic version of a multispecies annual plant population model. Our modified model, which turn interspecific competition coefficients into a function of multidimensional traits, includes trait mediated plant-plant interactions and population dynamics. Our evolutionary game reveals how stabilizing selection in evolutionary time acts as an equalizing mechanism in ecological time and disruptive selection in evolutionary time as a stabilizing mechanism in ecological time. The tension between these two selection types constrain trait evolution to make coexistence more likely. An advantage of the evolutionary game theory is that a community’s trait distribution and species diversity need not be known a priori but emerge from the evolutionarily stable strategy. Overall, we suggest evolutionary game theory is a powerful framework for linking ecological and evolutionary processes and can be used to create a predictive theory of coexistence.

10:15 AM Leveraging Parrondo's paradox for sustainable agriculture

Gokhale, Chaitanya

Crop rotation, a sustainable agricultural technique, has been at humanity's disposal since time immemorial. Switching between cover crops and cash crops helps avoid the adverse effects of intensive farming. Determining the optimum cash cover rotation schedule for maximising yield has been tackled on multiple fronts by agricultural scientists, economists, biologists and computer scientists, to name a few. However, considering the uncertainty due to diseases, pests, droughts, floods, and impending effects of climate change is essential when designing rotation strategies. Analysing this time-tested technique of crop rotations with a new lens of Parrondo's paradox allows us to optimally use the rotation technique in synchrony with uncertainty. While previous techniques are reactive to the diversity of crop types and environmental uncertainties, Parrondo's paradox leverages this uncertainty. We calculate optimum switching probabilities in a randomised cropping sequence and suggest optimum deterministic sequences and judicious use of fertilisers. Our methods demonstrate strategies to enhance crop yield and the eventual profit margins for farmers. Conforming to translational biology, we extend Parrondo's paradox, where two losing situations can be combined eventually into a winning scenario, to agriculture.

DGEM 5 Dynamic Games in Economics and Management 5

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Buratto, Alessandra

09:00 AM Enhancing Visibility In Circular Economy Systems Through Blockchain Technology

De Giovanni, Pietro

This paper proposes a dynamic game based on circular economy systems in which the value of the return flows is subject to uncertainty due to the absence of monitoring systems connected to the backward activities. The lack of visibility induces several issues like the wrong estimation of the returns' residual value, the uncertainty of quality, the approximate estimation of operations and tasks, and the questionable positioning within the circular economy cascade. Among the various digital technologies developed in the recent years, the blockchain technology can support the circular economy systems by enhancing transparency, visibility, and traceability of the return flows, leading to better estimation of the economic value of returns and to more efficient circular economy policies and strategies. Hence, this paper shows the advantages that blockchain grants to circular economy systems by using a dynamic game approach.

09:25 AM Dynamic Pricing and Advertising with Reference Price Effect When Envisioning Product-recall Crisis

Lu, Lijue; Mukherjee, Arka

In this paper, we study a stochastic optimal control problem where a company determines the pricing and advertising effort. We have introduced reference price - a memory-based information that can be updated every time a new observation is added. We have also considered a product-recall crisis that can happen at a random moment in the future. Naturally, the reference price slumps when the crisis occurs, which will translate into a loss of sales. Moreover, the product-recall crisis could also leave a long-term scar on the reference price, represented by the weakened advertisement effectiveness. Taking into account the effects of both reference price and potential product-recall crisis, we have characterized the optimal pricing and advertising strategies. We found that at the moment of crisis occurrence, the policy adjustment (increasing or decreasing price and advertisement effort) is dependent on the instantaneous reference price. If the price expectation is high enough at that time, it is wise to lower the retail price and maintain a low profile. Whereas increasing publicity would be a proper strategy for the case of low reference price.

09:50 AM Multi-Period Stackelberg Game with Dynamic, Price-dependent, and Distributionally Robust Demand

Fakhrabadi, Mahnaz; Sandal, Leif Kristoffer

We consider a game between two parties of a channel where the manufacturer is the leader, and the retailer follows him. The demand is dynamic and price-dependent for a perishable commodity. The problem is formulated as multi-period revenue management, whereby the parties decide on manufacturer price, order quantity, and retail price on the market in each period to optimize their holistic profits over all periods. The commodity is perishable and cannot be stored for the next period. Hence, any unsold items must be salvaged or discarded at a lower price/cost (s). The sequence of decisions is: In each period the manufacturer first declares his price (w) to the retailer which then decides the retailer's price (r) to the market (customers) and the order quantity (q) from the manufacturer. The retailer faces the market risk through the stochastic demand (D) and may suffer from not meeting the market by either missing an opportunity to sell more or discard/salvage too much. The demand (D) depends on the retailer's prices and time (periods). In each period the uncertainty is unveiled after the decisions are made. This work extends earlier works by assuming incomplete knowledge about the distributions of the demands. The only information on demand we have is the mean and variance, but not the actual distributions, and by incorporating the effects

of previous decisions. It is implemented by letting the means and standard deviations depend on all previous retail prices, such that previous prices scale the demand, but only the current retail price determines the current coefficient of variation (CV). Both parties are risk-neutral and desire to maximize their expected, discounted total profit. Solving such a problem with many periods, is in general, computationally hard. To circumvent this obstacle, one can apply a special type of multi-period game that can be decomposed into a sequence of dependent 1-period games. A special case of this approach covers the general case of a multi-period supply chain game, in which the demand in each period only depends on time and current price both when the distribution is known and in a distributionally robust approach.

10:15 AM A dynamic model for native advertising

Grosset, Luca; **Buratto, Alessandra**; Brambilla, Chiara

We consider a communication platform where different messages and content appear. In such a channel, advertising may take two different forms: the traditional one and the native one. Native advertising is a widely used marketing tool that aims to mimic the regular topics of the platform on which it is placed. The striking resemblance between the native ad and the platform content makes the former quite effective because consumers might not recognize it as a sponsored message. Nevertheless, once the consumers realize the real commercial scope of the native ad, they may feel deceived and react by losing their trust in the platform's credibility. Our model considers a firm that invests both in traditional and native advertising on a media platform characterized by high-quality content. The editor of the platform gains profit by publishing ads and, at the same time, has to consider the loss of credibility that native advertising may cause. He wants to maximize the quality of the platform's content, minimizing his efforts to maintain a high-quality level. We formalize the problem over an infinite time horizon as a linear state differential game played à la Stackelberg, where the editor acts as the leader and the firm as the follower. With the main objective of studying whether native advertising is long-term sustainable for the editor, we determine a time-consistent open-loop Stackelberg equilibrium. keywords: Marketing, Native advertising, Differential game, Open-Loop Stackelberg equilibrium.

DGM 4 Dynamic Games Methodology 4

Location: FC1 006 - FC1 Edifício de Matemática
Chair: Turova, Varvara

09:00 AM Turret Escape Differential Game

Von Moll, Alexander; Fuchs, Zachariah; Shishika, Daigo; Maity, Dipankar; Dorothy, Michael; Pachter, Meir

In this paper, a zero-sum differential game is formulated and solved in which a mobile Evader seeks to escape from within a circle at whose origin lies a stationary, turn-constrained Turret. The scenario is a variant of the famous Lady in the Lake game in which the shore-constrained Pursuer has been replaced with the Turret. As in the former, it is assumed that the Turret's maximum angular rate is greater than the linear velocity of the Evader. Since two outcomes are possible, a Game of Kind arises - either the Evader wins by reaching the perimeter of the circle, or the Turret wins by aligning with the latter's position. A barrier surface partitions the state space into two regions corresponding to these two outcomes and a Game of Degree is solved within each region. The solutions to the Games of Degree are comprised of the Value functions (i.e., the equilibrium value of the cost/utility as a function of the state) and the saddle-point equilibrium control policies for the two players. Like the Lady in the Lake game, the equilibrium policy of the Evader is not uniquely defined where it has angular rate advantage over the Turret. Unlike the Lady in the Lake game, the losing region for the Evader is present for all speed ratios, and there is an additional semi-permeable surface separating center- and shore-bound Evader trajectories.

09:25 AM Robust controllability of linear systems: extension to a non-scalarizable case

Turetsky, Vladimir; Glizer, Valery Y.

Key words: linear system, robust controllability, ellipsoidal constraints Previous results of the authors on robust controllability of linear systems are extended to the case of a non-scalarizable problem with ellipsoidal control constraints. Basic concepts, such as the robust transferring strategy and the robust controllability set, are revisited. Two main results are established. First, subject to a proper assumption, the necessary and sufficient conditions for the linear strategy to be robust transferring are established. Second, the robust controllability set is constructed based on the solution of the non-linear equation. The time cross-sections of the robust controllability set are ellipsoids with respect to the non-linear coordinate transformation in the corresponding Euclidean space. Numerical results for a three-dimensional pursuit-evasion problem with elliptical control constraints are presented.

09:50 AM Equilibrium in a queueing system with strategic users

Mazalov, Vladimir; Chirkova, Julia

We consider a single-server queueing system with strategic users. The customers (players) log into the service system with certain rules during a fixed time interval [0,T]. There are many such service systems in reality, it can be a bank, a ticket office, lunch in a cafe, etc. Queueing theory provides the methods for the evaluation of the waiting times, delays and other costs. At the same time, the flow of service requests is initially set in the form of some random process with some initially set intensity. We are considering a model where the queue itself is formed as a result of the actions of the visitors themselves. The customers choose the time of entry into the service system themselves. Thus, a game is formed between the visitors of the service system. We demonstrate how game theory can be applied to optimally select the instant of entry into the service system using the examples of a service with preemptive access and a queueing system with retrials.

10:15 AM Nikolai Botkin's contribution to differential games

Turova, Varvara

The most important results obtained by Nikolai Botkin (22.03.1956 - 14.09.2019) on numerical methods for solving differential games will be described. Interesting applied problems investigated using the developed numerical methods will be presented. Keywords: Numerical methods, Nonlinear dynamics, State constraints, Value function, Discriminating kernel, Aircraft control, Biomedical applications

10:40 AM Break

MFG 6 Mean Field Games 6

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Cirant, Marco

11:10 AM A potential approach for planning mean-field games in one dimension

Ferreira, Rita; Bakaryan, Tigran; Gomes, Diogo

In this talk, we address the study of planning problems for first- and second-order one-dimensional mean-field games (MFGs), with and without congestion. These games are comprised of a Hamilton–Jacobi equation coupled with a Fokker–Planck equation. Our study is based on a potential approach as follows. Applying Poincaré's Lemma to the Fokker–Planck equation, we deduce the existence of a potential, which allows us to rewrite the Hamilton–Jacobi equation in terms of the potential. We then obtain a system of Euler–Lagrange equations for certain variational problems. By the direct method in the calculus of variations, we prove the existence and uniqueness of solutions to these variational problems. One advantage of such variational approach is the elimination of the continuity equation.

11:35 AM **Weak and renormalized solutions to a hypoelliptic Mean Field Games system.**

Mimikos-Stamatopoulos, Nikiforos

In this talk we will present results about the well-posedness of a degenerate, hypoelliptic Mean Field Games system with local coupling and Hamiltonians which are either Lipschitz or grow quadratically in the gradient. In the former case, we will talk about existence and uniqueness of weak solutions while in the latter we look at the same question for renormalized solutions. The approach we present relies on the kinetic regularity of hypoelliptic equations obtained by Bouchut and the work of Porretta on the existence and uniqueness of renormalized solutions for the non-degenerate Mean Field Game system.

12:00 PM **LQG Mean Field Games with Entropy Regularization**

Firoozi, Dena; Jaimungal, Sebastian

We present a general class of entropy-regularized multi-variate LQG mean field games (MFGs) in continuous time with K distinct sub-population of agents. We extend the notion of actions to action distributions (exploratory actions), and explicitly derive the optimal action distributions for individual agents in the limiting MFG. We demonstrate that the optimal set of action distributions yields an ϵ-Nash equilibrium for the finite-population entropy-regularized MFG. Furthermore, we compare the resulting solutions with those of classical LQG MFGs and establish the equivalence of their existence.

12:25 PM **Stability in MFG, (a bit) beyond Lasry-Lions monotonicity**

Cirant, Marco

I will discuss some results on the long time behaviour in Mean Field Games, first under the assumption that the coupling is mildly nonmonotone in the Lasry-Lions sense, then in the displacement monotone setting. The talk will be based on joint works with A. Porretta and A. R. Meszaros.

EG 6 Evolutionary Games 6

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Broom, Mark

11:10 AM **A Markovian decision model of adaptive cancer treatment and quality of life**

Bayer, Péter; Broom, Mark; Brown, Joel; Dubbeldam, Johan

This paper develops and analyzes a Markov chain model for the treatment of cancer. Cancer therapy is modeled as the patient's Markov Decision Problem, with the objective of maximizing the patient's discounted expected quality of life years. Patients make decisions on the duration of therapy based on the progression of the disease as well as their own preferences. We obtain a powerful analytic decision tool through which patients may select their preferred treatment strategy. We illustrate the tradeoffs patients are facing in a numerical example and calculate the value lost to a cohort who choose suboptimal strategies. In a second model patients may make choices to include drug holidays. By delaying therapy, the patient temporarily forgoes the gains of therapy in order to delay its side effects. We obtain an analytic tool that allows numerical approximations of the optimal times of delay.

11:35 AM **Why Multilingual, and How to Keep It -- An Evolutionary Dynamics Perspective**

Wu, Zhijun

While many languages are in danger of extinction worldwide, multilingualism is being adopted for communication among different language groups, and playing a unique role of preserving language and cultural diversities. How multilingualism is developed and maintained therefore becomes an important interdisciplinary research subject for understanding complex social

changes of modern-day societies. Here we consider a mixed population of multilingual speakers, with multilingual defined broadly as zero, limited, or full uses of multiple languages or dialects, and propose an evolutionary dynamic model for its development and evolution. The model consists of two different parts, formulated as two different evolutionary games. The first part accounts for selection of languages based on competition for population and social or economic preferences. The second part relates to circumstances when selection of languages is altered, for better or worse, by forces other than competition such as public policies, education, or family influences. By combining competition with intervention, we show how multilingualism may evolve under these two different sources of influences. We show in particular that by choosing appropriate interventional strategies, stable co-existence of languages, especially in multilingual forms, is possible, and extinction can be prevented, which is in contrast with major predictions from previous studies that co-existence of languages is unstable in general, and one language will eventually dominate the population while all others extinct.

12:00 PM **One evolutionary model of investment decision making**

Gubar, Elena; Gerschuk, Tatyana; Sánchez Carrera, Edgar J.

We propose a model for an evolutionary framework for a hierarchical game. We consider two levels of hierarchy: at the first level we have an evolution game between two large populations consisting of several subpopulations. At the next level, we have an evolution game within the parent population. At the same time, we say that as a result of the game of the first level, the players of the second level get access to "signals" for making decisions in their game. We consider this model using the example of ETF trading, where investment funds play at the first level, and ETF holders play at the second level. And we propose, that as a result of the parent game of the first level, the players of the second level receive signals that can be interpreted as "buy", "sell", "hold". We analyse the properties of the strategies, formulate a dynamic model and run a series of numerical experiments.

12:25 PM **Evolution in a General Equilibrium framework**

Accinelli, Elvio; Muñiz, Humberto

In this paper, we consider a general equilibrium model in which the economies are characterized by the distribution of firms on a set of branches of production; we will show that based on the decisions of the managers of the firms, it is possible to build a dynamic system whose solutions reproduce the possible trajectories of the economy. Once the initial state of an economy is known, that is, the initial distribution of firms, we will have a unique solution for this dynamic system, which will coincide with the evolution of the economy, that is, the evolution of prices and equilibrium allocations. The investment decisions of the administrators of the companies will change the distribution over the set of existing productive branches, which in turn will produce changes in the wealth of consumers who are also shareholders of the companies and then as a consequence, their demand will change, and therefore the equilibrium allocations and prices will too. In most cases, these decisions lead to an improvement in the efficiency of the productive side of the economy and an increase in the welfare of the economy as a whole, but, as we will show, under some particular circumstances, even when it comes to rational decisions from the point of view of administrators, this can lead to undesirable repercussions on the welfare of consumers. Besides, in a neighborhood of a critical economy, even when these decisions may involve small changes in the distribution of companies, they can cause abrupt and unexpected changes in the behavior of the economy, or in other words, they can cause an economic crisis. These are characterized by large changes in the prices, in the demand, and in the supply of goods. In contrast, in a sufficiently small neighborhood of a regular economy, small changes in the distribution of firms produced by the investment decisions of managers do not lead to large changes in the subsequent behavior of the economy. We will exemplify these statements with several numerical examples.

ERE 1 Environmental and Resource Economics 1

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Sbragia, Lucia

11:10 AM Two dynamic programming approaches for finding optimal and Nash equilibrium harvesting strategies in predator-prey systems

Pająk, Michał; Ramsey, David

Lotka–Volterra equations give us a relatively simple representation of predator and prey interactions. Despite straightforward formulation, such a model can produce complex behavior that has been thoroughly studied since its conception. New avenues of research in this topic have expanded the scope of analysis by including an external control representing the human involvement connected with harvesting the modeled species. We propose two methods of analyzing predator-prey systems. Both enable us to find the optimal harvest and Nash equilibrium when multiple harvesting players compete with each other. The first operates on a version of the model with discrete time and populations. An extensive search for solutions is conducted using the Bellman equation. The second approach adopts approximate dynamic programming to find the social optimum and Nash equilibria for a model with continuous populations and discrete-time. We compare the results obtained by both methods in the context of a predator-prey system with a carrying capacity for both species. We analyze three parameter sets for the model, each of which results in specific behavior when there is no external control. The first, without human intervention, leads to the extinction of both populations. In the second, the system tends to a steady-state equilibrium. The final one corresponds to oscillatory population dynamics. Keywords: predator-prey system, dynamic programming, game theory, optimal harvest

11:35 AM The optimal time to join an International Environmental Agreement

De Giovanni, Domenico; Lamantia, Fabio

We analyze the circumstances under which a country has the interest in joining, immediately or at some future point in time, an already signed International Environmental Agreement. For this reason, we consider a binding International Environmental Agreement (IEA), initially agreed upon by countries (henceforth signatory countries) and a non-signatory country (henceforth defector), who contemplates when (and whether) it is optimal to join the agreement. With the IEA, signatory countries commit to choosing their emission levels cooperatively. Moreover, signatories punish defectors. Suppose that choice to adhere to the IEA is irreversible, meaning that, once entered, signatory countries are unable to quit the IEA. We want to address the following research questions: - Under which circumstances does the defector join immediately, delay adhesion, or remain the defector forever? If the defector delays adhesion, what are the factors influencing the delay? - What is the impact of the optimal decision on the signatory countries? The decision problem consists of a two-stage differential game in which the defector chooses the optimal time to switch. In particular, in the first stage the signatory countries choose their emission rate to maximize the aggregate objective function, punishing the defector and thus suffering the cost of punishing; the defector selects her stategy non-cooperatively the emission rate, and the optimal time to join the IEA. In the second stage, that is after (and if) the defector has joined the IEA, all countries coordinate their emissions to maximize the present value of future aggregate benefits. In terms of the damage caused by pollution, we analyze both the linear and the quadratic cases. The former enjoys analytical tractability, and several equilibrium quantities can be computed in closed form. In this setup, the special case in which pollution is irreversible has a nice interpretable solution. As the latter is less tractable, numerical evidence guides the solution and the interpretation of the results.

12:00 PM **A Common Resource Game with Quasi-hyperbolic Discounting and Uncertainty**

Marin-Solano, Jesus; **Alderborn, Joakim**

We consider, in a discrete time setting, the problem of exploitation of a common property resource by two players exhibiting different quasi-hyperbolic discount functions. In addition, the dynamics of the stock of the resource is governed by a stochastic equation. In the search of time-consistent (subgame perfect) decision rules, the dynamic programming equations to be solved, both under cooperation and under noncooperation, are presented. Two models are discussed in detail. First, for an infinite horizon model with power utilities with equal risk aversion of all players, and linear dynamics, time-consistent strategies are derived. Effects of the asymmetries of the discount factors on consumption (extraction) rates and welfare are illustrated. Results of the cooperative and noncooperative settings are compared. Finally, a renewable resource finite horizon model with power utilities, in which players have different risk aversion, and nonlinear dynamics, is discussed. It is assumed that, when the planning horizon has been reached, the stock of the resource must be at a given (and known in advance by all players) level. The problem is solved numerically, and the effects of the different asymmetries (in preferences for consumption and time preferences), as well as of uncertainty, are illustrated. Keywords: Common property, quasi-hyperbolic discounting, uncertainty, asymmetric players

12:25 PM **Self-image and the Stability of International Environmental Agreements**

Sbragia, Lucia; Breton, Michèle

In this paper we examine the stability of international environmental agreements about a (common) emissions target. By signing the agreement, the parties develop a sense of responsibility to the commitment made, gaining a self-image that contributes to their utility. We study a dynamic two-stage game where all countries act individualistically. We investigate how two fundamental components of the model, that is, the ambition of the pledge and the relative importance given to compliance to the commitment, affect the stability and efficiency of the agreement in terms of global welfare and total emissions. We find that participation is the key driver of all the results and that it is negatively related to the ambition of the pledge and positively related to countries' level of concern about environmental issues.

01:00 PM Meeting of the Executive Board of ISDG

July 28, 2022

MFG 7 Mean Field Games 7

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Malhamé, Roland

09:00 AM A Mean Field Game model in Economics with spatial interactions in the human capital

Ghilli, Daria

Keywords: Economics, spatial interactions, spatial spillovers, non-monotone cost, non-separable Hamiltonian. We study a Mean Field Game (MFG) model arising in Economics where each agent chooses its position in space and its level of human capital- i.e. the skills aimed at use in production possessed by each individual- and interacts with the other agents through the human capital. The main goal consists in studying the effect of spatial interaction terms (called in economics spatial spillovers) on the abilities of each individual, due to the proximity, in terms of spatial distance, of other individuals. The resulting MFG system has a non standard structure, that is, a non-separable Hamiltonian and a non-monotone cost. In the talk we describe the economic model, we prove existence to the associated MFG system by a fixed point argument, and we provide some numerical experience. This is a joint work with C. Ricci (Pisa) and G. Zanco (LUISS). The economic model has been developed through contributions by G. Fabbri (CNRS Grenoble), D. Fiaschi (Pisa), F. Gozzi (LUISS).

09:25 AM A mean field control approach for smart charging

Séguret, Adrien

The increase in electrical vehicles (EV for short) will raise some challenges in the management of the equilibrium production/consumption of the electrical network. To avoid congestion effects, the charging of large fleets of EVs must be optimized. In this talk, a mathematical model to manage the consumption of such a fleet is introduced, using the mean fields limit assumption. The existence of a solution to the associated mean field control problem is shown, as well as optimality conditions thanks to the Fenchel-Rockafellar duality. Our main result is the characterization of the optimality conditions as a system of two coupled PDEs. This system is composed of a continuity equation and of a Hamilton-Jacobi equation, similarly to systems obtained in Mean Field Games (MFG). In a second part, we compare the mean field control problem to an optimization problem with a finite number of EVs. We obtain Gamma-convergence results thanks to the regularity of the optimal controls obtained in the first part, and thanks to a superposition principle adapted to this context.

09:50 AM Hybrid control for optimal visiting problems

Festa, Adriano

In an optimal visiting problem, we want to control a trajectory that has to pass as close as possible to a collection of target points or regions. We introduce a hybrid control-based approach for the classic problem where the trajectory can switch between a group of discrete states related to the targets of the problem. The model is subsequently adapted to a mean-field game framework to study viability and crowd fluxes to model a multitude of indistinguishable players. This is a joint work with F. Bagagiolo and L. Marzufero.

10:15 AM A quantilized mean field game for a dynamic demand-sensitive pricing problem

Malhamé, Roland; Caines, Peter; Foguen-Tchuendom, Rinel

An electricity provider wishes to set electricity price as a time-varying function of a general quantile of total demand as opposed to simply its mean. This is to partially offset the typical extra cost of

above mean versus below mean demand excursions. A quantilized mean field pricing game (MFG) is formulated and solved for an infinite pool of linear stochastic homogeneous households. Households optimize a quadratic cost which is a trade-off between economy and comfort objectives. Existence of an MFG equilibrium associated with a time-varying price trajectory is established, as well as an epsilon-Nash property when limiting control laws are applied to the large but finite system. Furthermore, the rate of convergence of the epsilon deviation to zero as a function of the number N of households and the nature of the chosen quantile is characterized. This is joint work with Rinel Foguen and Peter Caines.

EG 7 Evolutionary Games 7

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Gubar, Elena

09:00 AM A Binary Signal Model for Herding Behavior with Imprecise Probabilities

Imma Lory, Aprea; **Armando, Sacco**

The paper addresses uncertainty analysis in decision theory, by applying Imprecise Probabilities to a Herding Behavior model, which describes imitative behavior and explains informational cascades phenomenon. In classic herding behavior models, agents act sequentially and observe independent private signals, each of which can be high or low. The application of the principle of rationality generates informational cascades, which are sequences of actions in which each agent makes his choice by taking into account the decisions taken by those who acted before him, regardless of the private information they own. In the classical approach, the uncertainty that agents have about the future value of a security is represented by a signal probability, later updated to a posterior probability by the Bayesian updating rule to allow the agent to choose whether to invest or not. Since the probability distribution of the signal may be hard to identify in some cases, this paper studies the herding behavior model by considering imprecise the signal probability, which is precise in the referring literature. The model tests the herding behavior model robustness when some assumptions no longer hold due to imprecise probabilities. In the simplest case of the binary signal model, the agent's private information is described by using a set of probability measures and assuming that the signal probability ranges in a probability interval. Through this representation, agents' decisions depend on the interval extreme values. The aim is to prove that an informational cascade may occur even with imprecise probabilities, as long as certain assumptions hold. Keywords: Imprecise Probabilities; Uncertainty; Herding Behavior; Informational Cascades.

09:25 AM The replicator dynamics for state based and age structured game theoretic models

Argasinski, Krzysztof

We present an attempt to integrate the classical evolutionary game theory based on replicator dynamics and the state-based approach of Houston and McNamara. In the new approach, individuals have different heritable strategies; however, individuals carrying the same strategy can differ in terms of state, role, or the situation in which they act. Thus, the classical replicator dynamics is completed by the additional subsystem of differential equations describing the dynamics of transitions between different states. In effect, the interactions described by the game structure, in addition to the demographic payoffs (constituted by births and deaths), can lead to a change in the state of the competing individuals. Special cases of reversible and irreversible incremental stage-structured models, where the state changes can describe energy accumulation, and developmental steps, are derived for discrete and continuous versions. The special limit cases of the switching dynamics, describing the age-structured models, are equivalent to Leslie Matrix and McKendrick von Foerster model. The new approach is illustrated using the example of the Owner-Intruder game with explicit dynamics of the role changes. The research was supported by grant OPUS 2020/39/B/NZ8/03485.

09:50 AM Considering interaction times leads to novel predictions in old models

Krivan, Vlastimil; Cressman, Ross

We have developed a new approach to classic matrix games that considers, besides payoffs, the time lost in various activities that depend on the particular model (e.g., Krivan and Cressman 2017, Cressman and Krivan 2019,2020). E.g., in the case of the Hawk-Dove model, this time can comprise the length of interactions between the two contestants. In the case of the repeated Prisoner's dilemma it can be the number of rounds two contestants stay together. In the case of the Battle of the Sexes game these times also comprise time for courtship, and the time to rear offsprings. Since these times are strategy dependent, distribution of pairs (and singles) is not given by the Hardy-Weinberg distribution as it is for the classic models. In fact, distributional dynamics are described by differential equations for which explicit calculation of equilibria can be impossible. In this talk we review some of these new evolutionary models and we also show how chemical reaction network theory can be applied to prove uniqueness and stability of distributional dynamics (Cressman and Krivan 2022). References: Krivan, V., Cressman, R. (2017) Interaction times change evolutionary outcomes: Two-player matrix games. Journal of theoretical biology, 416:199–207 Cressman, R., Krivan, V. (2019) Bimatrix games that include interaction times alter the evolutionary outcome: The owner-intruder game. Journal of Theoretical Biology 460:262-273 Cressman, R., Krivan, V. (2020) Reducing courtship time promotes marital bliss: The Battle of the Sexes game revisited with costs measured as time lost. Journal of Theoretical Biology 503:110382 Cressman, R, Krivan, V. (2022) Using chemical reaction network theory to show stability of distributional dynamics in game theory. Journal of Dynamics and Games. doi:10.3934/jdg.2021030

10:15 AM Generalised social dilemmas: the evolution of cooperation in populations with variable group size

Broom, Mark; Pattni, Karan; Rychtar, Jan

Evolutionary game theory is an important tool to model animal and human behaviour. A key class of games are the social dilemmas, where cooperation benefits the group but defection benefits the individual within any group. Previous works have considered which games qualify as social dilemmas, and different categories of dilemmas, but have generally concentrated on fixed sizes of interacting groups. In this talk we discuss a systematic investigation of social dilemmas on all group sizes. This allows for a richer definition of social dilemmas. For example, while increasing a group size to include another defector is always bad for all existing group members, extra cooperators can be good or bad, depending upon the particular dilemma and group size. We consider a number of commonly used social dilemmas in this context, and in particular show the effect of variability in group sizes for the example of a population comprising negative binomially distributed group sizes. The most striking effect is that increasing the variability in group sizes for non-threshold public goods games is favourable for the evolution of cooperation. The situation for threshold public goods games and commons dilemmas is more complex.

ERE 2 Environmental and Resource Economics 2

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Martín-Herrán, Guiomar

09:00 AM Stackelberg competition in groundwater resources with multiple uses

de Frutos Cachorro, Julia; Martín-Herrán, Guiomar; Tidball, Mabel

We study a problem of exploitation of a groundwater resource, mainly used for irrigation, in which a water agency is needed in order to manage an exceptional and priority extraction of water for an alternative/new use (e.g. domestic water). To this goal, we build a two-stage discrete Stackelberg game in which the leader (the water agency) just intervenes when the new use takes place (in the second stage) and the follower is a representative agent of the regular users of the aquifer, i.e. the agricultural users. We study two types of Stackelberg equilibrium, which can arise

depending on the agents' commitment behavior, namely open-loop (commitment) equilibrium and feedback (non-commitment) equilibrium. We analyze and compare extraction behaviors of the different agents for the different equilibria and the consequences of these extraction behaviors for the final state of the resource and the agents' profits. For some hypotheses on the parameters, theoretical results show that commitment strategies lead to higher stock levels than non-commitment strategies when the leader's weight assigned to the profits from the agricultural use is lower or equal than the one assigned to the profits from the non-agricultural use. However, performing numerical simulations relaxing previous economic assumptions, we show that there are situations in which non-commitment strategies could be more favorable than commitment strategies not only in terms of final stock of the resource but also in terms of users' profits.

09:25 AM **Transboundary pollution and New Economic Geography in a dynamic game framework**

Morales, José Rodolfo; Martín-Herrán, Guiomar; Martínez-García, María Pilar

This paper formulates and analyzes a dynamic game between two trading regions that face a transboundary pollution problem. We study how the distribution of firms and trade costs affect the optimal emission decision of governments and how the resulting environmental policy would alter the allocation of the industry between both countries. The underlying microeconomic behaviour of consumers and firms is framed within the Economic Geography literature, in particular within the Footlose Capital Model. The macroeconomic model that arises is a transboundary pollution linear-quadratic dynamic game. For the case of fully symmetric regions, we find that higher transport costs lead to higher industrial production and higher stock of pollution at the steady state. However, while domestic consumption increases, exports could shrink depending on how the representative consumer weighs the homogeneous goods and the pollution damage in his welfare. For the case of regions that differ in their shares of industrial activity, we find some preliminary results. Governments modify their production with respect to changes in the pollution stock in a similar way to the symmetric case, although the optimal emissions are different in both cases. In a second stage we observe that the production decisions of the governments can generate core-periphery structures, even in the initially symmetric region case, which is new in this type of linear models.

09:50 AM **Being Naïve or Sophisticated? Endogenous Growth, the Environment, Pollution and Non-Constant Discounting.**

Mañó-Cabello, Carles

In this paper we study an endogenous growth model, of the expanding variety class with exhaustible natural resources and non constant discounting. We analyse both extreme consumer behaviours. First, we study the naive agent (time inconsistent), who tends to procrastinate. Later, we explore the sophisticated player (time consistent). The non-renewable natural resource is extracted by a firm in the resource sector, and this is needed to produce the final good. Final producers use labour, non-renewable resource and different number of intermediate inputs (machines) produced by a continuum of monopolist. The economic activity generates pollution, which affects households negatively. We then compare the behaviours under different discount functions and its welfare implications.

10:15 AM **Sharks, squalene and COVID vaccines - When a scarce renewable resource becomes strategic in short run**

Wiszniewska-Matyszkiel, Agnieszka; Singh, Rajani

In this paper, we model the problem of "the tragedy of the commons" that appears in the context of current pandemic, describing one of the aspects whose potential consequences hasn't been investigated. It is related to the fact that some of COVID-19 vaccines at various stages of development use adjuvants based on squalene obtained from endangered deep-sea shark species. We model the squalene market, with such COVID-19 vaccines producers as a part of it, as a compound dynamic game taking into account various aspects of this market. One of them is the privileged position of relatively small number of those vaccine producers. The game describes

a market consisting of pharmaceutical, cosmetic and fishing sector. We calculate Nash and Stackelberg equilibria in which COVID-19 vaccine producers do not take into account their influence on the population of sharks. We prove the existence and uniqueness of equilibria together with deriving the formula to find it. We discuss the consequences of relaxing the assumption of vaccine producers myopia concerning sharks. "The tragedy of the commons" in both cases results in endangering of the vaccination programme, including COVID vaccines: either because of depletion of the shark population or its reduction to a level at which the cost of squalene production exceeds the maximal price that can be paid for it. So, the solution of "the tragedy" by a regulatory institution is suggested, with various remedies that can be used by it. Despite using terminology related to a specific problem of high current importance, this may be treated as a starting point to a general theory emphasizing the need to indicate other potential scarce renewable resources for which similar phenomena are likely to appear in the future, in order to counteract such risks a priori.

SPG 1 Search and Patrolling Games 1

Location: FC1 006 - FC1 Edifício de Matemática
Chair: Angelopoulos, Spyros

09:00 AM Social Distancing, Gathering, Search Games: Mobile Agents on Simple Networks

Zeng, Li; Alpern, Steve

During epidemics, the population is asked to socially distance, with pairs of individuals keeping two meters apart. We model this as a new optimization problem by considering a team of agents placed on the nodes of a network. Their common aim is to achieve pairwise graph distances of at least D, a state we call socially distanced. (If D=1, they want to be at distinct nodes; if D=2 they want to be non-adjacent.) We allow only a simple type of motion called a lazy random walk: with probability p (called the laziness parameter), they remain at their current node next period; with complementary probability 1−p, they move to a random adjacent node. The team seeks the common value of p which achieves social distance in the least expected time, which is the absorption time of a Markov chain. We observe that the same Markov chain, with different goals (absorbing states), models the gathering, or multi-rendezvous problem (all agents at the same node). Allowing distinct laziness for two types of agents (searchers and hider) extends the existing literature on predator–prey search games to multiple searchers. We consider only special networks: line, cycle and grid.

09:25 AM Learning in a game of search and pursuit

Brethouwer, Jan-Tino; Fokkink, Robbert

In 2014 Gal and Casas introduced a game combining search games and pursuit games. In this game the hider (a prey animal) can hide in one of several locations. The searcher (a predator animal) can search a number of these locations. If the hider isn't found, the hider wins. If the hider is found, a pursuit follows, where the chance of a successful pursuit is determined by the location. In 2018 Alpern and Lee expanded on this idea and introduced a repeated game of search and pursuit, where the capture probabilities only become known to the players over time, as each successful escape from a location lowers its perceived value capture probability. In this talk we expand upon the work of Alpern and Lee. We introduce a variant of the game where the capture probability of a location becomes known after a single pursuit and extend the number of locations and number of rounds from 2 to n. We investigate when learning the capture probability of a new location is important, and how this influences the strategies of the players.

09:50 AM A polyhedral approach to search games

Lidbetter, Thomas; Hellerstein, Lisa

10:15 AM Searching on the Line with Hints

Spyros, Angelopoulos; Diogo, Arsénio; Shahin, Kamali

We study the classic problem of searching for a hidden target in an unbounded line, in a setting in which the searcher has some information (or hint) on the hider's position. We consider three settings in regards to the nature of the hint: i) the hint suggests the position of the target on the line; ii) the hint suggests the direction of the optimal search (i.e., to the left or the right of the root); and iii) the hint is a general k-bit string that encodes some information concerning the target. Our objective is to study the Pareto-efficiency of strategies in this model. Namely, we seek optimal, or near-optimal tradeoffs between the searcher's performance if the hint is correct (i.e., provided by a trusted source) and if the hint is incorrect (i.e., provided by an adversary). We also demonstrate how the results can be extended to noisy hints, and thus incorporate a notion of error into the analysis of the strategies.

10:40 AM Break

Stochastic Games

Location: FC1 029 - FC1 Edifício de Matemática
Chair: Kleshnina, Maria

11:10 AM McKean stochastic game for a spectrally negative L\'evy process and a negative discount rate

Palmowski, Zbigniew; Palczewski, Jan

In this talk we consider the McKean stochastic game for a geometric spectrally negative L\'evy process of asset price which extends the American put option by introducing the possibility for the writer of the option to cancel the contract, at the expense of paying the intrinsic value plus an extra constant penalty. Additionally we consider a negative discount rate which produces double continuation region for holder. This is then different from the scenario when the discount rate is positive which was considered by Baurdoux and Kyprianou (2008). The main tool used in this paper is the so-called guess-and-verify method.

11:35 AM The effect of environmental information on cooperation in stochastic games

Kleshnina, Maria; Hilbe, Christian; Simsa, Stepan; Chatterjee, Krishnendu; Nowak, Martin

In stochastic games, the environment changes depending on the choices made by the interactants in iterated encounters. Stochastic games are therefore a generalisation of repeated games, where the environment is constant and the same payoff matrix applies in every round. Here we study evolution of cooperation in stochastic donation games with two states, each representing a different environment. We consider pure memory-1 strategies: the next move depends on the outcome of the previous round. However, we distinguish two cases: (i) players receive no information about the state; or (ii) players do receive information about the state. Analysing all possible two-state stochastic games we find and characterise settings where receiving information about the state of the environment is (i) better (ii) neutral or (iii) worse for evolution of cooperation.

12:00 PM Bilateral sequential selections with minimization of failure

Szajowski, Krzysztof

The discussion of an extension of the bilateral secretary problem in which the decision makers (DM) sequentially observes common up to n applicants whose values are different and rank-able. Each DM must select exactly one applicant, cannot recall released applicants, and receives a payoff of α_i choosing the best one, or he is penalized of $-\beta_i$ or $-\gamma_i$ when he makes other choices or nothing, respectively. For each encountered applicant, the DM only learns whether the applicant is the best so far. The optimal polices are threshold type. There

is no guarantee that the equilibra for these problems are unique. This ambiguity in the solution is particularly interesting when we analyze the time spent on recruitment.

Topics in Applied Models Involving Microeconomic Agents

Location: FC1 030 - FC1 Edifício de Matemática
Chair: Álvarez-López, Alberto A

11:10 AM KNPLS: measuring Consumer Behaviour latent data with PLS-PM along with Kohonen Networks

Sérgio, Nobre

PLS Path Modelling enables the estimation of complex latent models measured through a set of indicators. The assumption of the latent data structure is homogenous is often unrealistic. Unobserved heterogeneity discovery and classification is until now based on a) the use of sequential segmentation strategies such as dendrogram or k-means for the initial classes, b) hierarchical clustering on multi-group Structural Equation Models, c) units are assigned to a predetermined, arbitrary number of clusters to uncover group-specific inner and outer path model relationships, d) are based on the expectation-maximization algorithm. These methods have several assumptions that conflict with the Partial Least Squares Path Modelling un-parametric, soft modelling approach. A new method was presented called KNPLS that segments with out supervision the presented data. A literature review was made on the Partial Least Squares Path Modelling methods that segment latent data, and most of all fail to the soft approach of PLS. The mobi dataset was used for study and the results show very clear that there are two groups of consuming, one small another bigger for large consumers.

11:35 AM Climate change, evaluation tools for the analysis of local microeconomic measures: A practical application case. The decarbonization of public transport in Zaragoza

Martín García, Rodrigo; Arizcuren-Blasco, Javier; Ruiz-Rúa, Aurora

The transport of both people and goods has a high contribution to the negative effects of climate change. This is especially the case in cities where the mobility needs of their citizens have meant that, for example, air quality has substantially worsened in recent decades. Urban mobility plays a fundamental role not only from the economic or social point of view but also in relation to the environment, in this sense the greater use of public transport has been shown as a measure that allows improving environmental conditions in cities due to pollution or congestion in urban traffic, among others. The evaluation of public investment decisions to encourage the necessary changes is, in this case, a priority for public bodies. The use of appropriate economic models and evaluation tools is required in the decision-making process for an efficient use of public resources. In the evaluation of public infrastructure investments for a sustainable development of urban mobility, the cost-benefit analysis is considered as the most appropriate evaluation tool for prioritizing investments. However, its practical application must be adapted to the evaluated reality, even considering the timing of the evaluation (ex-ante ex-post). In our presentation we analyse all these points applied to a case study: The decarbonization process of the Zaragoza bus system.

12:00 PM Optimal sports pricing under uncertainty

Álvarez-López, Alberto A; Rodríguez-Puerta, Inmaculada

We consider some issues concerning sports pricing under uncertainty. We first present a model in which a risk-averse sports team must choose a value for the price of its ticket for a particular event, but the ticket demand is unknown at the moment of decision. We study some comparative-static effects related to this model: the effect of changes in the team's risk aversion, the influence of a variation in the risk of the random demand, and the effect of a variation in the rate of a proportional profit tax. Concerning the latter, we derive some conditions under which the sports team finds it optimal to reduce the ticket price as a consequence of a rise in the tax rate. In the

same context, we also consider the problem of the capacity for the stadium. Capacity is usually assumed to be unlimited, in order to concentrate in marginal decisions. This assumption may be enough for several useful purposes, but it should be dropped in the case of very relevant events. Finally we study the problem of the optimal number of partners of the sports team. We formulate a model with two types of tickets: those for partners (with absence of uncertainty about them), and those for direct sale (depending on an uncertain demand). Keywords: decision under uncertainty; risk; uncertain demand; pricing; sports economics.

ERE 3 Environmental and Resource Economics 3

Location: FC1 004 - FC1 Edifício de Matemática
Chair: Chenavaz, Régis

11:10 AM Great fish war with moratorium

Dahmoun, Ilyass; Parilina, Elena; **Zaccour, Georges**

We consider a discrete-time version of the fish war model, where a regulator imposes a moratorium on fishing activities whenever the stock reaches a predetermined critical low value. The moratorium will be in place until the fish stock recovers, that is, attains a desirable value. We obtain conditions on the parameter values such that a moratorium could be avoided, and its optimal duration when its imposition is deemed necessary. We propose a coordinated harvesting strategy profile and determine when it matches the Nash equilibrium in linear-state strategies. Numerical examples show the significant influence of the fish reproduction rate on the length of a moratorium regime and the equilibrium properties of the coordinated strategy profile.

11:35 PM Does the circular economy fuel the throwaway society? The role of opportunity costs.

Régis, Chenavaz; Schlosser, Rainer; Figge, Frank; Dimitrov, Stanko

The efficient use of natural resources is considered a necessary condition for their sustainable use. Extending the lifetime of products and using resources circularly are two popular strategies to increase the efficiency of resource use. Both strategies are usually assumed to contribute to the eco-efficiency of resource use independently. We argue that a move to a circular economy creates opportunity costs for consumers holding on to their products, due to the resource embedded in the product. Using an analytical model, we show that in a perfectly circular economy, consumers are incentivized to discard their products more quickly than in a perfectly linear economy. A direct consequence of our finding is that extending product use is in direct conflict with closing resource loops in the circular economy. We identify the salvage value of discarded products and technical progress as two factors that determine the impact that closing resource loops has on the duration of product use. The article highlights the risk that closing resource loops and moving to a more circular economy incentivizes more unsustainable behavior.

SPG 2 Search and Patrolling Games 2

Location: FC1 006 - FC1 Edifício de Matemática
Chair: Lidbetter, Thomas, Rutgers University

11:10 AM A Classical Search Game in Discrete Locations

Clarkson, Jake

Consider a two-person zero-sum search game between a hider and a searcher. The hider chooses where to hide among n discrete locations, and the searcher successively searches individual locations until finding the hider. A search of a location takes a known, location-dependent search time and will find the hider---if hidden there---independently with a known,

location-dependent detection probability. The hider aims to maximize the total expected length of the search, while the searcher aims to minimize it. Due to the infinite number of pure search strategies, this search game is difficult to solve, and hence most work in the current literature is limited to two locations or locations searched in unit time. We extend much of this existing work to the fully-general search game, uncovering new properties along the way. In particular, we prove the existence of an optimal strategy for each player, show that the hider's optimal mixed strategy hides in each location with a nonzero probability, and show that the searcher's optimal mixed strategy can be constructed with up to n simple sequences of locations. Keywords: Search games, Gittins index, semi-finite games, search and surveillance.

11:35 AM Search and rescue with a go/no-go

Fokkink, Robbert

Search and Rescue games were introduced by Lidbetter (EJOR 285, 2020) and have been applied to scheduling and optimization problems. In an S&R game, Searcher takes binary decisions that have independent probabilities of success. What happens if the probabilities are dependent? The simplest case is a go/no-go decision, after which part of the search locations can no longer be visited. In this talk I will show that with some additional effort Lidbetter's analysis of S&R games transfers in this setting.

12:00 PM Continuous Patrolling Games

Alpern, Steve; Bui, Thuy; Lidbetter, Thomas; **Papadaki, Katerina**

We study a patrolling game played on a network Q, considered as a metric space. The Attacker chooses a point of Q (not necessarily a node) to attack during a chosen time interval of fixed duration. The Patroller chooses a unit speed path on Q and intercepts the attack (and wins) if she visits the attacked point during the attack time interval. This zero-sum game models the problem of protecting roads or pipelines from an adversarial attack. The payoff to the maximizing Patroller is the probability that the attack is intercepted. Our results include the following: (i) a solution to the game for any network Q, as long as the time required to carry out the attack is sufficiently short, (ii) a solution to the game for all tree networks that satisfy a certain condition on their extremities, and (iii) a solution to the game for any attack duration for stars with one long arc and the remaining arcs equal in length. We present a conjecture on the solution of the game for arbitrary trees and establish it in certain cases.

12:25 PM Lunch

02:00 PM Isaacs Awards Presentations Room: FC1 027