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European Journal of Operational Research xxx (2007) xxx–xxx

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

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O.R. Applications

Basin-wide cooperative water resources allocation

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Received 8 February 2006; accepted 29 June 2007

Abstract

The Cooperative Water Allocation Model (CWAM) is designed within a general mathematical programming framework for modeling equitable and efficient water allocation among competing users at the basin level and applied to a large-scale water allocation problem in the South Saskatchewan River Basin located in southern Alberta, Canada. This comprehensive model consists of two main steps: initial water rights allocation and subsequent water and net benefits reallocation. Two mathematical programming approaches, called the priority-based maximal multiperiod network flow (PMMNF) method and the lexicographic minimax water shortage ratios (LMWSR) technique, are developed for use in the first step. Cooperative game theoretic approaches are utilized to investigate how the net benefits can be fairly reallocated to achieve optimal economic reallocation of water resources in the second step. The application of this methodology to the South Saskatchewan River Basin shows that CWAM can be utilized as a tool for promoting the understanding and cooperation of water users to achieve maximum welfare in a river basin and minimize the potential damage caused by water shortages, through water rights allocation, and water and net benefit transfers among water users under a regulated water market or administrative allocation mechanism.

Keywords: OR in natural resources; Resources allocation; Multiple objective optimization; Cooperative game theory; Water rights; Equity; Economic efficiency; South Saskatchewan River Basin

1. Introduction

Water scarcity is now a common occurrence in many countries. It has been estimated that today more than 2 billion people are affected by water shortages in over 40 countries among which 1.1 billion do not have sufficient drinking water. The situation is particularly grave in many cities located in developing countries (UNWWAP, 2003). The major reasons are high water demand from population growth, degraded water quality and pollution of surface and groundwater sources, and the loss of potential sources of fresh water supply because of old and unsustainable water management practices (UNCSD, 1994). Water shortages are exacerbated by the geographically and temporally uneven distribution of precipitation (Al Radif, 1999).

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2007.06.045

Conflicts often arise when different water users (including the environment) compete for a limited water supply. In order to achieve sustainable development and a secure society, institutions and methodologies for water allocation should be reformed, especially for regions having water resources shortages. Water allocation should consider three key principles: equity, efficiency and sustainability (UNESCAP, 2000). Equity means that water resources within a river basin should be fairly shared by all of the stakeholders. Efficiency concerns the economic use of water resources with respect to minimizing costs and maximizing benefits. Under sustainability, water is utilized economically both now and in the future such that the environment is not harmed. However, it is not easy to fulfill all three of the principles or goals for a water allocation problem at the basin scale.

Although many mathematical simulation and optimization models for water quantity, quality and economic management have been developed for use under various water rights systems (McKinney et al., 1999), most models and applications do not address the fairness issue, except for prior water allocation models (Fredericks et al., 1998; Wurbs, 2001) which interpret fairness such that senior users owning higher priorities have more privileges to withdraw water than junior users possessing lower priorities. In the literature, there are only a few studies that jointly consider both efficiency and equity in water allocation (Cai et al., 2002).

Due to differences in economic benefits that can be garnered by different users, water allocations merely based on a water rights approach usually do not make efficient use of water for the whole river basin. On the other hand, an economic efficient water allocation plan generally is not an equitable one for all water users or stakeholders, and an economic water allocation plan cannot be implemented if the involved participants or stakeholders do not regard it as being fair. Based upon the idea that the achievement of equitable and efficient water allocation requires all stakeholders' cooperation in sharing water resources, a modeling framework was proposed by Wang et al. (2003) for obtaining equitable, efficient and sustainable short-term water allocations among competing water uses and stakeholders in a river basin. In this methodology, water allocation is carried out in two steps based on a network representation of a river basin: (1) initial allocation of water rights to water stakeholders and users founded on legal water rights systems or agreements; and (2) reallocation of water to achieve efficient use of water and equitable redistribution of net benefits to promote cooperation of all stakeholders in a river basin by utilizing cooperative game theoretic approaches. Based on this framework, the cooperative water allocation model (CWAM) has been recently developed (Wang, 2005).

The objectives of this paper are to introduce CWAM and apply it to a complex water allocation problem in the South Saskatchewan River Basin in order to demonstrate how CWAM can be utilized to assist decision makers in evaluating fairness issues underlying water rights allocation and economic value of water uses under different scenarios of institutional and hydrological conditions, and to promote fair and efficient water allocation among competing users. In Section 2, the river basin network structure of CWAM and the formulations and algorithms of the initial water rights allocation approaches are described. The modeling approaches for water and net benefits reallocation are then introduced in Section 3. Next, a brief introduction to the South Saskatchewan River Basin is given in Section 4, including water availability, uses and allocation policies. Six case scenarios of water allocation under wet, normal, and assumed drought hydrologic conditions are designed. The major results of the initial water rights allocation as well as water and net benefits reallocation are presented in Sections 5 and 6, respectively, while in-depth explanations are available as technical reports (Wang et al., 2006a,b).

2. Mathematical programming approaches to initial water rights allocation

2.1. General formulation

The Cooperative Water Allocation Model (CWAM) is designed as a comprehensive approach for modeling equitable and efficient water resources allocation at the basin scale based on a node-link river basin network (Wang, 2005). As illustrated in Fig. 1a, a node-link river basin network is a graphical model describing a river basin or watershed. A node is symbolized as a dot, circle, triangle, or rectangle, representing a physical component of interest such as inflow, natural or man-made junction, intake structure, water or wastewater treatment plant, aquifer, reservoir, natural lake, dam, weir, or aggregate water demand site. A link represents a natural or man-made water conduit such as a river channel, canal or pipeline between two different nodes, but can also stand for any flow of water such as the seepage between a demand site and an aquifer. For a regional

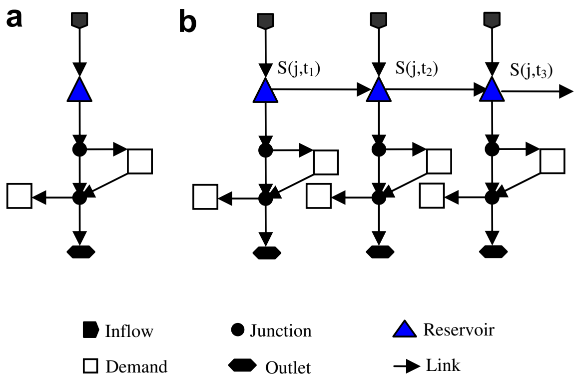

Fig. 1. Example river basin network (a) and multiperiod network (b).

water resources planning problem over multiple time periods, the river basin network can be represented as a multiperiod network configuration connected by the reservoir carry-over storage links as shown in Fig. 1b.

Constraints for water allocation formulated on a network structure are classified into three kinds: physical, policy and system control constraints. Physical constraints consist of mass balances and capacity limits. The water and pollutant balance equations for a general node k during each period t can be respectively written as

$$S(k,t) - S(k,t-1) = \sum_{(k_1,k)\in L} Q(k_1,k,t) - \sum_{(k_1,k)\in L} Q_l(k_1,k,t) + Q_g(k,t) - Q_c(k,t) - \sum_{(k,k_2)\in L} Q(k,k_2,t) \quad \forall k \in V, \tag{1}$$

$$C_p(k,t)S(k,t) - C_p(k,t-1)S(k,t-1) = \sum_{(k_1,k)\in L} C_p(k_1,k,t)Q(k_1,k,t) - \sum_{(k_1,k)\in L} Z_{pl}(k_1,k,t) + Z_{pg}(k,t) - Z_{pc}(k,t) - \sum_{(k,k_2)\in L} C_p(k,k_2,t)Q(k,k_2,t) \quad \forall k \in V, \tag{2}$$

where

V set of nodes,
L set of links,
t index of time periods (period length is Δt), $t \in T = \{1,2,\dots \tau\}$,
$S(k,t)$ storage volume for storage node (reservoir or aquifer)k, $k \in STO$, at the end of period t,
$Q(k_1,k,t)$ flow from node k_1 to k during period t,
$Q_l(k_1,k,t)$ conveyance losses because of evaporation, leakage and seepage of the flow from node k_1 to k,
$Q_g(k,t)$ gain of inflow adjustment at node k during period t for discharges from small tributaries, local catchment drainages, river reach seepages or flows from other sources,
$Q_c(k,t)$ water consumed at node k because of economic activities and evaporation,
p index of pollutant types, $p \in P = \{1,2,\dots,\xi\}$,
$C_p(k,t)$ concentration of pollutant p at storage node k at the end of period t,
$C_p(k_1,k,t)$ concentration of pollutant p in the water flow from node k_1 to k during period t,
$Z_{pl}(k_1,k,t)$ conveyance losses of pollutant p in the water flow from node k_1 to k,
$Z_{pg}(k,t)$ total amount of pollutant p added to node k during period t because of inflow adjustment Q_g (k,t) and of water use activities, and
$Z_{pc}(k,t)$ removal of pollutant p at node k.

Besides the general mass balance equations for each node, there are mass balance constraints for some natural physical response processes. These include link losses, node inflow adjustments, node losses, consumption and pollutant discharges, and outflows. For a typical water allocation problem, there are often several thousand constraints. Water allocation at the basin level is a generalized multiperiod network flow programming problem, which can be mathematically expressed as a multiple objective optimization problem:

$$\begin{aligned}
\max/\min \quad & \mathbf{f}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X}_s) \\
\text{subject to} \quad & \mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0}, \\
& \mathbf{h}_s(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X}_s) = \mathbf{0}, \\
& \mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geqslant \mathbf{0}, \\
& \mathbf{g}_s(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X}_s) \geqslant \mathbf{0}, \\
& \mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X}_s \geqslant \mathbf{0},
\end{aligned} \tag{3}$$

where $\mathbf{f}(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X}_s)$ is a vector of multiple objectives, $\mathbf{f} = (f_1, f_2, \ldots, f_m)$; $\mathbf{Q}$, $\mathbf{S}$ and $\mathbf{C}$ are, respectively, the vectors of network flow variables representing water flows, aquifer and reservoir storages, and pollution concentrations in link flows, aquifers or reservoirs; $\boldsymbol{X}_s$ is the vector of non-network type decision variables (side variables), which may be water prices, water transport costs, pollution control costs, crop types, irrigation areas, and product prices; $\mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0}$ (see, for example, Eqs. (1) and (2)) and $\mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geqslant \mathbf{0}$ represent the equality and non-equality constraints for network type variables $\mathbf{Q}$, $\mathbf{S}$ and $\mathbf{C}$, respectively; and $\mathbf{h}_s(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X}_s) = \mathbf{0}$ and $\mathbf{g}_s(\mathbf{Q},\mathbf{S},\mathbf{C},\mathbf{X}_s) \geqslant \mathbf{0}$ denote the equality and inequality constraints for both network type decision variables $\mathbf{Q}$, $\mathbf{S}$ and $\mathbf{C}$ and non-network type decision variables $\mathbf{X}_s$.

Depending on the purposes of water management planning, objective functions can be formulated in various forms. Some common types of objectives include: maximizing the flow to downstream nodes, maximizing the economic production, minimizing the differences in water deficits among all demand sites, or minimizing the pollutant concentrations at some locations. At the initial water allocation stage of CWAM, water rights is the focus and the objective is to satisfy water demands of all uses as fully as possible subject to water rights systems, water management agreements and policies, existing priorities, or economic factors. The decision variables of the problem are water flows and storages as well as pollutant concentrations, while non-network type factors are set as fixed. Two mathematical programming approaches are developed, namely the priority-based maximal multiperiod network flow (PMMNF) programming and lexicographic minimax water shortage ratios (LMWSR) methods. In CWAM, the water rights for a non-storage demand site are defined as a set of volume and pollutant concentration limits for all inflows and outflows; for a storage reservoir, water rights are defined as a set of reservoir storage and pollutant concentration limits.

2.2. Priority-based maximal multiperiod network flow (PMMNF) programming

In the PMMNF method, every inflow link to a demand node is viewed as consisting of one or several dummy sublinks and each sublink has a withdrawal demand and corresponding priority. Each inflow link to a stream flow requirement node is separated into a bypass sublink in addition to sublinks for stream flow requirements with various priorities. Note that no priority rank is assigned to a bypass sublink. The storage of every reservoir is also divided into several subzones each having a storage requirement and corresponding priority. PMMNF is formulated as the following problem with multiple ordered objectives:

$$\begin{aligned}
\max \quad & [\mathbf{f}^{(m)}(\mathbf{x})] \\
\text{subject to} \quad & \mathbf{h}(\mathbf{Q},\mathbf{S},\mathbf{C}) = \mathbf{0}, \\
& \mathbf{g}(\mathbf{Q},\mathbf{S},\mathbf{C}) \geqslant \mathbf{0}, \\
& Q(k,j,t) = \sum_{z=z_1}^{z_n} Q_z^r(k,j,t) \quad \forall j \in U, \\
& S(j,t) = \sum_{z=z_1}^{z_m} S_z^r(j,t) \quad \forall j \in RES, \\
& Q_z^r(k,j_1,t)/Q_z^r(k,j_2,t) = Q_{\mathrm{D},z}^r(k,j_1,t)/Q_{\mathrm{D},z}^r(k,j_2,t), \\
& \qquad \forall j_1,\ j_2 \in U,\ j_1 \neq j_2, Q_{\mathrm{D}}(j_1,t) = \emptyset,\ Q_{\mathrm{D}}(j_2,t) = \emptyset, \\
& \sum_{t\in T} \sum_{j\in U} \left[S_z^r(j,t) + (1 - e_L(k,j,t))Q_z^r(k,j,t)\right] = f_r(\mathbf{x}) \quad \forall j \in U \quad \forall r \in PR, \\
& 0 \leqslant Q_z^r(k,j,t) \leqslant Q_{\mathrm{D},z}^r(k,j,t) \quad \forall (k,j) \in L, \quad Q_{\mathrm{D},z}^r(k,j,t) > 0, \\
& 0 \leqslant S_z^r(j,t) \leqslant S_{\mathrm{D},z}^r(k,j,t) \quad \forall j \in RES, \\
& \mathbf{Q},\mathbf{S},\mathbf{C} \geqslant \mathbf{0},
\end{aligned} \tag{4}$$

where $\mathbf{f}^{(m)}(\mathbf{x}) = (f_{r_1}(\mathbf{x}), f_{r_2}(\mathbf{x}), \ldots, f_{r_m}(\mathbf{x}))$; $U = \{j \in V : j$ is a water demand node$\}$; $RES = \{j \in V : j$ is a reservoir$\}$; r is the priority assigned to a reservoir subzone or inflow sublink; and PR is the set of priority ranks whose elements are ordered from the highest to lowest as $r \in PR = \{r_1, r_2, \ldots, r_m\}$. Note that the priority ranks are positive integers, and the smaller the value the higher is the priority. $S(j,t)$ is the storage volume of reservoir $j \in RES$ at the end of period t; $S_z^r(j,t)$ and $S_{D,z}^r(j,t)$ are the storage variable and storage demand of the subzone z of reservoir j with a priority of r, respectively; $Q(k,j,t)$ is the flow from node k to demand j during period t; $Q_z^r(k,j,t)$ and $Q_{D,z}^r(k,j,t)$ are the inflow variable and withdrawal demand of the sublink z of link (k,j) with a priority of r, respectively; and $e_L\ (k,j,t)$ is the water loss coefficient for link (k,j) in period t.

The fifth constraint, for example, ensures that water is allocated proportionally in every time period to withdrawal demands with the same priority and directly connected to the same supply node, if no inflow demand is set for both demand nodes. The sixth constraint defines the objective function $f_r(\mathbf{x})$ as the total value of the storages and effective inflow volumes for all demands with the priority rank r during all time periods. If a vector $\mathbf{x}$ is used to represent all of the control variables and Ω is utilized to denote the feasible set defined by the constraints in the PMMNF problem, the problem can be expressed in a more compact form as

$$\max[\mathbf{f}^{(m)}(\mathbf{x}) : \mathbf{x} \in \Omega]. \tag{5}$$

If only linear water quantity constraints are included and all water demands are set to be the estimated constants, PMMNF is a linear program. It becomes a nonlinear program when water quality constraints are included, nonlinear area-storage relationships are applied to reservoirs, or demands of hydropower plants are allowed to vary with reservoir water heads. Sequential algorithms are built by solving the following maximal network flow program for each priority rank r', from the highest to lowest priority:

$$\begin{aligned} \max \quad & f_{r'}(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{x} \in \Omega, \\ & f_r(\mathbf{x}) \geqslant f_r^* \quad \forall r < r', \end{aligned} \tag{6}$$

where r' is the priority rank for which the current programming aims, and the f_r^* is the optimal value found for $f_r(\mathbf{x})$ in a previous solution loop. Without repeating again, all the algorithms in this paper are coded in GAMS and the solver MINOS is utilized (Murtagh et al., 2002) unless mentioned otherwise.

For a linear PMMNF, the primal simplex method can readily solve the problem. For a nonlinear PMMNF, a simple but effective domain decomposition approach called the ‘‘two-stage’’ approach is utilized by adopting a strategy of solving nonlinear programs from good starting points (Wang et al., 2007). Although it cannot be guaranteed that the two-stage approach will find the global solution, the ‘‘two-stage’’ algorithm can find an approximate global optimal solution. This is illustrated in the case study in Section 5. If multistart global optimization techniques for searching good starting points, such as the OQNLP solver (GAMS, 2005), are utilized in combination with the gradient-based nonlinear programming methods to solve the nonlinear programs in the second stage, the efficiency of the algorithm will achieve further improvement.

PMMNF is a flexible method that can be applied under prior, riparian and public water allocation regimes (Wang et al., 2007). Priorities may be assigned to uses according to the acquisition time of water rights based on the ‘‘first in time, first in right’’ rule under the prior water rights regime, or according to the relative locations under the riparian system, or according to the economic values and importance of water uses under the public rights system (Savenije and Van der Zaag, 2000). Note that the equity of water rights allocation is one of the main concerns of the first step of CWAM while economic efficiency is not. The solutions of PMMNF exhibit fairness to some degree because the PMMNF programming method searches for optimal allocations over the whole river basin and multiple periods as well as strictly preserves the priority order by sequential programming. If senior priorities are assigned to the basic water demands of all riparian landowners and junior priorities to their surplus demands, this special form of PMMNF leads to a third water rights allocations method called the modified riparian water rights allocation (MRWRA) within CWAM. Actually, MRWRA possesses the same mathematical formulation and algorithms as PMMNF. The MRWRA approach for application under a riparian rights system recognizes that an upstream riparian has the privilege to take water but should constrain its diversions for surplus demands to meet the basic needs of downstream riparian demands.

2.3. Lexicographic minimax water shortage ratios (LMWSR) programming

The lexicographic minimax water shortage ratios (LMWSR) method is a technique for water allocation under a public water rights regime adopting the lexicographic minimax fairness concept, which interprets fairness as minimizing weighted water shortage ratios of all water uses and the differences among them. The lexicographic minimax solution is always Pareto-optimal for a multiple objective problem and simultaneously satisfies equity principles (Luss, 1999). The equitable allocation of water storage, diversion and routing flow rights can be obtained through the following multiperiod lexicographic minimax program:

$$\mathrm{lexmin}[\mathbf{f}^{(\mu\tau)}(\mathbf{x}) : \mathbf{x} \in \Omega], \tag{7}$$

where μ is the number of uses, $\mu = |U|$; $\mathbf{f}^{(\mu\tau)}(\mathbf{x})$ is the vector of $\mu\tau$ elements $f_{jt}(\mathbf{x})$, whose elements are sorted in a non-increasing order; $f_{jt}(\mathbf{x}) = \omega(j,t) \cdot R(j,t)$ is the performance function of demand node j during period t, $\forall j \in U$, $\forall t \in T$; and $\omega(j,t)$ is the weight for the corresponding water shortage ratio. Note that Ω here is utilized to denote the feasible set defined by the general water quantity and quality constraints in the LMWSR problem. The demand shortage ratio, $R(j,t)$, is defined as

$$R(j,t) = \begin{cases} \frac{S_{\mathrm{D}}(j,t) - S(j,t)}{S_{\mathrm{D}}(j,t)} & \forall j \in RES \\ \frac{Q_{\mathrm{D}}(j,t) - \left(\sum_{(k,j)\in L}(1-e_L(k,j,t))Q(k,j,t) + Q_{\mathrm{g}}(j,t)\right)}{Q_{\mathrm{D}}(j,t)}, & \\ \forall j \in U \quad \forall j \notin RES & \end{cases} \tag{8}$$

where RES is the set of reservoirs; $S_{\mathrm{D}}(j,t)$ and $Q_{\mathrm{D}}(j,t)$ are the total demand of reservoir storage and off- or instream uses in time step t, respectively; and $Q_{\mathrm{g}}(k,t)$ represents the gain of inflow adjustment at node k during period t for discharges from small tributaries, local catchment drainages, river reach seepages or flows from other sources.

Similarly to PMMNF, the LMWSR programs can also be classified as linear and nonlinear. The generic solution approach for a lexicographic minimax program is to repeatedly solve a series of minimax problems:

$$\begin{aligned} \min \quad & M \\ \text{subject to} \quad & \mathbf{x} \in \Omega, \\ & \omega(j,t)R(j,t) \leqslant M \quad \forall (j,t) \in NR, \\ & \omega(j,t)R(j,t) \leqslant M^*_{jt} \quad \forall (j,t) \in FR, \end{aligned} \tag{9}$$

where NR is the set containing index pairs of (j,t) for which the corresponding upper bounds of $\omega(j,t)R(j,t)$ are not fixed, and, on the contrary, FR is the set containing index pairs of (j,t) for which the corresponding upper bounds of $\omega(j,t)R(j,t)$ are fixed to their optimal values M^*_{jt} found in the previous solution loops. Once a minimax problem is solved, constraints $\omega(j^*,t^*)R(j^*,t^*) = M^*$ are identified and the corresponding index pairs of (j,t) are removed from the set NR. At subsequent iterations, the upper bounds of these $\omega(j,t)R(j,t)$ are set to their optimal values. Iterations stop once the optimal values for all decision variables are identified. The number of iterations is normally much less than $|U| \times |T|$, and cannot exceed that. Note that the algorithm is well defined for linear problems, because Ω is a convex polyhedron at each iteration and a unique M^* can be easily found. A "two-stage" approach similar to that for nonlinear PMMNF programming is utilized to solve the nonlinear LMWSR problems.

It should be pointed out that, no matter which of PMMNF or LMWSR is utilized, the input parameters, especially the water availability, demands, priority or weights, affect the results of initial water rights allocation, and thus influence the subsequent reallocation of net benefits. The output of initial water rights allocation includes river flows, reservoir storages, offstream diversions, and pollutant concentrations in water for each time period. Periodical and annual statistics of water shortage ratios and satisfaction ratios are also calculated. A satisfaction ratio is determined by dividing the total value of inflow adjustment and effective inflows or storages by the node water demand during a specific time period.

Please cite this article in press as: Wang, L. et al., Basin-wide cooperative water resources allocation, European Journal of Operational Research (2007), doi:10.1016/j.ejor.2007.06.045

3. Equitable water and net benefits reallocation

3.1. Overview

To deal with the water shortage issue, water users may proceed to utilize their rights to pursue profits in a cooperate or non-cooperative way. Under a non-cooperative situation, conflict among stakeholders leads to a Nash equilibrium if they are rational players. However, a Nash equilibrium of a non-cooperative water allocation game may not necessarily be a Pareto optimal solution for the stakeholders, and a Pareto optimal solution may not be a Nash equilibrium. Even Pareto optimal solutions normally have total net benefits or social welfare less than the maximum that may be obtained by the cooperation of all players. This structural inefficiency of a non-cooperative equilibrium is interpreted as an incentive to promote stakeholders to cooperate in order to gain the maximum social welfare. Stakeholders are able to gain additional benefits if they cooperate to reallocate water properly based on their water rights, either through water markets or regulated water transfers.

The foregoing arguments explain why cooperative water allocation is attractive, but a key issue, fairness, still needs to be dealt with carefully in order to have cooperation. Theoretically, two main pathways are available for considering cooperation over water allocation: (1) the cooperation involves both water quota exchanges and side payments, i.e. direct income transfers; and (2) the cooperation is restricted to exchanges of water quotas only, and the distribution of income is determined solely by water transfers. Because water exchange generally causes the water giver to lose money, and the receiver to gain revenue, the second type of cooperative water allocation without side payments cannot provide incentive for a water user to participate in the cooperation. Thus, the first type of cooperative water allocation mechanism is incorporated into CWAM.

The second step of CWAM comprises three sub-models: the irrigation water planning model (IWPM), the hydrologic-economic river basin model (HERBM), and the cooperative reallocation game (CRG) of the net benefit of a given coalition. IWPM is a model for deriving benefit functions of irrigation water for all time periods, which maximizes the total profit of irrigated crop productions within an irrigation demand node by adopting quadratic empirical crop yield-water and salinity functions. HERBM is the core component of the coalition analysis, which is a tool for finding optimal water allocation schemes and net benefits of various coalitions of stakeholders. The inputs include hydrologic and water demand data, initial water rights, water demand curves and benefit functions, stakeholders, coalitions and owner-use relationships. CRG adopts cooperative game theoretic approaches to perform equitable allocation of the net benefits of a given coalition. The economically efficient use of water under a given coalition is achieved through water transfers (water reallocation) based on initial water rights.

3.2. Integrated hydrologic-economic river basin model

The most fundamental problems of hydrologic-economic modeling for water resources management are how to estimate the net benefits of water uses at demand sites and how to integrate the hydrologic and economic components. In CWAM, constant price-elasticity water demand functions are adopted to derive the monthly net benefit functions of municipal and industrial demand sites and hydropower stations, while quadratic gross benefit functions are used to find the monthly net benefit functions of agriculture water uses, stream flow demands and reservoir storage (Wang et al., 2006b). These functions are listed in Appendix A.

The integrated hydrologic-economic river basin model, or HERBM, is formulated as

$$\max\left(\sum_j \sum_t NB_{jt} : \mathbf{x} \in \Omega\right) \tag{10}$$

where NB_{jt} is the net benefit of demand j during period t; and $\mathbf{x} \in \Omega$ represents the hydrologic and economic constraints of the program. The objective of the model is to maximize the annual net benefit of water uses in the basin.

3.3. Reallocation of water and net benefits

Let $SH = \{1, 2, \ldots, sh\}$ be the set of all water stakeholders or players competing for water in the concerned river basin, $N = \{1, 2, \ldots, n\}$ be the set of stakeholders or players under consideration for reallocation, and $i \in N$ be a typical stakeholder. Obviously, $N \subseteq SH$. A group of stakeholders $S \subseteq N$ entering into a cooperative agreement and working together is called a coalition. N itself is called the grand coalition, the coalition consisting of all stakeholders under consideration. A coalition structure is a partition $\{S_1, S_2, \ldots, S_m\}$ of the n stakeholders, in which $\bigcup_1^m S_i = N$ and for all $i \neq j$, $S_i \cap S_j = \emptyset$. For a game with n players, 2^n coalitions are possible, or $2^n - 1$ if the null coalition is excluded. The expression $\upsilon(S)$ is used to represent the aggregate value gained by the members of coalition S, while the values of individual stakeholders acting in isolation are represented as $\upsilon(\{1\}), \upsilon(\{2\}), \ldots, \upsilon(\{n\})$.

Reallocation of water and net benefits through cooperation of all stakeholders in N can be viewed as an n-person *cooperative game* (N, υ), where N is the set of players, and υ is the characteristic function relating each coalition $S \subseteq N$ to a real number $\upsilon(S)$, representing the total value (net benefit) which S is able to generate through internal cooperation, with the convention that $\upsilon(\emptyset) = 0$. Since a coalition may correspond to a number of possible partitions containing it, a coalition S can have various values under different scenarios of partitions, in which the remaining stakeholders in the set $\{N - S\}$ may form a number of combinations of singletons or coalitions. It is extremely hard to find Nash equilibria for the large number of all possible partitions. Simplifications have to be made to reduce the problem size in order to keep the computational load within a reasonable limit. One may assume that all of the remaining stakeholders belonging to $\{N - S\}$ form another coalition outside of the coalition S. Then, the value of coalition S can be estimated as the Nash equilibrium found for the bargaining between the two coalitions. Another simple approach is to assume that the remaining stakeholders do not form any coalition and have no effect on the value of coalition S. The second approach is adopted in the current version of CWAM. Accordingly, the *value* $\upsilon(S)$ is defined as the maximum total net benefit that coalition S can gain based on the coalition members' water rights over the entire planning period, subject to not decreasing the water flows and not increasing the pollutant concentrations in the flows to other stakeholders not taking part in coalition S. Hence,

$$
\begin{aligned}
\upsilon(S) = \max \quad & NB(S) = \max\left(\sum_{i\in S}\sum_{j\in U_i}\sum_{t\in T} NB_{ijt}\right) \\
\text{subject to} \quad & \mathbf{x} \in \Omega, \sum_{(k,j)\in L} Q(k,j,t) \leqslant \sum_{(k,j)\in L} Q_R(k,j,t) \quad \forall j \in AGR \cup MI \cup HPP \text{ and } j \in U_S, \\
& Q(k,j,t) \geqslant Q_R(k,j,t) \quad \forall j \in U \setminus RES \text{ and } j \notin U_S, \\
& S(j,t) \geqslant S_R(j,t) \quad \forall j \in RES \text{ and } j \notin U_S, \\
& C_p(k,j,t) \leqslant C_{pR}(k,j,t) \quad \forall j \in U \setminus RES \text{ and } j \notin U_S, \\
& C_p(j,t) \leqslant C_{pR}(j,t) \quad \forall j \in RES \text{ and } j \notin U_S,
\end{aligned}
\tag{11}
$$

where $NB(S)$ is the net benefit function of coalition S, and $U_S = \bigcup_{i\in S} U_i$ is the set of water demand nodes of coalition S. The first inequality constraint ensures that the total diversion to all offstream demand sites and hydropower plants involved in a coalition (or water trade) does not exceed the total of their initial rights of diversion. To protect the water rights of water uses not participating in the coalition S, the next two inequalities set lower bounds of the water allocations while the last two inequalities set the upper bounds of pollution concentrations to corresponding initial rights. Under more cooperative environmental or regulated policies, the pollution constraints in the above definitions may be ignored.

A "solution" to a game is a vector of the payoffs or rewards received by each stakeholder after the reallocation or trade, which can be written as $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$. The payoff vector is normally called an imputation to the cooperative game. The central stability concept in cooperative game theory is the core, $C(N, \upsilon)$ (Young et al., 1982; Tisdell and Harrison, 1992). In particular, for a cooperative game (N, υ), a payoff vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ is in the *core* $C(N, \upsilon)$, if

$$\sum_{i \in N} x_i = \upsilon(N), \tag{12}$$

$$\sum_{i \in S} x_i \geqslant \upsilon(S) \quad \text{for all } S \subset N. \tag{13}$$

The first equality condition of this definition ensures that the payoff vector is feasible (the so-called joint efficiency condition) for the grand coalition N. The second inequality condition introduces a stability requirement which states that no coalition S by acting on its own can achieve an aggregate value higher than the share that it receives under the payoff vector. For coalitions with multiple players, it is called the group rationality condition; for singleton coalitions which consist of only one player, it is called the individual rationality requirement. If the core exists, a water and net benefits reallocation game may have an infinite number of solutions. If it is empty, there is no solution satisfying all of the rationality requirements.

Whether or not the core is empty, core-based and non-core-based concepts in cooperative game theory can be utilized to find a unique payoff vector equitable to all stakeholders (Owen, 1995). The frequently used nucleolus (core-based) (Owen, 1995) and the Shapley value (non-core-based) (Shapley, 1953) concepts are adopted in CWAM.

The nucleolus is the reward vector for which excesses for all coalitions are as small as possible, an excess being the amount by which the worth of a coalition exceeds the aggregate payoff to its members in isolation. The nucleolus satisfies the joint efficiency and individual rationality conditions. If the core is not empty, the nucleolus is in the core (Owen, 1995). Variations of the nucleolus are obtained by changing the definition of the excess function, while the optimization algorithm remains the same. The weak nucleolus concept (Young et al., 1982) replaces the excess with the average excess; proportional nucleolus (Young et al., 1982) substitutes the excess with the ratio of excess to net benefit of coalition S; normalized nucleolus (Lejano and Davos, 1995) replaces the excess with the ratio of excess to the summation of all individual payoffs of the members in coalition S.

Once the payoff vector is obtained, the values for players to participate in the grand coalition and associated side payments can be estimated. For stakeholder i, the *value* (or *gain*) *of participation* in the grand coalition equals $x(i) - \upsilon(\{i\})$. The *side payment* to other stakeholders is the difference of the value gained by participating in the grand coalition and allocated payoff, which is equal to $NB(i) - x(i)$. A negative side payment means receiving a payment from others.

The CWAM methodology presented above also has several notable implications for sustainability. Firstly, it can treat environmental requirements (stream flow and reservoir storage) as demands. For example, in the South Saskatchewan River Basin case study described in this paper, according to regulations, the model maintains certain minimum stream flows at several locations and discharges about 50% of the annual natural flow collected in the southern Alberta area to the downstream province of Saskatchewan to sustain its economy and the environment. Secondly, CWAM also considers water quality constraints. Thirdly, reallocation of water and net benefits encourages conservation of water. If an allocation only considers distributing water rights, water users have no motivation to improve their water use efficiency to save water for trading. Lastly, if a lexicographic approach is used for the initial water rights allocation, sharing of water shortages enforced by the method will pressure all water users to conserve water and enable all of their economies to have enhanced chances to be sustainable.

4. South Saskatchewan River Basin

The South Saskatchewan River Basin (SSRB) located in southern Alberta, Canada, comprises four sub-basins: the Red Deer, Bow and Oldman River sub-basins and the portion of the South Saskatchewan River sub-basin located within Alberta. The SSRB drains about 120,000 square kilometers and possesses a primarily semi-arid climate with the annual precipitation ranging from 300 to 450 millimeters for which less than half falls during the growing season (Dyson et al., 2004). More than 1.5 million people lived in the SSRB in 2004, about 81% living in urban centers including Canada's fifth-largest city, Calgary (about one million residents), Lethbridge (about 73,000 inhabitants), Red Deer (73,000 people), and Medicine Hat (51,000 people). Agriculture, petroleum, and manufacturing are major drivers of the economy in the SSRB (Dyson et al., 2004).

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Under the *Water Act*, the Government of Alberta owns the rights to all waters within its borders. Licenses assign to uses the maximum amounts of withdrawals and priorities on a first-in-time, first-in-right basis (Alberta Environment, 2003). According to the SSRB Water Management Plan approved in August 2006, Alberta Environment will no longer accept new water license applications for the Bow, Oldman, and South Saskatchewan sub-basin systems. New water allocations have to be obtained through water allocation transfers (Alberta Environment, 2006). Furthermore, water uses in the SSRB are subject to two water-sharing agreements: the *1909 Boundary Waters Treaty* between Canada and the USA and the *1969 Master Agreement on Apportionment* for the South Saskatchewan River between the Provinces of Alberta and Saskatchewan. The latter stipulates that at least 50% of the annual natural flow of the SSRB in southern Alberta shall be transferred to Saskatchewan, and the instantaneous flow to Saskatchewan should not be less than half of the instantaneous natural flow of the SSRB in southern Alberta or 42.5 m^3/s, whichever is less (Alberta Environment, 2002).

The SSRB network is depicted in Fig. 2. The network has 55 nodes in total, including 10 inflow (IN1–IN10), 1 outlet (O1), 17 reservoirs (R1–R17), 9 irrigation (A1–A9), 4 domestic (D1–D4), 4 general (G1–G4), 4 industrial (I1–I4), 2 hydropower plants (H1, H2), and 4 instream flow requirement (S1–S4)

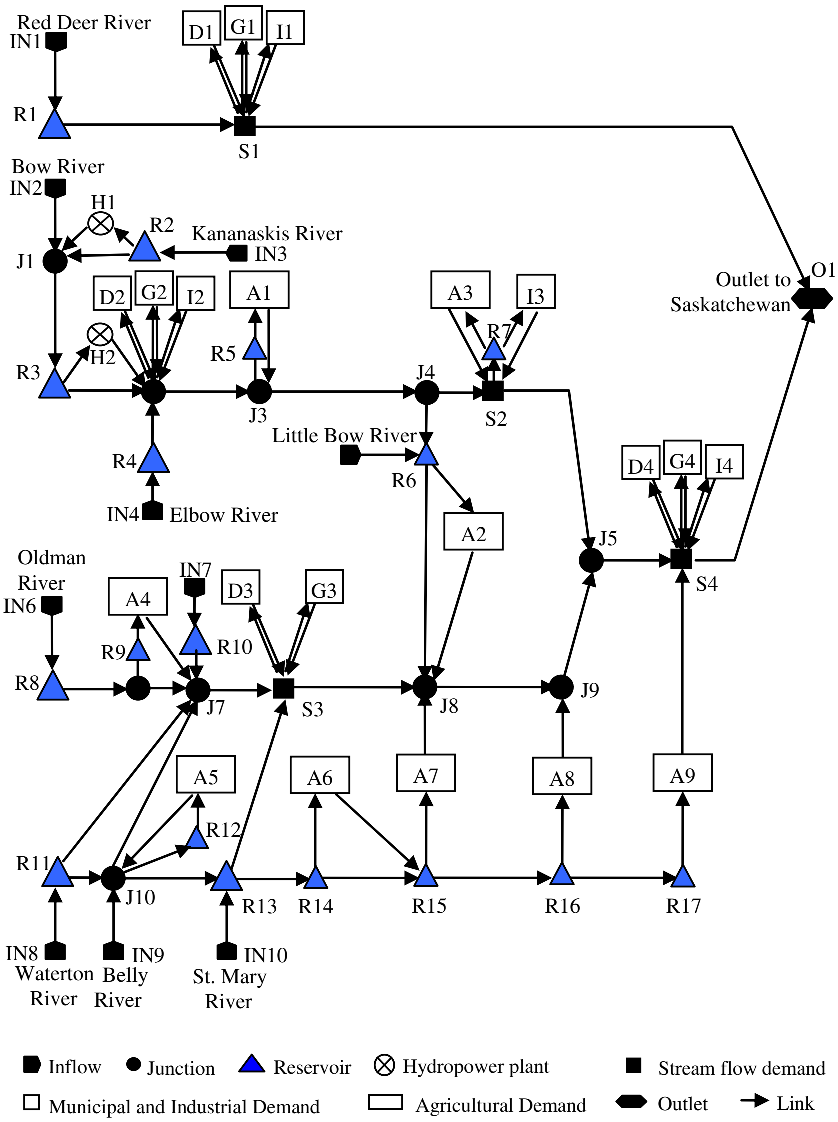

Fig. 2. Network of the South Saskatchewan River Basin in southern Alberta.

demand nodes. Note that the general demand refers to municipal, excluding domestic, demand. The directed links to offstream irrigation, domestic, general and industrial demand nodes are diversion canals, while the reversely directed links from them to nodes on streams represent the return flow routes. Thirteen irrigation districts and smaller privately-owned irrigation systems in the SSRB are aggregated into 9 irrigation nodes according to surface water sources and agroclimate zones. Groundwater sources are not considered.

The time horizon of the modeling in this study is one year having 12 monthly time periods. Input data are compiled from the Water Survey of Canada's HYDAT database (Environment Canada, 2002), reports of previous studies (Mitchell and Prepas, 1990; Mahan, 1997; AIPA, 2002; Golder Associates Ltd, 2003; Alberta Environment, 2004), and the online information system of Alberta's River Basins. Water demands of irrigation regions are assumed to occur during the growing season. Monthly irrigation demands are determined by the irrigation water planning model (IWPM), a sub-model of CWAM, as the difference between crop potential evapotranspiration and effective precipitation utilizing soft wheat, hard spring wheat, barley, canola, potatoes and alfalfa, as representative crops for the six crop categories. Salinity is considered in this study in order to explore the effects of water allocation on the salt concentrations in the river system and on the benefits of crop production.

Six case scenarios and related parameter sensitivity analyses are investigated according to the combinations of water demands, hydrologic conditions, and methods for initial water rights allocation. In Cases A, B, and C, initial water rights are allocated by the PMMNF method, reflecting Alberta's existing prior water rights system. In Cases D, E, and F, initial water rights are assigned by the LMWSR method, which reflects allocations under an assumed public regime. Case A (1995 wet & PMMNF) and Case D (1995 wet & LMWSR) represent the actual situations of water demands, tributary inflows and node adjustments in 1995. Case B (2021 normal & PMMNF) and Case E (2021 normal & LMWSR) consider the forecasted water demands in 2021, and the long term mean (1912–2001) tributary inflows and node adjustments. Case C (2021 drought & PMMNF) and Case F (2021 drought & LMWSR) explore water allocations under the forecasted water demands in 2021 and the hydrologic conditions of an assumed drought year. Since Case A represents the actual situation as of 1995, it is also used for calibrating model parameters such as water loss coefficients and node adjustments.

In the PMMNF method, 10 priority ranks are assigned to all the demands in the SSRB: all domestic water demands have the highest priority rank; licensed withdrawals of irrigation, municipal, and industrial demands with license application dates during the corresponding five time intervals, "Before 1982", "1982–1986", "1987–1991", "1992–1996" and "1997–2021" are assigned priority ranks 2, 3, 4, 7 and 8, respectively; hydropower generation water demands and stream flow requirements are set to priority ranks 5 and 6, respectively; and each reservoir is divided into two zones, the target and surplus storage zones, whose demands are assigned priority ranks 9 and 10, respectively.

In the LMWSR method, weights of water uses are set based on the "equivalent weighted shortages" rule, whereby water shortages are shared subject to equivalent weighted water shortage ratios. The higher the social utility or the lower the water-shortage endurance that the use has, the larger is the weight. Weights for demands are set as follows: domestic at 20, other offstream and hydropower generation water demands at 10, stream flow requirement at 3, and reservoir target storage at 1. This means that, without other constraints, if in a given month a reservoir storage has a shortage ratio of 90%, then the domestic, other offstream and hydropower generation water demands, and stream flow demands directly linking to and receiving outflows from it should share the shortage at ratios of 4.5%, 9% and 30%, respectively.

The modeling results show that in the wet and normal hydrologic years, all offstream and hydropower generation water demands are satisfied if PMMNF is used, and are nearly satisfied when LMWSR is utilized. Hence, there is little need or incentive for water reallocation. The upcoming discussion concentrates on the results of the two drought cases.

5. Initial water rights allocation in the South Saskatchewan River Basin

5.1. Priority-based water rights allocation

In Case C, the Bow River (A2), Lethbridge Northern (A4), St. Mary River-West (A7), Taber (A8) and St. Mary River-East (A9) irrigation regions, the general (G2) and industrial (I2) demands of Calgary, the hydropower plants on the Kananaskis River (H1), and the Oldman River at St. Mary River confluence (S3) will have water shortages, with the annual satisfaction ratios ranging from 0.966 (96.6%) to 0.475 (47.5%) as shown in Fig. 3. The stream flow demand sites usually receive more inflows than requirements since unused natural flows bypass them to the downstream, except that the annual flow at the Oldman River-St. Mary River confluence (S3) is 73.7% of its demand. Only two of the seventeen reservoirs, the Gleniffer Lake (R1) and the Crawling Valley, Lake Newell and Snake Lake Aggregate Reservoir (R7) are satisfied.

The difference in satisfaction ratios among demands reflects the ideals of the PMMNF method: the demands owning higher priorities have privileges to receive water, and an upstream demand has more advantage to take water than a downstream demand having the same priority rank. For the large municipality of Calgary, all monthly domestic demands are satisfied even if there is severe drought, because under Alberta's existing prior water rights system they are always assigned to the highest priority no matter when an application for withdrawals is submitted. The monthly satisfaction ratios of Calgary's general (G2) and industrial (I2) water demands are found to be constant, at 0.695 and 0.475, respectively, because the demands required by future development are licensed with lower priority ranks. Calgary would have to yield the rights to utilize these withdrawal licenses with lower priority in the drought scenario.

5.2. Lexicographic-based water rights allocation

Compared to the PMMNF method, the allocation by the LMWSR method under the drought Case F leads to more evenly distributed satisfaction ratios for offstream and the hydropower generation water demands varying from 0.802 to 1, as shown in Fig. 4. Similar to Case C, the stream flow demand sites normally have annual satisfaction ratios larger than one, except that the Bow River at Bassano Dam (S2) and Oldman River at St. Mary River confluence (S3) receive only 76% and 65%, respectively, of their required annual flows, and only the Gleniffer Lake (R1) and the Crawling Valley, Lake Newell and Snake Lake Aggregate Reservoir (R7) can store the target volumes.

As the LMWSR method for water rights allocation searches for the vector of lexicographic minimax fair distribution of weighted shortage ratios by iteratively finding the minimax water shortage ratios and then fixing their upper bounds, the demands possessing higher weights have the privilege to receive water, and all demands of the same weight are equitably treated no matter if they are located upstream or downstream. For example, the domestic use (D2) of Calgary usually has higher monthly satisfaction ratios than general

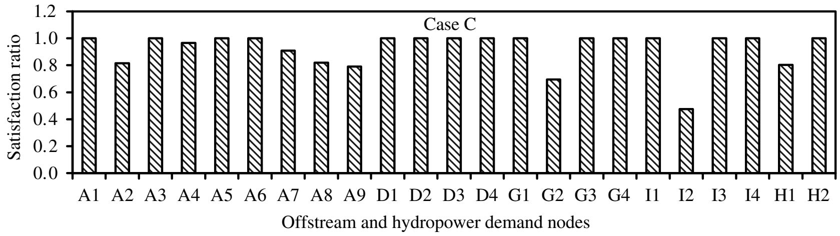

Fig. 3. Annual satisfaction ratios for initial water rights allocated by PMMNF (Case C).

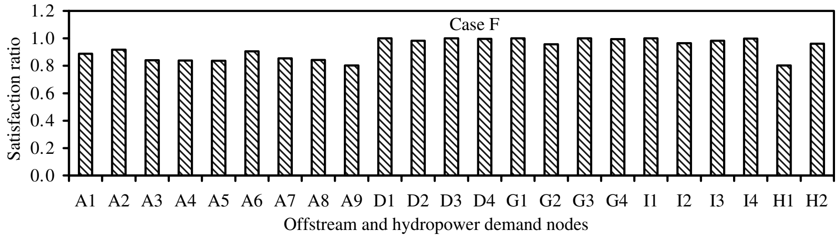

Fig. 4. Annual satisfaction ratios for initial water rights allocated by LMWSR (Case F).

(G2) and industrial (I2) uses under the drought year scenario, since the domestic use is assigned a larger weight. Information on reservoir releases and outflows to Saskatchewan are also obtained. The monthly demands of Raymond and Magrath Irrigation Region (A6) from May to August cannot be fully satisfied, while they are fully satisfied by the PMMNF method in Case C because of A6's advantageous upstream location.

6. Water and net benefits reallocation in the South Saskatchewan River Basin

6.1. Estimating water demand and benefit functions

The parameters of the monthly benefit functions for an agricultural demand node are estimated utilizing the Irrigation Water Planning Model (IWPM) (Wang, 2005). Note that the irrigation benefit is the remaining value of the crop production profit after deduction of the base benefit that could be gained with precipitation only. Thus, the demand and benefit functions for irrigation water vary with precipitation, crop pattern and areas. Each irrigation region has different irrigation benefit functions under different agroclimate and crop production conditions. A sample monthly benefit function of irrigation water at the Western Irrigation Region (A1) is shown in Fig. 5, assuming half of the long-term precipitation and 20% expansion of all crop areas over 1995 (the units for all monetary items in this study are in 1995 Canadian dollars).

Other investigations reveal that the municipal water demand elasticity differs by season, with elasticity being greater in summer than in other seasons (Diaz et al., 1997). For this research, monthly water demand functions of domestic and general (commercial and industrial) uses of municipal water supplies are assumed to be in the constant-elasticity form with an elasticity of -0.5 during the time period of May–September, and -0.4 for other months. All the monthly constant-elasticity water demand functions for all domestic and general demand sites are estimated from reference quantities, reference prices and constant-elasticity values. A reference price is defined as an observed price (marginal value) of water on a demand curve which corresponds to an observed quantity (i.e. reference quantity). Choke prices are arbitrarily set at \$5.00/m^3 for domestic water uses and \$3.50/m^3 for general water uses. Fig. 6 shows four examples of monthly domestic water demand functions for Calgary. Industrial water demand functions are estimated by the same methodology applied to municipal water demands. Choke prices are set at \$2.50/m^3 for all industrial water uses according to Mahan et al. (2002). It is assumed that hydropower generation has little effect on the power price in the regional power market. Thus, the hydropower demand curve remains at a constant price throughout the year. The unit hydropower value is estimated to be \$0.05/kW h according to Mahan (1997).

6.2. Net benefits of initial water rights

Water uses in the SSRB are grouped under the ownership of nine stakeholders: City of Red Deer (RD) – domestic, general and industrial; Bow River hydropower stations and associated reservoirs (BH); City of

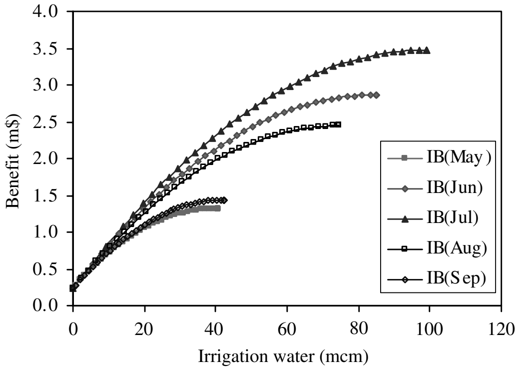

Fig. 5. Sample monthly benefit functions of irrigation at Western Irrigation Region (A1) (mcm means million cubic meters).

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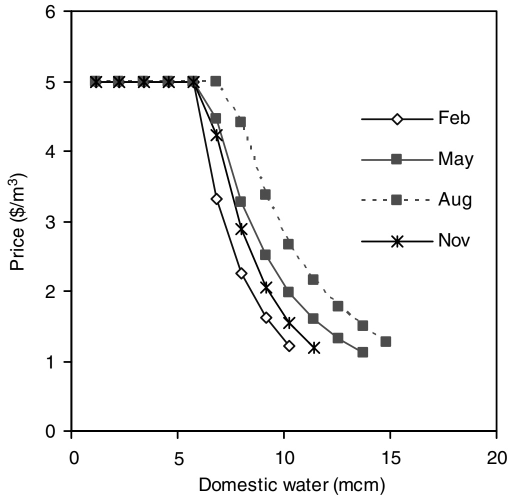

Fig. 6. Sample monthly domestic water demand curves for Calgary as of 2021.

Calgary (CA) – domestic, general and industrial; Eastern Industrial Region (EIN); Irrigation regions and associated offstream irrigation reservoirs in the Bow River sub-basin (BIR); City of Lethbridge (LB) – domestic and general; irrigation regions and associated offstream reservoirs in the Oldman River sub-basin (OIR); City of Medicine Hat (MH) – domestic, general and industrial; and Alberta Environment (AE) – stream flow demand sites and onstream reservoirs. Due to data limitations, the values of stream flows and reservoir storages are not explicitly included in the objective functions of the hydrologic-economic river basin modeling and coalition analysis. Instead their water rights are preserved through hydrologic constraints.

The economic analysis of the allocated initial rights demonstrates that for the drought Case C, the differences of water values among different water uses are shown by the different monthly marginal net benefits of raw water based on withdrawal rights to non-storage demand sites. Recall that the initial water rights are allocated by the priority-based maximal multiperiod network flow (PMMNF) method in this case. The marginal values of raw water withdrawn from the junction node J2 to the general (G2) and industrial (I2) demands of Calgary are more than $1.8 per cubic meter during all months, and are significantly higher than other uses, even the domestic use of Calgary. The domestic use of Calgary has the monthly marginal water value of $0.833 per cubic meter. Domestic, general and industrial water uses at other municipalities have marginal water values between $0.357 and $0.746 per cubic meter. The marginal values of irrigation withdrawals are between $0.015 and $0.051 per cubic meter, while those of hydropower stations are at a constant of 0.011 per cubic meter throughout the year.

For the drought Case F, the marginal values of raw water withdrawals based on the initial water rights allocated by the lexicographic minimax water shortage ratios (LMWSR) method are similar to those allocated by the PMMNF method, except that those of the domestic, general and industrial uses of Calgary are more evenly distributed among months between $0.745 and $1.312 per cubic meter. In the hypothetical drought year, the total annual inflow of non-storage demand sites in the SSRB based on the initial rights allocations by the LMWSR method is a little smaller than those allocated by the PMMNF method, and more water is left in the river system for satisfying stream flow requirements and is passed unto the downstream Province of Saskatchewan. However, the total net benefit based on the results of LMWSR is even greater than that by PMMNF. This means that the LMWSR method can produce water allocations that are not only equitable but also as economically efficient as PMMNF.

6.3. Basin-wide optimal water allocation

Under the basin-wide optimal allocation scenario in Case C, the irrigation regions A1, A2, A5, A6, and A7, and hydropower station H1, yield inflow amounts of 119.398, 5.158, 21.218, 2.006, 40.658, and 13.797 million cubic meters (mcm), respectively (202.235 mcm in total), while A8, A9, G2, and I2 receive an extra 9.103, 45.311, 24.314, and 80.954 mcm of water (159.682 mcm in total) in addition to their inflow rights, respectively. As noted, however, not all inflow yields by stakeholders can be received by others. The differences are lost during water transportation in the river system and diversion systems.

Annual net benefits of stakeholders under the initial and basin-wide optimal scenarios in Case C are summarized in Table 1, which shows water trade (including both intra- and inter-stakeholder trade) will lead to an increase in the total net benefit of the SSRB in the amount of 170.5 million dollars. Marginal net benefits of raw water withdrawals under the basin-wide optimal allocation scenario in Case C are evenly distributed among months due to the economic optimization covering all time periods. For example, the monthly marginal water values of the domestic, general and industrial uses at Calgary are $0.833, $0.745, and $0.745 per cubic meter, respectively. However, the marginal net benefits of raw water withdrawals at different locations and for different types of water uses may not be equal due to differences in their water demand or benefit functions, hydrologic constraints and the upper bounds set on monthly water demands.

Under the basin-wide optimal allocation scenario in Case F, large amounts of water are exchanged among irrigation regions, rather than to a more valuable user, the City of Calgary. Although the irrigation regions A5 and A7, as well as hydropower stations H1 and H2, yield inflow in the total amount of 272.126 mcm, while A1, A2, A3, A4, A6, A8, A9, D2, G2, and I2 receive an extra 263.925 mcm in total in addition to their inflow rights, the gain of water trade is only $15.53 million. This is caused by the more evenly-spread water satisfaction ratios of initial water rights obtained by the LMWSR method.

Table 1
Annual net benefits of water use by stakeholders under Case C (million $)

Stakeholder	Demand site	Water use net benefit		Stakeholder net benefit	
		Initial rights	Basin optimal	Initial rights	Basin optimal
RD (City of Red Deer)	D1	18.520	18.520	208.851	208.851
	G1	12.825	12.825		
	I1	177.506	177.506		
BH (Hydrostations on Bow River)	H1	2.157	2.002	4.847	4.692
	H2	2.690	2.690		
CA (City of Calgary)	D2	525.112	525.112	857.918	1030.963
	G2	172.774	207.975		
	I2	160.032	297.877		
EIN (Eastern Industrial Region)	I3	30.761	30.761	30.761	30.761
BIR (Irrigation regions in Bow River sub-basin)	A1	5.567	2.427	88.423	85.328
	A2	42.341	42.386		
	A3	40.515	40.515		
LB (City of Lethbridge)	D3	47.454	47.454	87.046	87.046
	G3	39.592	39.592		
OIR (Irrigation regions in Oldman River sub-basin)	A4	19.092	19.092	127.499	128.205
	A5	1.764	1.313		
	A6	5.511	5.463		
	A7	29.024	27.781		
	A8	21.673	22.080		
	A9	50.436	52.476		
MH (City of Medicine Hat)	D4	21.431	21.431	112.212	112.212
	G4	18.454	18.454		
	I4	72.327	72.327		
Total		1517.557	1688.059	1517.557	1688.059

6.4. Reallocation of net benefits among stakeholders

The eight stakeholders considered in the hydrologic-economic river basin modeling and coalition analysis lead to 255 non-empty coalitions. It would require extensive computational effort to solve so many large-scale nonlinear programming problems. However, the comparative analysis of the initial rights and basin-wide optimal scenarios under both Cases C and F demonstrates that only four stakeholders (BH, CA, BIR, and OIR) have significant changes of inflows and net benefits, while others are either nil or very small. Thus, the coalition analysis just needs to consider these four players. The reduction of stakeholders drastically decreases the coalition number to only 15. For every coalition, the multistart global optimization algorithm for coalition analysis utilizes the OQNLP solver to generate hundreds of scatter trial points, select good starting points, and summon the gradient-based nonlinear solver MINOS for further optimization and final solution determination.

Stakeholders may have different capacities of water withdrawal and different gains of water uses when participating in different coalitions. For example, under Case C, if BH and CA pursue the intra-optimal allocation, they may only get nil and \$28.110 million more than those obtained by their initial rights, respectively. But they can gain an increase of \$38.642 million if they work cooperatively. The different capability of gains for a stakeholder involved in different coalitions makes it necessary to analyze all the possible coalitions of stakeholders in order to promote the grand coalition and equitably allocate the net benefits (side payments). The cores of the cooperative net benefit reallocation games under both Cases C and F are non-empty, which means there are infinite possible allocations satisfying the equity rationalities as long as they are located in the cores. The allocations by various nucleolus and Shapley values can be used for reference and comparison purposes.

Value of participation in the grand coalition for each stakeholder is represented as the additional gain over the independent optimal (intra-stakeholder optimal) net benefit that can be produced based on his or her initial water rights. The additional gains under Case C after reallocation of water and net benefits with different cooperative game solution concepts are summarized in Fig. 7. Under Case C, Calgary is normally allocated

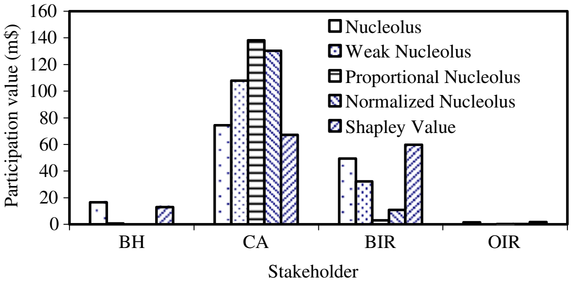

Fig. 7. Values of participation in the grand coalition for stakeholders under Case C reallocated with different cooperative game solution concepts.

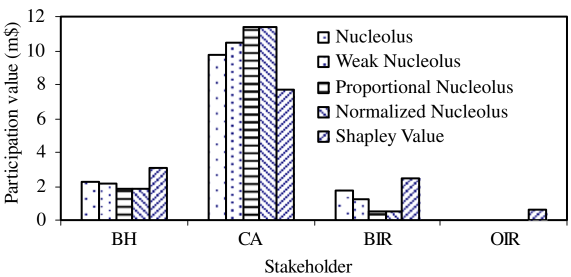

Fig. 8. Values of participation in the grand coalition for stakeholders under Case F reallocated with different cooperative game solution concepts.

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most of the additional gain over the intra-optimal scenario benefit, ranging from 67 to 138 million dollars, since it is the major contributor to the grand coalition. The Bow River Irrigation Regions (BIR) makes additional gains from about 3 to 60 million dollars, by receiving a side payment from Calgary for the water trade among them. The values of participation in the grand coalition under Case F are plotted in Fig. 8. The total additional gain over the intra-optimal allocation is only \$13.815 million. Most of the gain is allocated to Calgary due to its significant contribution to the grand coalition. BIR's gains range from \$0.553 to \$2.421 million, since it withdraws more water under the grand coalition scenario than its initial water rights. Bow River hydropower stations (BH) make additional gains of about 2 to 3 million dollars, by receiving side payments from Calgary and the Bow River Irrigation Regions (BIR) for water rights transferred to them.

7. Conclusions

The cooperative water allocation model (CWAM) is purposefully constructed for pursuing fair and efficient water resources allocation among competing users at the basin scale, taking into account hydrologic, economic and environmental interrelationships. CWAM consists of two main steps: initial water rights allocation, and equitable reallocation of water and net benefits. Multiple objective optimization techniques, consisting of the priority-based maximal multiperiod network flow (PMMNF) programming and lexicographic minimax water shortage ratios (LMWSR) programming, constitute realistic methods for initial water rights allocation under different water rights systems, which are the major components of the first step. The second step pursues optimal economic use of water at the basin scale through the fair reallocation of water and associated net benefits. The optimization model for accomplishing this contains three key integrated sub-models: the irrigation water planning model for estimating benefit functions of irrigation water, an extended hydrologic-economic river basin model for computing optimal net benefits for possible coalitions of stakeholders through water transfers, and cooperative game theoretic constructs for determining the equitable redistribution of the net benefits. The South Saskatchewan River Basin (SSRB) case study demonstrates that this novel methodology can be applied to complex real-world problems to suggest equitable resolutions which can be used as a basis for supporting informed decision making and negotiation. Besides employment in water allocation, the basic idea underlying CWAM can be appropriately modified for tackling resources allocation problems in other contexts.

Acknowledgements

The authors would like to express their appreciation to the anonymous referees and the Editor for their constructive comments which enhanced the quality of the paper.

Appendix A. Net benefit functions

A.1. Net benefit functions of municipal and industrial demand sites

The constant price-elasticity water demand function is:

$$Q(j,t) = \alpha(j,t)P(j,t)^{\beta(j,t)},$$

where

$Q(j,t)$ $\quad Q(j,t) = Q_a(j,t) + \sum_{(k,j)\in L} Q(k,j,t)(1 - e_L(k,j,t))$, total inflow to demand node j during period t (10^6 m^3),

$Q_a(j,t)$ inflow adjustment at node k during period t (10^6 m^3),

$Q(k,j,t)$ water quantity of inflow from link (k,j) during period t (10^6 m^3),

$e_L(k,j,t)$ water loss coefficient for link (k,j) in period t,

$P(j,t)$ price of willingness to pay for additional water at full use (\$/m^3),

$\alpha(j,t)$ parameter for the constant-elasticity demand function ($\alpha(j,t) > 0$), and

$\beta(j,t)$ price-elasticity of demand ($\beta(j,t) < 0$), $\varepsilon = (\partial Q/Q)/(\partial P/P) = \beta(j,t)$.

The inverse water demand function with constant price-elasticity and choke price is:

$$P(j,t) = \begin{cases} P_0(j,t), & 0 \leqslant Q(j,t) \leqslant Q_0(j,t), \\ (Q(j,t)/\alpha(j,t))^{1/\beta(j,t)}, & Q(j,t) > Q_0(j,t), \end{cases}$$

where
P_0 choke price ($\$/m^3$), and
Q_0 choke quantity ($10^6\ m^3$).

The net benefit function of the total inflow to a municipal and industrial demand site is:

$$NB_{jt} = B_{jt} - \sum_{(k,j)\in L} Q(k,j,t)wc(k,j,t) \quad \forall j \in MI,$$

where
B_{jt} gross benefit of the total inflow to a demand site j (10^6\$),
$wc(k,j,t)$ water supply cost ($\$/m^3$) of diversion, and

MI $MI = \{j \in V : j$ is a municipal and industrial demand node$\}$.

A.2. Net benefit functions of hydropower plants

The power generated at a hydropower station is:

$$POW(j,t) = \sigma\eta Q(j,t)\Delta H(j,t),$$

where
$POW(j,t)$ power generated (10^6 kW h)
σ $\sigma = 0.00273$, numerical coefficient to conserve units (10^6 kW h/$10^6\ m^3$m), and
η turbine efficiency (%),
$Q(j,t)$ rate of discharge ($10^6\ m^3$), and
$\Delta H(j,t)$ effective water head of hydropower generation (m).

The inverse demand function for hydropower with constant price-elasticity and choke price is:

$$P(j,t) = \begin{cases} P_0(j,t), & 0 \leqslant POW(j,t) \leqslant POW_0(j,t), \\ (POW(j,t)/\alpha(j,t))^{1/\beta(j,t)}, & POW(j,t) > POW_0(j,t), \end{cases}$$

where
P_0 choke price (\$/kW h) for hydropower, and
POW_0 choke quantity (10^6kW h) for hydropower.

The net benefit function of the total inflow to a hydropower plant is:

$$NB_{jt} = B_{jt} - POW(j,t)pc(j,t) - \sum_{(k,j)\in L} Q(k,j,t)wc(k,j,t), \forall j \in HPP$$

where
$pc(j,t)$ power production cost (\$/kW h) for hydropower station j during period t, and
HPP $HPP = \{j \in V : j$ is a hydropower plant$\}$.

A.3. Net benefit functions of agricultural demand sites

The quadratic form gross benefit function is:

$$B_{jt} = b_0(j,t) + b_1(j,t)Q(j,t) + b_2(j,t)Q(j,t)^2 + \sum_p [b_{3p}(j,t)C_{pN}(j,t) + b_{4p}(j,t)C_{pN}(j,t)^2 + b_{5p}(j,t)Q(j,t)C_{pN}(j,t)], \quad \forall j \in AGR$$

where

$C_{pN}(j,t)$ $C_{pN}(j,t) = \frac{1}{Q(j,t)}\left[Q_a(j,t)C_{pa}(j,t) + \sum_{(k,j)\in L} Q(k,j,t)C_p(k,j,t)(1 - e_{pL}(k,j,t))\right]$, mixed concentration of pollutant p in the total inflow to demand node j (g/l),

$C_{pa}(j,t)$ pollutant p concentration in node adjustment (g/l),

$C_p(k,j,t)$ concentration of pollutant p in link flow (g/l),

$e_{pL}(k,j,t)$ pollutant p loss coefficient for link (k,j) in period t,

AGR $AGR = \{j \in V$: j is an agriculture demand node$\}$, and

b_0 to b_{5p} coefficients obtained by econometric methods or by regression analysis from the output of external simulation or optimization models.

The net benefit function is:

$$NB_{jt} = B_{jt} - \sum_{(k,j)\in L} Q(k,j,t)wc(k,j,t) \quad \forall j \in AGR.$$

A.4. Net benefit functions of stream flow requirement demand and reservoirs

Because instream recreational opportunities, aquatic ecology and environment quality are not generally goods sold in a market, estimating the benefits from water use for stream flow demands and reservoir storages requires unique economic valuation approaches, such as the travel cost method and the contingent valuation method (Brown et al., 1991; Diaz et al., 1997). The gross benefit function is represented as a quadratic function, which is the same as that of an agricultural demand, except that the items of flow volumes are changed to average storages for a reservoir. The net benefit function is also same as that of an agricultural demand.

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