Dynamic Competition in Electricity Markets: Hydropower and Thermal Generation

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Abstract

We study dynamic duopolistic competition between hydropower and thermal generation plants under demand uncertainty. We characterize (S-adapted open-loop and Markov perfect) equilibrium investments, hydropower scheduling, and thermal outputs. We compare duopoly equilibrium outcomes with the socially efficient outcomes. In particular, the degree of investment in thermal-capacity and the efficiency of water-use are analyzed.

Keywords: Dynamic game; capacity investment; demand uncertainty; electricity markets

1 Introduction

It is common to find alternative electricity generation technologies coexisting in a market. An interesting mix from the point of view of the dynamics of competition is when a hydroelectric generator coexists with a thermal generator. In many jurisdictions, hydroelectric power generation is the dominant source of electricity. It accounts for 80% of generation in New Zealand, 97% in Brazil, 90% in Quebec, and 98% in Norway. In other jursdictions, such as Ontario and California, it is a significant source of electricity, but not as dominant.

Hydroelectric generation can be characterized by low marginal cost when operating, but subject to the availability of water to drive the turbines. In contrast, thermal generation units have more flexibity in the sense that their inputs (gas, coal, etc.) are not subject to the same contraints as water in a resevoir, however the marginal cost of generation is higher.

The dynamics of hydroelectric generation become important for the analysis of competitive behaviour in a market with mixed generation technologies because the availability of water can constrain the use of hydroelectric generation. The objective of this paper is to analye the dynamic interaction between hydro and thermal generation when they coexist in a market. We model a market for electricity with two producers: one with a hydroelectric power generation facility and the other with a thermal electric power generation facility. We are interested in the following questions:

- What sort of competitive behaviour do we expect to see in such a market? Does the hydroelectric producer have incentives to strategically withhold or release water?
- What are the implications for investment in additional generation facilities? In particular, does strategic behaviour by the hydroelectric producer inhibit capacity investment by the thermal generator?

We turn next to a review of the literature regarding hydroelectric power generation in a market setting. We then turn to the analysis of a finite horizon game played between a hydroelectric generator and an thermal generator. Finally, we investigate an alternative, infinite horizon version of the game.

1.1 Literature Review

We now discuss the existing work that has been done regarding the behaviour of hydroelectric power generators operating in market settings. The papers most relevant to ours are Crampes and Moreaux [3] and Bushnell [1]

Scott and Read [13] apply a dual dynamic programming methodology to a short term market simulation model to analyze the effects of contracts and market structure in the New Zealand wholesale electricity market. They compare the simulation market outcomes of Cournot duopolists (comprised of hydro and thermal players each with several stations) and competitive firms. In particular, they optimize hydro reservoir management in competition settings in which hydro generation supplies the bulk of electricity consumption. They find that high amounts of (physical) contracts along with high demand elasticity reduce gaming activities and increase both productive efficiency and allocative efficiency. In addition, production withholding incentives such as deliberately spilling water or pushing storage toward shortage do not occur contrary to the predictions of some critics of reform in NZ.

Haurie and Moresino [7] study a discrete-time concave stochastic dynamic oligopoly in which they characterize S-adapted (open loop) equilibrium as the solution of a variational inequality (VI) problem. They propose an approximation method to solve for large scale VI problems in which the scale of the problem increases as the number of scenarios (i.e., possible demand states) and players increase. The approximation method uses Monte Carlo sampling technique which reduces the size of the VI problem, and then they compute the equilibrium of the VI problem by using an algorithm due to Konnov (1993). They illustrate their methodology on two examples in which they find that the equilibrium outcomes approach to turnpike steady states as the sample size increases.

Crampes and Moreaux [3] study a short run electricity market with different structures, (monopoy, duopoly, social planner) in which hydropower and thermal generators compete for electricity production in two periods subject to a deterministic demand. In the Markov perfect equilibrium duopoly, as a result of scarce water resource (i.e., final period water constraint binds) thermal output in the final period is affected by both initial and final period decisions of the hydro output, but this does not hold in the open loop duopoly equilibrium.

Bushnell [1] analyzes a multi period (daily) short run (a month horizon) Cournot oligopoly with fringe competition between multiple firms in the Western United States. All firms have both thermal and hydro production technologies and compete for production quantities in the market with linear and deterministic demand, and piecewise linear marginal costs. He provides equilibrium conditions in the form of linear complementarity conditions, which are solved by means of a solver, here it is PATH solver. By using a data from the western states regions, he solves the equilibrium production units of suppliers in a month horizon. He finds that some larger firms may find it profitable to allocate more water in off-peak periods of oligopoly than in off-peak periods of perfect competition, and cause allocative and productive inefficiencies.

Pritchard and Zakeri [12] use a Stochastic Dynamic Programming approach to solve for optimum offer curves of hydroelectric generators, which are price takers, in the simplified version of the New Zealand electricity spot market. They assume neither strategic generators nor other production technologies in the market. Specifically, they model the uncertain electricity spot prices, the only random term in the whole model, based on a first-order Markov process. They present construction of optimal offer curves, which may be used in offering electricity in the market, based on numerical examples.

Chaton and Doucet [2] extend the model proposed by Chaton (1997) to three periods and study Hydro-Quebecs capacity expansion planning in a stochastic linear programming model. Hydro-Quebec is provincially owned monopoly with hydroelectric capacity close to 90% of the total available capacity in the province. The objective function is minimization of total expected costs subject to market clearing constraint, and transmission and production constraints. The uncertainty stems from fuel costs and demand growth. The aim is to meet the final period demand by capacity additions (with option values) made in earlier periods. They calibrate the model, using the GAMS software, with the data from Hydro-Quebec and neighboring jurisdictions to forecast investment behavior of Hydro-Quebec.

Pritchard et. al. [11] formulate a dynamic programming model to optimize offer stacks (supply functions) of a price taking generator operating a hydro reservoir. In the model the hydro generator can adopt his supply function decisions to finer time scale market trading periods given that the reservoir levels are subject to longer time horizon.

2 A Finite-Horizon Model

We assume that demand for electricity is an affine function and stochastic. Specifically, the inverse demand is $P(Q_n) = D - \beta Q_n + \tilde{\delta_n}$, where n is the node number on the event tree, $D > 0$ is the intercept, $\beta > 0$ is the slope, Q_n is the quantity (e.g., MWh) at node n, $\tilde{\delta_n}$ is the random variable, and P is the price. We assume a long run analysis, in which demand is price responsive, i.e., the length of a time period is relatively long. We use a

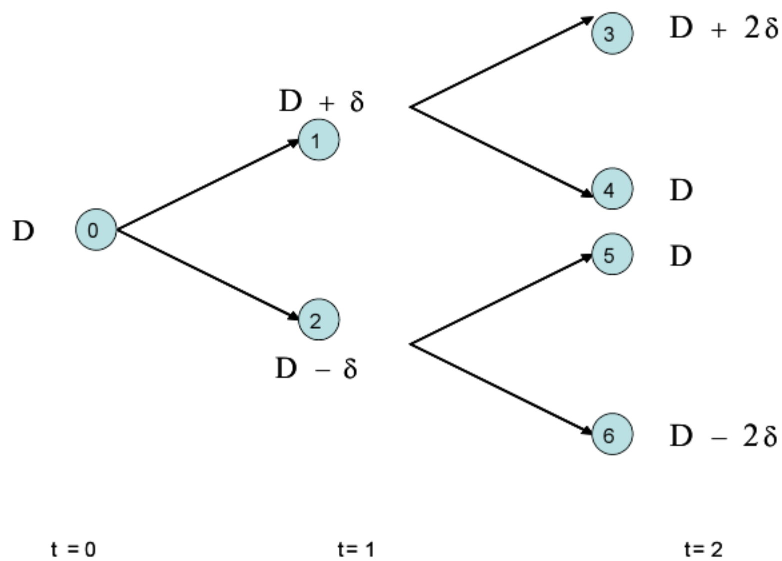

Figure 1: Event tree of demand states for 3 periods.

tree representation of demand, and let $\tilde{\delta_n}$ follow a random walk process. It is Markovian and independent of suppliers' decisions. We depict the event tree for three periods below in Figure 1, in which we assume two demand states at time 1: "upstate" and "downstate"; four demand states at time 2: "up-upstate", "down-upstate", "down-upstate", and "down-downstate". In short, the number of states at time t is 2^t. From any node to the following node, demand increases by δ amount with probability u or decreases by δ amount with probability d. These probabilities are exogenous and known by all suppliers, and of course $u + d = 1$. Let the initial period root node denote node 0, and denote upstate demand node 1 and downstate demand node 2. From any node in period 1 to time 2 nodes demand can take either upstate (i.e., demand shifts upward by δ) or downstate (i.e., demand shifts downward by δ). Let nodes offspring from node 1 be node 3 and node 4 for the upstate and downstate, respectively, with the same probabilities above. Let nodes offspring from node 2 denote node 5 and node 6 for the up and down states, respectively. The other nodes in the following periods ($t = 3, 4, ...$) follow the similar order as above.

We assume that there are two players (i.e., suppliers): a hydro player (with hydropower units that use water in dam(s) to spin the electric generators) and a thermal player (with thermal units that burn some fossil fuel(s)). [1] The marginal cost of production for hydro units is zero. The hydro player is not involved in capacity investments, and doesn't expand the production capacity by any means. [2] We let W_0 represent the initial stock of water available to the hydro producer. In each period, there is an inflow of water into the resevoir that can be either "high" or "low", denoted f_h and f_l. The probabilities of the high and low flows occuring are p_h and p_l.

The notation for the thermal production and investment costs is as follows. $F(I_n^T)$ stands for capacity investment cost of thermal unit at node n and is a quadratic function of investment I_n^T, where the superscript T denotes thermal. The marginal cost of investment at node n is linear and equal to $e_1 I_n^T$, where $e_1 > 0$. $C(q_n^T)$ denotes production cost of thermal generation at node n and is a quadratic function of production q_n^T. The marginal cost of thermal production at node n is affine and equal to $c_1 q_n^T + c_2$ where $c_1 > 0$, $c_2 \geq 0$.

2.1 Duopoly

We study a three period/stage ($t \in${0,1,2}) dynamic stochastic game [3]. We assume that the thermal player can invest in capacity and the hydro player is not able to investment on the hydropower units. Let $K_0 > 0$ be the initial thermal capacity. For the thermal producer the state variable is the capacity, and the control varible is the investment. Assume that the capacity accumulation dynamics is linear and is as follows. The state at node 1 is $K_1 = K_0 + I_0^T$, the state at node 2 is $K_2 = K_0 + I_0^T$. The other state equations are $K_3 = K_0 + I_0^T + I_1^T$, $K_4 = K_0 + I_0^T + I_1^T$, $K_5 = K_0 + I_0^T + I_2^T$,

[1] The thermal cost structure may incorporate a vector of technologies that burn different kinds of fossil fuels.

[2] Construction of a new river dam, depending on the production capacity, to produce electricity can take upto fifteen years. e.g., Three Gorges Dam in China, the largest dam on earth, completed in about 14 years. Also because of environmental and several other reasons (e.g., irrigational, recreational), it may not be possible to expand the available production capacity of the hydro player. Marginal cost of production is generally assumed to be zero, since the water turning the turbines is a free good.

[3] The game could be extented to more than three, but finite, periods, but then variance of the demand would explode. We could overcome this issue by considering a different type of random variable with finite and bounded variance. However, it still would be hard to analytically solve equilibrium strategies at the final stage. Note that, the number of nodes exponentially grows as the number of stages increases. Therefore, for the sake of tractability we focus on three periods that we show below capture some dynamics of the capital accumulation.

and $K_6 = K_0 + I_0^T + I_2^T$ at the nodes 3, 4, 5, and 6, respectively. We will assume that I_0^T, I_1^T, $I_2^T \geq 0$, hence we do not consider reducing the existing capacity. Also we assume no capacity depreciations, although it is easy to incorporate into the above states. On the other hand, the hydro player has no state state trajectories since it is not involved in capacity expansion.

The thermal player solves the following maximization problem:

$$
\begin{array}{rl}
\max & P(Q_0)q_0^T - C(q_0^T) - F(I_0^T) \\
+ & u[P(Q_1)q_1^T - C(q_1^T) - F(I_1^T)] + d[P(Q_2)q_2^T - C(q_2^T) - F(I_2^T)] \\
+ & uu[P(Q_3)q_3^T - C(q_3^T)] + ud[P(Q_4)q_4^T - C(q_4^T)] \\
+ & du[P(Q_5)q_5^T - C(q_5^T)] + dd[P(Q_6)q_6^T - C(q_6^T)] \\
s.t. & q_0^T \leq K_0, q_1^T \leq K_0 + I_0^T \\
 & q_2^T \leq K_0 + I_0^T, q_3^T \leq K_0 + I_0^T + I_1^T, q_4^T \leq K_0 + I_0^T + I_1^T \\
 & q_5^T \leq K_0 + I_0^T + I_2^T, q_6^T \leq K_0 + I_0^T + I_2^T \\
 & I_0^T, I_1^T, I_2^T \geq 0, q_n^T \geq 0, n = 0, 1, ..., 6
\end{array}
$$

where $P(Q_n) = D + \tilde{\delta_n} - Q_n$, and we normalized the slope. $P(Q_n)$ is the inverse demand at nodes $n = 0, 1, ..., 6$ and where $Q_n = q_n^T + q_n^H$is the total quantity supplied by the thermal generator (q_n^T) and the hydropower generator (q_n^H). The expected profit function is a function of its own control variables, its own outputs, (and available capacity) and the rival's outputs. It is twice continuosly differentiable.

The hydropower player solves the following maximization problem:

$$
\begin{array}{rl}
\max & P(Q_0)q_0^H - C(q_0^H) \\
+ & u[P(Q_1)q_1^H - C(q_1^H)] + d[P(Q_2)q_2^H - C(q_2^H)] \\
+ & uu[P(Q_3)q_3^H - C(q_3^H)] + ud[P(Q_4)q_4^H - C(q_4^H)] \\
+ & du[P(Q_5)q_5^H - C(q_5^H)] + dd[P(Q_6)q_6^H - C(q_6^H)] \\
s.t. & q_0^H \leq W_0, q_1^H \leq W_0 + p_h f_h + p_l f_l - q_0^H \\
 & q_2^H \leq W_0 + p_h f_h + p_l f_l - q_0^H, q_3^H \leq W_0 + 2(p_h f_h + p_l f_l) - q_0^H - q_1^H \\
 & q_4^H \leq W_0 + 2(p_h f_h + p_l f_l) - q_0^H - q_1^H \\
 & q_5^H \leq W_0 + 2(p_h f_h + p_l f_l) - q_0^H - q_2^H \\
 & q_6^H \leq W_0 + 2(p_h f_h + p_l f_l) - q_0^H - q_2^H \\
 & q_n^H \geq 0, n = 0, 1, ..., 6
\end{array}
$$

where W_0 is the initial water amount (equivalent of MW) in dam, which may increase by inflows. p_h, p_l, f_h, f_l denote probability of high water flow, probability of low water flow, high water flow quantity and low water flow quantity, respectively. We assume that these flow probabilities are known, and the flow quantities represent the historical averages.

Above we adopt a node formulation (or known as recursive formulation) of the maximization problem as opposed to scenario formulation (or known as non-recursive formulation) of the problem. Genc, Reynolds, and Sen [5] (hereafter, GRS) uses scenario formulation to show how to numerically solve equilibrium outcomes of a large-scale stochastic oligopoly game. The examples of node formulation of stochastic dynamic games include Haurie, Smeers, and Zaccour [8] for a deterministic game (??? check) , Haurie and Moresino [7] for a stochastic game.These formulations, each has its own advantages, are different but yield to the identical solutions (see GRS). The main advantage of scenario formulation is that it allows to solve the equilibrium conditions when the objective function is non-separable by time periods, and it permits construction time lags between technologies. In the scenario formulation, branches of the demand tree are split from initial/root node to final node, and each brach from root node to the final node represents a scenario. In this formulation, each scenario problem (or called 'subproblem' which consists of profit maximizing problem subject to scenario constraints, such as demand at each node in this scenario and capacity constraints) is solved separately but connected with a constraint called nonanticipativity or nonclairvoyance condition that means that (investment) decisions should be made before the realization of (demand) uncertainty.

In the objective functions we assume away discount factor, and demand growth rates since the number of periods is relatively small. We also discard salvage value at the final period of the planning horizon. However, in the following section (in the infinite horizon model) we will take them into account. [4]

The timing of the game is as follows. In the first stage players choose production quantities simultaneously to maximize their profits. At the same stage, thermal player who makes capacity investment under uncertainty understands that there are two possible demand levels at the next stage and investment in this period will become available for production in the following period. In the second stage, depending on which state the players are

[4] The salvage value, discount factor, demand growth and capacity depreciation affect the optimal market outcomes and investments. For example, the higher the salvage value, the higher the capacity investment is. Genc and Sen [6] numerically analyze the effects of these factors in a model applied to the Ontario wholesale electricity market.

at, given that demand uncertainty unfolds at this stage, they make their production decisions optimally and the thermal player also makes capacity investments under uncertainty to meet the future demand for the next period. In the third stage, players make their optimal production decisions conditional on the demand state that reveals, and no investments will take place since it is the final stage.

The players have complete information on the model parameters, costs and market demand, but have imperfect information about the actions of rivals and know each other's payoff function. They share the common information about the demand uncertainty and the demand functional form. Since we study a dynamic stochastic game, one needs a richer information structure. A natural candidate information structure is S-adapted information (or S-adapted open loop information), which is commonly studied in the recent stochastic dynamic games literature. With this information structure players adapt their decisions to the sample paths of the stochastic demand scenarios. That is, at the initial node and time each player observes the likely demand patterns of the future and makes the (production and/or investment) decisions for the present and the future without observing rivals' decisions. However, the decisions are time consistent. Moreover, players know each other's initial production capacity, but do not know the state equations (i.e., capacity accumalation process) of rivals. We call the game strategies played under this information structure as S-adapted open loop strategies, and call an equilibrium as the S-adapted open loop Nash equilibrium.

As an equilibrium type, a S-adapted open loop Nash equilibrium is a Nash equilibrium in S-adapted strategies. [5] This equilibrium concept first introduced by Zaccour [14] and Haurie, Zaccour, and Smeers [8]. It is extended and employed for large-scale oligopolies by Haurie and Moresino [7], GRS, and Genc and Sen [6]. In this equilibrium, players condition their decisions on time period, demand state and initial capacity levels. Players choose their profit maximizing strategies given the rivals' strategies. This equilibrium concept is a half way between closed loop and open loop equilibrium paradigms (see, e.g., GRS and Pineau and Murto [10]. In the decision making process players condition their strategies on the collection of probabilistic demand scenarios, but not on a single path (e.g., expected demand path) which involves certainty.

[5] 'S' in S-adapted refers to 'sample' which, in turn, refers to random outcomes in the uncertainty model. Uncertainty, in the current model, is due to the probabilistic demand states.

Formally, S-adapted open loop Nash equilibrium strategies are (q_n^T, q_n^H, I_n^T), for each node $n = 0, 1, ..., 6$ on the event tree, that satisfy Karush-Kuhn-Tucker (KKT) conditions. The equilibrium strategies will be unique (see GRS (2007)).

We assume that at the initial stage the total capacity of players can meet the highest level of quantity demanded. The total hydro capacity including infows are equal to three times the monopoly output. That is, $W_0 + 2(p_h f_h + p_l f_l) = 3(D + 2\delta)/2$. For later periods we also assume that the demand levels (i.e., $D + \tilde{\delta}$) are high enough so that thermal capacity investmets are weakly positive. We make this assumption to better analyze the dynamics of capacity investments so that we will be able to to answer the following questions: How will investments of the thermal generator affect the behavior of the hydro generator? How will thermal investments at earlier stages affect the distribution of investments at later stages? How will these investments affect equilibrium market outcomes such as prices, outputs and social welfare? If we would assume the model parameters in a different region then it is possible that thermal player does not make any capacity investments, therefore we would not be able to investigate the investment dynamics.

Lemma 1: Given the above market demand and cost assumptions, for each player 'upstate' production is (weakly) greater than the 'downstate' production.

The proof of this Lemma is similar to the proof of Lemma 1 in GRS (2007).

Corollary 1: Lagrange multiplier of the upstate production constraint is greater than that of the downstate one. That is, $a_1 > a_2$, $a_3 > a_4$, $a_5 > a_6$.

This Corollary is a direct result of the above Lemma.

Proposition 1: Assume nonbinding hydro-flow-contraints [6] and positive capacity investments by thermal player. In the duopoly dynamic game the S-adapted open-loop Nash equilibrium strategies solve the following output, investment and (shadow) price equations:

Outputs:

[6] The condition that enforces this constraint is that the total water level over three periods is less than or equal to the three times the monopoly output of hydro player.

$(q_0^T, q_0^H) = (\frac{D-2c_2}{3+2c_1}, \frac{D(1+c_1)+c_2}{3+2c_1})$ or $(K_0, \frac{D-K_0}{2})$ [7]
$(q_1^T, q_1^H) = (K_0 + I_0^T, \frac{D+\delta-K_0-I_0^T}{2})$
$(q_2^T, q_2^H) = (\frac{D-\delta-2c_2}{3+2c_1}, \frac{(D-\delta)(1+c_1)+c_2}{3+2c_1})$
$(q_3^T, q_3^H) = (K_0 + I_0^T + I_1^T, \frac{D+2\delta-K_0-I_0^T-I_1^T}{2})$
$(q_4^T, q_4^H) = (\frac{D-2c_2}{3+2c_1}, \frac{D(1+c_1)+c_2}{3+2c_1})$
$(q_5^T, q_5^H) = (K_0 + I_0^T + I_2^T, \frac{D-K_0-I_0^T-I_2^T}{2})$
$(q_6^T, q_6^H) = (\frac{D-2\delta-2c_2}{3+2c_1}, \frac{(D-2\delta)(1+c_1)+c_2}{3+2c_1})$.

Investments: $I_0^T = \frac{a_1+a_3+a_5}{e_1}$, $I_1^T = \frac{a_3}{ue_1}$, $I_2^T = \frac{a_5}{de_1}$, where **[???]**

Shadow prices: $a_n = pr_n[-q_n^T + (D + \tilde{\delta_n} - q_n^T - q_n^H) - (c_1 q_n^T + c_2)]$, where $pr_n \in \{u, uu, du\}$ and $\tilde{\delta_n} \in \{0, \delta, 2\delta\}$ and $n = 1, 3, 5$.

Proof: The thermal player solves the following Lagrangian function:

$$\begin{aligned}
\max L^T &= P(Q_0)q_0^T - C(q_0^T) - F(I_0^T) \\
&+ u[P(Q_1)q_1^T - C(q_1^T) - F(I_1^T)] + d[P(Q_2)q_2^T - C(q_2^T) - F(I_2^T)] \\
&+ uu[P(Q_3)q_3^T - C(q_3^T)] + ud[P(Q_4)q_4^T - C(q_4^T)] \\
&+ du[P(Q_5)q_5^T - C(q_5^T)] + dd[P(Q_6)q_6^T - C(q_6^T)] \\
&+ (K_0 - q_0^T)a_0 + (K_0 + I_0^T - q_1^T)a_1 + (K_0 + I_0^T - q_2^T)a_2 \\
&+ (K_0 + I_0^T + I_1^T - q_3^T)a_3 + (K_0 + I_0^T + I_1^T - q_4^T)a_4 \\
&+ (K_0 + I_0^T + I_2^T - q_5^T)a_5 + (K_0 + I_0^T + I_2^T - q_6^T)a_6,
\end{aligned}$$

where for $n = 0, 1, 2, ..., 6$, $a_n \geq 0$ are the Lagrange multipliers of thermal constraints.
The hydropower player solves the following Lagrangian function:

$$\begin{aligned}
\max L^H = P(Q_0)q_0^H - C(q_0^H) + u[P(Q_1)q_1^H - C(q_1^H)] \\
+d[P(Q_2)q_2^H - C(q_2^H)] + uu[P(Q_3)q_3^H - C(q_3^H)] \\
+ud[P(Q_4)q_4^H - C(q_4^H)] + du[P(Q_5)q_5^H - C(q_5^H)] \\
+dd[P(Q_6)q_6^H - C(q_6^H)] + (W_0 - q_0^H)b_0 \\
+(W_0 + p_h f_h + p_l f_l - q_0^H - q_1^H)b_1 \\
+(W_0 + p_h f_h + p_l f_l - q_0^H - q_2^H)b_2 \\
+(W_0 + 2(p_h f_h + p_l f_l) - q_0^H - q_1^H - q_3^H)b_3
\end{aligned}$$

[7] We will assume interior solution for thermal output at node 0. Since it is an initialization assumption, the results in the paper will hold without loss of generality.

$$+(W_0+2(p_hf_h+p_lf_l)-q_0^H-q_1^H-q_4^H)b_4$$
$$+(W_0+2(p_hf_h+p_lf_l)-q_0^H-q_2^H-q_5^H)b_5$$
$$+(W_0+2(p_hf_h+p_lf_l)-q_0^H-q_2^H-q_6^H)b_6$$

where for $n=0,1,2,...,6$, $b_n\geq 0$ are the Lagrange multipliers of hydropower constraints.

For hydro outputs KKT conditions satisfy :
$\frac{\partial L^H}{\partial q_n^H}q_n^H=0$, $\frac{\partial L^H}{\partial b_n}b_n=0$, $(n=0,1,2,...,6)$

Since we assume that hydro-flow-consraints do not bind in all periods and nodes (that is $W_0+2(p_hf_h+p_lf_l)\leq 3(D+2\delta)/2)$ then $b_n=0$ for $n=0,1,2,...,6$. Then KKT conditions for the hydro player imply that the marginal revenues are equal in all nodes, since marginal costs are equal to 0 in all nodes. Hence, $\frac{\partial L^H}{\partial q_n^H}=0$ implies $q_n^H=\frac{D+\tilde{\delta_n}-q_n^T}{2}$, where $\tilde{\delta_n}\in\{0,\delta,\ 2\delta,$ $-\delta,\ -2\delta\}$. Specifically $q_0^H=\frac{D-q_0^T}{2}$, $q_1^H=\frac{D+\delta-q_1^T}{2}$, $q_2^H=\frac{D-\delta-q_2^T}{2}$, for the first three nodes, and the other response functions follow as stated in q_n^H.

For thermal outputs and investments KKT conditions are the following necessary and sufficient conditions for $(q_n^T,I_n^T>0)$:
$\frac{\partial L^T}{\partial q_n^T}q_n^T=0$, $\frac{\partial L^T}{\partial a_n}a_n=0$, $(n=0,1,2,...,6)$, $\frac{\partial L^T}{\partial I_n^T}I_n^T=0$, $(n=0,1,2)$.
$\frac{\partial L^T}{\partial q_n^T}q_n^T=0$ imply $[pr_n((D+\tilde{\delta_n}-2q_n^T-q_n^H)-(c_1q_n^T+c_2))-a_n]q_n^T=0$, where pr_n is the probability of node n in the event tree.

Based on the Lemma 1 and Corollary 1, it clear that $a_1>a_2=0$, $a_3>a_4=0$, $a_5>a_6=0$. Then it follows that $q_1^T=K_0+I_0^T$, $q_3^T=K_0+I_0^T+I_1^T$, $q_5^T=K_0+I_0^T+I_2^T$, and $q_n^T=\frac{D+\tilde{\delta_n}-q_n^H-c_2}{2+c_1}$ hold for nodes $n=2,4,6$, and $\tilde{\delta_2}=-\delta$, $\tilde{\delta_4}=0$, $\tilde{\delta_6}=-2\delta$. Also at node 0, since we assume interior solution, $a_0=0$ holds, hence $q_0^T=\frac{D-q_0^H-c_2}{2+c_1}$ (otherwise $a_0>0$ would hold, and $q_0^T=K_0$, $q_0^H=\frac{D-K_0}{2}$ would be attained).

Next we solve response functions for the equilibrium points. By substituting one player's response functions into other's functions we obtain that $q_n^T=\frac{(D+\tilde{\delta_n})-2c_2}{3+2c_1}$ and $q_n^H=\frac{(D+\tilde{\delta_n})(1+c_1)+c_2}{3+2c_1}$ for nodes $n=2,4,6$, and $\tilde{\delta_2}=-\delta$, $\tilde{\delta_4}=0$, $\tilde{\delta_6}=-2\delta$. The hydro outputs at other nodes satisfy, $q_1^H=\frac{D+\delta-K_0-I_0^T}{2}$, $q_3^H=\frac{D+2\delta-K_0-I_0^T-I_1^T}{2}$, $q_5^H=\frac{D-K_0-I_0^T-I_2^T}{2}$. Also at the node 0, $q_0^T=\frac{D-2c_2}{3+2c_1}$ and $q_0^H=\frac{D(1+c_1)+c_2}{3+2c_1}$ hold for the interior solution.

For optimal investment outcomes we solve $\frac{\partial L^T}{\partial a_n}a_n=0$, $(n=0,1,2,...,6)$, $\frac{\partial L^T}{\partial I_n^T}I_n^T=0$, $(n=0,1,2)$ and obtain that $a_n=pr_n[-q_n^T+(D+\tilde{\delta_n}-q_n^T-q_n^H)-(c_1q_n^T+c_2)]$, $I_0^T=\frac{\sum_n a_n}{e_1}$, where $pr_n\in\{u,uu,du\}$, $\tilde{\delta_n}\in\{0,\delta,\ 2\delta\}$, $n=1,3,5$, and $I_1^T=\frac{a_3}{ue_1}$, $I_2^T=\frac{a_5}{de_1}$. One can substitute thermal and hydro outputs into these investment equations to characterize them as functions of the model parameters. □

Lemma 2: For each player expected total profit is increasing in demand volatility δ. Moreover, the total expected investment of thermal player is increasing in demand volatility δ.[8]

Corollary 2: Thermal equilibrium investments are functions of hydro outputs q_1^H, q_3^H, and q_5^H. The rate of change of optimal investment levels with respect to the hydro outputs in duopoly are: $\frac{\partial I_0^T}{\partial q_1^H} < 0$, $\frac{\partial I_0^T}{\partial q_3^H} < 0$, $\frac{\partial I_0^T}{\partial q_5^H} < 0$, $\frac{\partial I_1^T}{\partial q_1^H} > 0$, $\frac{\partial I_1^T}{\partial q_3^H} < 0$, $\frac{\partial I_1^T}{\partial q_5^H} > 0$, $\frac{\partial I_2^T}{\partial q_1^H} > 0$, $\frac{\partial I_2^T}{\partial q_3^H} > 0$, $\frac{\partial I_2^T}{\partial q_5^H} < 0$.

Proof: Based on the proposition 1, we have $I_0^T e_1 = a_1 + a_3 + a_5 = u[D + \delta - K_0 - I_0^T - q_1^H - c_2 - (1 + c_1)(K_0 + I_0^T)] + uu[D + 2\delta - K_0 - I_0^T - I_1^T - q_3^H - c_2 - (1 + c_1)(K_0 + I_0^T + I_1^T)] + du[D - K_0 - I_0^T - I_2^T - q_5^H - c_2 - (1 + c_1)(K_0 + I_0^T + I_2^T)]$; $I_1^T e_1 = u[D + 2\delta - K_0 - I_0^T - I_1^T - q_3^H - c_2 - (1 + c_1)(K_0 + I_0^T + I_1^T)]$, $I_2^T e_1 = u[D - K_0 - I_0^T - I_2^T - q_5^H - c_2 - (1 + c_1)(K_0 + I_0^T + I_2^T)]$. We can solve these three equations for investments as functions of the hydro outputs q_1^H, q_3^H, q_5^H. The expressions are lengthy and messy, hence we report only the derivatives. Now drop the term e_1 in each investment expression above, since it will not change the derivatives qualitatively. The partial derivatives are, $\frac{\partial I_0^T}{\partial q_1^H} = -[u[1 + u(2 + c_1)]]/[1 + (2 + c_1)(2 + d)u + (6 + 5c_1 + c_1^2)u^2] < 0$, $\frac{\partial I_0^T}{\partial q_3^H} = -[u[u]]/[1 + (2 + c_1)(2 + d)u + (6 + 5c_1 + c_1^2)u^2] < 0$, $\frac{\partial I_0^T}{\partial q_5^H} = -[u[d]]/[1 + (2 + c_1)(2 + d)u + (6 + 5c_1 + c_1^2)u^2] < 0$. $\frac{\partial I_1^T}{\partial q_1^H} = [u[u(2 + c_1) + u^2(4 + 4c_1 + c_1^2)]]/[(1 + u(2 + c_1))(1 + (2 + c_1)(2 + d)u + (6 + 5c_1 + c_1^2)u^2)] > 0$, $\frac{\partial I_1^T}{\partial q_3^H} = [u[-u(4 + 2c_1 + 2d + c_1 d) - u^2(4 + 4c_1 + c_1^2) - 1]]/[(1 + u(2 + c_1))(1 + (2 + c_1)(2 + d)u + (6 + 5c_1 + c_1^2)u^2)] < 0$, $\frac{\partial I_1^T}{\partial q_5^H} = [u[ud(2 + c_1)]]/[(1 + u(2 + c_1))(1 + (2 + c_1)(2 + d)u + (6 + 5c_1 + c_1^2)u^2)] > 0$. $\frac{\partial I_2^T}{\partial q_1^H} = -[u[-u(2 + c_1) - u^2(4 + 4c_1 + c_1^2)]]/[(1 + u(2 + c_1))(1 + (2 + c_1)(2 + d)u + (6 + 5c_1 + c_1^2)u^2)] > 0$, $\frac{\partial I_2^T}{\partial q_3^H} =$

[8] The proof of the first statement of this Lemma is similar to the proof of Proposition 1 in Genc, Reynolds, and Sen (2007), who used the sufficiency condition $u \geq d$, (but it is not a necessary condition), to complete the proof. The proof of the second statement is very lengthy, hence we do not report it here. However, it can be obtained from the authors.

$-[u[-u^2(2+c_1)]]/[(1+u(2+c_1))(1+(2+c_1)(2+d)u+(6+5c_1+c_1^2)u^2)] > 0$, $\frac{\partial I_2^T}{\partial q_5^H} = -[u[1+u(4+2c_1)+u^2(6+5c_1+c_1^2)]]/[(1+u(2+c_1))(1+(2+c_1)(2+d)u+(6+5c_1+c_1^2)u^2)] < 0$. The signs of the above expressions are readily obtained given that all of the above parameters are nonnegative. $\square$

The intuition of this corollary is as follows. If the thermal player at node 0 thinks that the hydro player will produce more than optimum than the thermal player will reduce the investment in the initial stage. If the thermal player thinks or believes that the hydro will increase the production at node 1, which is a high demand node, then it means that he will have low water in the following stage that encompasses the highest demand stage. Therefore, the thermal will increase the investment at node 1. If the thermal at node 1 thinks that the hydro will increase the production at node 3 then it implies that thermal player's investment incentive decreases, since its investment will be effective at the next stage, time 2. If the thermal player at node 1 believes that the hydro will increase the production at node 5 (at time 2) and given that hydro does not increase the production in other nodes at the final period , then the thermal will increase the investment at node 1, which will be effective at the final period and at the highest demand nodes including node 3, and vice versa. For the investment at node 2, if the thermal believes that the hydro will increase the production at node 5, then the thermal has no incentive to increase the investment at node 2, since its investment will be effective in the following stage. Hence, the investment incentive at node 2 will decrease as hydro production at node 5 will increase. If hydro player increases its production at node 3, then the distribution of equilibrium hydro outputs change. The hydro output at node 0 will reduce, for instance. This will affect the investment behavior at node 2 with respect to the other hydro outputs in a way similar to the ones discussed above.

Corollary 3: Expected investment at any node (n) at time period t is equal to the expected future investments plus the imputed investments in the following immediate descendant nodes (n_+) at time $t+1$.

This corollary is a direct result of Proposition 1. The investment equations are functions of shadow prices, hence substituting one equation for another results in the relationship between investments over time. The imputed investment is defined as the ratio of the shadow price of capacity to the average marginal cost of investment. Alternatively, the imputed investments are the difference between the current actual investment and the expected future investments. The investment at node 0 is equal to the expected future investments plus the sum of the imputed investments at the nodes right after the ancestor node 0. For instance, in the *three* period game, investment at node 0 will be equal to the expected investments at nodes 1 and 2 plus the imputed investments at nodes 1 and 2, that is $I_0^T = uI_1^T + dI_2^T + (a_1 + a_2)/e_1$. Note that the unit of shadow prices a_1 and a_2 is price, the unit of e_1 is price divided by quantity, where e_1 is, as defined at the very beginning, the slope term of the marginal cost of investment and converts (investment) quanitites into dollars (marginal costs). Hence the unit of both sides of the above equality is quantity. The expected investment at node 1 will be equal to imputed investments at nodes 3 and 4 plus the expected investments at nodes 3 and 4, (which are zero since these nodes belong to the final period), that is $uI_1^T = (a_3 + a_4)/e_1$. Similarly, $dI_2^T = (a_5 + a_6)/e_1$, that is the expected investment at node 2 will be equal to the expected future investments (which are zero) plus the imputed investments at the nodes 5 and 6. Say, if we would extend the game to *four* periods, for another example, investment at node 0 will be equal to the expected investments at nodes 1, 2, 3, 4, 5, 6 plus the imputed capacity investments at nodes 1 and 2, that is $I_0^T = uI_1^T + dI_2^T + uuI_3^T + udI_4^T + duI_5^T + ddI_6^T + (a_1 + a_2)/e_1$. Also, the expected investment at node 1 would be equal to the expected investments at nodes 3 and 4 plus the imputed investments at nodes 3 and 4, that is $uI_1^T = uuI_3^T + udI_4^T + (a_3 + a_4)/e_1$. In other nodes investments will follow the pattern as stated in Corollary 3.

Given the above duopoly market analysis, however from the policy point of view the natural question arises: How does investment behavior of power generators change if the industry would transform from one market structure to another? How is investment distorted by oligopoly? In the following section, we will investigate efficient investment patterns of power generators to compare and contrast duopoly market outcomes with that of the socially optimum. We will also touch upon overinvestment and underinvestment issues in the duopoly market structure.

2.2 Social Planner's Problem

Perfectly competitive equilibrium is obtained by solving the social planner's problem which chooses production and investment quantities to maximize the sum of the expected welfare (the consumer surplus less production and investment costs) subject to production constraints.

$$max \sum_n E_n[\int_0^{Q_n^w} P_n(q)\mathrm{d}q - C(q_n^{w,H}) - C(q_n^{w,T}) - F(I_n^{w,T})]$$

$$
\begin{aligned}
&\textit{subject to} \\
q_0^{w,T} \leq K_0,\ q_1^{w,T} &\leq K_0 + I_0^{w,T} \\
q_2^{w,T} \leq K_0 + I_0^{w,T}, q_3^{w,T} &\leq K_0 + I_0^{w,T} + I_1^{w,T} \\
q_4^{w,T} &\leq K_0 + I_0^{w,T} + I_1^{w,T} \\
q_5^{w,T} &\leq K_0 + I_0^{w,T} + I_2^{w,T} \\
q_6^{w,T} &\leq K_0 + I_0^{w,T} + I_2^{w,T} \\
q_0^{w,H} \leq W_0, q_1^{w,H} &\leq W_0 + p_h f_h + p_l f_l - q_0^{w,H} \\
q_2^{w,H} &\leq W_0 + p_h f_h + p_l f_l - q_0^{w,H} \\
q_3^{w,H} &\leq W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_1^{w,H} \\
q_4^{w,H} &\leq W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_1^{w,H} \\
q_5^{w,H} &\leq W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_2^{w,H} \\
q_6^{w,H} &\leq W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_2^{w,H} \\
q_n^{w,H}, q_n^{w,T} &\geq 0, n = 0, 1, ..., 6
\end{aligned}
$$

where E is the expectation operator, the first argument of which is the consumer surplus, and all the rest are the costs. Above we use superscript w which denotes 'welfare' differentiates socially optimum quantities from the duopoly quantities. Note that hydropower production cost is zero as in duopoly model.

To be able to compare duopoly outcomes with the planner's outcomes, we assume the same paramater region for the planner as we assumed for the duopolists. We assume that the planner makes capacity investments in thermal units and uses water 'efficiently'. Specifically we let $\underline{W} \equiv W_0 + 2(p_h f_h + p_l f_l) = 3(D + 2\delta)/2$, that is total available water will be equal to three (the time horizon) times the monopoly output of the hydropower player. Observe that if the total water level would be $3(D + \delta)$ then it is clear that all demand levels could be met by only using the hydropower units, hence $q_n^{w,T} = 0 = I_n^{w,T}$ would satisfy. Also, the condition that the total water level is in $[3(D + 2\delta)/2, 3(D + 2\delta))$ will guarantee that $q_n^{w,T} \geq 0$ for all n.

Proposition 2: S-adapted open-loop competitive equilibrium characterization of the social planner's problem is such that

i) Thermal outputs satisfy, $q_0^{w,T} = \frac{(D - q_0^{w,H} - c_2)}{1 + c_1}$, $q_1^{w,T} = K_0 + I_0^{w,T}$, $q_2^{w,T} = \frac{(D - \delta - q_2^{w,H} - c_2)}{1 + c_1}$, $q_3^{w,T} = K_0 + I_0^{w,T} + I_1^{w,T}$, $q_4^{w,T} = \frac{(D - q_4^{w,H} - c_2)}{1 + c_1}$, $q_5^{w,T} = K_0 +$

$I_0^{w,T} + I_2^{w,T}$ when $a_5^w > 0$ or $q_5^{w,T} = \frac{(D-q_5^{w,H}-c_2)}{1+c_1}$ when $a_5^w = 0$, $q_6^{w,T} = \frac{(D-2\delta-q_6^{w,H}-c_2)}{1+c_1}$ or $q_6^{w,T} = 0$ when $a_6^w \geq 0$. Thermal shadow prices satisfy, $a_1^w = u[D+\delta-c_2-q_1^{w,H}-(1+c_1)(K_0+I_0^{w,T})]$, $a_3^w = uu[D+2\delta-c_2-q_3^{w,H}-(1+c_1)(K_0+I_0^{w,T}+I_1^{w,T})]$, $a_5^w = du[D-c_2-q_5^{w,H}-(1+c_1)(K_0+I_0^{w,T}+I_2^{w,T})]$.

ii) Hydropower outputs satisfy, $q_3^{w,H} = \underline{W} - q_0^{w,H} - q_1^{w,H} = q_4^{w,H}$, $q_5^{w,H} = \underline{W} - q_0^{w,H} - q_2^{w,H} = q_6^{w,H}$, $q_3^{w,H} - q_1^{w,H} = \delta(2u-1) + q_1^{w,T} - uq_3^{w,T} - dq_4^{w,T}$, $q_5^{w,H} - q_2^{w,H} = \delta(1-2d) - uq_5^{w,T} - dq_6^{w,T} + q_2^{w,T}$, $uq_1^{w,H} + dq_2^{w,H} - q_0^{w,H} = \delta(u-d) + q_0^{w,T} - uq_1^{w,T} - dq_2^{w,T}$.

iii) Thermal capacity investments will satisfy,

$I_0^{w,T} = (a_1^w + a_3^w + a_5^w)/e_1$,

$I_1^{w,T} = a_3^w/ue_1$, $I_2^{w,T} = a_5^w/de_1$ (when $a_5^w > 0$), where a_1^w, a_3^w, a_5^w are as defined above in i).

Proof: The planner maximizes the Lagrangian function L^w:

$$
\begin{aligned}
\mathrm{Max} L^w = \sum_n E_n[\int_0^{Q_n^w} P_n(q)\mathrm{d}q - C(q_n^{w,H}) - C(q_n^{w,T}) - F(I_n^{w,T})] \\
+(K_0 - q_0^{w,T})a_0^w \\
+(K_0 + I_0^{w,T} - q_1^{w,T})a_1^w + (K_0 + I_0^{w,T} - q_2^{w,T})a_2^w \\
+(K_0 + I_0^{w,T} + I_1^{w,T} - q_3^{w,T})a_3^w + (K_0 + I_0^{w,T} + I_1^{w,T} - q_4^{w,T})a_4^w \\
+(K_0 + I_0^{w,T} + I_2^{w,T} - q_5^{w,T})a_5^w + (K_0 + I_0^{w,T} + I_2^{w,T} - q_6^{w,T})a_6^w \\
+(W_0 - q_0^{w,H})b_0^w + (W_0 + p_h f_h + p_l f_l - q_0^{w,H} - q_1^{w,H})b_1^w \\
+(W_0 + p_h f_h + p_l f_l - q_0^{w,H} - q_2^{w,H})b_2^w \\
+(W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_1^{w,H} - q_3^{w,H})b_3^w \\
+(W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_1^{w,H} - q_4^{w,H})b_4^w \\
+(W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_2^{w,H} - q_5^{w,H})b_5^w \\
+(W_0 + 2(p_h f_h + p_l f_l) - q_0^{w,H} - q_2^{w,H} - q_6^{w,H})b_6^w
\end{aligned}
$$

The first order conditions of the above Lagrangian with respect to the hydro quantities will satisfy the followings. $\partial L^w/q_0^{w,H} = 0$ implies $D - (q_0^{w,H} + q_0^{w,T}) - b_3^w - b_4^w - b_5^w - b_6^w = 0$. $\partial L^w/q_1^{w,H} = 0$ implies $u[D + \delta - (q_1^{w,H} + q_1^{w,T})] - b_3^w - b_4^w = 0$. $\partial L^w/q_2^{w,H} = 0$ implies $d[D - \delta - (q_2^{w,H} + q_2^{w,T})] - b_5^w - b_6^w = 0$. Similarly, for other nodes, $uu[D + 2\delta - (q_3^{w,H} + q_3^{w,T})] - b_3^w = 0$, $ud[D - (q_4^{w,H} + q_4^{w,T})] - b_4^w = 0$, $du[D - (q_5^{w,H} + q_5^{w,T})] - b_5^w = 0$, $dd[D -$

$2\delta-(q_6^{w,H}+q_6^{w,T})]-b_6^w=0$ hold respectively. Appropriate substitution of these equations and given the fact that at the last period all water will be utilized will yield to the hydropower output relations in ii).

The first order conditions for thermal units satisfy,

at n=0: price minus marginal cost equals shadow price, that is $a_0^w = D-q_0^{w,T}-q_0^{w,H}-c_1q_0^{w,T}-c_2$ which implies $q_0^{w,T}=\frac{(D-q_0^{w,H}-c_2-a_0^w)}{1+c_1}$, in which without loss of generality we assume nonbinding initial stage production constraint so that $a_0^w=0$. For other nodes one of the followings will hold: a) $q_n^{w,T}=\frac{(D+\tilde{\delta_n}-q_n^{w,H}-c_2)}{1+c_1}$, when a_n^w= 0. b) $q_n^{w,T}=K_0+I_0^{w,T}+I_j^{w,T}$, when $a_n^w>0$, where $I_j^{w,T}\in\{0,I_1^{w,T},I_2^{w,T}\}$. c) $q_n^{w,T}=0$. By the result of Lemma 1 we obtain the thermal outputs in i). The derivative of the Lagrangian with respect to investments will yield to $I_0^{w,T}=(a_1^w+a_3^w+a_5^w)/e_1$, $I_1^{w,T}=a_3^w/ue_1$, and $I_2^{w,T}=a_5^w/de_1$ if $a_5^w>0$. By taking dervative of the Lagrangian with respect to the thermal outputs we obtain the equations for thermal shadow prices as in i). We substitute them into the investment equations to solve for optimal investments in iii). □

Lemma 2: The planner's expected total profit is increasing in demand volatility δ. Also, the total expected investment on thermal capacity is also increasing in demand volatility δ.

This Lemma is similar to the Lemma 1 in the duopoly model.

Corollary 4: Equilibrium investments are functions of hydro outputs $q_1^{w,H}$, $q_3^{w,H}$, and $q_5^{w,H}$. The rate of change of optimal investment levels with respect to the hydro outputs in the planner's problem are: $\frac{\partial I_0^{w,T}}{\partial q_1^{w,H}}<0$, $\frac{\partial I_0^{w,T}}{\partial q_3^{w,H}}<0$, $\frac{\partial I_0^{w,T}}{\partial q_5^{w,H}}<0$,$\frac{\partial I_1^{w,T}}{\partial q_1^{w,H}}>0$, $\frac{\partial I_1^{w,T}}{\partial q_3^{w,H}}<0$, $\frac{\partial I_1^{w,T}}{\partial q_5^{w,H}}>0$, $\frac{\partial I_2^{w,T}}{\partial q_1^{w,H}}>0$, $\frac{\partial I_2^{w,T}}{\partial q_3^{w,H}}>0$, $\frac{\partial I_2^{w,T}}{\partial q_5^{w,H}}<0$.

Proof: Based on the Proposition 2, $I_0^{w,T}e_1=a_1^w+a_3^w+a_5^w=u[D+\delta-q_1^{w,H}-c_2-(1+c_1)(K_0+I_0^{w,T})]+uu[D+2\delta-q_3^{w,H}-c_2-(1+c_1)(K_0+I_0^{w,T}+I_1^{w,T})]+du[D-q_5^{w,H}-c_2-(1+c_1)(K_0+I_0^{w,T}+I_2^{w,T})]$; $I_1^{w,T}e_1=u[D+2\delta-q_3^{w,H}-c_2-(1+c_1)(K_0+I_0^{w,T}+I_1^{w,T})]$; $I_2^{w,T}e_1=u[D-q_5^{w,H}-c_2-(1+c_1)(K_0+I_0^{w,T}+I_2^{w,T})]$. We can solve these equations for investments as functions of the hydro outputs. Since the expressions are lengthy, we only report the derivatives. We calculate that $\frac{\partial I_0^{w,T}}{\partial q_1^{w,H}}=-[u[1+u(1+c_1)]]/[1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2]<0$, $\frac{\partial I_0^{w,T}}{\partial q_3^{w,H}}=-[u[u]]/[1+(1+c_1)(2+d)u+(2+$

$3c_1+c_1^2)u^2] < 0$, $\frac{\partial I_0^{w,T}}{\partial q_5^{w,H}} = -[u[d]]/[1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2] < 0$. $\frac{\partial I_1^{w,T}}{\partial q_1^{w,H}} = [u[u(1+c_1)+u^2(1+2c_1+c_1^2)]]/[(1+u(1+c_1))(1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2)] > 0$, $\frac{\partial I_1^{w,T}}{\partial q_3^{w,H}} = [u[-u(2+2c_1+d+c_1d)-u^2(1+2c_1+c_1^2)-1]]/[(1+u(1+c_1))(1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2)] < 0$, $\frac{\partial I_1^{w,T}}{\partial q_5^{w,H}} = [u[ud(1+c_1)]]/[(1+u(1+c_1))(1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2)] > 0$. $\frac{\partial I_2^{w,T}}{\partial q_1^{w,H}} = -[u[-u(1+c_1)-u^2(1+2c_1+c_1^2)]]/[(1+u(1+c_1))(1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2)] > 0$, $\frac{\partial I_2^{w,T}}{\partial q_3^{w,H}} = -[u[-u^2(1+c_1)]]/[(1+u(1+c_1))(1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2)] > 0$, $\frac{\partial I_2^{w,T}}{\partial q_5^{w,H}} = -[u[1+u(2+2c_1)+u^2(2+3c_1+c_1^2)]]/[(1+u(1+c_1))(1+(1+c_1)(2+d)u+(2+3c_1+c_1^2)u^2)] < 0$. The signs of the above expressions are readily obtained given that all of the above parameters are nonnegative. $\square$

An economic intuition for this corollary is in the same vein of that for Corollary 2.

Corollary 5: Expected investment at any node (n) at time period t is equal to the expected future investments plus the imputed investments in the following descendant nodes (n_+) at time $t+1$.

This Corollary is similar to Corollary 3, and a proof of it is obtained directly from Proposition 2.

We also conjecture but not prove, since proofs are lengthy and cumbersome, that total expected investment is increasing in market demand level D, and decreasing in water level W under both market structures. However the intuitions are straightforward. As demand level increases, keeping other parameters constant, the planner or the thermal duopolist has an incentive to increase the total investments as long as it is profitable. As the water level increases, hydro units increase production since it is costless, and the thermal production decreases, hence investment incentives of the thermal player declines.

2.3 Examples

In Example 1 we calculate the S-adapted open loop equilibrium outcomes based on Proposition 1. We will compare these outcomes with the outcomes of socially optimum in the following examples.

Example 1 (duopoly): Let $K_0 = 30$, $W_0 + 2(p_h f_h + p_l f_l) = 450$, $c_1 = c_2 = e_1 = 1$, $D = 250$, $\delta = 25$, $u = d = 1/2$. The equilibrium outcomes at each node are,
Hydro outputs: $q_0^H = 110$, $q_1^H = 114.58$, $q_2^H = 90.2$, $q_3^H = 123.26$, $q_4^H = 100.2$, $q_5^H = 101.04$, $q_6^H = 80.2$.
Thermal outputs: $q_0^T = 30$, $q_1^T = 45.83$, $q_2^T = 44.6$, $q_3^T = 53.48, q_4^T = 49.6$, $q_5^T = 47.92, q_6^T = 39.6$.
Thermal investments: $I_0^T = 15.83, I_1^T = 7.65, I_2^T = 2.09$, hence the total expected investment over three periods is 20.7.
Thermal shadow prices: $a_0 = 49, a_1 = 10.96, a_3 = 3.82, a_5 = 1.05, a_2 = a_4 = a_6 = 0$.
Market prices: $P_0 = 110, P_1 = 114.58, P_2 = 90.2, P_3 = 123.26, P_4 = 100.2, P_5 = 101.04$, $P_6 = 80.2$.
The total expected consumer surplus over three periods is 32,053. The total expected profits for thermal and hydro players are 9,948 and 33,200, respectively. Note that by assumption the cost of production of the hydro player is zero, and he does not undertake any investment decisions.

Before we solve for socially optimum outputs, scheduling and investments, based on the Proposition 2, we first implement a hypothetical policy, which we call 'naive policy' that requires the planner to release the same amount of water in every period. We analyze the implications of this policy and compare the equilibrium outcomes with that of duopoly, and then discuss why this policy cannot be an optimal one. This naive policy might be attractive for a planner since decision making process under uncertainty in the investment model could be rather complex and intractable. This type of policy is not uncommon in some industries especially when the number of decision variables and state variables are many and subject to uncertainty. In that case, the decison maker might optimize the production plan only for more complex technologies such as thermal generators.

Example 2 ('naive policy'): Assuming that the the planner releases equal amount of water in each period, it is clear that $q_n^{w,H} = \frac{(D+2\delta)}{2}$ for each $n = 0, 1, 2, ..., 6$. Then the equilibrium thermal outputs with positive investments will be, based on the proof of the above proposition, $q_0^{w,T} = \frac{(D-q_0^{w,H}-c_2)}{1+c_1}$, $q_1^{w,T} = K + I_0^{w,T}$, $q_2^{w,T} = \frac{(D-\delta-q_2^{w,H}-c_2)}{1+c_1}$, $q_3^{w,T} = K_0 + I_0^{w,T} + I_1^{w,T}$, $q_4^{w,T} = \frac{(D-q_4^{w,H}-c_2)}{1+c_1}$, $q_5^{w,T} = K_0 + I_0^{w,T} + I_2^{w,T}$, $q_6^{w,T} = \frac{(D-2\delta-q_6^{w,H}-c_2)}{1+c_1}$.
The shadow prices will be, $a_1^w = u[D+\delta-c_2-(D+2\delta)/2-(1+c_1)(K_0+I_0^{w,T})]$,

$a_3^w = uu[D+2\delta - c_2 - (D+2\delta)/2 - (1+c_1)(K_0 + I_0^{w,T} + I_1^{w,T})]$, $a_5^w = du[D - c_2 - (D+2\delta)/2 - (1+c_1)(K_0 + I_0^{w,T} + I_2^{w,T})]$, shadow prices at other nodes equal zero. The equilibrium investments will be, $I_0^{w,T} = (a_1^w + a_3^w + a_5^w)/e_1$, $I_1^{w,T} = a_3^w/(ue_1)$, $I_2^{w,T} = a_5^w/(de_1)$.
Now we apply the paramaters in Example 1 ($K_0 = 30$, $W = 450$, $c_1 = c_2 = e_1 = 1$, $D = 250$, $\delta = 25$, $u = d = 1/2$) to compute the equilibrium outcomes of planner's problem. We calculate that hydro outputs are, $q_n^{w,H} = 150$, for each n, thermal outputs are, $q_0^{w,T} = 30$, $q_1^{w,T} = 49.2$, $q_2^{w,T} = 37$, $q_3^{w,T} = 61.85$, $q_4^{w,T} = 49.5$, $q_5^{w,T} = 49.35$, $q_6^{w,T} = 24.5$. Thermal investments are, $I_0^{w,T} = 19.2$, $I_1^{w,T} = 12.65$, $I_2^{w,T} = 0.15$. Total expected equilibrium investments over three periods is, $E(I^{w,T}) = 19.2 + 6.4 = 25.6$, which is higher than the total equilibrium duopoly investments ($E(I^T) = 15.83 + 4.87 = 20.7$), as calculated in Example 1. Under the naive policy total expected consumer surplus over three periods is 54,215. The total expected production and investment costs are 2,679.3 and 224.3, respectively. Hence, the total expected welfare is 51,311.

Obviously duopoly underinvests relative to the naive policy of the planner. However, it can be shown by constructing an example that this policy is not optimal. Instead of releasing equal amount of water over time, the planner could increase the social welfare by witholding some water in the initial stage and use the rest of the water when he faces high demand stages. For example, by releasing 25 units of water at the initial node and releasing the rest of the water equally in other nodes result in consumer surplus increase (it will increase from 54,215 to 54,468).
We now calculate the optimal outputs and invetsments based on the above proposition.

Example 3 ('planner's optimal policy'): Given the parameters in Example 1, we calculate socially optimum output quantities and investments at each node;
Hydro outputs: $q_0^{w,H} = 161.5$, $q_1^{w,H} = 149.2$, $q_2^{w,H} = 143.75$, $q_3^{w,H} = 139.27$, $q_4^{w,H} = 139.27$, $q_5^{w,H} = 144.74$, $q_6^{w,H} = 144.74$.
Thermal outputs: $q_0^{w,T} = 30$, $q_1^{w,T} = 50.16$, $q_2^{w,T} = 40.13$, $q_3^{w,T} = 65.01$, $q_4^{w,T} = 54.86$, $q_5^{w,T} = 51.15$, $q_6^{w,T} = 27.13$
Thermal investments: $I_0^{w,T} = 20.16$, $I_1^{w,T} = 14.85$, $I_2^{w,T} = 0.99$. Hence the total expected investment is 28.08.
Thermal shadow prices: $a_1^w = 12.24$, $a_3^w = 7.43$, $a_5^w = 0.49$, $a_0^w = a_2^w = a_4^w = a_6^w = 0$.
Hydro shadow prices: $b_3^w = 23.93$, $b_4^w = 13.97$, $b_5^w = 13.53$, $b_6^w = 7.04$,

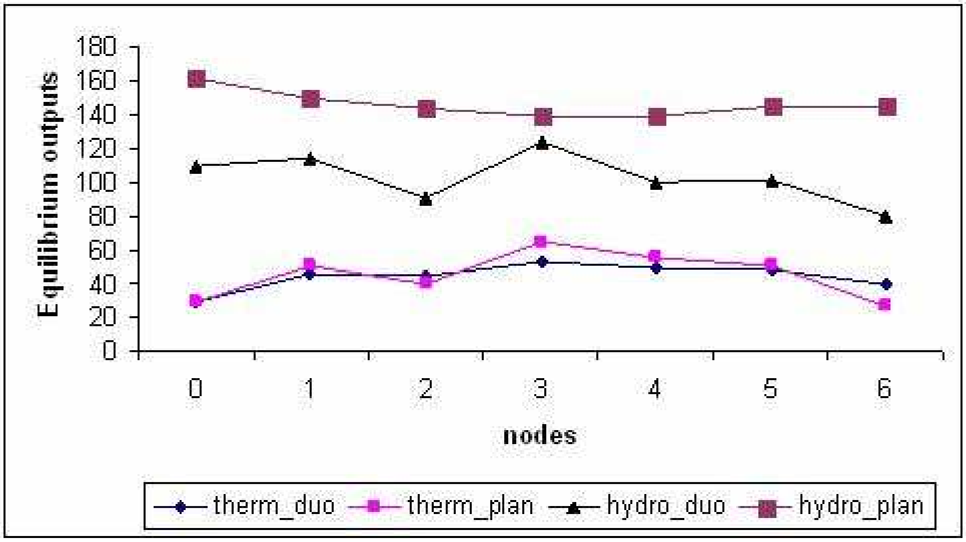

Figure 2: Comparing market outputs of duopoly and socially optimum.

$b_0^w = b_1^w = b_2^w = 0$.
Market prices: $P_0 = 58.5, P_1 = 75.64, P_2 = 36.52, P_3 = 95.72, P_4 = 55.87, P_5 = 54.11,\ P_6 = 28.13$.

Total expected consumer surplus over three periods is 55,145. Total expected production and investments costs are 2929.7 and 258.6, respectively. The difference will yield to total expected welfare which is 51,956.7.

Clearly, the total expected investment exceeds the one in duopoly, hence duopoly underinvests relative to socially optimum . This example is interesting itself; since even though the planner utilizes all the water at the last period, it still invests more than what duopoly invests, in expectation, at the equilibrium.

In Figure 2 we plot the equilibrium production outputs of the social planner (with optimal policy) and duopoly. The planner produces more electricity than duopoly at each node from hydro units. Node 0 production of the planner by hydro units is higher than the production at other nodes since production by thermal units at node 0 is limited by the capacity. For both market structures, production from hydro units is higher than that from thermal units for all nodes since the cost of production is zero and available water capacity is higher than available thermal capacity. Moreover, even though at some nodes the thermal production of duopoly exceeds the thermal production of the planner, expected total production from thermal units are higher by the planner than by duopoly, since expected investments under the duopoly is lower than that under the planner.

3 Infinite horizon

In this section we present a discrete-time infinite horizon version of the model in order to analyse the stationary distribution of hydro and thermal production. The model differs from that of the previous section in a couple of ways. The demand function has the same form:

$$P_t = D_t - \beta(q_t^H + q_t^T) \tag{1}$$

however we now assume that $D_t \sim N(\mu, \sigma^2)$, rather than following a random walk. Furthermore, the support of the demand state distribution is a continuum, as opposed to the discrete support assumed in the previous section.

As before, the direct costs of hydroelectric generation are zero. The flow of water follows:

$$W_{t+1} = (1-\gamma)W_t + \omega - q_t^H, \tag{2}$$

where W_t is the level of the reservoir at the beginning of period t, γ is a parameter that determines the rate of evaporation in the reservoir over an interval of time, and ω is the rate of net inflow into the reservoir over an interval of time. We are assuming now that water flows are deterministic. Stochastic water flows can be added to this model, but to keep the initial analysis simple and focussed on competition between the two producers, we assume deterministic water flows.

Thermal generation costs are given by $C(q^T) = (c/2)(q^T)^2$.

Although the ultimate objective is to analyse the dynamic game played between the hydro and thermal producers, we start with an analysis of the efficient outcome in this market in order to provide a comparison with the results of the previous section.

3.1 The planner

The planner chooses thermal and hydro generation with the objective of maximizing the expected present value of the stream of consumer surplus less generation costs:

$$\sum_{t=1}^{\infty} \delta \left(D_t(q_t^H + q_t^T) - \frac{\beta}{2}(q_t^H + q_t^T)^2 - \frac{c}{2} {q_t^T}^2 \right) \tag{3}$$

The state vector at time t consists of the current demand state, D_t, and the current reservoir level, W_t.

The planner's value function then satisfies the Bellman equation:

$$V(D_t, W_t) = \max_{q^H, q^T} D_t(q_t^H + q_t^T) - \frac{\beta}{2}(q_t^H + q_t^T)^2 - \frac{c}{2}{q_t^T}^2 + \delta E_t V(D_{t+1}, W_{t+1}) \tag{4}$$

subject to

$$W_{t+1} = (1-\gamma)(W_t - q_t^H) + \omega, \tag{5}$$

$$0 \leq q_t^H \leq W_t, \tag{6}$$

$$0 \leq q_t^T \leq K, \tag{7}$$

and

$$D_{t+1} \sim N(\mu, \sigma^2). \tag{8}$$

The necessary conditions for the maximization problem are

$$D_t - \beta(q_t^H + q_t^T) - \delta(1-\gamma)\frac{\partial[E_t V(D_{t+1}, W_{t+1})]}{\partial W_{t+1}} + b_W - b_0 = 0 \tag{9}$$

and

$$D_t - \beta(q_t^H + q_t^T) - cq^T + a_K - a_0 = 0 \tag{10}$$

where b_W and b_0 are the Lagrange multipliers on hydro production's capacity and non-negatitivity constriants and a_K and a_0 are the multipliers on thermal production's capacity and non-negativity constraints. Equations (9) and (10) imply

$$a_K - a_0 - cq^T = \delta(1-\gamma)\frac{\partial[E_t V(D_{t+1}, W_{t+1})]}{\partial W_{t+1}} - b_W + b_0 \tag{11}$$

which for an interior solution simplifies to

$$\delta(1-\gamma)\frac{\partial[E_t V(D_{t+1}, W_{t+1})]}{\partial W_{t+1}} = cq^T. \tag{12}$$

For an interior solution, the marginal value of water is equalized with the marginal cost of thermal production.

Since the value function is not smooth when the control constraints bind, we solve the problem by approximating $E_t V(D_{t+1}, W_{t+1})$, which will generally be smoother than V. This approach has the additional benefit of allowing us to approximate a function of one state variable only (W_{t+1}) since the future demand shock is integrated out.[9]

[9] This is a consequence of the assumption that the demand states are i.i.d. If we were to allow any serial correlation in this process, the expected value function would be a function of two state variables.

3.1.1 Numerical algorithm

We approximate the expected value function using the collocation method[10]. In particular,

$$E_t V(D_{t+1}, W_{t+1}) \approx \sum_{i=1}^{n} d_i \phi(W_{t+1}) \equiv \tilde{V}(W_{t+1}) \tag{13}$$

where the ϕ_i are known basis functions. Collocation proceeds by determining the $d_i,\ i = 1, ...n$, in order for the approximation to hold exactly at n collocation nodes, $W_+^1, W_+^2, ..., W_+^n$. The algorithm we use to solve the problem is described as follows:

0. Choose a starting approximation of $\tilde{V}^0(W_{t+1})$, i.e., starting values $d_i^0,\ i = 1, 2, ..., n$.

1. Given the current approximation, $\tilde{V}^k(W_{t+1})$ compute the value function at the collocation nodes, $W_+^1, W_+^2, ..., W_+^n$. We simply compute the value for each possibility of binding/non-binding controls at every node i, conditional on the demand state, D_+:

   a) For $q_+^T = 0$, find the value for $q_+^H = 0$, $q_+^H = W_+^i$, and q_+^H interior. Choose the solution giving the largest V.
   b) For $q_+^T = K$, find the value for $q_+^H = 0$, $q_+^H = W_+^i$, and q_+^H interior. Choose the solution giving the largest V.
   c) For q_+^T interior, find the value for $q_+^H = 0$, $q_+^H = W_+^i$, and q_+^H interior. Choose the solution giving the largest V.

   The largest of these three values yields $V^k(D_+, W_+^i)$. This step yields the value in the next period as a function of the demand state for each W_+^i.

2. Integrate the new value function numerically over demand states to update $\tilde{V}(W_t^{k+1})$, i.e. find new values $d_i^1,\ i = 1, ..., n$.

3. If convergence achieved, stop. Else, return to step 1.

[10] See Judd [9]

3.1.2 Example

We present the solution to the planner's problem for the following set of parameter values: $c = 1.0$, $K = 4.0$, $\delta = 0.9$, $\gamma = 0.3$, $\omega = 5.0$, $\mu = 10.0$, $\sigma = 1.0$, $\beta = 1.0$. A 61 node Chebyshev approximation[11] is used ($n = 61$, and the ϕ_i functions are the Chebyshev polynomials). We present the solution as a series of plots over a range of D_t and W_t values. The range of D_t is given by $\pm$ 2.5 standard deviations around the mean, while the range of W_t is between zero and the "natural" steady state water level, ω/γ.

The value function is plotted in Figure 3. It is smoothly increasing in both the demand state and the water level. Output of the hydro producer is presented in Figure 4. For low water levels, q_t^H increases one for one with the available water and does not vary with the demand state. In this region, the constraint that $q_t^H \leq W_t$ binds and the only water available in the next period is due to the inflow, ω.

Figure 5 contains the optimal thermal output. There are three "regimes" characterizing the optimal thermal output. For low water levels and high demand states, thermal output is constrained by capacity. For somewhat higher water levels, the capacity constraint no longer binds, but hydro output is still constrained by W_t, so thermal output falls quickly as water levels increase. Finally for values of the state in which hydro production is not constrained, optimal thermal output increases less quickly with water levels.

Finally, we present the marginal value of water, $V_W(D_t, W_t)$, in Figure 6. For values of the state vector in which hydro production is constrained by the water levels, the marginal value of water is highest and constant. In this region, the water available in period $t+1$ is constant resulting in a constant V_W. Once the water level is sufficient for the water contraint to not bind, the marginal value of water falls with the water level and increases with the demand state. The marginal value of water does not fall to zero for any feasible value of the state vector. This implies that thermal output is also not zero since it is optimal to equate the marginal value of water to the marginal cost of thermal production in an interior solution.

4 Conclusion

We have studied dynamic competition between thermal and hydroelectric producers under uncertainty in finite and infinite time horizon models. In

[11] The computations are done with C++ and make use of routines for Chebyshev approximation, numerical integration, and root finding from the Gnu Scientific Library. [4]

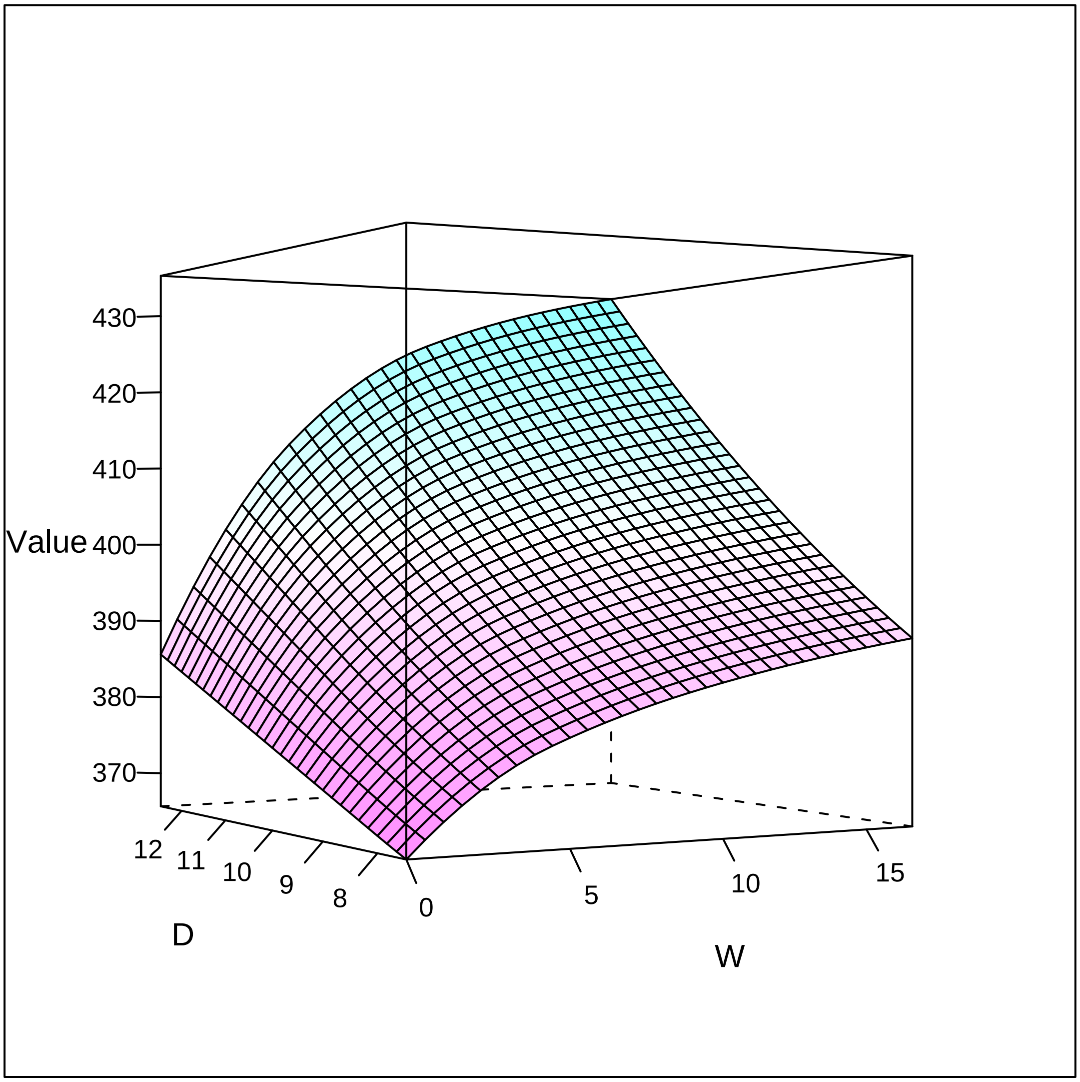

Figure 3: Planner's Value Function

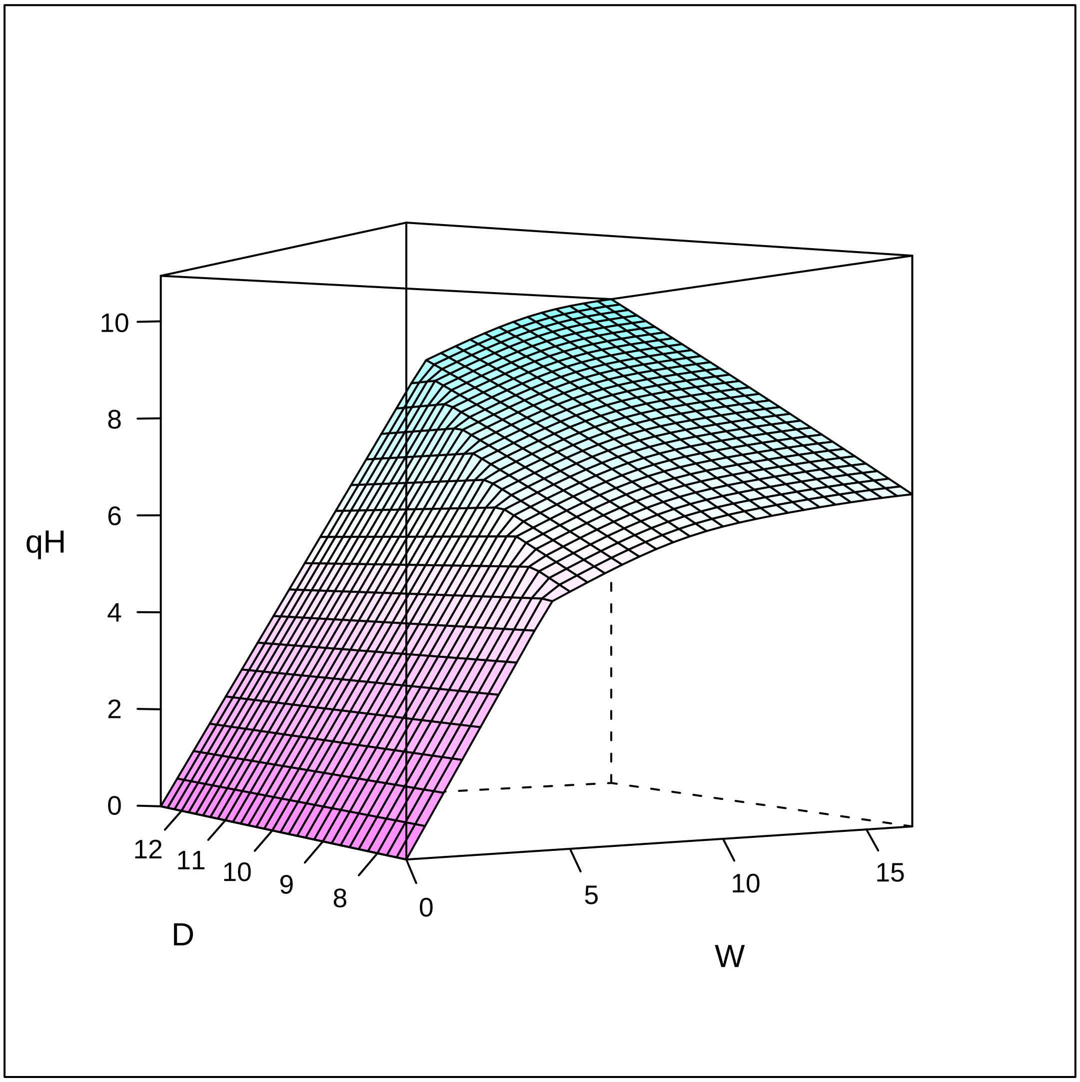

Figure 4: Efficient Hydro Generation

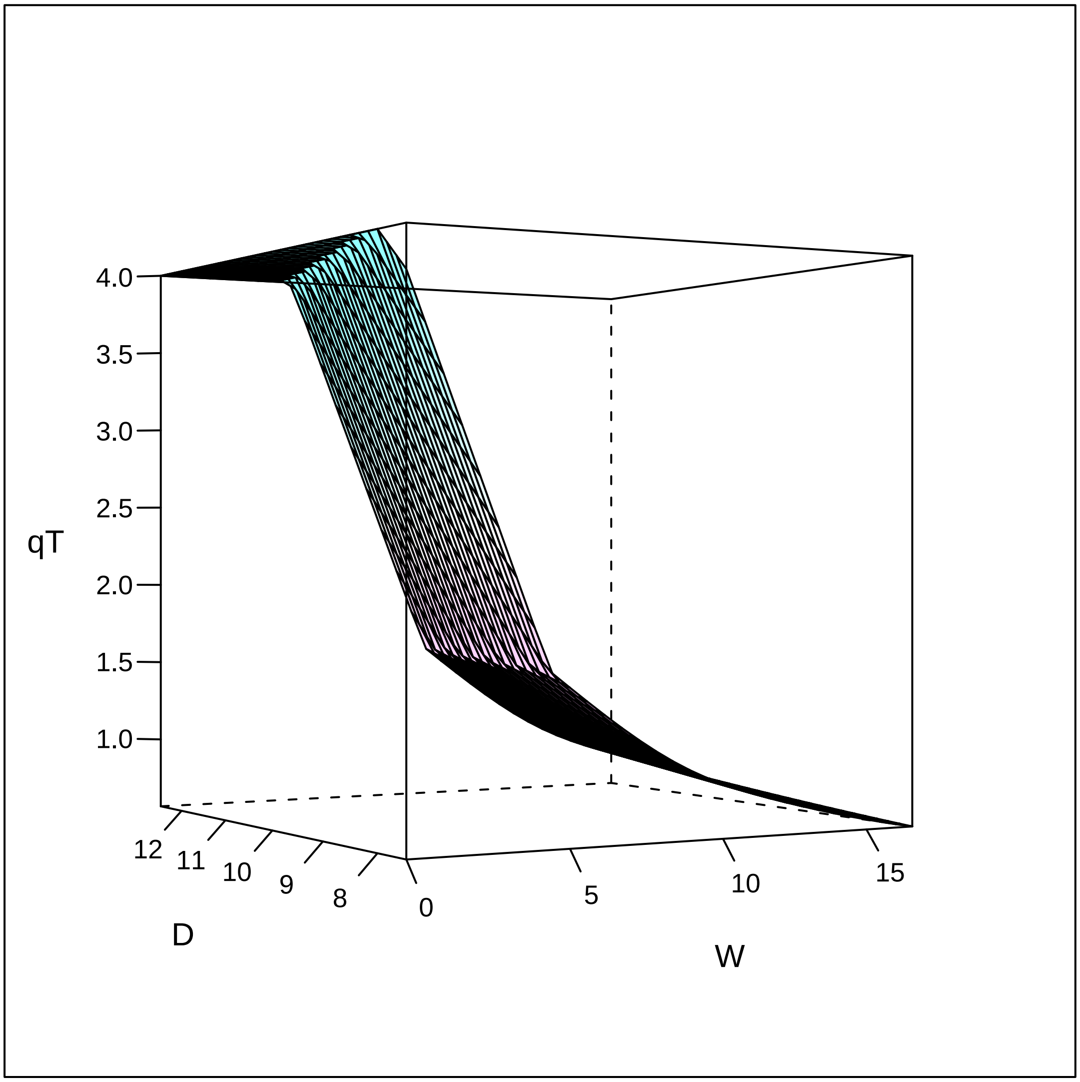

Figure 5: Efficient Thermal Generation

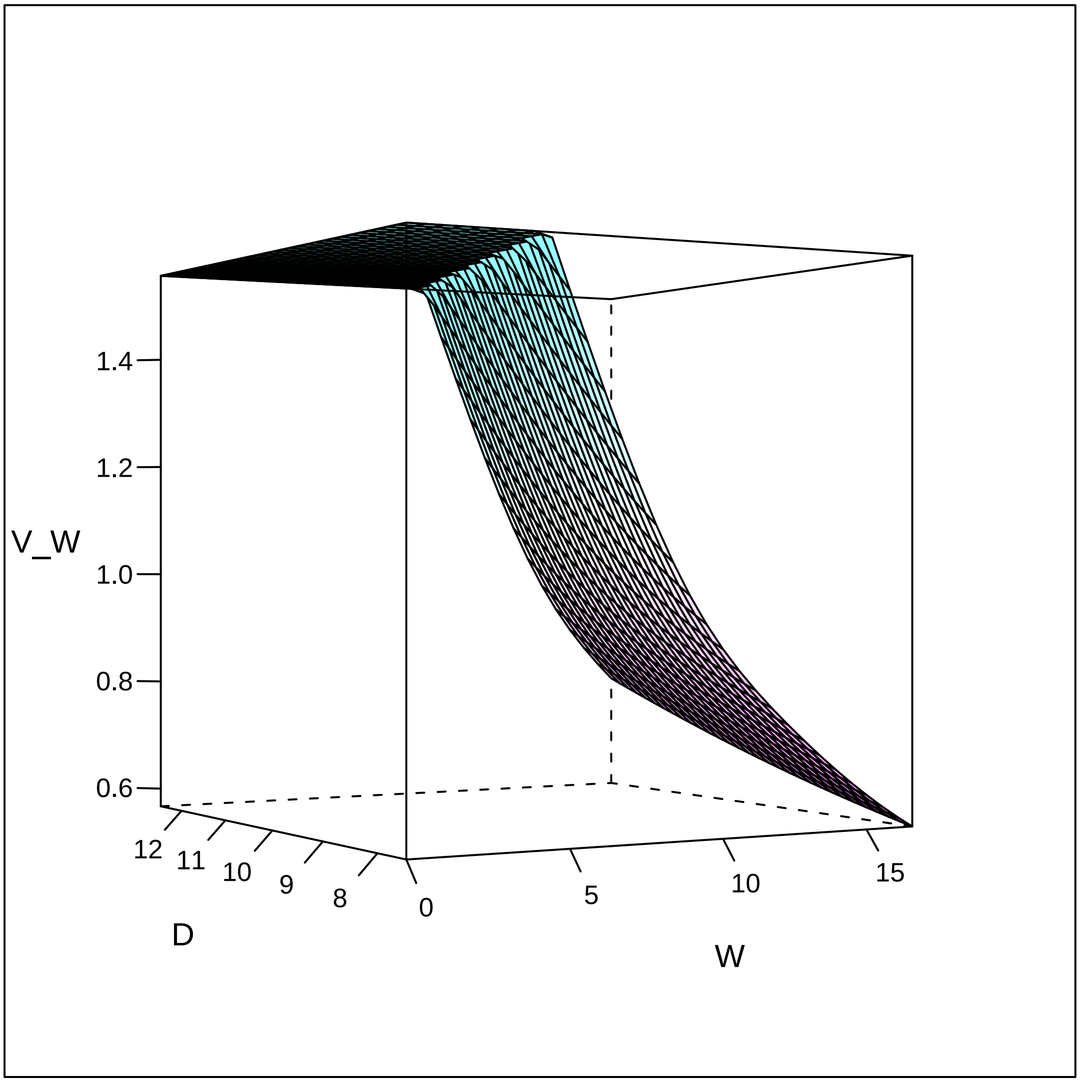

Figure 6: Marginal Value of Water

the finite horizon model, we have analytically characterized equilibrium output strategies of thermal and hydro players. We also characterized the optimal capital investment strategies of thermal player under uncertainty. We find that investment at any time will be equal to the imputed investments in the following periods. In other words, expected investment at any node in a time period equals summation of expected future investments plus the imputed investments in the next period. This result shows the dynamics of investment pattern of the thermal player. We analyze how strategic behavior by the hydro producer inhibits capacity investment by the thermal producer. We also compare duopoly competition market outcomes with perfect competition outcomes in several examples, and discuss the degree of underinvestment in the duopoly.

In the infinite time horizon model we have presented computational solutions to the social planner's problem. For the chosen example, even though marginal production cost is higher for the thermal generator, it is efficient to always use some thermal power. This is because the shadow price of water is never zero. Extending these results, we plan to compute the optimal capacity level for the planner and to compute the outcome of the non-cooperative game.

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