Storage and Market Power: The influence of speculation on monopoly.

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Abstract

We analyze the effects of competitive storage when the production of the good is controlled by a monopolist. The existence of competitive storers serves to reduce the monopolist's effective demand when speculators are selling and to increase it when they are buying. This results in the monopolist manipulating the frequency of stockouts, and hence, the price-smoothing effects of competitive storage. We use a two-period model to show that there is a lower probability of a stockout under a monopolist than in a perfectly competitive market. Consequently, high-price regimes occur less frequently under monopoly. We then extend the model to an infinite horizon to examine the implications for price volatility using collocation methods to approximate both the expected future price and the expected value function. We confirm that stockouts occur less frequently under the monopolist, even though price is more volatile. We also demonstrate that while free entry by speculators does reduce the gap in price volatility, it does not remove it.

1 Introduction

Competitive storage has the potential to be an important influence on the behaviour of firms with market power. For many storable commodities, production is undertaken in concentrated industries. Examples include petroleum, natural gas, nickel, tin[1], and aluminum. Although the storage technology differs across these commodities, they generally share the feature that storage by intermediaries is not precluded. For the non-ferrous metals traded on the London Metal Exchange, it is relatively simple to store the commodity as

[1] The International Tin Agreement collapse in 1985 was partly attributable to speculators' behaviour (Anderson and Gilbert [1])

the exchange organizes recognized storage facilities. In effect, any individual can purchase and store these metals. Storage of natural gas is more complicated, although deregulation in the U.S.A. over the past couple of decades has resulted in entry of independent storage firms.[2] For all these commodities, it is natural to ask how speculative storage will affect the use of market power. We analyze this question by examining the effects that competitive storage has on the behaviour of a monopolist/cartel in order to highlight any incentives that firms with market power have to influence speculative activity, and consequently affect the distribution of prices.

One can view production and storage as sequential stages of activity for which agents that store the good need not be the same as those that produce it, in this way allowing for different degrees of market power at different points in the supply chain. Much of the work that has been done on the effects of speculative storage imposes perfect competition at both the production and storage stages. The theory under these conditions has been well established by Samuelson [11], Newbery and Stiglitz [10], Scheinkman and Schechtman [12], Williams and Wright [13], and Deaton and Laroque [2]. As this work demonstrates, the constraint that inventories be non-negative causes the distribution of price to have two regimes: one corresponding to positive inventories being held, and one corresponding to stockouts.

The effects of market power in the storage activity has been examined by Newbery [9], Williams and Wright [13], and McLaren [7]. Newbery [9] shows that a firm with monopoly power over storage will smooth harvest fluctuations more than a competitive storage sector would. In an extension to his basic model, he allows the monopolist to control the harvest and demonstrates that the monopolist's residual demand is kinked when the price causes competitive storage to occur, although he does not solve this version of model. The model we develop in this paper also generates the this type of residual demand for the monopolist. McLaren [7] examines an oligopoly in the storage activity and finds that in a Markov-perfect equilibrium, the oligopoly smooths price less than would occur under competitive storage.

In contrast to these papers that examine market power in storage with competitive production, we examine market power in production with competitive storage. We separate the two activities completely by not allowing the monopolist to store the good, so that any storage that occurs must be done by the competitive storers. However, the monopolist is able to induce storage by manipulating price to induce speculative purchases or sales. There are two broad questions that we ask. First, how does the frequency of stockouts and the distribution of price differ under monopoly as opposed to competitive production? Second, how does speculation affect the behaviour of a monopolist? The answers to these questions has implications for the distribution of price that we expect to observe. In what follows, we first examine a two period model for which we can get a closed form solution, after which we analyze an infinite horizon model using numerical techniques.

[2] As of 2005, independent storage operators accounted for 13% of storage capacity in the U.S.A.(Energy Information Association [4]). This represented a substantial increase over just a few years prior.

2 The Model

We consider a discrete time economy with an horizon T which may be finite or not. In every period $t = 1, ..., T$, consumers have a demand D_t for an homogenous and non-perishable product they can buy on a spot market. We assume that consumers cannot store, and therefore cannot sell the product. Consumers's demand in period t, D_t, is a decreasing function of the spot price p_t, and is an increasing function of a random state a_t which represents consumers' maximum willingness to pay for the product in this period. We assume that consumers' demand is a linear function of p_t and a_t, given by

$$D_t = \max\{a_t - p_t, 0\}, \tag{1}$$

where the random state a_t is drawn by Nature at the beginning of period t and known to every participant of the spot market before decisions are made in this period. All market participants have rational expectations over future demand conditions, but these conditions are not known before they are realized: only the distribution of the future random states is known. We assume that random states $\{a_t\}_{t=1,...,T}$ are independently and identically drawn according to a time-invariant cumulative distribution function $F(.)$ with support $[\underline{a}, \overline{a}]$ on the positive real line, where the upper bound $\overline{a}$ can be infinite.

In every period t a fringe of independent and atomless storers, the *competitive speculators*, are able to buy or sell on the spot market, and are able to store the product. Let x_t denote the position of speculators on the spot market of period t: if x_t is positive, then speculators selling the product, while if x_t is negative speculators are buying the product. Speculators are able to store the product at a unit cost of w per period and we denote h_t the amount of storage available at the beginning of period t. Inventories do not depreciate. The transition equations for inventories is then

$$h_{t+1} = h_t - x_t. \tag{2}$$

Negative inventories are not allowed and an aggregate storage capacity of $\bar{h}$ that cannot be overcome is available. Therefore in every period speculators position must satisfy

$$x_t \in [h_t - \bar{h}, h_t]. \tag{3}$$

Finally we assume that and that final inventories, h_{T+1}, can be destroyed at no cost. Let the discount factor be $\delta \leq 1$, and let E_0 denote the expectation operator conditional to the information available ex-ante, the payoffs to competitive speculators from the sequence of aggregate sales are equal to

$$\Pi_0^S = E_0 \sum_{t=0}^{T} \delta^t (p_t x_t - w h_t). \tag{4}$$

In every period t, a monopolist sets a price p_t and is producing its output q_t using a decreasing returns to scale technology: the production technology is described by a convex cost function, assumed to be

$$C(q_t) = \frac{c}{2} \cdot q_t^2, \tag{5}$$

where the rate of increase of the marginal cost c is constant and equal in every period. The monopolist cannot store its output. At the price he chooses, the monopolist has to serve the demand addressed to him. Its aggregate profit ex-ante is equal to

$$\Pi_0^m = E_0 \sum_{t=0}^{T} \delta^t \left(p_t q_t - \frac{c}{2} q_t^2 \right) \tag{6}$$

where the quantity produced q_t must be equal to the total quantity demanded,

$$q_t = -x_t + D_t. \tag{7}$$

3 Analysis

As it is well known (see for example Willams and Wright [13]), in most cases, it is not possible to obtain closed form expressions for the equilibrium outcomes due to the dependence of the solution on the expected future price. In this section, we discuss the general form of the solution, which forms the base underlying our solutions to the two-period and infinite horizon cases.

Price-taking speculators determine their position as follows. If the current spot price is strictly lower than the discounted expected price of next period minus the discounted storage cost, speculators wish to store as much as possible, resulting in storage equal to their capacity, $\bar{h}$. As they are capacity constrained, equilibrium prices and speculative sales satisfy the following pair of complementarity conditions:

$$p_t - \delta E_t[p_{t+1}] + \delta w < 0 \qquad x_t = h_t - \bar{h}. \tag{8}$$

In this case we will say that the market is in a *capacity regime*.

Similarly if the current spot price is strictly higher than the discounted expected price of next period minus the discounted storage cost, then speculators wish to sell as much as possible. As negative inventories are not allowed, equilibrium prices and speculative sales satisfy therefore the following pair of complementarity conditions:

$$p_t - \delta E_t[p_{t+1}] + \delta w > 0 \qquad x_t = h_t \tag{9}$$

In this case we will say that the market is in a *stock-out regime*.

Finally if the current spot price is exactly equal to the discounted expected future price minus the discounted storage cost, that is if $p_t = \delta E_t[p_{t+1}] - \delta w$, then speculative

sales are determined to ensure market clearing at this price, i.e., $q_t = a_t - p_t - x_t$ and $p_t = \delta E_t[p_{t+1}] - \delta w$. As the expected future price is a function of the state variables (a_t, h_t), speculative sales in this *smoothing regime* are a function of the state and of the quantity supplied q_t. This dependence on q_t implies that future inventories are influenced by q_t. By our assumption of IID demand shocks, expected next-period price is independent of the current realization of the demand shock, consequently, we define $g(h_{t+1}) \equiv E_t(p_{t+1})$. Using this notation we have

$$h_t - h_{t+1} = a_t - q_t - \delta g(h_{t+1}) + \delta w, \tag{10}$$

which defines h_{t+1} as an implict function of q_t. Define this function as $h_{t+1} = \psi(q_t; a_t, h_t)$.

Continuity ...

The behaviour of speculators results in two threshold prices.

$$\begin{aligned} p_t > p_u \quad &\Leftrightarrow \quad x_t = h_t \\ p_t < p_\ell \quad &\Leftrightarrow \quad x_t = h_t - \bar{h} \end{aligned} \tag{11}$$

Continuity implies that

$$p_u = \delta g(0) - \delta w \tag{12}$$

and

$$p_\ell = \delta g(\bar{h}) - \delta w \tag{13}$$

The quantities that correspond to these two bounds are

$$q_{\ell t} = a_t - h_t - p_u = a_t - h_t - \delta g(0) + \delta w \tag{14}$$

and

$$q_{ut} = a_t - h_t + \bar{h} - p_\ell = a_t - h_t + \bar{h} - \delta g(\bar{h}) + \delta w \tag{15}$$

We can now express inverse residual demand faced by the monopolist as

$$P(q_t; a_t, h_t) = \begin{cases} a_t - q_t - h_t + \bar{h} & \text{if } q_t > q_u \\ \delta g(\psi(q_t; a_t, h_t)) - \delta w & \text{if } q_t \in [q_\ell, q_u] \\ a_t - q_t - h_t & \text{if } p_t < q_\ell \end{cases} \tag{16}$$

Given $g(h_{t+1}) = E_t[p_{t+1}]$, we can solve for the monopolist's optimal output in period t, also a function of the state vector. Define this function as $f(a_t, h_t)$. Even though the monopolist has no direct control over the future state, it still faces a dynamic optimization problem since it can influence the level of inventories that speculators carry forward into the next period via its influence on price. The Bellman equation for the monopolist's problem is

$$V^m(a_t, h_t) = \max_{q_t} \left\{ P(q_t; a_t, h_t) q_t - \frac{c}{2} q_t^2 + \delta E_t V^m(a_{t+1}, h_{t+1}) \right\} \tag{17}$$

subject to $h_{t+1} = h_t - x_t$. There are three cases to be analyzed corresponding to the three cases in (16).

"Stock-out" regime, $q_t^* < q_{\ell t}$ or $p_t^* > p_u$: Here, speculators sell their entire stock in t, so $h_{t+1} = 0$ and the monopolist's choice of output has no influence on next period's state. Hence, the optimal output is that which maximizes its static profit with $x_t = h_t$:

$$q_t^* = \frac{a_t - h_t}{2 + c} \equiv f_1(a_t, h_t) \tag{18}$$

If q_t is chosen to be in this region, the value for the monopolist is

$$v_1(a_t, h_t) = \frac{(a_t - h_t)^2}{2(2 + c)} + \delta E_t V^m(a_{t+1}, 0) \tag{19}$$

"Capacity" regime, $q_t^* > q_{ut}$ or $p_t^* < p_\ell$: In this case, speculators purchase and carry the maximal amount of stocks into the next period: $h_{t+1} = \bar{h}$. Since this means that next periods speculative stocks are unaffected by changes in the monopolist's output, the optimal output for the monopolist is that which maximizes its static profit, given $x_t = h_t - \bar{h}$:

$$q_t^* = \frac{a_t - (h_t - \bar{h})}{2 + c} \equiv f_3(a_t, h_t) \tag{20}$$

If q_t is chosen to be in this region, the value for the monopolist is

$$v_3(a_t, h_t) = \frac{(a_t - (h_t - \bar{h}))^2}{2(2 + c)} + \delta E_t V^m(a_{t+1}, \bar{h}) \tag{21}$$

"Smoothing" regime, $q_t^* \in [q_{\ell t}, q_{ut}]$ or $p_t^* \in [p_\ell, p_u]$: For this intermediate case, $p_t = \delta g(\psi(q_t; a_t, h_t)) - \delta w$, so q_t^* solves

$$\max_{q_t} \{\delta(g(\psi(q_t; a_t, h_t)) - w)q_t - \frac{c}{2}q_t^2 + \delta E_t V^m(a_{t+1}, h_{t+1})\} \tag{22}$$

subject to $h_{t+1} = h_t - x_t$. In this region, q_t and x_t are related by the requirement that $p_t = \delta(g(\psi(q_t; a_t, h_t)) - w)$ or

$$a_t - q_t - x_t = \delta(g(\psi(q_t; a_t, h_t)) - w). \tag{23}$$

Solving for x_t, we can write the evolution of inventories as

$$h_{t+1} = h_t - a_t + q_t + \delta(g(\psi(q_t; a_t, h_t)) - w) \tag{24}$$

which highlights the influence that the monopolist now has on the future state. For an interior optimum in this region, the following necessary condition must hold:

$$\delta g(\psi(q_t; a_t, h_t)) - \delta w - cq_t + \delta q_t \frac{\partial g(\psi(q_t; a_t, h_t))}{\partial q_t} + \delta \frac{\partial E_t V^m(a_{t+1}, h_{t+1})}{\partial q_t} = 0 \tag{25}$$

with h_{t+1} given by (24). The last two terms on the left hand side are the effects that variation in output have on expected future price and value due to the influence on future inventories. Let $f_2(a_t, h_t)$ denote optimal production and $v_2(a_t, h_t)$ the corresponding value in this region.

To summarize this analysis, the monopolist's production in t will be given by one of the three functions, f_1, f_2 or f_3. Which function is optimal is determined by the value obtained under each one. Consequently

$$V^m(a_t, h_t) = \max[v_1(a_t, h_t), v_2(a_t, h_t), v_3(a_t, h_t)]. \tag{26}$$

Note that the functional form of v_1 and v_3 are known. Since a_t is independently distributed, they are quadratic functions of a_t and h_t that are known up to a constant term, $E_t V^m(a_{t+1}, 0)$ for v_1 and $E_t V^m(a_{t+1}, \bar{h})$ for v_3.

Clearly, the difficulty lies in determining the solution in the smoothing region as it requires the evalution of the composition of two unknown functions: the price expectation function, $g()$ and the equilibrium carry-out function, $\psi()$. In the next section, we restrict the time horizon to two periods for which we can obtain a closed-form solution. Following that, we examine numerical solutions to the infinite horizon version of the model.

4 Two period horizon

As a first step in our analysis of the model described in the previous section we examine the equilibrium when there are only two periods. Although restrictive, this exercise has the advantage that a closed-form solution is feasible, which allows us to gain some intuition for the consequences of imperfect competition on the behaviour of competitive storers and in turn on the distribution of prices and on the overall welfare properties of the equilibrium allocation. As we wish to focus on the determination of prices, we maintain the following assumption:

Assumption 1 *Speculators are small:* $\bar{h} < \underline{a}$.

Assuming that speculators storage capacity is limited compared to the minimal demand seems realistic in practice[3], and is made to ensure that selling the maximal amount of inventories cannot imply that consumers are satiated, in which case the market price, and therefore the gross return of speculation, would be equal to zero, making any investment on $\bar{h}$ unprofitable.

In the second period speculators have inventories of h_2 available. Since the storage cost is sunk at this point and since they are price-takers, they will sell their entire stock as long as the market price for the product is non-negative. Under assumption 1, speculative

[3]For example on the natural gas market, underground gas storage capacity is equal to x % of annual consumption at a price equal to ...

inventories can never be such that an entire stock-out causes the spot price in second period to be equal to 0, no matter the realized demand a_2. Profits to speculators in period two are thus $\pi_2^s = (p_2 - w) \cdot h_2$, the difference between the revenue obtained on the market and the cost to carry inventories forward. In the first period, denoting E_1 the expectation operator conditional to the information available in this period, and denoting δ the discount factor, speculators have h_1 stocks available and choose sales to maximize total payoffs given by

$$\Pi_1^s = p_1 \cdot x_1 - w \cdot h_1 + \delta \; E_1[(p_2 - w) \cdot (h_1 - x_1)] \tag{27}$$

where second period inventories have been replaced by $h_1 - x_1$ and the first period position must satisfy the capacity and the non negativity constraints. The position of speculators as a function of first period price p_1 and the state (a_1, h_1) obeys exactly the same rule than what we described at the beginning of the previous section.

4.1 Perfect competition

Given the cost function of the industry, the competitive supply as a function of market price p_t in every period is equal to $S_t(p_t) = p_t/c$. As speculators sell h_2 on the second period market, aggregate demand in this period is $D_2(p_2; a_2, h_2) = a_2 - p_2 - h_2$. Under the assumption $\overline{h} \leq \underline{a}$ we are sure that a positive equilibrium price may exist. The intersection between the competitive demand and the competitive supply gives

$$p_2^c = \beta^c \cdot (a_2 - h_2) \quad q_2^c = (1 - \beta^c) \cdot (a_2 - h_2) \quad \text{where } \beta^c = c/(1+c) \tag{28}$$

Given the expression of the second period price, it is possible to derive the expected price consistent with the position taken in period 1.

- If $x_1 = h_1 - \overline{h}$, then $h_2 = \overline{h}$ and the expected second period price is equal to $E_1(p_2) = \beta^c \cdot (E(a) - \overline{h})$. For this position to be taken in period 1, it must be the case that $p_1 < \delta \; \beta^c \cdot (E(a) - \overline{h}) - \delta \; w$ and we define

$$p_l^c \equiv \delta \; \beta^c \cdot (E(a) - \overline{h}) - \delta \; w, \tag{29}$$

- If $x_1 = h_1$ then $h_2 = 0$ and the expected second period price is equal to $E_1(p_2) = \beta^c \; E(a)$. For this position to be taken in period 1, it must be the case that $p_1 > \delta \; \beta^c \; E(a) - \delta \; w$, and we define

$$p_u^c \equiv p_1 > \delta \; \beta^c \; E(a) - \delta \; w, \tag{30}$$

- If $p_1 \in [p_l^c, p_u^c]$, then the position x_1 taken by speculators must be consistent with the expected second period price that results. Since $p_1 = \delta(E_1(p_2) - w)$ then $E_1(p_2) = w + p_1/\delta$. Replacing $E_1(p_2)$ by is expression as a function of h_1 at the competitive equilibrium, $E_1(p_2) = \beta^c \; (E(a) - h_1 + x_1)$, and solving for x_1 gives

$$x_1 = h_1 - E(a) + \left(w + \frac{p_1}{\delta}\right) / \beta^c. \tag{31}$$

The competitive demand net of speculation is consequently equal to

$$D_1(p_1; a_1, h_1) = \begin{cases} a_1 - h_1 - p_1 + \overline{h} & \text{if } p_1 < p_l^c \\ a_1 - h_1 - p_1 + E(a) - \left(w + \frac{p_1}{\delta}\right)/\beta^c & \text{if } p_1 \in [p_l^c, p_u^c] \\ a_1 - h_1 - p_1 & \text{if } p_1 > p_u^c \end{cases} \tag{32}$$

The competitive equilibrium can be obtained from the intersection of the competitive demand with the competitive supply, as proposition 1 below states:

Proposition 1 *Given the first period demand a_1, the equilibrium under perfect competition is characterized as follows:*

1. *If selling inventories at the competitive price in period 1 is more profitable than selling them at the expected competitive price in period 2, $\beta^c\ (a_1 - h_1) > \delta\ \beta^c\ E(a) - \delta\ w$, then speculators sell their entire initial inventories in period 1 and a "stock-out" occurs. Prices are:*
$$p_1^c = \beta^c \cdot (a_1 - h_1), \quad p_2^c = \beta^c \cdot a_2$$

2. *If selling inventories at the competitive price in period 1 is less profitable than storing the maximal quantity and selling it at the expected competitive price in period 2, $\beta^c\ (a_1 - h_1 + \bar{h}) < \delta\ \beta^c\ (E(a) - \bar{h}) - \delta\ w$, then a "capacity" regime occurs in period 1. Prices are:*
$$p_1^c = \beta^c \cdot (a_1 - h_1 + \overline{h}), \quad p_2^c = \beta^c \cdot (a_2 - \overline{h})$$

3. *Else if neither a "stock-out" nor a "capacity" regime is profitable, $\beta^c\ (a_1 - h_1) \leq \delta\ \beta^c\ E(a) - \delta\ w$ and $\beta^c\ (a_1 - h_1 + \bar{h}) \geq \delta\ \beta^c\ (E(a) - \bar{h}) - \delta\ w$, then speculators store and smooth prices:*
$$p_1^c = \frac{\delta\ \beta^c\ (a_1 - h_1 + E(a)) - \delta\ w}{1+\delta}, \quad p_2^c = \beta^c\ a_2 + \frac{\beta^c\ (a_1 - h_1 - \delta\ E(a)) + \delta\ w}{1+\delta}$$

Proof: See appendix.‖

4.2 Monopolistic production, competitive speculation

The objective of the monopoly is to maximize its expected total profit integrating the impact of speculation on it when choosing its period 1 price:

$$\max_{p_1} \left\{ p_1 \cdot D_1(p_1; a_1, h_1) - \frac{c}{2} \cdot (D_1(p_1; a_1, h_1))^2 + \delta\ E_1(\pi_2^m(p_1; a_1, h_1)) \right\} \tag{33}$$

The price and the quantity sold in second period result from the maximization of the second period monopoly profit $\pi_2^m = (a_2 - h_2 - p_2) \cdot p_2 - c \cdot (a_2 - h_2 - p_2)^2/2$ with respect to p_2, giving

$$p_2^m = \beta^m \cdot (a_2 - h_2) \quad q_2^m = (1 - \beta^m) \cdot (a_2 - h_2) \text{ where } \beta^m = (1 + c)/(2 + c). \tag{34}$$

Replacing the slope of the marginal cost of production c by its expression as a function of β^m, $c = \frac{2\beta^m - 1}{1 - \beta^m}$, the equilibrium second period profit is equal to

$$\pi_2^m = \frac{(1 - \beta^m)}{2} \cdot (a_2 - h_2)^2. \tag{35}$$

In period 1, the monopoly must integrate his conjecture on the behaviour of competitive speculators into his computation in order to determine the price he wants to charge in this period. For any given price p_1, speculators determine a position x_1 that must be consistent in equilibrium with the expected second period price that results:

- if the price p_1 is strictly lower than $\delta\beta^m(E(a) - \overline{h}) - \delta w$, then the market is in a "capacity" regime. Indeed at the price p_1 the expected net return of speculative storage, equal to the difference between the expected price at which inventories are sold in second period and the cost of buying and storing the product in first period, is strictly positive no matter how large inventories are. Consequently speculators store the product up to their capacity by buying $x_1 = h_1 - \overline{h}$ and speculative inventories sold on the market in second period are $h_2 = \overline{h}$. The expected second period price is equal to $E_1(p_2) = \beta^m \ (E(a) - \overline{h})$. We define the price at which speculators are indifferent between storing the product up to their capacity and storing slightly less by:
$$p_l^m \equiv \delta \ \beta^m \ (E(a) - \overline{h}) - \delta \ w. \tag{36}$$

- if the price p_1 is strictly higher than $\delta\beta^m E(a) - \delta w$, then the market is in a "stock-out" regime. At the price p_1, the expected net return of speculative storage is strictly negative no matter how small speculative inventories are. Consequently speculators sell their entire initial inventories, $x_1 = h_1$ and no inventories are carried forward, $h_2 = 0$. The expected second period price is equal to $E_1(p_2) = \beta^m \ E(a)$. We define the price at which speculators are indifferent between selling their initial inventories entirely and storing a very small quantity by:
$$p_u^m \equiv \delta \ \beta^m \ E(a) - \delta \ w. \tag{37}$$

- if the price p_1 belongs to $[p_l^m, p_u^m]$, then the market is in a "smoothing" regime. Speculative inventories must be such that the expected net return of speculation is equal to 0, and the position taken by speculators must be consistent with $p_1 = \delta \ E_1(p_2) - \delta \ w$. Extracting $E_1(p_2)$ from this equation gives $E_1(p_2) = w + p_1/\delta$. Since

$p_2 = \beta^m\ (a_2 - h_2)$ and $h_2 = h_1 - x_1$ then $E_1(p_2) = \beta^m\ (E(a) - h_1 + x_1)$. Equating both definitions of $E_1(p_2)$ and solving for x_1 gives the position of speculators compatible with the smoothing regime,

$$x_1 = h_1 - E(a) + \left(w + \frac{p_1}{\delta}\right)/\beta^m. \tag{38}$$

The effect of changes in first period monopoly price on the second period monopoly profit through changes in the position of speculators is immediate to derive: if p_1 is lower than p_l^m, $\bar{h}$ is sold in second period by speculators, leaving the monopoly with the lowest residual demand, if p_1 is higher than p_u^m then no inventories are carried forward and the largest residual demand is left to the monopoly, and in between smoothing occurs. Therefore the second period profit as a function of first period price and initial inventories is equal to:

$$\pi_2^m(p_1, h_1) = \begin{cases} \frac{1}{2}(1-\beta^m)(a_2 - \overline{h})^2 & \text{if } p_1 < p_l^m \\ \frac{1}{2}(1-\beta^m)\left(a_2 - E(a) + \frac{w}{\beta^m} + \frac{p_1}{\delta\beta^m}\right)^2 & \text{if } p_1 \in [p_l^m, p_u^m] \\ \frac{1}{2}(1-\beta^m)(a_2)^2 & \text{if } p_1 > p_u^m \end{cases} \tag{39}$$

This profit is a continuous function of p_1, and is constant for period 1 prices such that the market is in a "stock-out" ($p_1 > p_u^m$) or a "stocks up-to-capacity" ($p_1 < p_l^m$) regime. However it is strictly increasing in p_1 in the region where smoothing occurs[4]: when the price p_1 increases from the "stocks up-to-capacity" to the "smoothing" regime, the total quantity brought forward by speculators decreases and the monopoly faces less competition from the speculators in second period. Its second period profit increases. By linearity of the expectation operator, the expected second period marginal profit of increasing p_1 is equal to

$$E_1\left(\frac{\partial \pi_2^m}{\partial p_1}\right) = \frac{1-\beta^m}{\delta\ \beta^m} \cdot \left(\frac{w}{\beta^m} + \frac{p_1}{\delta\ \beta^m}\right). \tag{40}$$

The second period expected marginal profitability is consequently upward jumping at the price p_l^m such that speculators are indifferent between storing the product up to their capacity and smoothing prices, and downward jumping at the price p_u^m such that speculators are indifferent between smoothing prices and selling their initial inventories entirely. An increase in price reduces speculative inventories when speculators capacity is fully used and causes a price increase in second period: it improves marginally the expected second period profit. Conversely, an increase in price that reduces speculative inventories carried

[4]Indeed the first order derivative of the profit when $p_1 \in [p_l^m, p_u^m]$ is equal to $\frac{\partial \pi_2^m}{\partial p_1} = \frac{1-\beta^m}{\delta\ \beta^m} \cdot \left(a_2 - E(a) + \frac{w}{\beta^m} + \frac{p_1}{\delta\ \beta^m}\right)$ which is strictly positive if and only if $p_1 > \delta\beta^m(E(a) - a_2) - \delta\ w$. Since $a_2 > \bar{h}$, then $\delta\ \beta^m\ (E(a) - a_2) - \delta\ w < \delta\ \beta^m(E(a) - \bar{h}) - \delta\ w = p_l^m$. As period 1 price must be higher than p_l^m for smoothing to occur, the price p_1 is always such that the derivative above is strictly positive.

to the second period down to 0 forces the monopolist to support an increase in its marginal cost of production in second period and reduces its profit.

Using the derivation of the speculative position x_1 as a function of p_1, it is also possible to obtain the expression of the first period demand net of speculation. It is a continuous function of p_1, given by

$$D_1(p_1; a_1, h_1) = \begin{cases} a_1 - h_1 - p_1 + \overline{h} & \text{if } p_1 < p_l^m \\ a_1 - h_1 - p_1 + E(a) - \frac{w}{\beta^m} - \frac{p_1}{\delta\beta^m} & \text{if } p_1 \in [p_l^m, p_u^m] \\ a_1 - h_1 - p_1 & \text{if } p_1 > p_u^m. \end{cases} \tag{41}$$

The first period marginal profitability of increasing the price in period 1 is consequently given by:

$$\frac{\partial \pi_1^m}{\partial p_1} = p_1 \frac{\partial D_1}{\partial p_1} + D_1(p_1; a_1, h_1) - \left(\frac{2\beta^m - 1}{1 - \beta^m}\right) \frac{\partial D_1}{\partial p_1} D_1(p_1; a_1, h_1), \tag{42}$$

and the total marginal profitability is equal to the discounted sum of the first and the expected second period marginal profits. We leave its characterization for the proof of proposition 2 below, but we characterize some critical values that will allow us to present our results. Let the limit of the marginal profit when the price p_1 goes to p_l^m by upper values be:

$$\frac{\partial \pi^m}{\partial p_1}(p_l^{m+}) = \frac{\delta\ (\beta^m)^2 + 2\beta^m - 1}{\delta\ \beta^m(1 - \beta^m)} \left(a_1 - h_1 - \delta\ E(a) + (1 + \delta)\ \bar{h}\right) + \frac{1 + \delta}{1 - \beta^m} w. \tag{43}$$

This marginal profit measures the variation in profit resulting from an increase in price that causes speculators to store slightly less than their capacity and therefore that leaves a residual demand slightly higher than the minimal one to the monopoly in second period. As we shall establish in the proof of proposition 2, when this marginal profit is negative then the monopoly always prefer speculators to store up to their capacity. Let the limit of the marginal profit when p_1 goes to p_u^m by lower values be

$$\frac{\partial \pi^m}{\partial p_1}(p_u^{m-}) = \frac{\delta\ (\beta^m)^2 + 2\beta^m - 1}{\delta\ \beta^m(1 - \beta^m)} \left(a_1 - h_1 - \delta\ E(a)\right) + \frac{1 + \delta}{1 - \beta^m} w \tag{44}$$

and the when it goes to p_u^m by upper values be

$$\frac{\partial \pi^m}{\partial p_1}(p_u^{m+}) = \frac{\beta^m}{1 - \beta^m} \left(a_1 - h_1 - \delta E(a)\right) + \frac{\delta}{1 - \beta^m} w. \tag{45}$$

The first limit can be interpreted as the variation of profit obtained by the monopoly when the price set is such that speculators keep slightly positive inventories instead of nothing, while the second can be interpreted as the profit variation obtained by the monopoly when

the price is such that speculators sell their initial inventories entirely in first period. Finally, let us introduce the additional condition (C1) on speculators' capacity $\overline{h}$,

$$\text{(C1)} \quad \frac{\delta\ (1-\beta^m)^2\ w}{\beta^m(\delta(\beta^m)^2+2\beta^m-1)} \leq (1+\delta)\ \overline{h}.$$

When (C1) holds, speculative capacity is not too small, so that the differential between the boundary prices p_l^m and p_u^m (which is equal to $\delta\ \beta^m\ \overline{h}$) is large enough. As the proof of proposition 2 shows, if the capacity of storage of speculators is large enough, the monopolist has never to choose between a price inducing a stocks up-to-capacity regime and a stock-out regime, that is between the two extreme price regimes available. We can establish the following proposition.

Proposition 2 *When (C1) holds, given the first period demand a_1, the equilibrium under monopoly is characterized as follows :*

1. *If the marginal profit of setting the lowest price such that a stock-out occurs is positive then the monopoly sets a price such that speculators sell their entire initial inventories in period 1:*
$$\frac{\partial \pi^m}{\partial p_1}(p_u^{m+}) \geq 0 \ \Rightarrow\ p_1^* = \beta^m(a_1-h_1),\ p_2^* = \beta^m\ a_2$$
2. *If letting a stock-out induces marginal losses, and if the marginal profit of setting a price such that speculators keep an infinitesimal unit in inventories is positive, then the monopoly sets the lowest price such that a stock-out occurs:*
$$\frac{\partial \pi^m}{\partial p_1}(p_u^{m+}) < 0,\ \frac{\partial \pi^m}{\partial p_1}(p_u^{m-}) \geq 0 \ \Rightarrow\ p_1^* = \delta\beta^m E(a) - \delta w,\ p_2^* = \beta^m\ a_2$$
3. *If allowing speculators to keep small inventories induces marginal losses, and if the marginal profit of setting a price such that speculators store slightly less than their capacity is positive, then the monopoly sets a price such that smoothing occurs:*
$$\frac{\partial \pi^m}{\partial p_1}(p_u^{m-}) < 0,\ \frac{\partial \pi^m}{\partial p_1}(p_l^{m+}) \geq 0 \ \Rightarrow\ p_1^* = \frac{\delta\beta^m}{1+\delta}(a_1-h_1+E(a)) + \frac{\delta(1-2\beta^m)\ w}{\delta(\beta^m)^2+2\beta^m-1},$$
$$p_2^* = \beta^m\left(a_2 + \frac{a_1-h_1}{1+\delta} - \frac{\delta}{1+\delta}E(a)\right) + \frac{\delta\ (\beta^m)^2\ w}{\delta\ (\beta^m)^2+2\ \beta^m-1}$$
4. *Finally if the marginal profit of setting a price such that speculators store slightly less than their capacity is negative, then the monopoly chooses the price such that speculators store up to capacity:*
$$\frac{\partial \pi^m}{\partial p_1}(p_l^{m+}) < 0 \ \Rightarrow\ p_1^* = \beta^m(a_1-h_1+\overline{h}),\ p_2^* = \beta^m(a_2-\overline{h}).$$

Proof: See appendix.‖

We leave the discussion of the results to the next paragraph.

4.3 Comparison between perfect competition and monopoly

The likelihood of the different price regimes as a function of the state (a_1, h_1) varies by market structure. To investigate this issue, let us rank the bounds of all the regions presented in propositions 1 and 2. From the proofs of these two propositions, we know that belonging to each of the regions depends on the comparison between $a_1 - h_1 - \delta\ E(a)$ and a function of c, δ, w and $\bar{h}$. Ranking the different bounds is straightforward: we can summarize the different price regimes in perfect competition and in monopoly by Figure 1 When the industry is in monopoly, four different market regimes are possible, while there are only 3 regimes under perfect competition. The area hatched horizontally presents the values of (a_1, h_1) such that speculators store the product up to their capacity. Given initial inventories h_1, this regime occurs when a_1 is sufficiently lower than the average $E(a)$. Note that the area of this region is larger than the area of the region filled in dark grey, representing the region in which speculators are in the "capacity" regime under perfect competition. Given h_1, the likelihood of being in a capacity regime is therefore larger under a monopoly than under perfect competition.

The area hatched diagonally shows the values of (a_1, h_1) such that the market is in a "smoothing" regime under a monopoly. This occurs for intermediate/small values of a_1 compared to $E(a)$. The region in which smoothing occurs under perfect competition is filled in light grey. Although the area of these two regions in the graph (a_1, h_1) is the same, due to the fact that the monopoly is charging a higher price, smoothing occurs for higher values of a_1 (given h_1) under a monopoly compared to perfect competition.

The area hatched vertically shows the values of (a_1, h_1) such that speculators are in a "stock-out" regime under a monopoly. The area of this region is higher than under perfect competition, area filled in white, and again occurs for higher values of first period demand, given initial inventories under a monopoly compared to perfect competition. An interesting sub-region is the one in which the monopoly sets the lowest price in which a stock-out occurs, filled in yellow and red. For these values of first period demand and initial inventories, the monopoly prefers to forbid speculators to carry inventories forward by sticking to a price level sufficiently high.

It is finally interesting to compare the level of prices charged by a monopoly which faces speculation with the level of prices of a static monopoly which is equal to $p_t^m = \beta^m a_t$ in any period. Because of the speculative activity, prices are often higher compared to the static monopoly in one period and lower in the other. There is however an interesting case: when a stock-out occurs at price $p_1^* = \delta\beta^m E(a) - \delta w$, for some values of the state (a_1, h_1) the price may be higher than what a static monopoly would charge in period 1, and equal in second period. To see this, remark that the conditions leading to case 2 of proposition 2 can be re-written $a_1 - h_1 - \delta E(a) + \delta w/\beta^m < 0$ and $a_1 - h_1 - \delta E(a) + (1+\delta)\delta\beta^m w/(\delta(\beta^m)^2 + 2\beta^m - 1) \geq 0$. The price p_1^* is strictly higher than $\beta^m a_1$ if and only if $a_1 - \delta E(a) - \delta w < 0$: this bound on a_1 obviously belongs to the region of case 2 for initial inventories sufficiently low. For these values of first period demand and initial inventories, the monopolist facing speculation

prefers to charge a higher price than a static monopoly in first period to force speculators to stay out in order to charge the static monopoly price in second period. This region is filled in red in the graph.

So far we have dealt with the case in which h_1 is exogenous. Let us consider the infinite horizon case to see how the level of inventories and hence the distribution of prices differs between the two market structures.

5 Infinite horizon analysis

We now turn to an analysis of the model in an infinite horizon setting. This will relax two limitations of the two-period model. First, the equilibrium price in the previous section was a function of initial stocks, h_1. Since there is no way to distiguish the different levels of stocks expected to be held in the monopoly versus the competitive case in the two-period setting, we were not able to fully characterize the price distribution. Second, speculators sell their stocks in the second period as long as price is positive in the two-period model. This likely exaggerates the desire of the monopolist to limit storage.

It is well known that there is no closed-form solution to the infinite horizon storage problem, even in the case of competitive production (Williams and Wright [13]). Consequently, the analysis of this section proceeds by numerical solutions to the problem under the alternative conditions of competitive and monopolistic production. We will describe the solution method that we use for the two market structures next. Following that we provide a comparison of the equilibrium under the two market structures for a particular set of values for the parameters.

5.1 Monopoly production and competitive storage

In the model with monopoly production, there are two unknown functions that need to be determined in order to solve the problem: the monopolist's value function, $V^m(a_t, h_t)$, and speculators' price expectations, $E_t[p_{t+1}] \equiv g(h_{t+1})$. Since the value function described in (26) is not expected to be smooth, we choose instead to approximate the expectation of next period's value function, $E_t[V^m(a_{t+1}, h_{t+1})] \equiv \tilde{V}^m(h_{t+1})$. This has the added advantage that the function being approximated depends on only one variable.

We will proceed by using the collocation method[5] to compute approximate solutions for $g(h_+)$ and $\tilde{V}^m(h_+)$. In particular, we use

$$\tilde{V}^m(h_+) \approx \sum_{i=0}^{n} c_i \phi_i(h_+) \tag{46}$$

[5]See Judd [6], Chapter 11.

where c_i are coefficients and the $\phi_i()$ are known basis functions. We use Chebyshev polynomials for the $\phi_i()$ functions in what follows. Similarly

$$g(h_+) \approx \sum_{i=0}^{n'} d_i \phi_i(h_+) \tag{47}$$

The collocation method forces these approximations to be exact at the n and n' collocation nodes.

5.1.1 Numerical Algorithm

Three numerical routines are required to solve the model: a routine to compute Chebyshev approximations, a routine to integrate the price and value functions, and a routine to solve the fixed point problem in the smoothing region. We use the gsl_cheb, gsl_integrate_qag, and gsl_root_fsolver_brent routines from the GNU Scientific Library[6] for these tasks.

The algorithm is

Step 0. Choose degrees of approximation, n and n'. Initialize starting values d^0, c^0. These are chosen as Chebychev approximations to decreasing linear functions since both $g(h_{t+1})$ and $\tilde{V}^m(h_{t+1})$ are expected to be decreasing.

Step 1. Update the price expectations equation by

$$\sum_{i=0}^{n'} d_i^1 \phi_i(h'_{+i}) = \int p(a, h'_{+i}) \theta(a) da \tag{48}$$

where $\theta(a)$ is the density of a_t. Here the new d^1 are found by forcing this to hold at the n' collocation nodes and the price is evaluated using the approximations defined by d^0 and c^0. The price function on the right-hand side is found by computing the feasible choice for the monopolist that yields the highest value and using the resulting price. In other words, given the current approximation, we solve (18), (20), and (25) and the corresponding value for each to get the optimal production and price choice. This optimal price is used in the numerical integration of the right-side of (48).

Step 2. Update the value function expectation by

$$\sum_{i=0}^{n} c_i^1 \phi_i(h_+) = \int V^m(a, h_+) \theta(a) da \tag{49}$$

Here the new c^1 are found by forcing this to hold at the n collocation nodes and value function is evaluated using the approximations defined by d^0 and c^0. The value

[6] Galassi [5]

function on the right-hand side is found by computing the feasible choice for the monopolist that yields the highest value. In the same manner as for the previous step, we solve (18), (20), and (25) and use the corresponding value for each in (26) to get the values used in the right-side of (49).

Step 3. Stop if $||d^1 - d^0||$ is smaller than the convergence criterion. Otherwise set $d^0 = d^1$ and return to Step 1.

5.2 Competitive production and storage

The model with competitive production and storage is the benchmark case that has seen substantial analysis in previous work (for example see Williams and Wright [13]). The only difference between our model with competitive production and the standard treatment is the addition of a capacity constraint[7]

Given the quadratic production costs we are using, competitive supply is simply p_t/c. Stockouts occur if $p_t > \delta(E[p_{t+1}] - w)$, which, using the demand function, reduces to $a_t > h_t + \delta(E[p_{t+1}] - w)(1 + c)/c$. Similarly, storage to capacity occurs if $a_t < h_t - \bar{h} + \delta(E[p_{t+1}] - w)(1 + c)/c$. Hence equilibrium price for any a_t, h_t combination is

$$p_t = \begin{cases} (a_t - h_t + \bar{h})c/(1+c) & \text{if } a_t < h_t + (\delta(E[p_{t+1}] - w))(1+c)/c \\ (a_t - h_t)c/(1+c) & \text{if } a_t > h_t - \bar{h} + (\delta(E[p_{t+1}] - w))(1+c)/c \\ \delta E_t[p_{t+1}] - \delta w & \text{otherwise} \end{cases} \tag{50}$$

This problem is simpler than the one in the case of monopoly production since there is only one unknown function to approximate. We proceed by approximating $E[p_{t+1}]$ with a Chebychev polynomial $\sum_{i=0}^{n} d_i\phi_i(x)$ and finding the d_i that result in a close approximation to the expectation of (50) when evaluated at period $t+1$. The main difficulty is to find the value of h_{t+1} to use in the last case of (50). In particular, since $p_t = \delta E_t[p_{t+1}|h_{t+1}] - \delta w$ in this case, we must have

$$a_t - h_t + h_{t+1} - (\delta \sum_{i=0}^{n} d_i\phi_i(h_{t+1}) - \delta w)/c = \delta \sum_{i=0}^{n} d_i\phi_i(h_{t+1}) - \delta w \tag{51}$$

which we solve with the root-finding algorithm. The solution algorithm is similar to the one used in the monopoly case, but without Step 2.

5.3 Example

We present the solution to the problem for parameter values of $\delta = 0.95, c = 0.5, w = 0.1, \bar{h} = 1$. The demand intercept is assumed to be normally distributed with $E[a_t] = 5$ and variance of 1.0. With these parameters, the average price charged by a static monopoly

[7] Miranda and Fackler[8] apply a constraint in their example of this problem.

is equal to 3. The orders of the Chebyshev polynomials are $n = n' = 8$ for the expected price and expected value functions in the monopoly case and $n = 6$ in the competitive case.[8] Figure 2 plots the equilibrium price function for both market structures: p_t versus a_t and h_t. The upper surface is the monopolist's price function, while the lower surface is that for the perfectly competitive market. The relatively flat portion of each plot is where smoothing occurs. Kinks occur at the boundaries to the smoothing region. We see that as was the case in the two-period model, for a given level of inventories, the threshold demand shock at which stockouts occur is higher for the monopolist than for the perfectly competitive market. This corroborates our findings on the two periods case in the sense that a stockout is more likely in a perfectly competitive market than a monopoly at a given level of inventories.[9] One final point to make about Figure 2 is that the monopoly price function has a steeper slope as a_t varies than the competitive one does for any value of h_t. This implies a higher price variance under monopoly, and consequently a higher return to storage so we would expect to see more inventories carried under monopoly production.

Figure 2 shows price for a given state. However, it is likely that the distribution of inventories will differ under the different market structures. This implies that to know what the actual differences in the price distribution will be, we need to compute the distribution of inventories under the different market structures. In order to see the effects of market structure on the equilibrium distributions of price and inventories, we generate series for a_t that are 1000 periods long, computing prices and inventories levels for each period, begining with $h_0 = 0$. We construct 500 such samples to generate averages over the 500 samples of some statistics of interest, which are presented in the first two rows of Table 1.

Not surprisingly, the monopoly price is generally higher than that of the competitive industry. Also, prices under monopoly are significantly more volatile than is efficient. Monopoly prices are also slightly more skewed and autocorrelated than competitive ones. The higher price volatility results in storers holding more inventories under a monopolist, which combined with the lower output results in a substantially higher inventory/production ratio. Also, stockouts occur less frequently under the monopolist.

To demonstrate the robustness of these results, Figures 3 and 4 plot price variance and inventory levels for a range of values of $\bar{h}$. The maximum value of $\bar{h} = 3.0$ represents an effectively non-binding constraint for these parameter values so those values are representative of the model without a capacity constraint. Descriptive statistics for $\bar{h} = 3.0$ are provided in the last two rows of Table 1. The qualitative results are robust to variations in $\bar{h}$. The difference between price variance under monopoly versus perfect competition is somewhat reduced as capacity increases, but monopoly price variance remains substantially higher even when capacity no longer binds. Furthermore, Figure 4 suggests that the constraint has more effect on storage under a monopoly, as storers increase average stocks

[8] The expected price and expected value functions are much smoother than the non-differentiable price and value functions, which permits a relatively low degree approximation.

[9] Of course, the equilibrium level of inventories held under the different market structures will not necessarily be the same.

Table 1: Simulated statistics

		E[p]	Var[p]	Skew[p]	Cor[p]	E[h]	E[h/q]	Stockout
$\bar{h} = 1$								
	P.C.	1.67	0.08	0.62	0.17	0.16	0.05	0.60
	Monopoly	3.02	0.22	0.65	0.19	0.20	0.10	0.55
$\bar{h} = 1.5$								
	Monopoly	3.02	0.20	0.87	0.20	0.23	0.12	0.54
$\bar{h} = 3.0$								
	P.C.	1.67	0.07	0.79	0.18	0.18	0.06	0.59
	Monopoly	3.03	0.20	1.01	0.20	0.24	0.12	0.53

held more quickly as the constraint is relaxed. This later result is again due to the fact that prices are more volatile under the monopoly so speculators have a stronger desire to increase inventories and hence, more likely to be capacity contrained when $\bar{h}$ is small.

In order to get a sense of the overall distibution of prices, in Figure 5 we plot histograms of prices from a single long simulation (100,000 observations) using a fine cell width. This figure highlights that the mass point in prices that was found in the two-period model also occurs in the infinite horizon solution. Under monopoly, the price equals the upper bound (p_u) about 2.6% of the time, while under perfect competition it essentially never occurs. Hence the qualitative difference in equilibrium prices that occured in the two-period model continues to exist in the long-horizon case.

5.4 The returns to speculation

Since price volatility differs across market structures, it is natural to ask what the effect of free entry in speculation would be. In particular, we can examine the payoff to speculators (in aggregate) for a given storage capacity. Since prices are more volatile under monopoly production, we would expect speculative returns to be higher for a given level of capacity, and consequently more capacity to appear in a long-run equilibrium.

Rather than finding the speculators' value function explicitly, we compute the discounted present value of the returns to speculators (equation (4)) over the 1000 periods of

the simulated series[10] and average over our 500 samples. Speculative returns under the two market structures are plotted in Figure 6. Due to the higher price volatility speculation is substantially more profitable under monopoly production. If one were to superimpose a plot of the cost of capacity on this figure (say due to the opportunity cost of space for storage), then the intersection of the two would depict the long-run level of storage capacity. Clearly, for any cost of capacity low enough to allow speculation, a free entry equilibrium will result in more storage capacity under monopoly production than would occur under perfectly competitive production.

We can get some idea of the difference in long-run equilibrium between the monopoly and the perfectly competitive markets by normalizing capacity in the competitive industry to $\bar{h} = 1.0$. From Figure 6, if capacity costs were linear and generated $\bar{h} = 1.0$ for competitive production, zero-profit capacity for the case of monopoly production would occur at approximately $\bar{h} = 1.5$. Statistics for this capacity with monopoly production are presented in the third row of Table 1. Comparisons of the first and third rows of this table then reflect comparisons of long run equilibrium under the two market structures given the imputed cost of capacity. Entry of speculative capacity does narrow the gap in price variance between the two market structures, but price remains significantly more volatile under monopoly, even with the larger amount of inventories being held. In addition, the higher level of inventories held does generate more serial correlation in the monopoly case, which is interesting given that Deaton and Laroque [3, 2] find that the model with competitive supply does not generate as much serial correlation as is found in the data. However, the increase in serial correlation in prices is not so large as to fully explain the extent of serial correlation found in the data.

6 Conclusion

This paper represents a first attempt to analyze the effectiveness of competitive storage in the presence of imperfectly competitive production of a commodity. Our two-period model showed that stock-outs occur less frequently and that storage to capacity occurs more frequently under monopoly than under perfect competition. We confirmed this result using numerical solutions to the infinite horizon version of the model. In addition, even though the long-run equilibrium has more inventories carried under monopoly production, this extra storage is not enough to remove the significant difference in price volatility under the two market structures.

The fact that competitive storage introduces kinks into the monopolist's residual demand curve results in a mass point in the price distribution. There exists a non-trivial range of demand states over which the monopolist wishes to hold price constant at the level that just induces a stock-out. The advantage of this is that there are no stocks to

[10] Given the discount factor we use, the trucation to 1000 periods has no discernable impact on the calculation of (4)

compete with the monopolist's production in the next period. This effect is completely absent under perfectly competitive production.

Appendix

A Proof of proposition 1

Let us study of the intersection of $D_1(p_1, h_1)$ and $S_1(p_1)$. Two cases are trivial: the "stocks up to capacity" and the "stock-out" regimes. In the first case, demand is given by $D_1(p_1, h_1) = a_1 - p_1 - h_1 + \overline{h}$, and the intersection with the competitive supply gives a market price equal to $p_1^c = \beta^c \cdot (a_1 - h_1 + \overline{h})$. Inventories carried forward are $h_2^c = \bar{h}$, and the competitive price in equilibrium must be such that speculators indeed find profitable to pile stocks up to their storage capacity, that is p_1^c must be strictly lower than p_l^c, giving a condition on first period demand:

$$\beta^c \cdot (a_1 - h_1 + \overline{h}) < \delta\ \beta^c\ (E(a) - \overline{h}) - \delta\ w \Leftrightarrow a_1 < \delta\ E(a) + h_1 - (1+\delta)\overline{h} - \delta\ w/\beta^c.$$

In the case where a "stock-out" occurs, demand is given by $D_1(p_1, h_1) = a_1 - p_1 - h_1$, and the intersection with the competitive supply gives a market price equal to $p_1^c = \beta^c \cdot (a_1 - h_1)$. Inventories carried forward are $h_2^c = 0$, and again there is a condition on parameters,

$$p_1^c > p_u^c \Leftrightarrow \beta^c(a_1 - h_1) > \delta\ \beta^c\ E(a) - \delta\ w \Leftrightarrow a_1 > \delta\ E(a) + h_1 - \delta\ w/\beta^c.$$

In the case in which "smoothing" occurs, substituting speculators x_1 with the position derived before compatible with smoothing, the quantity brought forward h_2 is equal to

$$h_2 = h_1 - x_1 = E(a) - \left(w + \frac{p_1}{\delta}\right)/\beta^c.$$

Let us derive the first period price from the intersection between the competitive supply and $D(p_1, h_1)$. It comes

$$p_1/c = a_1 + E(a) - w/\beta^c - p_1/(\delta\ \beta^c) - p_1 - h_1$$

giving

$$p_1^c = \frac{\delta\ \beta^c}{1+\delta}\left(a_1 + E(a) - h_1 - \frac{w}{\beta^c}\right)$$

The quantity carried forward by speculators is then equal to

$$h_2^c = E(a) - \left(w + \frac{\beta^c}{1+\delta}\left(a_1 + E(a) - h_1 - w/\beta^c\right)\right)/\beta^c = \frac{\delta\ E(a)}{1+\delta} - \frac{a_1}{1+\delta} + \frac{h_1}{1+\delta} - \frac{\delta}{1+\delta}\frac{w}{\beta^c}$$

and the second period price follows,

$$p_2^c = \beta^c \cdot \left(a_2 - \frac{\delta}{1+\delta}E(a) + \frac{a_1}{1+\delta} - \frac{h_1}{1+\delta} + \frac{\delta}{1+\delta}\frac{w}{\beta^c}\right).$$

B Proof of proposition 2

The derivative of π_1^m with respect to p_1 is given by equation (42). Since depending on p_1, the expression of the total profit differs, we study part by part the derivative of first period profit with respect to p_1.

- If $p_1 \leq p_l^m$, then

$$\frac{\partial \pi_1^m}{\partial p_1} = a_1 - h_1 + \overline{h} - 2p_1 + \left(\frac{2\beta^m - 1}{1 - \beta^m}\right)(a_1 - h_1 + \overline{h} - p_1)$$

  giving

$$\frac{\partial \pi_1^m}{\partial p_1} = \frac{\beta^m\ (a_1 - h_1 + \overline{h})}{1 - \beta^m} - \frac{p_1}{1 - \beta^m}.$$

- If $p_1 \in]p_l^m, p_u^m[$, then

$$\frac{\partial \pi_1^m}{\partial p_1} = -\left(1 + \frac{1}{\delta\beta^m}\right)p_1 + a_1 - h_1 + E(a) - \frac{w}{\beta^m} - \left(1 + \frac{1}{\delta\beta^m}\right)p_1$$
$$+\frac{2\beta^m - 1}{1 - \beta^m}\left(1 + \frac{1}{\delta\beta^m}\right)\left(a_1 - h_1 + E(a) - \frac{w}{\beta^m} - \left(1 + \frac{1}{\delta\beta^m}\right)p_1\right)$$

  which simplifies into

$$\frac{\partial \pi_1^m}{\partial p_1} = \frac{\delta(\beta^m)^2 + 2\beta^m - 1}{(1 - \beta^m)\delta\beta^m}\left(a_1 - h_1 + E(a) - \frac{w}{\beta^m}\right) - \frac{(\delta\beta^m + 1)(\delta\beta^m + 2\beta^m - 1)}{(1 - \beta^m)(\delta\beta^m)^2}p_1.$$

- If $p_1 \geq p_u^m$, then the derivative is similar to the case $p_1 \leq p_l^m$

$$\frac{\partial \pi_1^m}{\partial p_1} = \frac{\beta^m}{1 - \beta^m}(a_1 - h_1) - \frac{1}{1 - \beta^m}p_1.$$

We may now derive local maxima of the profit region by region by equating the total marginal profit to 0, and establish conditions under which these local maxima are indeed interior to the region that generated the expression of the profit. Let us consider the first order condition of profit maximization:

$$\frac{\partial \pi_1^m}{\partial p_1} + \delta E_1\left(\frac{\partial \pi_2^m}{\partial p_1}\right) = 0.$$

Then:

- if $p_1 \leq p_l^m$, then speculators store up to capacity and a marginal change in price does not affect second period profit. The price that should be chosen by the monopoly is

$$p_1^1 = \beta^m(a_1 - h_1 + \overline{h}).$$

This price belongs to the region i.e. is lower than p_l^m if and only if $\beta^m(a_1 - h_1 + \overline{h}) \leq \delta\beta^m E(a) - \delta\beta^m\overline{h} - \delta w$ that is

$$a_1 - h_1 + (1+\delta)\overline{h} - \delta E(a) + \delta\frac{w}{\beta^m} \leq 0. \tag{52}$$

If condition (52) is satisfied, then there is a local maximum of the total profit in which speculators store up to capacity in period 1.

- If $p_1 \in]p_l^m, p_u^m[$, then speculators store and smooth prices. The first order condition gives

$$\frac{\delta(\beta^m)^2 + 2\beta^m - 1}{(1-\beta^m)\delta\beta^m}(a_1 - h_1 + E(a)) + \frac{(1+\delta)(1-2\beta^m)}{(1-\beta^m)\delta(\beta^m)^2}w - \frac{(1+\delta)(\delta(\beta^m)^2 + 2\beta^m - 1)}{(1-\beta^m)(\delta\beta^m)^2}p_1 = 0$$

and therefore the local optimum is

$$p_1^2 = \frac{\delta\beta^m}{1+\delta}(a_1 - h_1 + E(a)) + \frac{\delta(1-2\beta^m)}{\delta(\beta^m)^2 + 2\beta^m - 1}w.$$

This price belongs to $[p_l^m, p_u^m]$ if and only if

$$p_1^2 \leq p_u^m \Leftrightarrow a_1 - h_1 - \delta E(a) + \frac{\delta\beta^m(1+\delta)}{\delta(\beta^m)^2 + 2\beta^m - 1}w \leq 0 \tag{53}$$

and

$$p_1^2 \geq p_l^m \Leftrightarrow a_1 - h_1 - \delta E(a) + (1+\delta)\overline{h} + \frac{\delta\beta^m(1+\delta)}{\delta(\beta^m)^2 + 2\beta^m - 1}w \geq 0. \tag{54}$$

If conditions (53) and (54) are satisfied, then there is a local maximum of the profit such that smoothing occurs and prices are linked between periods.

- Finally if $p_1 \geq p_u^m$, then speculators sell their inventories out, and therefore the price charged by the monopoly should be

$$p_1^3 = \beta^m(a_1 - h_1).$$

This price is higher or equal to p_u^m if and only if $\beta^m(a_1 - h_1) \geq \delta\beta^m E(a) - \delta w$ that is

$$a_1 - h_1 - \delta E(a) + \delta\frac{w}{\beta^m} \geq 0 \tag{55}$$

If condition (55) is satisfied, then there is a local maximum of the profit such that a stock-out occurs in period 1.

Due to the continuity of the total profit, and to the linearity of the marginal profit in each of the three regions, finding the global optimum of this profit can be done by inspecting the position of each local optimum with respect to the bounds of its region, p_l^m and p_u^m. However due to the discontinuity of the marginal profit, it is possible for p_l^m or p_u^m to be the global optimum of the profit. Condition (C1) rules out some of the potential cases, as we shall now establish. To simplify the analysis, let us rewrite conditions (52), (53), (54) and (55) using the following notations. Let

$$A \equiv a_1 - h_1 - \delta E(a) \text{ and } K(\beta^m, \delta) \equiv \frac{\delta\beta^m(1+\delta)}{\delta(\beta^m)^2 + 2\beta^m - 1}, \tag{56}$$

then condition (52) becomes

$$A + (1+\delta)\bar{h} + \frac{\delta}{\beta^m} w \leq 0,$$

condition (53) becomes

$$A + K(\beta^m, \delta) w \leq 0,$$

condition (54) becomes

$$A + (1+\delta)\bar{h} + K(\beta^m, \delta) w \geq 0,$$

and condition (55) becomes

$$A + \frac{\delta}{\beta^m} w \geq 0.$$

Since $\beta^m \in [1/2, 1]$, it is possible to verify (for example numerically with Mathematica) that the denominator of $K(\beta^m, \delta)$ is strictly positive for any value $\delta \in [0, 1]$. Consequently,

$$\frac{\delta\beta^m(1+\delta)}{\delta(\beta^m)^2 + 2\beta^m - 1} \geq \frac{\delta}{\beta^m} \Leftrightarrow (\beta^m)^2(1+\delta) \geq \delta(\beta^m)^2 + 2\beta^m - 1 \Leftrightarrow (\beta^m - 1)^2 \geq 0$$

which always holds. Therefore $K(\beta^m, \delta) \geq \frac{\delta}{\beta^m}$ and we can obtain the following ranking

$$A + \frac{\delta}{\beta^m} w \leq A + K(\beta^m, \delta) w \leq A + (1+\delta)\overline{h} + K(\beta^m, \delta) w.$$

Similarly

$$A + \frac{\delta}{\beta^m} w \leq A + (1+\delta)\overline{h} + \frac{\delta}{\beta^m} w \leq A + (1+\delta)\overline{h} + K(\beta^m, \delta) w$$

and to rank conditions (52) to (55) it remains to compare $A + (1+\delta)\overline{h} + \frac{\delta}{\beta^m} w$ with $A + K(\beta^m, \delta) w$. We have

$$A + K(\beta^m, \delta) w \leq A + (1+\delta)\overline{h} + \tfrac{\delta}{\beta^m} w \Leftrightarrow \left(\tfrac{\delta\beta^m(1+\delta)}{\delta(\beta^m)^2 + 2\beta^m - 1} - \tfrac{\delta}{\beta^m} \right) w \leq (1+\delta)\overline{h}.$$

Let us introduce the following condition:

$$\frac{\delta(1-\beta^m)^2}{(\delta(\beta^m)^2+2\beta^m-1)\beta^m}w \leq (1+\delta)\overline{h} \quad \text{(C1)}.$$

If (C1) holds then

$$A+\frac{\delta}{\beta^m}w \leq A+K(\beta^m,\delta)w \leq A+(1+\delta)\overline{h}+\frac{\delta}{\beta^m}w \leq A+(1+\delta)\overline{h}+K(\beta^m,\delta)w$$

and if (C1) fails to hold then

$$A+\frac{\delta}{\beta^m}w \leq A+(1+\delta)\overline{h}+\frac{\delta}{\beta^m}w \leq A+K(\beta^m,\delta)w \leq A+(1+\delta)\overline{h}+K(\beta^m,\delta)w$$

We can extract directly the global optimum depending on whether (C1) holds. Consider first the case where (C1) holds, then

1. If $0 \leq A+\frac{\delta}{\beta^m}w$ then the profit is strictly increasing for $p_1 \leq p_l^m$, strictly increasing for $p_1 \in [p_l^m, p_u^m]$ and has a local maximum at $p_1^3 > p_u^m$. By continuity of the profit, the price chosen by the monopoly is

   $$p_1^* = p_1^3 = \beta^m(a_1-h_1).$$

   The condition under which this price is the optimum of the monopolist can be rewritten

   $$0 \leq A+\frac{\delta}{\beta^m}w \Leftrightarrow \beta^m\left(a_1-h_1-\delta\ E(a)\right)+\delta\ w \geq 0 \Leftrightarrow \frac{\partial \pi^m}{\partial p_1}(p_u^{m+}) \geq 0.$$

2. If $A+\frac{\delta}{\beta^m}w \leq 0 \leq A+K(\beta^m,\delta)w$, then the profit is strictly increasing for $p_1 \leq p_u^m$ and strictly decreasing for $p_1 \geq p_u^m$. The global optimum is therefore

   $$p_1^* = p_u^m = \delta\beta^m E(a)-\delta w.$$

   The first condition under which this price is the optimum is equivalent to $\frac{\partial \pi^m}{\partial p_1}(p_u^{m+}) < 0$, and it is immediate to verify that the second one is equivalent to $\frac{\partial \pi^m}{\partial p_1}(p_u^{m-}) \geq 0$.

3. If $A+K(\beta^m,\delta)w \leq 0 \leq A+(1+\delta)\overline{h}+\frac{\delta}{\beta^m}w$ then the profit is strictly increasing for $p_1 \leq p_l^m$, strictly decreasing for $p_1 \geq p_u^m$ and has a global maximum at

   $$p_1^* = p_1^2 = \frac{\delta\beta^m}{1+\delta}(a_1-h_1+E(a))+\frac{\delta(1-2\beta^m)}{\delta(\beta^m)^2+2\beta^m-1}w.$$

   The first condition is equivalent to $\frac{\partial \pi^m}{\partial p_1}(p_u^{m-}) < 0$. We leave the second condition for next case.

4. If $A+(1+\delta)\overline{h}+\frac{\delta}{\beta^m}w \leq 0 \leq A+(1+\delta)\overline{h}+K(\beta^m,\delta)w$ then two solutions must be compared, namely p_1^1 and p_1^2. When the price is p_1^1, first period demand is equal to $D_1(p_1^1,h_1)=(1-\beta^m)\ (a_1-h_1+\bar{h})$ and therefore

$$\Pi(p_1^1)=\beta^m\ (1-\beta^m)(a_1-h_1+\bar{h})^2-\frac{2\ \beta^m-1}{2\ (1-\beta^m)}(1-\beta^m)^2\ (a_1-h_1+\bar{h})^2$$
$$+\delta\ E_1\left(\frac{1-\beta^m}{2}(a_2-\bar{h})^2\right)=\frac{1-\beta^m}{2}\left((a_1-h_1+\bar{h})^2+\delta\ V(a)+\delta\ (E(a)-\bar{h})^2\right).$$

When the price is equal to p_1^2, first period demand is equal to

$$D_1(p_1,h_1)=\frac{\delta\ (1-\beta^m)}{1+\delta}(a_1-h_1+E(a))-\frac{\delta\ (1-\beta^m)}{\delta\ (\beta^m)^2+2\ \beta^m-1}\ w.$$

The first period profit is equal to

$$\pi_1(p_1^2)=\frac{\delta^2}{(1+\delta)^2}\frac{1-\beta^m}{2}(a_1-h_1+E(a))^2$$
$$-\frac{\delta^2(1-\beta^m)\beta^m}{(1+\delta)(\delta\ (\beta^m)^2+2\beta^m-1)}\ (a_1-h_1+E(a))\ w-\frac{\delta^2\ (1-\beta^m)(1-2\beta^m)}{2\ (\delta\ (\beta^m)^2+2\beta^m-1)^2}w^2$$

and the expected second period profit is equal to

$$E_1\left(\frac{1-\beta^m}{2}\left(a_2+\frac{a_1-h_1}{1+\delta}-\frac{\delta}{1+\delta}E(a)+\frac{\delta\ \beta^m}{\delta\ (\beta^m)^2+2\ \beta^m-1}w\right)^2\right)$$
$$=\frac{1-\beta^m}{2}\left(\left(\frac{a_1-h_1+E(a)}{1+\delta}++\frac{\delta\ \beta^m}{\delta\ (\beta^m)^2+2\ \beta^m-1}w\right)^2+V(a)\right)$$

The total profit is equal to

$$\Pi(p_1^2)=\frac{\delta}{1+\delta}\frac{1-\beta^m}{2}(a_1-h_1+E(a))^2+\frac{\delta^2\ (1-\beta^m)}{2(\delta\ (\beta^m)^2+2\ \beta^m-1)}w^2+\frac{\delta(1-\beta^m)}{2}V(a).$$

Simplifying the common factor $\frac{1-\beta^m}{2}$, the sign of the difference between $\Pi(p_1^1)$ and $\Pi(p_1^2)$ can be studied:

$$\Pi(p_1^1)-\Pi(p_1^2)\geq 0 \Leftrightarrow (a_1-h_1+\bar{h})^2+\delta\ V(a)+\delta\ (E(a)-\bar{h})^2$$
$$\geq\frac{\delta}{1+\delta}(a_1-h_1+E(a))^2+\delta\ V(a)+\frac{\delta^2}{\delta\ (\beta^m)^2+2\ \beta^m-1}w^2.$$

Simplifying $\delta\ V(a)$ in the left and right-hand-side in the inequality above, replacing $a_1-h_1+\bar{h}$ by $a_1-h_1+E(a)-E(a)+\bar{h}$ in the left-hand-side and developing gives:

$$(a_1-h_1+E(a))^2-2(a_1-h_1+E(a))(E(a)-\bar{h})+(E(a)-\bar{h})^2+\delta\ (E(a)-\bar{h})^2$$
$$\geq\frac{\delta}{1+\delta}(a_1-h_1+E(a))^2+\frac{\delta^2}{\delta\ (\beta^m)^2+2\ \beta^m-1}w^2$$

which is equivalent to

$$\frac{(a_1 - h_1 + E(a))^2}{1+\delta} - 2(a_1 - h_1 + E(a))(E(a) - \bar{h}) + (1+\delta)(E(a) - \bar{h})^2 \geq \frac{\delta^2}{\delta\ (\beta^m)^2 + 2\ \beta^m - 1} w^2$$

giving

$$\left(\frac{a_1 - h_1 + E(a)}{\sqrt{1+\delta}} - \sqrt{1+\delta}(E(a) - \bar{h})\right)^2 \geq \frac{\delta^2}{\delta\ (\beta^m)^2 + 2\ \beta^m - 1} w^2.$$

The difference between $\Pi(p_1^1)$ and $\Pi(p_1^2)$ is positive if and only if

$$\left(\frac{a_1 - h_1 + E(a)}{\sqrt{1+\delta}} - \sqrt{1+\delta}(E(a) - \bar{h}) - \frac{\delta\ w}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}\right) \times \left(\frac{a_1 - h_1 + E(a)}{\sqrt{1+\delta}} - \sqrt{1+\delta}(E(a) - \bar{h}) + \frac{\delta\ w}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}}\right) \geq 0$$

that is if $a_1 - h_1 + E(a) \leq (1+\delta)(E(a) - \bar{h}) - \frac{\delta\ \sqrt{1+\delta}}{\sqrt{\delta(\beta^m)^2+2\beta^m-1}} w$, then both terms of the product above are negative and the product is positive, if $a_1 - h_1 + E(a) \geq (1+\delta)(E(a) - \bar{h}) - \frac{\delta\ \sqrt{1+\delta}}{\sqrt{\delta(\beta^m)^2+2\beta^m-1}} w$ and $a_1 - h_1 + E(a) \leq (1+\delta)(E(a) - \bar{h}) + \frac{\delta\ \sqrt{1+\delta}}{\sqrt{\delta(\beta^m)^2+2\beta^m-1}} w$, then one term is positive, the other is negative, and the product is negative. Finally if $a_1 - h_1 + E(a) \geq (1+\delta)(E(a) - \bar{h}) + \frac{\delta\ \sqrt{1+\delta}}{\sqrt{\delta(\beta^m)^2+2\beta^m-1}} w$ then both terms of the product are positive and the product is positive. The initial conditions on parameters such that two local maxima co-exist can be rewritten:

$$A + (1+\delta)\bar{h} + \frac{\delta}{\beta^m} w \leq 0 \Leftrightarrow a_1 - h_1 + E(a) \leq (1+\delta)(E(a) - \bar{h}) - \frac{\delta}{\beta^m} w$$

and

$$0 \leq A + (1+\delta)\bar{h} + K(\beta^m, \delta) w \Leftrightarrow a_1 - h_1 + E(a) \geq (1+\delta)(E(a) - \bar{h}) - K(\beta^m, \delta) w.$$

Since $K(\beta^m, \delta)$ is positive, $(1+\delta)(E(a) - \bar{h}) - K(\beta^m, \delta) w < (1+\delta)(E(a) - \bar{h}) + \frac{\delta\ \sqrt{1+\delta}}{\sqrt{\delta(\beta^m)^2+2\beta^m-1}} w$, and it suffices to compare the lower bound of the two inequalities above, $(1+\delta)(E(a) - \bar{h}) - \frac{\delta}{\beta^m} w$, with the lowest root of the product $(1+\delta)(E(a) - \bar{h}) - \frac{\delta\ \sqrt{1+\delta}}{\sqrt{\delta(\beta^m)^2+2\beta^m-1}} w$ to end up the characterization of the global maximum. Here,

$$-\frac{\delta}{\beta^m} w \geq -\frac{\delta\sqrt{1+\delta}}{\sqrt{\delta(\beta^m)^2 + 2\beta^m - 1}} w$$

is equivalent to

$$\beta^m\sqrt{1+\delta} \geq \sqrt{\delta(\beta^m)^2 + 2\beta^m - 1} \Leftrightarrow (\beta^m)^2 - 2\beta^m + 1 \geq 0$$

which always holds. Therefore the sign of the difference of the profits is always negative in the region considered and the global optimum is p_1^2. The monopoly always prefers speculators to smooth prices in that case. Remark that the solution in the same than in the previous case. Therefore the first condition leading to this case does not matter, while the second one is equivalent to $\frac{\partial \pi^m}{\partial p_1}(p_l^{m+}) \geq 0$.

5. Finally if $A + (1+\delta)\overline{h} + K(\beta^m, \delta)w \leq 0$ then the profit has a global maximum for

$$p_1^* = p_1^1 = \beta^m(a_1 - h_1 + \overline{h}).$$

The condition leading to this case is equivalent to $\frac{\partial \pi^m}{\partial p_1}(p_l^{m+}) < 0$.

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Figure 1: Equilibrium regimes: Monopoly and Perfect Competition

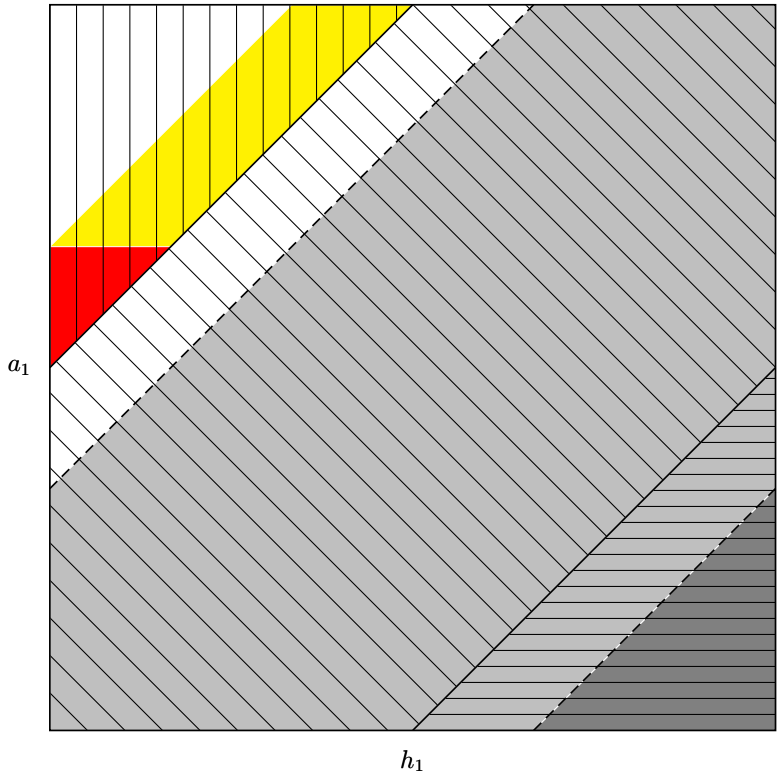

Figure 2: Equilibrium price: Monopoly (top) and Competitive (bottom)

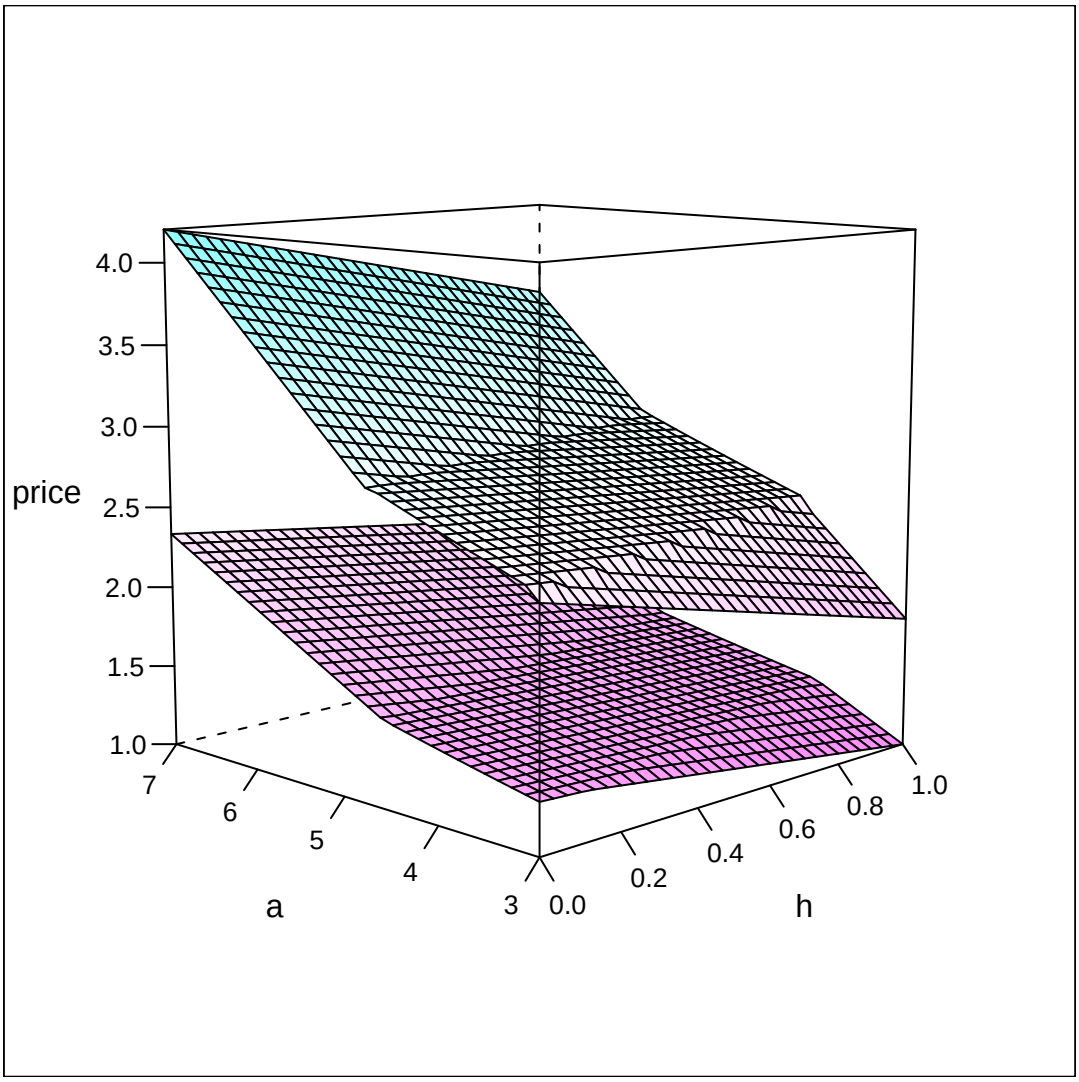

Figure 3: Price variance: Competitive vs. Monopoly production

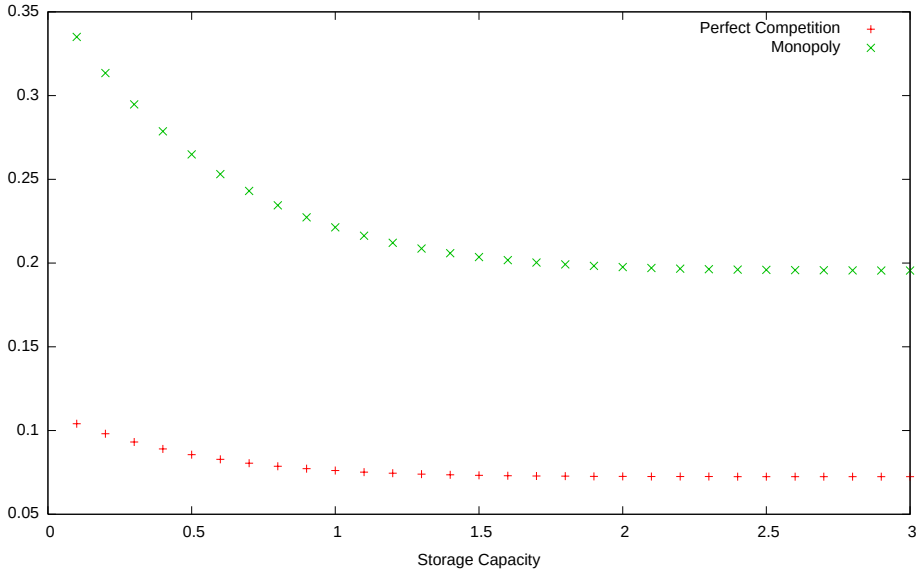

Figure 4: Inventory levels: Competitive vs. Monopoly production

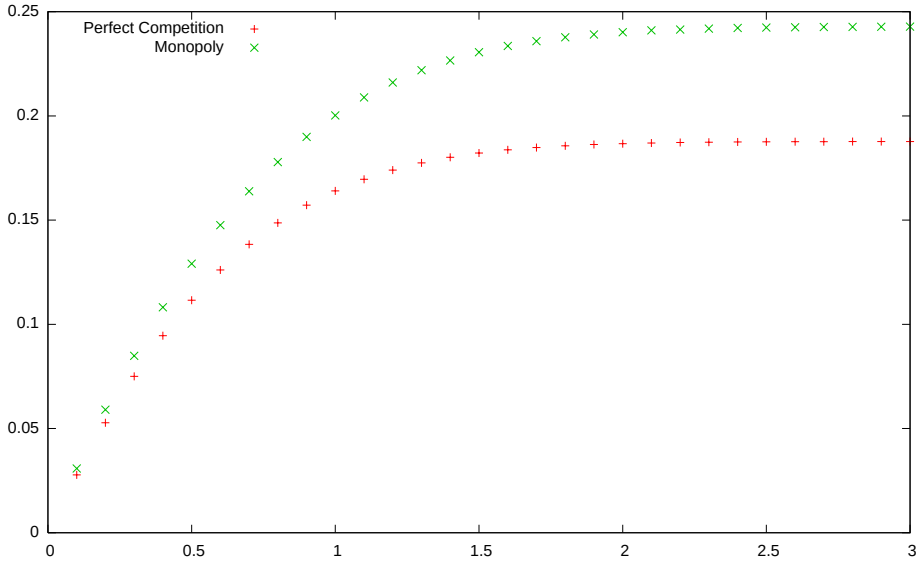

Figure 5:

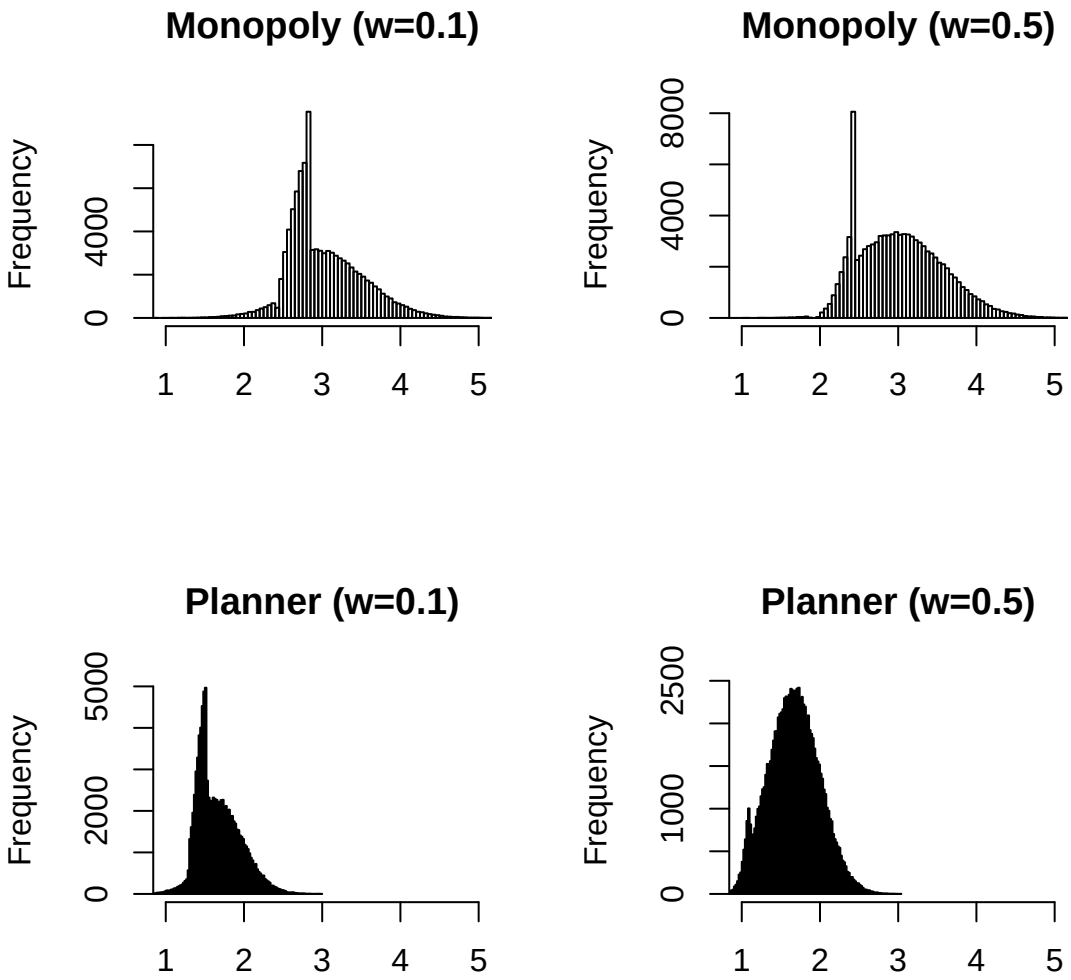
Monopoly (w=0.1)
Frequency
0
4000
1
2
3
4
5
Monopoly (w=0.5)
Frequency
0
4000
8000
1
2
3
4
5
Planner (w=0.1)
Frequency
0
2000
5000
1
2
3
4
5
Planner (w=0.5)
Frequency
0
1000
2500
1
2
3
4
5

Figure 6: Returns to speculation: Competitive vs. Monopoly production